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Autor: Mazzolani, Federico M. / Piluso, Vincenzo

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PLASTIC DESIGN OF SEMIRIGID FRAMES FOR FAILURE MODE CONTROL

Federico M. Mazzolani
Department of Structural Analysis and Design, University of Naples, Italy

Federico M. Mazzolani is Full Professor of Structural Engineering at the University of Naples (Italy). Author of more than 350 papers and 12 books in the field of metal structures, seismic design and rehabilitation. Member of many national and international organizations. Presently Chairman of: UNI-CIS/SC3 Steel and Composite Structures, CNR Fire Protection, ECCS-TC13 Seismic Design and CEN-TC250/SC9 Aluminium Alloy Structures.

Vincenzo Piluso
Department of Civil Engineering, University of Salerno, Italy

Vincenzo Piluso, born 1962, graduated in Civil Engineering in 1987 from the Naples University (Italy) where also received his PhD degree in Structural Engineering in 1992. Full member and secretary of ECCS TC13 on Seismic Design and member of COST C1 Seismic Working Group, presently he is researcher at the Department of Civil Engineering of Salerno University. His activity is mainly devoted to steel structures and seismic engineering.

Summary

A new method for the plastic design of moment resisting frames with semirigid connections is presented in this paper. The method is the extension to the case of semirigid frames of a procedure for the failure mode control already proposed by the authors with reference to rigid frames with full-strength beam-to-column connections. Starting from the analysis of the typical collapse mechanisms of frames subjected to horizontal forces, the method is based on the application of the kinematic theorem of plastic collapse. The beam section and the connection details are preliminary designed to resist vertical loads. As a consequence, the unknowns of the design problem are the column sections. They are determined by means of design conditions expressing that the kinematically admissible multiplier of the horizontal forces corresponding to the global mechanism has to be the smallest among all kinematically admissible multipliers. The preliminary design of beams and connections can be accepted provided that checks against the serviceability limit states are satisfied. Therefore, the complete design procedure includes also an iterations to fulfil serviceability requirements. In addition, second order plastic analysis is applied to account for the influence of P-Δ effects through linearized mechanism equilibrium curves.

1. Introduction

The simple design criteria, suggested by modern seismic codes, do not always lead to structural schemes failing in global mode. For this reason, a more sophisticated design procedure, assuring the development of a collapse mechanism of global type, has been recently proposed [1,2,3] and its reliability has been verified on a large number of structural schemes, leading in all cases to the fulfilment of the design requirement [4].

The method is based on the observation that the collapse mechanisms of frames under horizontal forces can be considered belonging to three main typologies (Fig.1). The collapse mechanism of the global type is a particular case of type-2 mechanism. The control of the failure mode can be performed through the analysis of $3n_s$ mechanisms (where n_s is the number of storeys). It is assumed that the beam sections and beam-to-column connections are preliminary designed to resist vertical loads. With reference to extended end plate connections, this preliminary design can be carried out through the procedure suggested in reference [5,6] which is able to guide the designer up to the complete detailing of beam-to-column joints. As a result of this preliminary design, only the column sections have to be determined. Aiming at the failure mode control, the values of the plastic section modulus of columns have to be defined so that the kinematically admissible multiplier of the horizontal forces corresponding to the global mechanism is less than those corresponding to the other

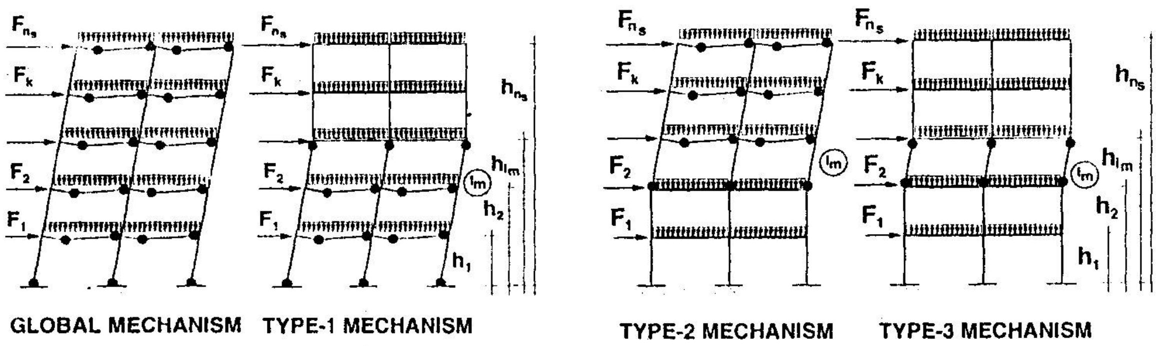

Fig.1 - Analysed collapse mechanism typologies

$3n_s$-1 kinematically admissibile mechanisms. It means that, according to the upper bound theorem, the above stated multiplier is the true collapse multiplier and, therefore, the true collapse mechanism is the global failure mode.

