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## **Retrofitting of Prestressed Concrete Beams with Exterior Post-Tensioned CFRP Tendons**

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### **Summary**

Tests were conducted on 200 x 400 x 5500 mm steel prestressed concrete beams strengthened with exterior post-tensioned Carbon Fiber Reinforced Polymer (CFRP) tendons. External tendons were harped at two locations and beams were tested under short-term four-point loading. The study included development of an analytical model that accurately predicts the ultimate load, midspan deflection at ultimate load and external CFRP tendon load at ultimate load. The analytical model was used in a parametric study of externally post-tensioned concrete beams.

## **1. Experimental Program**

The experimental program consisted of two "control" prestressed beams and four prestressed beams with external post-tensioned CFRP tendons. The beams were tested to failure. Details are provided elsewhere [1].

### **1.1 Test Set-up and Procedure**

The cross-sectional details of the test beams prior to addition of external tendons are shown in Figure 1. A profile of the test specimens with the CFRP exterior tendons is shown in Figure 2. Steel stirrups of #2 wires were provided as reinforcement in the shear span of the beams, placed at 200 mm spacing, excluding the center 900 mm of the beams. Average concrete strength at time of testing was 44.2 MPa.

For prestressed beams strengthened by external CFRP tendons, one CFRP tendon was placed on each side of the beam. Attachment of the CFRP tendons to the ends of the concrete beam was accomplished using a steel U-shaped device attached to the end of the beams at the midheight of the beam (200 mm from top). Harping of each of the CFRP tendons at the midspan of the beams was provided at two locations spaced 710 mm apart and symmetric about the midspan of the beam. At midspan, the tendons were located a depth of 390 mm from the top of the beam, which was approximately the bottom of the beam. Harping points consisted of aluminum plates cut to a radius of 510 mm, with a semicircular groove cut along the arch with a diameter of 8 mm. The initial harping angle of the tendon at each harping point was 4.8 degrees. The average post-

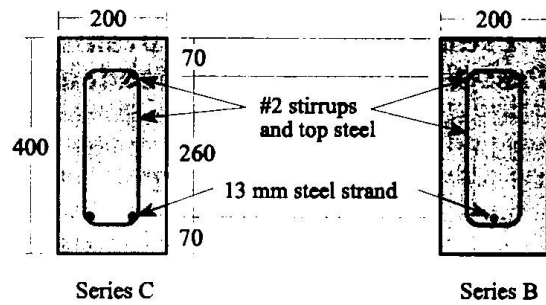


Fig. 1 Cross-section of beam specimens

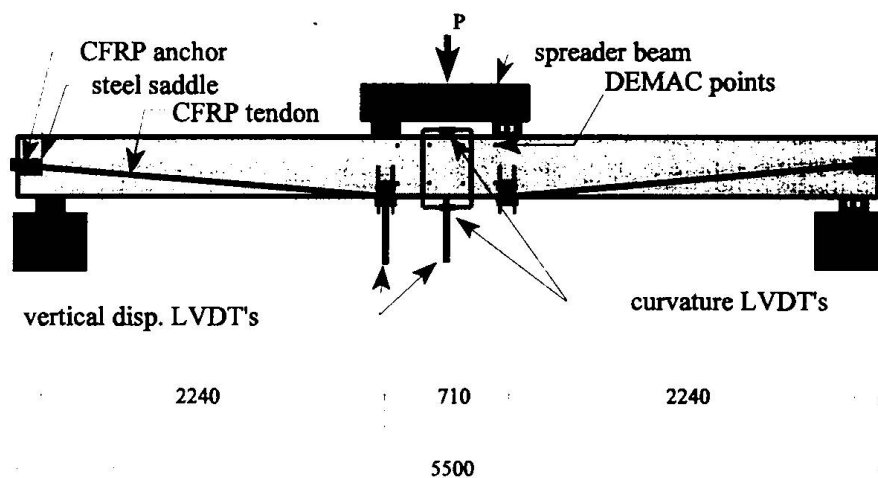


Fig. 2 Schematic diagram of the prestressed beams strengthened by external CFRP tendons

tensioning force per CFRP tendon was 124 kN.

The beam span was 5190 mm with point loads spaced 710 mm apart and symmetric about the midspan of the beam. Vertical deflections of the beams were measured at two locations: midspan of the beams and at one of the two loading points. Strains in the CFRP tendons were measured using surface mounted strain gages.

