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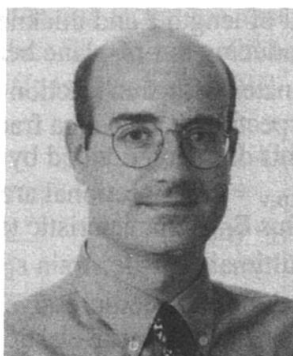
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Behaviour of Masonry Structures Strengthened with Composites

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Summary

The paper presents a systematic analysis for the short-term strength of masonry walls strengthened with externally bonded fibre reinforced polymer (FRP) laminates, under monotonic out-of-plane bending, in-plane bending and in-plane shear, all combined with axial load, within the framework of modern design codes such as Eurocode 6. The results are presented in the form of both design equations and normalized interaction diagrams.

1. Introduction and background

Many of the masonry structures throughout the world (including several of considerable historical and architectural importance) have suffered from the accumulated effects of inadequate construction techniques and materials, seismic and wind loads, foundation settlements and environmental deterioration, and are structurally deficient or marginal for current use. In addition to these factors, changed usage and more stringent seismic design requirements have resulted in many masonry structures being designated in need of upgrading through strengthening. Traditional methods, such as reinforced concrete or shotcrete jacketing, for strengthening of masonry structures, suffer from a few disadvantages (considerable additional weight, change of dimensions, increased labour costs, obstruction of occupancy and violation of aesthetics), so that researchers have recently looked at other techniques, involving the use of fibre reinforced polymer (FRP) composites. These materials, which are typically made of carbon, glass or aramid fibres, bonded together with a polymeric matrix (epoxy, polyester, vinylester), offer the designer an outstanding combination of properties, including high strength and stiffness in the direction of the fibres, immunity to corrosion, low weight and availability in the form of laminates, fabrics and tendons of practically unlimited lengths.

Past studies related to the use of composites as strengthening materials of masonry are reported in [1-7]. These materials have been examined in the form of either unbonded tendons [1-2] or epoxy-bonded laminates or fabrics [3-7]. The last concept involves bonding of FRP strips or fabrics to the surface of masonry walls, playing the role of tensile reinforcement; the concept has been verified experimentally and applied successfully to strengthen some of the load-bearing walls of a six storey residential building in Zurich [4]. The results obtained from the above studies point to the conclusion that for the sake of both economy and optimum mechanical response, unidirectional FRP reinforcement in the form of laminates (or fabric strips) is preferable than two-dimensional fabrics which cover the whole surface of masonry walls.

In this study, the author aims at contributing to the development of a basic understanding of the mechanical behaviour of unreinforced masonry walls strengthened with externally bonded composite laminates (or fabric strips) using simple modelling, consistent with the approach of Eurocode 6 for masonry structures. The three most common cases of masonry loading are analyzed, namely: out-of-plane bending, in-plane bending and in-plane shear (with axial force in all cases).

2. Analysis

2.1 Out-of-Plane Bending With Axial Force

Consider first the case of a masonry wall of length ℓ and thickness t , subjected to compressive force N_{Rd} and bending moment $M_{0,Rd}$ inducing out-of-plane bending. The wall's tensile face is reinforced with epoxy-bonded FRP laminates with area fraction equal to Q_v and Q_h in the longitudinal and transverse direction, respectively. The area fraction in one direction is defined as the total cross-sectional area of FRP in this direction divided by the corresponding area of the wall. Hence Q_v is equal to $A_{frp,v}/\ell t$, where $A_{frp,v}$ = cross-sectional area of FRP in longitudinal direction. The FRP laminates have Young's modulus E_{frp} , characteristic tensile strength (that is, with a 5% probability of under-strength) $f_{frp,k}$ and ultimate tensile strain $\epsilon_{frp,u}$; and the masonry has characteristic compressive strength f_k and ultimate compressive strain $\epsilon_{M,u}$. As far as stress-strain relationships are concerned, the FRP is considered linear-elastic to failure and the masonry is idealized according to the rectangular compressive stress block approach. The partial safety factors for masonry and FRP are denoted as γ_M and γ_{frp} , respectively. Further assumptions are that plane sections before bending remain plane after bending and that the tensile resistance of the masonry, the adhesive and the FRP in the transverse direction may be neglected.

