A simplified Model for the Reliability-Based Evaluation of Fatigue in Existing Bridges

Joan R. CASAS
Associate Professor
School of Civil Eng.
Barcelona, Spain

Joan R. Casas, born in 1960, received his civil eng. degree from the Tech. Univ. of Catalunya (UPC) in 1984, where he completed his doctorate in 1988. His research includes dynamics and field testing of bridges as well as bridge reliability
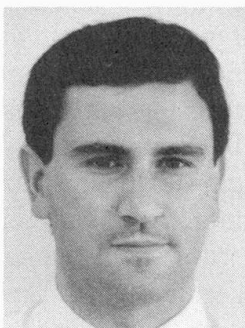
César CRESPO
Research Assistant
School of Civil Eng.
Barcelona, Spain

César Crespo-Minguillón, born 1969, got his civil eng. degree from the Tech. Univ. of Catalunya (UPC) in 1992. He completed his doctorate at UPC in 1996. His research field is the reliability of partially prestressed bridges.

Summary

The paper presents a simplified probabilistic live load model based on the results of a more general model for the simulation of traffic flow over highway bridges developed by the authors [1]. The model, suitable for the fatigue evaluation due to traffic actions, takes advantage of the fact that the rainflow method only uses information about local extremes. The comparison of the results of this simplified model with the general model for a wide range of bridge types, demonstrates its sufficient accuracy and feasibility for practical reliability evaluation purposes.

1. Introduction

The process of evaluation of a bridge clearly involves two separate parts: the updating of the actual resistance and of the loading characteristics, as they can be different from those assumed in the design. Concerning the load part and because the traffic is the most important external action leading to fatigue in short and medium span bridges, to perform a fatigue evaluation it is of principal importance to know the actual stress increments caused by this action during the service life. This can be only achieved via a global model for the continuous traffic flow simulation over the bridge and using the site-specific characteristic of the traffic in or close to the bridge location. However, such a model, formulated in probabilistic terms, is quite complicated and very costly in computational terms. As a consequence, its use in the practical reliability based evaluation of a bridge can be excesively cumbersome and, therefore, rejected. Because of that, a simplified probabilistic live load model, based on the results of a global model for the simulation of the traffic fatigue effects in simply supported and continuous bridges was developed. In this way, a practical and easy application of the fatigue evaluation methods based on Structural Reliability can be performed .

2. Theoretical basis and description of the model

Traffic action on bridges represents a continuous effect in time. In [1] a complete model for continuous traffic flow simulation was developed. To take into account the most important

uncertainties present and to obtain an almost continuous in time history of the traffic effects in the bridge, the general model requires an important computational effort that makes it not suitable for practical evaluation purposes. However, for the traditional fatigue analysis based on the number and magnitude of the increments of the studied effects, the temporal scale provides no information. In fact, the most common method used for cycle counting, the rainflow method, only needs information about the local extremes (local maximums and minimums, Figure 1).
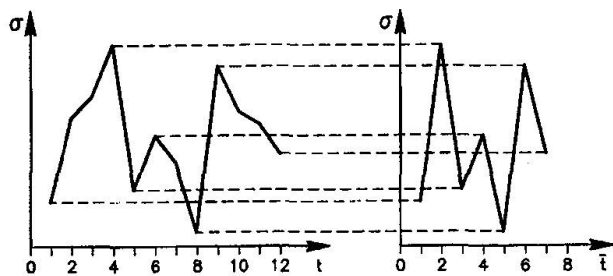
Fig. 1 *Two equivalent diagrams of effects*

Since the sequence of these diagrams of effects that are the base for the later fatigue reliability analysis, is always the same: a local maximum followed by a local minimum, it was thought that it might be worth to study the possibility of deriving an algorithm for the simulation of these special histories of effects. The analysis of the stochastic process"maximum local extremes" [1] showed an immediate drop in the autocorrelation function and that there will not be any dominant frequency in the spectral density function of the process [1]. Thus, it can be concluded that the analysed process, may be treated as a simply random variable. If the Cumulative Distribution Function (CDF) of the extreme maximum effect value were known, it would be very simple to generate a simulated history of this variable.