All tests were conducted under displacement control. Displacements were applied at a rate of approximately 5 mm/minute. To simulate damaged beams in service, all beams were initially loaded to just beyond their cracking load and then unloaded to 19 kN. This initial limit load for the beams was 37 kN and 64 kN for the B series beams and C series beams, respectively. For the control beams, after reducing the load to 19 kN, the load was increased until failure, which was initiated by crushing of the concrete. For the strengthened beams, after reducing the load to 19 kN, the CFRP tendons were post-tensioned. The deflection associated with the 19 kN load was maintained on the beam throughout the post-tensioning process. At the completion of post-tensioning, the vertical load on the beam increased due to the camber affect induced by post-tensioning. The load on the beam was again reduced to 19 kN before loading the beam to failure.

## 1.2 Test Results

Load-midspan deflection curves for beams B-0, B-1, C-0 and C-1 are shown in Figures 3 and 4. Results for beams B-2 and C-2 were similar to results for B-1 and C-1, respectively. The figures also show the results of the analytical analysis discussed later. A summary of the test results for the beam tests is presented in Table 1. Table 1 does not include results for the analysis including tension stiffening affects (TS analysis).

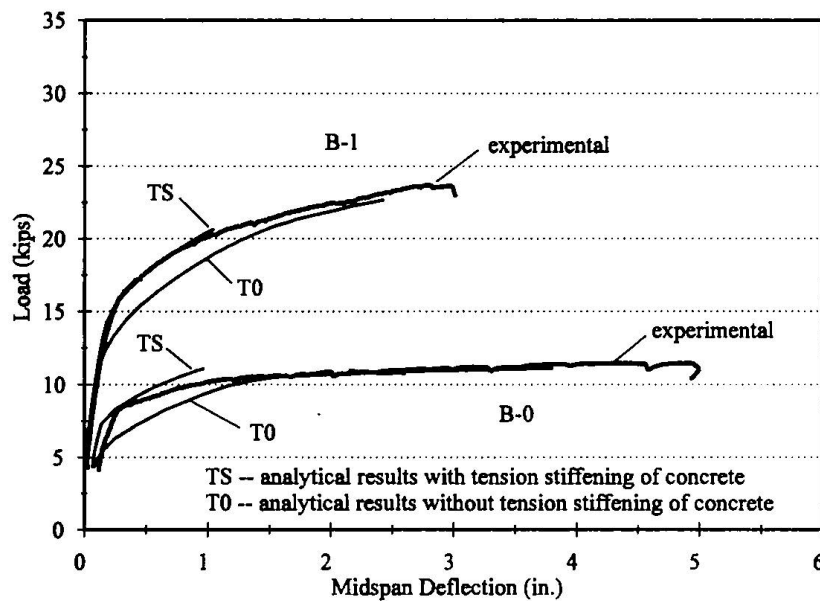


Fig. 3 Experimentally observed and predicted load-midspan deflection for B-0 and B-1

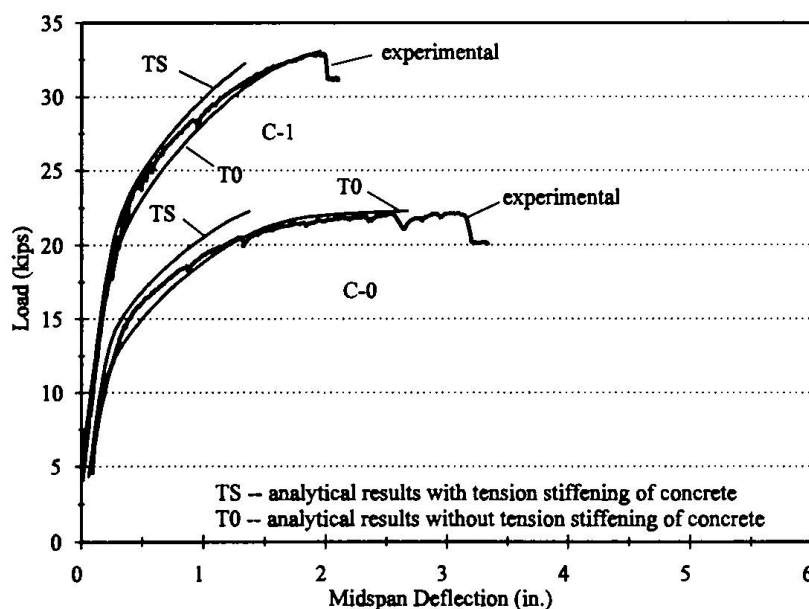


Fig. 4 Experimentally observed and predicted load-midspan deflection for C-0 and C-1