Typically, failure of the FRP-strengthened masonry will be due to compressive crushing, unless the longitudinal reinforcement area fraction, Q_v , is very small. In the latter case, FRP fracture will precede masonry crushing, and thereafter the wall will behave as unreinforced. The limiting Q_v value, $Q_{v,lim}$, for such a mechanism to be avoided, is obtained by considering the strain and stress distribution in the cross section, as shown in Fig. 1, with $\epsilon_{frp} = \epsilon_{frp,u}$ and $E_{frp}\epsilon_{frp} = f_{frp,k}/\gamma_{frp}$. Force equilibrium and strain compatibility give the following equation for the limiting FRP area fraction:

$$\omega_{lim} = \frac{\epsilon_{M,u} E_{frp}}{f_k} Q_{v,lim} = \frac{\epsilon_{M,u}}{\epsilon_{frp,u}} \left[\frac{0.8}{\gamma_M} \frac{1}{(1 + \epsilon_{frp,u} / \epsilon_{M,u})} - \frac{N_{Rd}}{\ell t f_k} \right] \quad (1)$$

Next, provided that $Q_v \geq Q_{v,lim}$, the bending capacity of the cross section can be obtained by considering compatibility of strains and equilibrium of internal forces and moments, as shown in Fig. 1. The result is given in the following form:

$$\frac{M_{0,Rd}}{\ell t^2 f_k} = \frac{1}{2} \omega \frac{(1 - x/t)}{x/t} + \frac{0.4}{\gamma_M} \frac{x}{t} \left(1 - 0.8 \frac{x}{t} \right) \quad (2)$$

where

$$\frac{x}{t} = \frac{\gamma_M}{1.6} \left[-\omega + \sqrt{\omega^2 + \frac{3.2}{\gamma_M} \left(\omega + \frac{N_{Rd}}{\ell t f_k} \right)} \right] \quad (3)$$

and

$$\omega = \frac{\epsilon_{M,u} E_{frp}}{f_k} Q_v \quad (4)$$

As seen in Fig. 2 (based on $\gamma_M = 2.5$ and $\epsilon_{frp,u}/\epsilon_{M,u} = 4$), for low to moderate axial load levels, the bending capacity increases with the normalized FRP area fraction ω . Such an increase may vary from quite dramatic to negligible, depending on the axial load; and for values of ω exceeding approximately 0.5 it is, in most cases, negligible. It is also quite interesting to note that for high axial load ratios (exceeding approximately 0.3) the bending capacity decreases as ω increases. It may be observed in Fig. 2 that the upper curve, corresponding to zero axial load, does not continue all the way to $\omega = 0$. The missing part is associated with FRP fracture before crushing of

the masonry. The limiting value of ω at the transition between the two failure modes is 0.016. As given by (1), such limiting values do not exist for $N_{Rd}/\ell t f_k \geq 0.064$. Hence it may be concluded that, for practical axial load levels and FRP area fractions, premature FRP fracture is highly unlikely to occur. In addition, Fig. 2 in combination with (4) reveal that for a given masonry material the effectiveness of the strengthening technique, that is the increase in out-of-plane bending capacity, depends on the product $E_{frp} Q_v$; very stiff laminates, such as CFRP, are much more efficient than others of lower stiffness, such as GFRP.

