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## Crack Formation in Bending Composite Elements

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### Summary

The paper examines behaviour of composite structures, i. e. bending reinforced concrete elements, armoured externally under short - term static loadings the influence of external reinforcement and its anchorage towards resistance of crack formation, rigidity and strength has been analysed.

On the basis of the test results the methods for calculation of strength, cracking and rigidity have been compared.

### 1. Introduction

At present external glued reinforcement is widely used in the world for practical restoration of serviceability condition of structures. Strips or sheets of steel, glass fiber, asbestos cement are glued to the concrete surface of structures. Sheets and strips can be readily adjusted to various cross - sectional shapes, permanent load and depth of the structures practically are not increased, the job is quite simple and speedy.

External bonded reinforcement and the layer of polymer glue reduce concentration of stress in the tension zone, increase ultimate extensibility and tension strength of concrete, increase resistance to cracking and stiffness of members.

Resistance to cracking stiffness and carrying capacity of flexural members with external reinforcement depend substantially on reliability of bond between concrete and external reinforcement. In the case of inadequate anchorage of the external reinforcement a crack may appear along the joint and consequently effect of external reinforcement reduces. Therefore it is essential to ensure reliability of bond between reinforcement and surface of the structure to be strengthened.

The best anchorage of glued reinforcement is achieved when this reinforcement is extended behind the supports. It is complicated to do so in strengthening of flexural members in service and it leads to the use of additional anchorage by the means of special anchors. It is associated with supplementary work and material expenditure. Thus the

problem of required length of extension of glued reinforcement behind the critical cross - section, i.e. determination of the development length is urgent.

This article deals with influence of external glued reinforcement on crackresistance and strength of normal sections of flexural members - beams.

## 2. Experimental investigations in members strengthened with glued external reinforcement

Behaviour of flexural members - strengthened with glued external reinforcement have been investigated in the department of reinforced concrete structures of Vilnius Gediminas technical university.

Four series of beams have been tested under short time static load. A series consisted of 2 beams. Beams tested were of  $70 \times 100$  mm in cross - section and length of the beams was  $l=970$  mm. The beams were made of normal weight concrete of cube strength  $R_{15,m}=41,5$  MPa, reinforced with  $2\phi 5$  Bp I (reinforcement ratio  $\mu_s=0,00696$ ). In addition beams were reinforced with the following external glued reinforcement:

- non - metallic - 3 layer of glass fabric impregnated with epoxy resin paste (series S1 and S3);
- metallic - 0,72 mm thick steel strip of grade ST 3 (series S2).

The external reinforcement of beams of series S3 were lapped over external ribs of the beams.

Diagrams of control beams S0 and those with glued external reinforcement S1, S2 and S3 are shown in Figure 1.

In beam tests moment of crack formation  $M_{crc,obs}$ , failure moment  $M_{u,obs}$ , distance between normal cracks and limit deformations of concrete in Tension zone were determined.

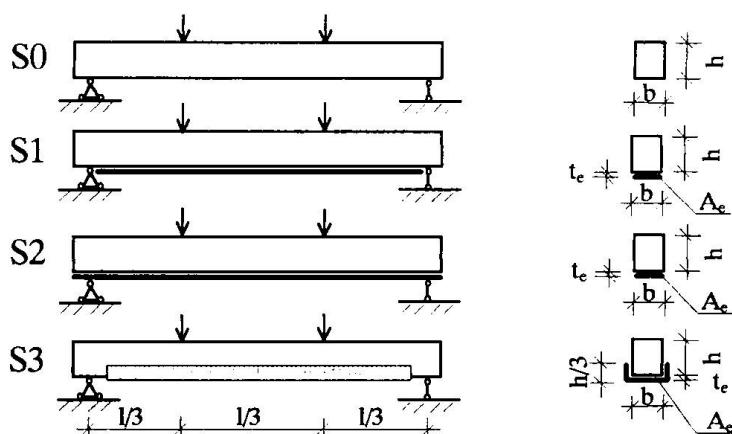


Fig. 1 Diagrams of reinforcing of beams with external glued reinforcement

The main characteristics of tested beams and the main test results are presented in Table 1.

No of sample series	Cube strength of concrete in compression	External reinforcement				Test results			
		Type	Strength $R_e$ , MPa	$A_e$ , $\text{cm}^2$	$\varepsilon_{e,u} \times 10^5$	$M_{crc,obs}$ kNm	$\frac{M_{crc,obs,i}}{M_{crc,obs,o}}$	$M_{u,obs}$ kNm	$\frac{M_{u,obs,i}}{M_{u,obs,o}}$
S0		—	—	—		0,39	1	2,05	1
S1		glass fabric	55,6 2,625	2,775 2,625	1013	0,78	2	2,77	1,35
S2	41,5	metal strip	$R_y = 287,7$	0,504	400	1,3	3,33	2,21	1,08
S3		glass fabric	76,48	3,85 2,56	963	0,78	2	2,78	1,35

Table 1 The main characteristics of samples and test results

According to the test results (see Table 1) resistance to normal crack formation of beams strengthened with external non - metallic and metallic glued reinforcement increased by 2 and 3,33 times, respectively, in comparison with control beams.