In figure 2, the pairs (maximum effect, difference between maximum and next minimum effect), for a simply supported and for a continuous bridges are presented. In both cases, a clear linear relationship between the two variables can be seen. It follows that from the history of maximums effects, it should not be very complicated to determine the minimum associated to each maximum. The same effect was observed in many other simply supported and continuous bridges analysed [1].
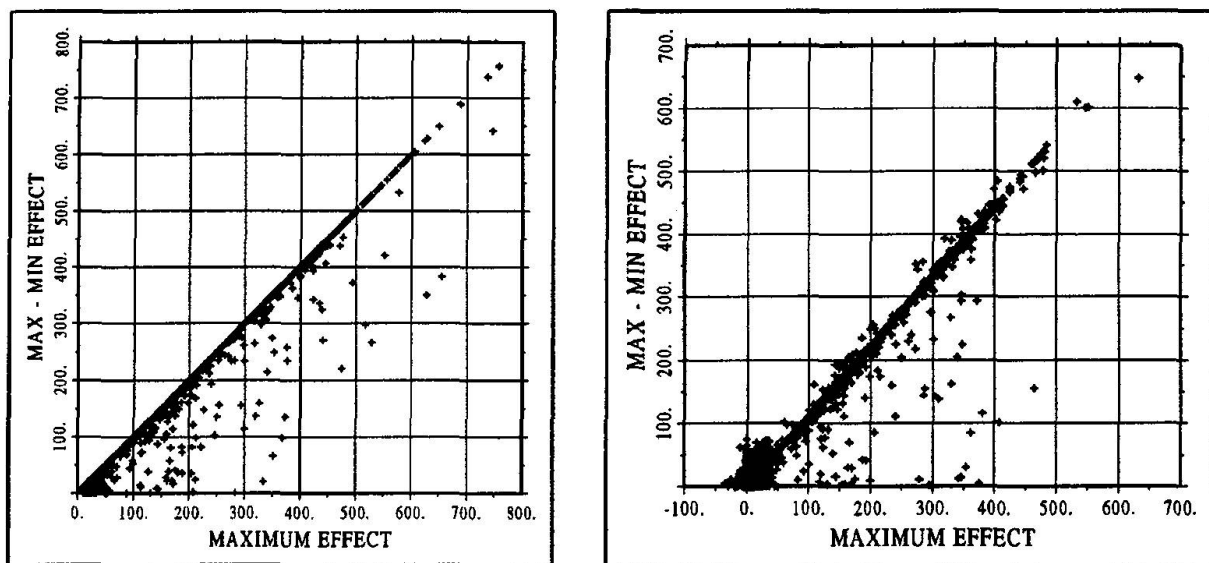
Fig. 2 *Plot of increments of effect vs maximum effect for a simply supported (left) and a continuous (right) bridges*

Next step in the construction of the model consists of the elimination of the transverse structural behavior of each particular deck from the data that will be the base for the model. After several studies, it was decided to use a vehicle type for taking out the dimensions of all recorded histories of traffic. The vehicle was also used for the calculation of the slope of the straight line reflecting the linear relationship between maximum local effects and corresponding increments of effects (Figure 2). The comparison between the real measured slopes and that calculated with this procedure for five continuous bridges with maximum spans from 22 to 150 m, is presented in table 1.

Bridge	Real slope	Calc. slope
S1622	1.149	1.153
S2033	1.201	1.195
B4256	1.167	1.151
C4080	1.107	1.103
C75150	1.103	1.096

Table 1 Relation between the increment of effects and maximum effects

Looking at figure 2, it comes out that the relation between maximums effects and increments of effects, is not exactly linear and deterministic. The deviations of the increments variable around the value given by the deterministic linear relation have been studied. The conclusion is that these deviations can be modelled as a random normally distributed variable with mean equal to 1.0 and COV of 5% in all cases.