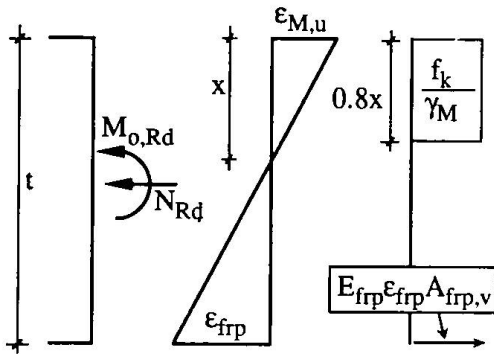


Fig. 1 Strain and stress distribution at out-of-plane flexural failure

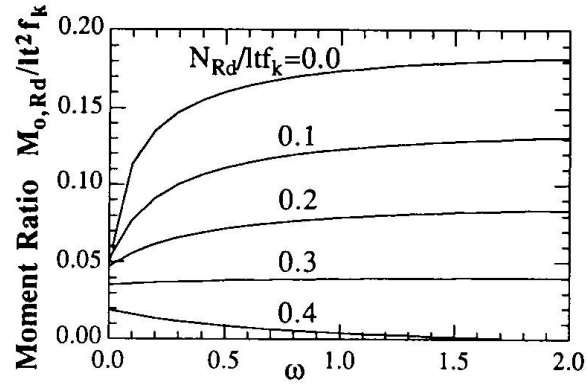


Fig. 2 Out-of-plane moment capacity versus normalized FRP area fraction

2.2 In-Plane Bending With Axial Force

Consider next the case of the FRP-strengthened masonry wall under in-plane bending moment $M_{i,Rd}$ with axial force N_{Rd} . The longitudinal tensile reinforcement is assumed to be in the form of n laminates of cross-sectional area A_i each, at an equal spacing s . Here, too, failure will be due to compressive crushing, unless: (a) Q_v is very small, which will result in premature FRP fracture (this is highly unlikely); (b) the laminates' bond development length is too short, which will result in shearing of the FRP in the tension zone (peeling-off) directly beneath the bond. Quantification of the peeling-off failure mechanism is not attempted here. Note that for the rather limited cases where in-plane flexural failure precedes in-plane shear failure (long and narrow, as opposed to squat elements), the geometry of masonry walls will most likely be such that the development length will be sufficiently large, so that failure will be dominated by compressive crushing.

The limiting Q_v value, $Q_{v,lim}$, for premature FRP fracture to be avoided, is obtained by considering the strain and stress distribution in the cross section, as shown in Fig. 3 (with $\epsilon_n = \epsilon_{frp,u}$). It can be shown that force equilibrium and strain compatibility give the following equation for the limiting FRP area fraction:

$$\omega_{lim} = \frac{\epsilon_{M,u} E_{frp}}{f_k} Q_{v,lim} = \frac{(g+1)}{(\epsilon_{frp,u}/\epsilon_{M,u} - g)} \left[\frac{0.4(g+1)}{\gamma_M} \frac{1}{(1 + \epsilon_{frp,u}/\epsilon_{M,u})} - \frac{N_{Rd}}{\ell t f_k} \right] \quad (5)$$

Under the assumption that $Q_v \geq Q_{v,lim}$, the bending capacity of the cross section can be obtained from strain compatibility and equilibrium of internal forces and moments, according to Fig. 3. After proper manipulations, the result is obtained in the following form:

$$\frac{M_{i,Rd}}{\ell^2 f_k} = \frac{(n+1)g^2}{12(n-1)} \omega \frac{1}{x/\ell} + \frac{0.4}{\gamma_M} \frac{x}{\ell} \left(1 - 0.8 \frac{x}{\ell} \right) \quad (6)$$

where

$$\frac{x}{\ell} = \frac{\gamma_M}{1.6} \left[-\omega + \frac{N_{Rd}}{\ell t f_k} + \sqrt{\left(\omega - \frac{N_{Rd}}{\ell t f_k} \right)^2 + \frac{1.6}{\gamma_M} \omega} \right] \quad (7)$$

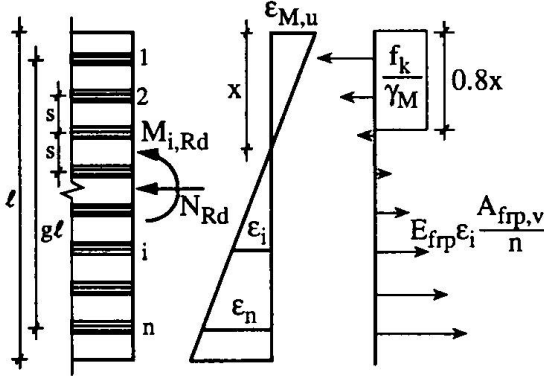


Fig. 3 Strain and stress distribution at in-plane flexural failure

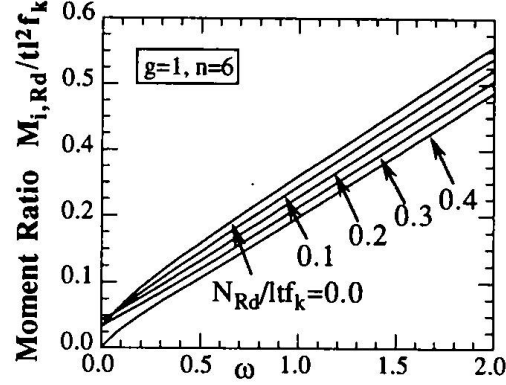


Fig. 4 In-plane moment capacity versus normalized FRP area fraction

As Fig. 4 shows, the bending capacity, in general, increases almost linearly with the normalized FRP area fraction ω (and, for a given masonry material, almost linearly with $E_{frp}Q_v$), and is considerable, regardless of the axial load level. For a given ω , higher values of moment capacity are possible as the axial load increases, but this dependence is weak.