The results of the above design procedure, oriented only to the failure mode control, can be accepted provided that the checks against serviceability limit states are satisfied. In the opposite case, the rotational stiffness of beam-to-column joints or the beam sections have to be increased and the design procedure for failure mode control has to be repeated. Convergence is achieved when both failure mode control and fulfilment of serviceability requirements are obtained.

2. Location of plastic hinges in beams with semirigid connections

The rotational stiffness and the flexural resistance of beam-to-column joints are strictly related. In particular, this has been evidenced with reference to extended end plate connections showing how, decreasing the joint rotational deformability, the joint flexural resistance increases [5,6]. Therefore, depending on the structural detail of the connection, semirigidity can lead to full-strength or to partial-strength joints. In the first case, yielding is located at the member ends so that plastic hinges develop the beam plastic moment. On the contrary, in the second case, yielding occurs in the connecting elements so that plastic hinges develop the joint flexural resistance whose magnitude is less than the beam plastic moment. However, it is important to stress that the location of the plastic hinges depend on the magnitude of vertical loads acting on the beams as well as on the degree of flexural resistance of the beam-to-column connections. In the following, for seak of simplicity, reference will be made only to the case of uniform loads acting on the beams. The results for other beam loading conditions can be similarly derived.

In addition, the case of non-proportional loading will be considered, because failure mode control assumes primary importance in seismic design. The seismic action is modelled through a system of horizontal forces whose distribution can be selected according to a proper combination of the eigenmodes. The magnitude of these horizontal forces is governed by the multiplier α, while the vertical loads are assumed to be constant. For this reason, at any loading stage characterized by a given value of the horizontal force multiplier α, the bending moment diagram of the beams is the result of the superposition of those due to both vertical and horizontal forces. It means that, increasing the horizontal forces (i.e. the multiplier α), the first plastic hinge is always developed at the beam end or the con-

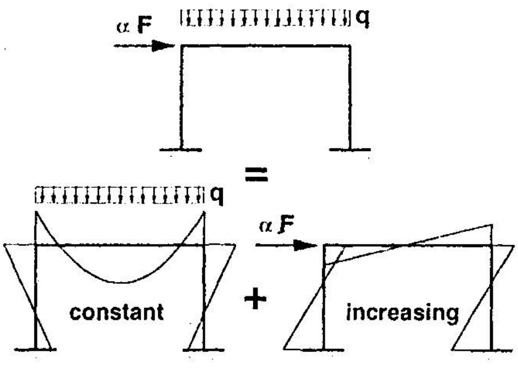

Fig.2 - Plastic hinge location

nection opposite to the horizontal forces (Fig.2).

Regarding the location of the second plastic hinge, it is strictly dependent on the magnitude of vertical loads and on the flexural resistance of connections.

The flexural resistance of connections is expressed through the following nondimensional parameters:

$$\overline{m}_l = \frac{M_{j.Rd}^{(left)}}{M_b} \qquad \overline{m}_r = \frac{M_{j.Rd}^{(right)}}{M_b} \tag{1}$$

where $M_{j.Rd}^{(left)}$ and $M_{j.Rd}^{(right)}$ are the design flexural resistance of left and right beam-to-column joints, respectively; M_b is the design plastic moment of the beam section.

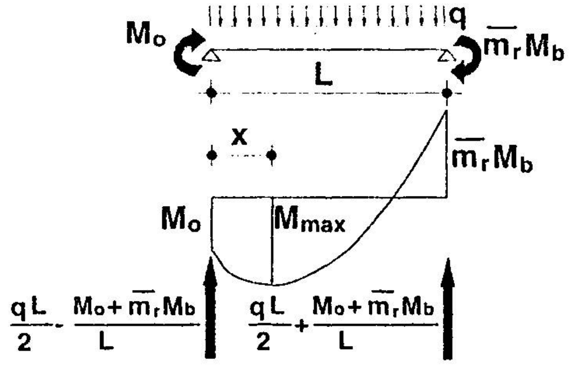

Fig.3 - Analysis of plastic hinge location

The location of plastic hinges can be determined taking into account that the plastic moment $\overline{m}_r M_b$ acts at one end, where the first plastic hinge is formed, while at the second end there is a bending moment M_o which progressively increases due to the progressive increase of the horizontal forces (Fig.3). The maximum bending moment is attained at the abscissa given by:

$$x = \frac{L}{2} - \frac{M_o + \overline{m}_r M_b}{q\,L} \tag{2}$$

where L is the beam length and q is the uniform load acting on the beam.