The fatigue studies performed showed that only the ranges in the highest levels of stress do really condition the results of the final external fatigue solicitation. Therefore, it was decided to concentrate all efforts in a good definition of the CDF of the non-dimensional local maximum effect, but eliminating the effects caused by light vehicles. The reference value for the effect of a light vehicle was set as a proportion of the 11% of the effect caused by the vehicle type. Next step consisted of studying the extreme maximums higher than the light vehicle limit effect and accomplishing the condition of having minimums after them that lead to effect ranges on the fictitious straight line, (max. effect- range of effect), commented above. The result of this analysis was a CDF of the non-dimensional maximum effects greater than the limit, and a proportion of the number of maximums accomplishing the condition over the total number of lorries expected to cross over the structure. This was done for all bridges, analysing the results from the original model for the simulation of traffic flows. Eleven weeks of simulated traffic results were used.The results for a particular traffic condition are presented in table 2 and figure 3

The average number of lorries per simulated week was calculated to be 41585. So the number of maximum peaks could be thought as a proportion of this average number of expected trucks. It is interesting to realise that the number of maximum extreme situations in the bridges decreases with the length of the deck. This effect is because the probabilities of having situations of several vehicles over the same span of the bridge are higher in long span than in short span bridges.

After seeing the shape of the cumulative distributions functions of maximum non-dimensional peaks (figure 3), it was decided to approximate them in several intervals. The definition of the points of the curves limiting the intervals would be done based on the length of the main spans of the bridges. Given that the curves for the simply supported bridges of lengths 27 and 40 m did not alternate well with the curves of the continuous bridges of close span lengths, it was decided to study both structural types by separate. The study is also divided in two cases of traffic: heavy (Average Daily Traffic of 20,000 vehicles in two lanes with 30 % of trucks) or light (ADT= 10,000 and 15 % of trucks) traffic conditions. In the following, the different parts and steps for the definition of the simplified model are presented.

Bridge	Num of spans	Max. span (m.)	Weekly average number of max. peaks
S1622	3	22	40362
S2033	4	33	39812
B4256	3	56	34303
C4080	3	80	33173
C75150	3	150	27733
S27	1	27	39375
B40	1	40	37939

Table 2 *Non-dimensional maximum peaks higher than light vehicle effect limit*
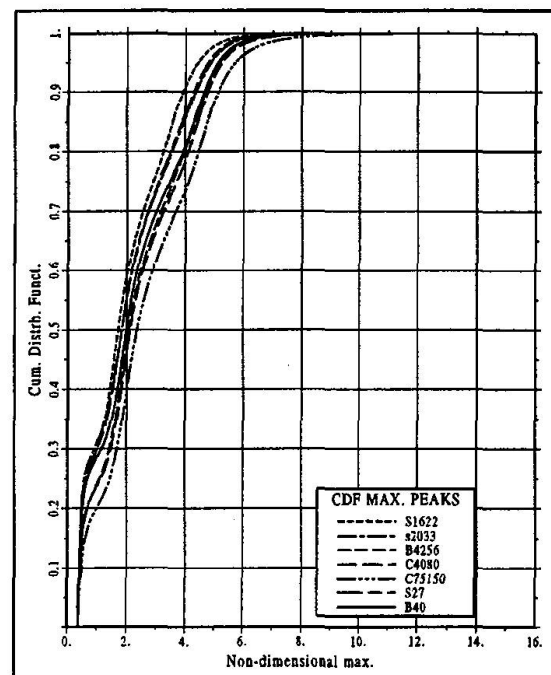
Fig 3 *Cumulative distribution functions of the non-dimensional effect at midspan .*

2.1   Definition of number of simulations

The first step consists of the definition of the number of maximum effects to be simulated. It is given as a proportion of the average number of expected lorries in a week. In table 3, the results of the simulation of traffic flow are presented for the two traffic conditions.