Many fine cracks appeared in beams strengthened with external reinforcement. Their width  $a_{crc}=0,025 - 0,05$  mm and average spacing  $l_{crc}=55 - 60$  mm. Crack width in non - strengthened (control) beams was larger  $a_{crc}=0,15$  mm and spacing was  $l_{crc}=85$  mm.

Strength of cross - section increased 1,35 and 1,08 times respectively. Lower increase in strength of cross - sections of beams strengthened with metallic external reinforcement is due to the fact, that the metallic reinforcement was not fully exploited, because of inadequate strength of joint between reinforcement and concrete surfaces, reinforcement at support regions broke off and the bond between reinforcement and concrete surfaces was damaged.

### 3. Calculation of crack formation moment

Resistance of crack formation of flexural members with glued external reinforcement may increase due stress redistribution between strengthening structure and external reinforcement and also due to increased extensibility limit of concrete layers saturated with glue. Efficiency of exploitation of external reinforcement depends on deformability (yielding) of links, the function of which is performed by glue seam. Results of calculation of crack formation moment depend on the accuracy of assessment of yielding in glue seam. Quite a few number of methods for calculation of cracking formation moments proposed by various authors are known. Some of them are analysed and calculation results compared below.

Crack formation moment in reference [1] is recommended to calculate as follows:

$$M_{crc} = \frac{W_{pl} \cdot R_{bt} \cdot k_p}{\left[ 1 - \frac{k_z \cdot z \cdot (z + r)}{\gamma \cdot E_b \cdot I_{red}} \right]}, \quad (1)$$

where:  $k_p$  is a coefficient to allow for tensile strength of concrete covered by polymer,  $k_z \leq 1$  and it is coefficient to account for deformability (yielding) of the links;  $z$  is distance between external reinforcement and the centroid of the cross - section,  $r$  = distance between the centroid of the cross - section and the upper kern point.

In reference [2] a general method for calculation of crack formation bending moments of flexural reinforced concrete members with additional external non - metallic reinforcement is proposed. A triangular stress diagram in concrete under compression is recommended by the authors for the case when compressive stress in the extreme compressive layer of the cross - section does not exceed the limit of microcracking of concrete, and in the opposite case this diagram is assumed to be trapezium. Then  $M_{crc}$  is calculated as follows:

$$M_{crc} = \frac{c \cdot R_{bt}}{h_t} \left( I_{b,el} + \alpha_s \cdot I_s + \alpha_p \cdot I_p + \alpha_s \cdot I_s + \alpha_e \cdot I_e \right) + R_{bt} (S_{bt} + \varphi \cdot S_{b,pl}) + P \cdot e_n, \quad (2)$$

where:  $c = \varepsilon_{bt,u} / \varepsilon_{bt,el}$ ,  $I_s, I_p, I_e, I_s$  are second moments of area of cross - sections of adequate compressive and tensile reinforcing bars;  $I_e, I_{b,el}$  are such moments but of external reinforcement and of concrete compression zone the depth of which is equals to  $x(1 - \eta)$ ;  $S_{b,pl}$  - the first moment of area of concrete compression zone the depth of which is  $\eta x$ ,  $S_{bt}$  = the same but for tension zone in depth of  $h_t$ ;  $P$  = concrete precompression force;  $e_n$  = distance from the point of application of force  $P$  up to the neutral axis;  $h_t$  = tension zone depth in a section with crack; coefficient  $\varphi = \sigma_{b,max} / R_{bt}$ ;  $\alpha$  = reinforcement to concrete modular ratio

The method of calculation of crack formation moment proposed in reference [3] also allows for plastic deformations of concrete in tension. the following formula is proposed for cracking moment calculation:

$$M_{crc} = \frac{\varepsilon_{btu}}{h - x} \left[ \frac{E_{bk} \cdot b \cdot x^3}{2 + n_\sigma} + E_s \cdot A_s (h_0 - x)^2 + E_e \cdot b_e \cdot t_e (h - x)^2 + E'_{btu} \cdot \frac{b(h - x)^3}{2 + f_t} \right], \quad (3)$$

where:  $\varepsilon_{btu}$  is ultimate deformations of extreme fibers of concrete in tension;  $E_{bk}$  is deformation modulus of glues;  $E'_{btu}$  is the secant deformation modulus of tension concrete at failure;  $f_t = E_b / E'_{btu}$  and is characteristic of non - linear behaviour of tension concrete.