Examination of (6) shows that the number of laminates plays an important role in mechanical response. For the same area fraction, the reinforcement's effectiveness increases by decreasing the number of laminates; using two laminates as far apart as possible results in the highest increase in bending capacity. Note that the last statement is not valid in the case of steel-reinforced masonry, where the in-plane moment capacity is almost independent of whether the reinforcement is uniformly distributed along the wall or concentrated near the ends.

2.3 In-Plane Shear With Axial Force

Last, we examine the case of FRP-strengthened masonry walls under in-plane shear V_{Rd} with axial force N_{Rd} . According to Eurocode 6 [8], the analysis and design of reinforced masonry in shear is based on the assumption that the total contribution to shear capacity is given as the sum of two terms, similarly to reinforced concrete. The first term, V_{Rd1} , accounts primarily for the contribution of uncracked masonry, while the second term, V_{Rd2} , accounts for the effect of shear reinforcement, which is usually modeled by the well-known truss analogy:

$$V_{Rd} = V_{Rd1} + V_{Rd2} \leq \frac{0.3 f_{vk} t d}{\gamma_M} \quad (8)$$

where $V_{Rd1} = f_{vk} t d / \gamma_M$ and d is the effective depth. For masonry walls with several layers of reinforcement, as in our case, d can be taken approximately equal to 0.8ℓ [9]. f_{vk} is the characteristic shear strength of masonry, given as:

$$\begin{aligned} f_{vk} &= \min \left[f_{vko} + 0.4 \frac{N_{Rd}}{\ell t}, 0.7 f_{vk,lim}, 0.7 \max(0.065 f_b, f_{vko}) \right] \\ &= \min \left(f_{vko} + 0.4 \frac{N_{Rd}}{\ell t}, f_{vk,max} \right) \end{aligned} \quad (9)$$

where: f_{vko} , the characteristic shear strength of masonry under zero compressive stress, is between 0.1-0.3 MPa (the lower limit applies in the absence of experimental data), depending on the type of masonry units and the mortar strength; $f_{vk,lim}$, the limiting value of f_{vk} , is in the order of 1.0-1.7 MPa, depending on the type of masonry units and the mortar strength; f_b , the normalized compressive strength of masonry units, is equal to a size factor (between 0.65-1.55) times the mean compressive strength of masonry units; and the factor 0.7 applies only in the (usual) case of seismic design. Note that if strengthening is applied in the absence of full repair, that is in the case of damaged (diagonally cracked) masonry walls, the value of f_{vk} should be taken lower than that given by (9). Such a reduction depends on the degree of damage, and can only be estimated on a case by case basis.

The contribution of FRP reinforcement to shear capacity is more difficult to quantify. One assumption made here is that the contribution of vertical FRP reinforcement, which provides mainly a dowel action effect, is negligible. This can be justified by the high flexibility of the laminates, in combination with their local debonding in the vicinity of shear cracks. The only shear resistance mechanism left is associated with the action of transverse laminates, which can be modeled in analogy to the action of stirrups in reinforced concrete beams. Adopting the classical truss analogy, it can be shown that the contribution of transverse FRP to shear capacity is:

$$V_{Rd2} = Q_h E_{frp} \left(r \frac{\epsilon_{frp,u}}{\gamma_{frp}} \right) t 0.9d = \frac{0.7}{\gamma_{frp}} Q_h E_{frp} \epsilon_{frp,e} t \quad (10)$$

where r is a reinforcement efficiency factor, depending on the exact FRP failure mechanism (FRP debonding or tensile fracture), γ_{frp} , the partial safety factor for FRP in uniaxial tension is approximately equal to 1.15, 1.20 and 1.25 for CFRP, AFRP and GFRP, respectively [2], and $\epsilon_{frp,e}$ is an effective FRP strain, the only unknown yet to be determined for completing the analysis on FRP contribution to shear capacity. Qualitatively, one may argue that $\epsilon_{frp,e}$ depends heavily on the area of the FRP-masonry debonded interfaces, or, in other words, on the FRP development length, defined as that necessary to reach FRP tensile fracture before debonding. Apart from the bond conditions, the development length depends (almost proportionally) on the FRP axial rigidity (area times elastic modulus), expressed by the product $Q_h E_{frp}$. Hence, one would roughly expect $\epsilon_{frp,e}$ to be inversely proportional to $Q_h E_{frp}$. The implication of this argument is that as the FRP laminates become stiffer and thicker debonding dominates over tensile fracture and the effective FRP strain is reduced.