Bridge	Type	Span length (m.)	Heavy traffic ($N_{trucks} = 41585$)		Light traffic ($N_{trucks} = 10384$)	
			Weekly N. of peaks	Proportion over N. of trucks	Weekly N. of peaks	Proportion over N. of trucks
S1622	C	22	40362	0.9706	14461	1.3927
S2033	C	33	39812	0.9574	14971	1.4418
B4256	C	56	34303	0.8249	12859	1.2384
C4080	C	80	33173	0.7977	12619	1.2153
C75150	C	150	27733	0.6669	11655	1.1224
B19	SS	19.6	39934	0.9603	14580	1.4041
S27	SS	27	39375	0.9469	13987	1.3470
B40	SS	40	37939	0.9123	13385	1.2890

C= continuous, SS = simply supported

Table 3 *Relation between the number of peaks and the number of expected trucks*

With the results in table 3, the total number of major maximum extremes to simulate (N_{peaks}) as a function of the total number of expected trucks in a week (N_{trucks}) and the maximum span length of the bridge, can be derived using the expression:

$$\frac{N_{peaks}}{N_{trucks}} = a + b\ L + c\ L^2 \tag{1}$$

The coefficients in equation (1) are shown in table 4 depending on the traffic conditions and bridge type.

Bridge type	Traffic	a	b	c
SS	Heavy	0.9698	3.78e-4	-4.54e-5
	Ligth	1.6159	-1.37e-2	1.37e-4
C	Heavy	1.0772	-4.77e-3	1.36e-5
	Ligth	1.5522	-6.06e-3	2.13e-5

Table 4 *Values of parameters in equation 1. (SS= simply supported, C=continuous)*

2.2   Obtention of the CDF

Once the number of peaks to simulate in each case was known, the next step consisted on the definition of the Cumulative Distribution Function of the variable "non-dimensional local maximum" to be used in the simulation. It was decided to split the function in different intervals. According to figure 3, where the real CDF for the different cases are presented, and to better approximate each case, the limits of the intervals are different depending on the bridge type and traffic conditions. From the different cases studied, it was derived that less of the 10% of the increments of effects had an effective and practical influence on the final fatigue damage. Therefore, it was decided to give the low parts of the CDF through some straight lines, and to concentrate all efforts in the correct definition of its highest part, causing the relevant values. In this way, in tables 5 to 8, the analytical expressions that best fit the real non-dimensional maximums corresponding to the limits of the different intervals in the CDF are given. As an example, in figure 4 the shape of the analytical expression for the case of continuous bridges and heavy traffic conditions is presented and compared to the values of the real CDF.
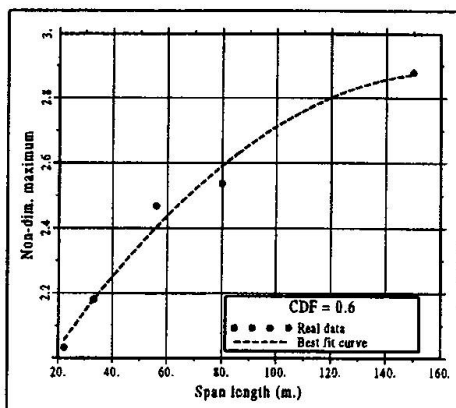
As it was previously explained, the highest efforts were put in the study of the upper tail of the CDF because these are the effects that mostly contribute to the final fatigue damage. The main problem was to choose an analytical expression that were easy to deal with and also, accurate enough to represent the highest effects of traffic. Related to this accuracy, it was intended that the simulated CDF led not only to good estimations of the fatigue life of all studied structures, but also that predicted a histogram of high effects similar to that given by the   true simulation of traffic flow.