Sample code	Method 1 [1]	Method 2 [2]	Method 3 [3]	Method 4 [4]
	$\frac{M_{crc,obs}}{M_{crc,cal}}$	$\frac{M_{crc,obs}}{M_{crc,cal}}$	$\frac{M_{crc,obs}}{M_{crc,cal}}$	$\frac{M_{crc,obs}}{M_{crc,cal}}$
S1-1	0,948	1,413	1,03	1,50
S1-2	0,929	1,376	1,03	1,469
S2-1	1,363	1,854	0,755	1,903
S2-2	1,30	1,774	0,748	1,823
S3-1	0,91	1,340	0,941	1,432
S3-2	0,908	1,340	0,948	1,432
Bondarenko V.M. [3]	0,997	1,541	1,03	1,657
	1,144	1,10	1,05	1,937
	1,176	1,894	1,054	2,044
	0,858	1,217	1,08	1,315
	0,970	1,395	1,071	1,508
	1,060	1,538	1,084	1,666
Voronkov R. V. [6]	0,993	0,719	-	1,33
	1,03	0,745	-	1,63
	1,00	0,730	-	1,33
	0,719	0,615	-	1,11
	1,09	0,759	-	1,01
	1,189	0,922	-	0,835
	1,2	0,931	-	0,835
*	0,832	1,214	-	1,324
*	0,894	1,39	-	1,518
*	1,142	1,649	-	1,795

\* The beams tested at reinforced concrete department of VGTU

Table 2 Results of calculation of crack formation moments  $M_{crc}$

Calculation methods	Number of samples	The mean value $\gamma_m$	Standard deviation $S_\gamma$	Coefficient of variation $\delta_\gamma$	$k_{min}$ , at $\beta=2$
1	22	1,03	0,158	0,153	1,4
2	22	1,248	0,388	0,311	2,12
3	12	0,985	0,118	0,12	1,34
4	22	1,47	0,33	0,225	1,24

Table 3 Results of statistical analysis of ratio  $\gamma = M_{crc,obs}/M_{crc,cal}$  moments of crack formation

Values of crack formation moments are calculated according the methods discussed and compared with test results on beams. Results of comparison are presented in Tables 2 and 3. The average value of ratio at crack formation moments  $\gamma_m = M_{crc,obs}/M_{crc,cal}$  characterizes accuracy of method of calculation, while the average square standard deviation of this ratio  $S_\gamma$  and coefficient of variation  $\delta_\gamma$  define reliability of the method (see Table 3). It was assessed by verification that distribution of the ratio  $\gamma = M_{crc,obs}/M_{crc,cal}$  agrees with the normal law of distribution. Then according to [5] the minimum conventional reliability coefficient  $k_{min}$  of a calculation method can be determined:

$$k_{min} = 1/\gamma_m(1 - \delta_\gamma \cdot \beta) \quad (4)$$

where:  $\beta$  is number of standard deviations in accordance with assumed reliability.

The minimum conventional reliability coefficient  $k_{min}$  of calculation method makes a part of general safety coefficient. Its values are presented in Table 3.

After comparing results of analysis presented in Table 3 it is easy to discover, that calculation methods 1 and 3 can give quite accurate and sufficiently reliable results of calculation of normal crack formation moments of beams strengthened with external glued reinforcement.

## References

1. Kamaitis Z. Vostanovlenie usilenije konstrukcij i sooruzhenij sintetytsheskimi smolami. - Vilnius. Technika, 1992. - 280 p.
2. Valivonis J., Jurksha A. Obobshchionyj metod raszheta izgibajemych zhelezobetonnych elementov s dopolnytelnym wneshnym nemetalitsheskim armirovaniem. Trudy VTU "Zhelezobetonnyje konstrukcij", 1995, № 17. - p 43 - 48.
3. Bondarenko V. M., Schagin A. L. Raschiot efektivnych mnogokomponentnych konstrukcij. - M.: Strojizdat, 1987. - 176 p.
4. Posobje po projektirovaniyu betonnych i zhelezobetonnych konstrukcij iz tiazhiykh i liogkoch betonow bez predvaritel'nogo napriazhenija armatury (k SNiP 2.03.01 - 84). - M.: Strojizdat, 1986.
5. Vadtuga R. R., Kudzis A.P. Analiz tochnosti i nadiozhnosti raschiota po prochnosti izgibajemych zhelezobetonnych elementow kolcewogo setchenija. Trudy VISI, "Zhelezobetonnyje konstrukcij", Vilnius 1976, № 7, p 155 - 162.
6. Voronkov R. V. Zhelezobetonnyje konstrukcij s listowej armaturoi. L.: Strojizdat, 1975. - 145 p.