Table 1 Summary of experimental and analytical beam test results

Beam No.	Ultimate load, kN		Midspan deflection at ultimate load, mm		Initial total force of external CFRP tendons, kN	Total force of external CFRP tendons at ultimate, kN	
	Exp.	Analytical	Exp.	Analytical	Exp.	Exp.	Analytical
B-0	51.2	49.4 (0.97)	113	96.8 (0.86)	--	--	--
B-1	105	101 (0.96)	71.4	61.7 (0.86)	117	169	164 (0.97)
B-2	108	103 (0.95)	68.8	57.9 (0.84)	127	179	171 (0.96)
C-0	98.7	99.2 (1.00)	78.5	68.1 (0.87)	--	--	--
C-1	147	147 (1.00)	50.5	50.0 (0.99)	128	164	164 (1.00)
C-2	147	145 (0.99)	50.5	49.5 (0.98)	124	161	158 (0.98)

( ) numbers in parenthesis indicate the percent (%) of experimental value

Failure of the beams for all tests was due to crushing of concrete at the top of the beam. Ultimate loads for beams post-tensioned with external CFRP tendons averaged 55 and 48 kN higher than their companion non-post-tensioned beams (Table 1). This corresponds to an ultimate strength of 209% and 149% of the companion non-post-tensioned beam strength for the B series and C series beams, respectively. Midspan deflections at ultimate for the externally post-tensioned beams averaged approximately 64% of the midspan deflection at ultimate for the companion non-post-tensioned beams for both B and C series beams.

As shown in Figures 3 and 4, the exterior post-tensioned beams had a positive tangential stiffness, defined as the change in load divided by the associated change in midspan deflection, up to failure. This was not evident for the non-post-tensioned beams B-0 and C-0. It was found in more detailed analysis that this positive tangential stiffness at failure was due to the increased upward forces at the harping points. The increased upward forces at the harping points was due to the beam deflections which caused larger bend angles of the tendons at the harping points and caused increased forces in the external tendons due to additional strains in the tendons.

Figure 5 shows the variation in total external CFRP tendon force with the midspan deflection. It is evident in the figure that the external CFRP tendon strains, and therefore tendon forces, varied linearly with the midspan deflection of the beams. The response for beams C-1 and C-2 varied in a similar manner. The external CFRP tendon forces at ultimate averaged 143% and 129% of the initial external CFRP tendon forces for the B series and C series beams, respectively (Table 1).

## 2. Analytical Model

As part of this study, a computerized analytical model was developed that accurately predicts the ultimate load, midspan deflection and external tendon force at ultimate for externally post-tensioned, simply supported beams loaded symmetrically about the midspan with two point

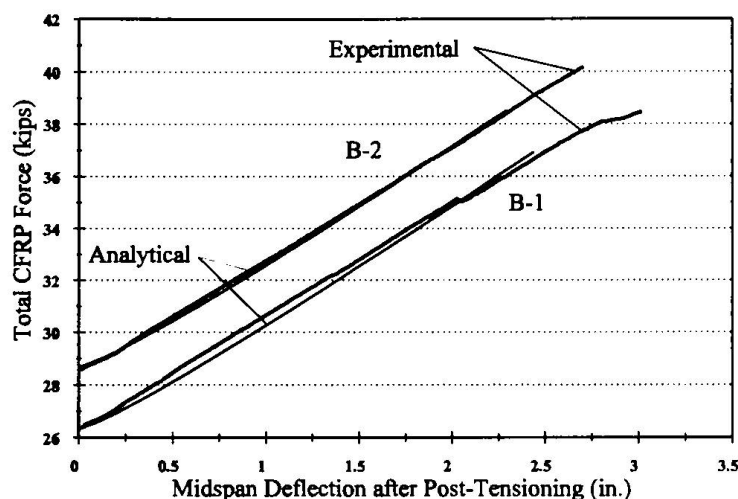


Fig. 5 Total CFRP tendon force versus midspan deflection after post-tensioning, B-1 and B-2

loads. External tendons are harped at two locations symmetric about the midspan of the beam and no friction is assumed at the harping points of the tendons. The analytical model was included in a computer program called EXPOST.