Fig. 4 *Shape of expression CDF = 0.6*

CDF	Analytical expression
$0.2^{(1)}$	$0.3799+3.9593 \cdot 10^{-3} \cdot L$
0.4	$01.1928+1.1355 \cdot 10^{-2} \cdot L - 3.7492 \cdot 10^{5} \cdot L^{2}$
0.6	$1.7802+1.3392 \cdot 10^{-2} \cdot L - 4.0710 \cdot 10^{-5} \cdot L^{2}$
$0.925^{(2)}$	$3.1652+5.8923 \cdot 10^{-2} \cdot L - 6.2092 \cdot 10^{-4} \cdot L^{2}$ $+2.1735 \cdot 10^{-6} \cdot L^{3}$
0.999997	$13.8168-8.0269 \cdot 10^{-4} \cdot L + 9.9336 \cdot 10^{-5} \cdot L^{2}$

CDF	Analytical expression
$0.3^{(1)}$	$- 0.369+7.8954 \cdot 10^{-2} \cdot L - 1.0234 \cdot 10^{-3} \cdot L^{2}$
0.7	$0.2358 + 0.1389 \cdot L - 1.7454 \cdot 10^{-3} \cdot L^{2}$
$0.925^{(2)}$	$2.4488 + 0.1100 \cdot L - 1.2480 \cdot 10^{-3} \cdot L^{2}$
0.9999 97	$12.0463+2.4992 \cdot 10^{-2} \cdot L + 5.3353 \cdot 10^{-4} \cdot L^{2}$

Table 5 *Analytical expressions of the NDMs as a function of the main span length (L).Heavy traffic* and *continuous bridges.*

Table 6 *Analytical expressions of the Non-Dimensional Maximums (NDMs). Heavy traffic and simply supported bridges*

CDF	Analytical expression
$0.3^{(1)}$	$0.3080 + 6.6277 \cdot 10^{-3} \cdot L - 1.9895 \cdot 10^{-5} \cdot L^{2}$
0.7	$1.8373 + 1.7929 \cdot 10^{-2} \cdot L - 6.5435 \cdot 10^{-5} \cdot L^{2}$
$0.95^{(2)}$	$2.3728 + 0.1201 \cdot L - 2.0051 \cdot 10^{-3} \cdot L^{2} +$ $1.4020 \cdot 10^{-5} \cdot L^{3} - 3.4392 \cdot 10^{-8} \cdot L^{4}$
0.999994	$13.4615 - 7.8838 \cdot 10^{-3} \cdot L + 7.0042 \cdot 10^{-5} \cdot L^{2}$

CDF	Analytical expression
$0.4^{(1)}$	$- 1.1182 + 0.1292 \cdot L - 1.6605 \cdot 10^{-3} \cdot L^{2}$
0.7	$0.7150 + 8.9375 \cdot 10^{-2} \cdot L - 1.0441 \cdot 10^{-3} \cdot L^{2}$
$0.95^{(2)}$	$3.0665 + 7.7582 \cdot 0^{-2} \cdot L - 8.1213 \cdot 10^{-4} \cdot L^{2}$
0.999994	$12.194 + 9.2991 \cdot 10^{-3} \cdot L$

Table 7 *Analytical expressions of the NDMs. Light traffic and continuous bridges*

Table 8 *Analytical expressions of the NDMs. Light traffic and simply supported bridges*

Finally it was decided to study the problem from the rigorous statistical tail approximation theory. The proposal given in [2], for the analysis of the excesses of a variable over a certain threshold, was used. From the simulation results of 200 weeks of traffic, eleven of them (weeks 1,20,40,...,200), were chosen. After taking the transverse effects of each corresponding surface of influence out with the procedure explained before, the final true-assumed Cumulative Distribution Function of the variable non-dimensional maximum local extreme was obtained in a format given by several thousand points. Then, the excesses over the threshold u corresponding to a CDF of 0.925 (heavy traffic)or 0.95 (light) were analysed in a plot representing the function $-log(1. - F_S(s))$ versus the excess s $(s = x-u)$, over the threshold u. S is a new variable: excess of the studied effect X over the threshold u, and $F_S(s)$ is the Cumulative Distribution Function of this new variable. Knowing the CDF of X, $F_X(x)$, the values of $F_S(s)$ are immediate:

$$F_S (s = x - u) = \frac{F_X (x) - F_X (u)}{1.0 - F_X (u)}$$

(2)

For each case, the threshold u is decided based on the value of the initial point of the interval (a $^{(2)}$ in tables 5 to 8), in the way that $F_X(u) = a$. The expression for calculating its simulated value for each main span length, and traffic conditions is given in tables 5 to 8. In this way, any generated random number higher than "a" will be assumed as a realization of $F_X(x)$. With the expression given in equation 2, it will be transformed into a realization of $F_S(s)$. Then, if the analytical

expression of $F_S(s)$ is known, the corresponding value of s could be calculated. Finally, adding the value of s to the threshold u, $x = u + s$, the value of X corresponding to the original random number assumed for $F_X(x)$ will be obtained.