From all the above, we can finally write the shear capacity of FRP-strengthened masonry as:

$$\frac{V_{Rd}}{f_k t t} = \frac{0.8}{\gamma_M} \min \left(\frac{f_{vko}}{f_k} + 0.4 \frac{N_{Rd}}{f_k t t}, \frac{f_{vk,max}}{f_k} \right) + \frac{0.7}{\gamma_{frp}} \omega_h \frac{\epsilon_{frp,e}}{\epsilon_{M,u}} \leq \frac{0.25}{\gamma_M} \quad (11)$$

where $\omega_h = \epsilon_{M,u} E_{frp} Q_h / f_k$. The expression for $\epsilon_{frp,e}$ has recently been obtained for concrete members strengthened with FRP in shear in [10]. The same expression is adopted here for masonry structures, and given below:

$$\epsilon_{frp,e} = 0.0124 - 0.0214 (Q_h E_{frp}) + 0.0107 (Q_h E_{frp})^2 \quad (12)$$

where E_{frp} is in GPa.

To obtain a better insight of the FRP contribution to the shear capacity of masonry walls, the results given above are presented in Fig. 5 for typical cases of material properties, as follows: $\epsilon_{M,u} = 0.0035$, $\gamma_M = 2.5$, $f_k = 5$ MPa, $\gamma_{frp} = 1.15$, $f_{vko} = 0.2$ MPa, $f_{vk,max} = 0.5$ MPa (Fig. 5a) and $f_{vk,max} = 1.0$ MPa (Fig. 5b). It is demonstrated that, depending on the axial load level, the

increase in shear capacity due to the external reinforcement can be high, and that it reaches a cut-off value at relatively low values of ω_h , corresponding to very low values of FRP area fractions.

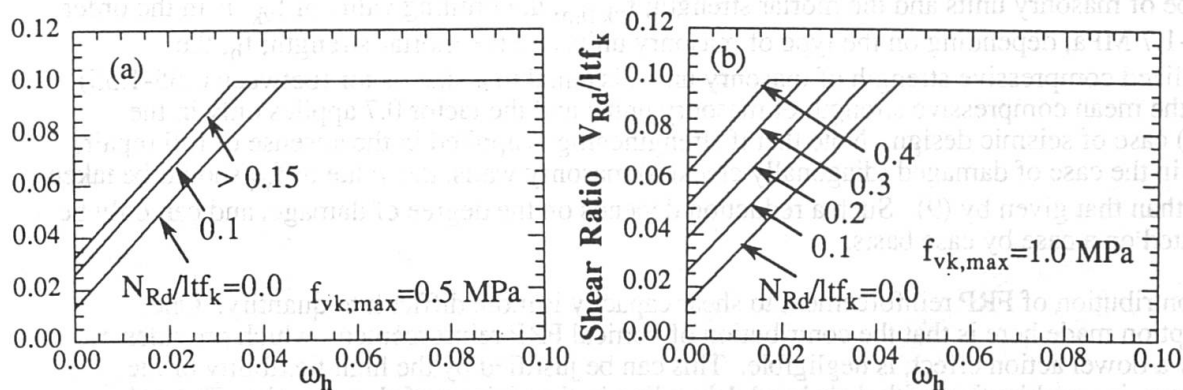


Fig. 5 In-plane shear capacity versus normalized FRP area fraction

3. Conclusions

Strengthening of masonry walls with externally bonded FRP laminates appears to be an attractive alternative to traditional retrofit techniques, especially in cases where implementation of such techniques is impractical. The present study focused on establishing a systematic analysis procedure for the short-term strength of FRP-strengthened masonry walls under monotonic out-of-plane bending, in-plane bending and in-plane shear, all combined with axial load, within the framework of Eurocode 6. It was shown that when out-of-plane bending response dominates, which is typically the case in the upper levels of masonry structures (where axial loads are low), the increase in bending capacity is quite high. Most important in the case of in-plane bending is the area fraction and distribution of reinforcement: high area fractions of reinforcement placed near the highly stressed zones give considerable strength increases. Finally, the in-plane shear capacity of FRP-strengthened walls can be quite high, too, especially in the case of low axial loads.

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