## 2.1 Development of Analytical Model

The analytical model uses a constitutive relationship for concrete developed by Ahmad [2]. Constitutive relationship for the prestressing steel was modelled as linear up to yielding, represented by a fourth-order polynomial expression at yielding, and then linear prior to reaching ultimate strain. Non-prestressed steel reinforcement was model as linear up to yielding with no strain hardening. CFRP tendons were modelled as linearly elastic.

At each load increment, the model assumes an external CFRP tendon force and determines beam segment curvatures. Curvatures are integrated over the span length to determine displacements. After displacements are determined, a CFRP tendon force is predicted using the initial CFRP tendon force plus additional forces due to straining of the external tendons resulting from vertical displacements at the harping points and horizontal extension of the beam at the height of the tendon anchorage. If the predicted and assumed CFRP tendon forces are within 0.05%, then the beam load is increased to the next load level, otherwise, the analysis is repeated with a new assumed CFRP tendon force. If the predicted and assumed CFRP tendon forces are within a tolerance (assumed to be 0.05%) and the maximum load on the beam is greater than the maximum allowable load of a beam section, the beam is considered to have failed.

## 2.2 Comparison Between Analytical and Experimental Results

As shown in Figures 3, 4 and 5 and in Table 1, the computerized analytical model developed in this study accurately predicts the ultimate load, ultimate midspan deflection and the CFRP tendon force at ultimate for the beams tested as a part of this study. Predictions of ultimate load ranged between 95% and 100% of the experimentally observed ultimate load. Midspan deflections were slightly under predicted (84% to 99% of the experimentally observed deflections). Predictions of the CFRP tendon forces at ultimate ranged between 96% and 100%

of the experimentally observed CFRP tendon forces.

### 2.3 Results of Parametric Study

The computer program EXPOST was used in a parametric study of steel prestressed concrete beams externally post-tensioned with CFRP tendons. Two types of beams were considered -- rectangular and T-beams, with two amounts of effective prestress force for each type of beam. The dimensions of the rectangular beams were 152 mm x 406 mm and of the T-beams were 51 mm x 610 mm top flange and 356 mm x 102 mm web. The effective prestress force was varied by changing the area of prestressing steel from 77 to 161 sq.-mm for the rectangular beams and from 174 to 374 sq.-mm for the T-beams, with the effective steel prestress remaining 1030 MPa. The depth of prestressing steel for both beams was 330 mm. Span lengths for the beams were 6.1 m and 9.1 m for the rectangular and T-beams, respectively. Variables for the parametric study were the initial total load of the external tendons (53 to 160 kN), the harping point location for the external tendons (either at the midspan or at 1/3 span of the beam), and the axial stiffness of the external CFRP tendons (7120 to 24000 kN/m/m).

Results of the parametric study indicate that the ultimate load of externally post-tensioned beams increases with increases in initial force of the external tendons, harping at third-points versus midpoints of the beam, and with increases in the stiffness of the external tendons. Increases in ultimate loads averaged 172% of the companion non-post-tensioned reference beam ultimate load. Beams with external tendons harped at third-points showed an average increase in ultimate load 15% higher than beams with external tendons harped at midspan. Midspan deflections at ultimate for the beams reduced an average of 46% due to the addition of external post-tensioning.

Increases in the external CFRP tendon forces at ultimate was significant. For the rectangular and T-beams, forces in the external CFRP tendons at ultimate averaged 158% and 205% of the initial tendon forces, respectively. For each of the four types of beams investigated in the study, little difference was observed in the energy absorption capacity of the beams at failure.

## 3. Summary and Conclusions

Results of this study showed that externally post-tensioned CFRP tendons can be effectively used for strengthening of prestressed concrete beams. The behavior of externally post-tensioned prestressed beams can be predicted using the computerized model developed in this study.

## 4. References

1. Jerrett, C. V., "Performance of Carbon Fiber Reinforced Polymer (CFRP) Tendons and their use for Strengthening of Prestressed Concrete Beams," PhD Dissertation, North Carolina State University, 1996.
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