The plots of $-\log(1.- F_S(s))$ versus s show that a parabolic curve fitting is indicated. Therefore, the next step is to obtain the parameters a1 and a2 of the parabolic curve $a1 \cdot s + a2 \cdot s^2$, that best fits the functions $-\log(1.- F_S(s))$ (s = excess over the threshold), in each case. The analytical expressions to obtain the values of a1 and a2 to define the upper part of the simulated CDF, as a function of the main span length (L) for the 4 cases considered, are as follows:

$$al = p + qL + rL^2 \qquad\qquad a2 = s + tL + wL^2 \qquad (3)$$

Bridge type	Traffic	p	q	r	s	t	w
SS	Heavy	2.686	-5.313e-2	5.508e-4	-0.172	6.787e-3	-8.526e-5
	Ligth	2.379	6.204e-2	-4732e-4	-0.153	-9.362e-4	5.997e-5
C	Heavy	1.740	-5.229e-3	0	-7.837	8.267e-4	-1.991e-6
	Ligth	2.126	-5.736e-3	3.994e-6	-0.126	6.726e-4	-7.170e-7

Table 9 Values of parameters in equation 3. (SS= simply supported, C=continuous)

In figure 5, plots to illustrate the goodness of the analytical expression for a1 and a2 as a function of the main span length, are presented .
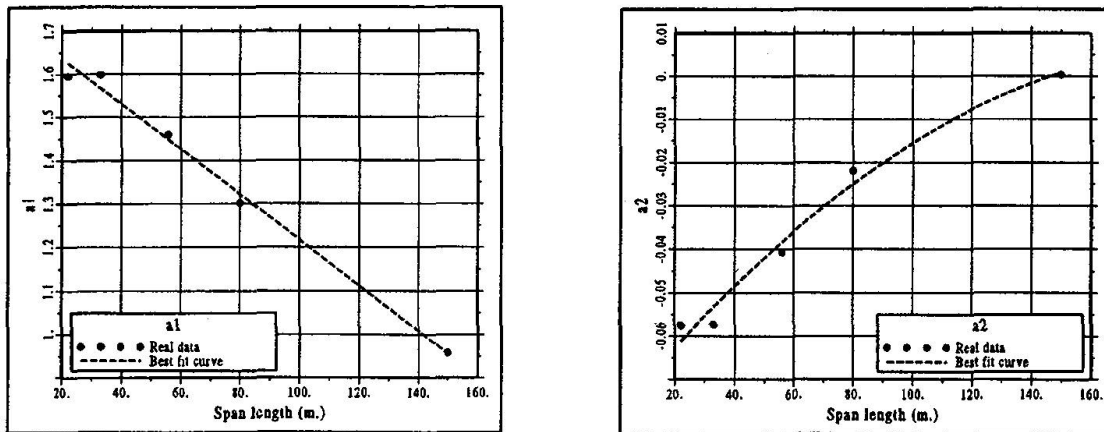
Fig. 5 Plot of real values of a1 and a2 and best fit curve for the case of continuous bridges and heavy traffic conditions

Summarizing, the final simulated CDF of the maximum non-dimensional peaks will have five (table 5) or four parts (tables 6, 7 and 8). The first one will have a parabolic shape, the following three (or two) will be straight lines and the last one will have an exponential shape. The simulation algorithm of peaks becomes then, as follows:

1) The first step consists of computing the points and parameters defining the simulated CDF. So the first interval of the simulated CDF will range from CDF = 0.0 to CDF = [1] (see tables 5 to 8).

The value of the variable corresponding to CDF = 0.0, will be $NDM_0 = 0.367$ constant for all span lengths, traffic conditions and bridge type. The Non-Dimensional Maximum (NDM) corresponding to the value at CDF = [1] will be computed with the corresponding expressions in tables 5 to 8. From the plot of the true CDF in figure 3, it can be deduced that a parabola between these two limits of the first interval would fit the real curve better than a straight line. The slope of the CDF function in the origin, has been found to correspond to a value of 0.524, in all cases.

The values of the studied non-dimensional variable corresponding to the rest of milestone points in the simulated CDF, can be easily computed through the expressions given in tables 5 to 8. Then, for the intervals of CDF between the values [1] and [2], straight lines will be assumed.

2) Definition of the parameters governing the shape of the CDF in its upper part. The threshold value, u, of the variable, (the value corresponding to a true CDF of [2] in tables 5 to 8), can be calculated through the expressions given in tables 5 to 8. The parameters of the parabolic curve: $a1 \cdot s + a2 \cdot s^2$ that best fits the function: $-\log(1.-F_S(s))$, where S represents "the excess of X over the threshold", can be calculated with the expression given in equation (3) and table 9. Then, if a randomly generated number greater than $F_X(u) = $ [2] and less than 1, is thought as a realization of $F_X(x)$, the value of its corresponding $F_S(s)$, $(s = x-u)$, will be easily calculated through equation (2). Knowing the analytical expression of $F_S(s) = 1.0 - \text{EXP}(- a1 \cdot s - a2 \cdot s^2)$, it just rests to solve the second order equation (4) to calculate the value of s (where Fs(s) is already known):

$$a2 \cdot s^2 + a1 \cdot s + \log(1.0 - F_S(s)) = 0. \tag{4}$$

The final inverse to the generated value of the CDF, will be x = u + s.

2.3   Obtention of extreme traffic effects

At this point, all the information needed for the simulation of a fictitious history of extreme traffic effects at the midspan section has already been given.Summarizing, the steps are:

1)Determination of the total number of major maximum extremes as a function of the total number of expected trucks in a week as described in section 2.1 (use equation (1)).

2) Obtention of the non-dimensional maximum peaks using the proposed CDF, as described in section 2.2, in the simulation process.

3) Multiplication of the non-dimensional peaks by the reference constant effect corresponding to the specific surface of influence for each bridge and the assumed vehicle type. In this way, the dimensional effects are obtained. The reference constant is obtained as the maximum effect that the vehicle type, placed on the axle of the slow lane, would cause when crossing a lone over the surface of influence of the bridge. The vehicle type should be chosen so that the range of nondimensional effects was wide enough as to easily detect the differences between the results from the application of the true simulation model and the simplified model.

4) Building up of the final history of simulated traffic effects following the algorithm explained in figure 2 and using the relationship: slope= (max-min)/max, and considering the randomness reflected in figure 2 through a Normal variable with mean 1 and COV of 5 %. The slope will be calculated from the surface of influence of the bridge and the vehicle type, obtaining the maximum and minimum effects of the vehicle when crossing the bridge.

The simulation algorithm to build the final history of peaks in the case of simply supported bridges is similar to the case of the continuous bridges. In this case, however, because of the shape of the surface of influence of the studied effect, each local maximum leads only to two points in the whole history. These two points are the simulated maximum and the corresponding minimum, this last is set at a value of 0.0.

3.   Verification of the model

To check the reliability of the proposed method, the Reliability Index (β) in front of fatigue, following the methodology presented in [1,3], for different bridges with different typologies, span-lenghts and amounts of prestressing is evaluated using the traffic effects obtained from the complete simulation model of traffic flow and those obtained with the simplified model presented in the paper. The results are summarized in table 10.

Bridge and Section	Heavy traffic		Light traffic	
	β_{comp}	β_{simp}	β_{comp}	β_{simp}
S162201	6.48	6.32	7.16	6.82
S162202	6.78	6.62		
S162203	7.55	7.22		
S203301	6.88	6.91	7.39	7.26
S203302	7.05	7.05		
S203303	7.38	7.27		
B425600	4.63	4.58	5.58	5.39
B425603	6.12	6.14		
B425604	6.40	6.30		
C408000	4.38	4.68	5.36	5.31
C408001	6.37	6.55		
C408002	6.75	6.89		
C7515000	4.25	4.56	5.39	5.52
C7515001	5.22	5.34		
C7515002	6.15	6.22		
S2700	4.30	4.04	5.34	5.21
B4000	4.79	4.71	5.69	5.59

Table 10   Comparison of Reliability Indexes in front of fatigue using a complete traffic flow model and the simplified model

In table 10, the letter S in the bridge definitions stands for slab, B for box-girder and C for box girder bridge built by the balanced cantilever method. The results show the good accuracy got by the application of the simplified model. In the case of continuous bridges and hevay traffic, only in three out of the fifteen cases studied, the differences between the results given by the model for simulation of real traffic flows over bridges and those from the simplified model, are greater than 3%. In the case of simply supported bridges, the slight deviation towards lower safety indexes, is probably due to the fact that no factor has been adopted for correcting the relation between the maximum effect and the corresponding increment. In figure 2, it can be seen that for several cases of high maximums, their following minimum did not take a 0.0 value. So, the final increment of effect was lower than the value adopted by the local maximum. The inclusion of this effect into the simulation would add some difficulties to its application. In the current format, the model is accurate, simple and just slightly conservative.

4.   Conclusions

From the aplication of the proposed model for traffic action, the following conclusions are drawn:

1) The simplified model can be used by researchers and professionals that wish to approximate, in an accurate but not cumbersone and way needing high computational resources and time , the Reliability Index in fron of fatigue of a bridge. The model proposed allows for creating, in a very simple and practical way, fictitiuous histories of traffic effects that can be directly used in the rain-flow algorithm. The relation between the accuracy and the savings in terms of computational costs, provided by this simplified model is very high. Its simpleness makes it very suitable for use in both evaluation and design stages. Furthermore, given the special sensitivity of the studied

problem to the actual values of the external effects, (the stress increments are raised to high power in the corresponding S-N curves for the material), the high accuracy reached with this simplified model indicates that the sequence of high extreme maximums resulting from the application of the model, must be very similar to that from the complete traffic simulation.

2) Because the model is based in the results of a complete traffic flow model over bridges, it includes the two-dimensional effect of bridge decks, the effects of possible multiple presence of vehicles in one and/or several lanes, correlations between vehicles,...In the paper, the parameters of the model for continuous and simply supported bridges are presented. Two extreme cases of traffic are also presented. Other traffic conditions can be easily obtained based on the methodology presented in the paper.

3) Although the model has been derived for the study of fatigue effects in partially postensioned concrete bridges, the methodology outlined makes also extensible the results to bridges with other materials (steel, composite,....) since only the geometry of the bridge is used via its surface of influence.

4) An interesting point of the model is that it automatically introduces the situations of multipresence of vehicles on the bridge. This effect is only included in the most complete and detailed traffic model for fatigue checking given in the Eurocode 1- Part 3., which proposes the use of real records of traffic. The simplified model proposed herein is much simpler and cheaper.

Acknowledgments

References

1.   CRESPO-MINGUILLON, C. A reliability based methodology to define the limit state of decompression in prestressed concrete bridges. Ph.D. Thesis. Technical University of Catalunya. Barcelona, 1996.

2.   MAES, M.A. Tail heaviness in Structural Reliability. Proceedings of Applications of Statistics and Probability ICASP, Paris 1995, 997-1002.

3.   CASAS, J.R.; CRESPO-MINGUILLON, C. Serviceability and fatigue evaluation of existing bridges. Proceedings of the US-Europe Workshop on Bridge Engineeering (Casas, Klaiber and Mari Eds.), Barcelona 1996, 441- 464.