The maximum bending moment, which occurs at the abscissa provided by equation (2), is given by:

$$M_{max} = \frac{M_o - \overline{m}_r M_b}{2} + \frac{q\,L^2}{8} + \frac{(M_o + \overline{m}_r M_b)^2}{2\,q\,L^2} \tag{3}$$

It can be observed that the second plastic hinge can develop in an intermediate beam section provided that the yielding condition $M_{max} = M_b$ and the limitation $M_o < \overline{m}_l M_b$ are contemporaneously satisfied. The yielding condition $M_{max} = M_b$ gives, through equation (3), a second order equation whose positive solution is given by:

$$M_o = \left[2\,M_b\,q\,L^2\,(\,\overline{m}_r + 1\,)\right]^{1/2} - \overline{m}_r M_b - \frac{q\,L^2}{2} \tag{4}$$

which represents the value of the end moment M_o corresponding to the occurrence of the second plastic hinge at the abscissa provided by equation (2).

By imposing the limitation $M_o < \overline{m}_l M_b$, a limit value is found for the magnitude of the vertical load acting on the beams:

$$q > \frac{2\,M_b}{L^2}\left\{(\,2 + \overline{m}_r - \overline{m}_l\,) + 2\,[\,(\,\overline{m}_r + 1\,)(1 - \overline{m}_l)\,]^{1/2}\right\} \tag{5}$$

which, in the case of full-strength joints, provides $q > 4\,M_b/L^2$.

This means that the second plastic hinge develops in an intermediate beam section provided that relationship (5) is satisfied. In the opposite case, the two beam ends or connections are involved.

The abscissa of the intermediate section where the second plastic hinge forms, provided that condition (5) is satisfied, can be computed by combining equation (4) with equation (2). This gives:

$$x = L - \left(\frac{2\,M_b\,(\,\overline{m}_r + 1\,)}{q}\right)^{1/2} \tag{6}$$

where, obviously, the limit case of full-strength joints is obtained for $\overline{m}_r = 1$.

3. Second order plastic analysis

3.1 Notation

The following notation is adopted:

- n_s is the number of storeys;
- n_b is the number of bays;
- i is the column index;
- i_m is the mechanism index;
- n_c is the number of columns;
- k is the storey index;
- j is the bay index;
- L_j is the span of the jth bay;
- $M_{c,ik}$ is the plastic moment, reduced for the presence of the axial internal force, of the ith column of the kth storey;
- $M_{b,jk}$ is the plastic moment of the jth beam of the kth storey;
- $\overline{m}_{r,jk}$ is the nondimensional plastic moment of the right end beam-to-column joint of the jth bay of the kth storey;
- $\overline{m}_{l,jk}$ is the nondimensional plastic moment of the left end beam-to-column joint of the jth bay of the kth storey;
- q_{jk} is the uniform vertical load acting on the jth beam of the kth storey;
- x_{jk} is the abscissa of the second plastic hinge of the jth beam of the kth storey, given by:

$$x_{jk} = L_j - \left(\frac{2\, M_{b,jk} \left(\overline{m}_{r,jk} + 1 \right)}{q_{jk}} \right)^{1/2} \tag{7}$$

$$\text{for} \quad q_{jk} > \frac{2\, M_{b,jk}}{L_j^2} \left\{ \left(2 + \overline{m}_{r,jk} - \overline{m}_{l,jk} \right) + 2 \left[\left(\overline{m}_{r,jk} + 1 \right) \left(1 - \overline{m}_{l,jk} \right) \right]^{1/2} \right\}$$

while $x_{jk} = 0$ in the opposite case;
- $R_{b,jk}$ is a coefficient related to the participation of the jth beam of the kth storey to the collapse mechanism; in addition, this coefficient accounts for the magnitude of the rotations of the plastic hinges resulting:

$$R_{b,jk} = \frac{L_j}{L_j - x_{jk}} \tag{8}$$

when the jth beam of the kth storey participate to the collapse mechanism and $R_{b,jk} = 0$ in the opposite case;
- $R_{c,ik}$ is a coefficient accounting for the participation of the ith column of the kth storey to the collapse mechanism, being:
  $R_{c,ik} = 2$ when the column is yielded at both ends
  $R_{c,ik} = 1$ when only one column end is yielded
  $R_{c,ik} = 0$ when the column does not participate to the collapse mechanism;
- $D_{v,jk}$ is a coefficient, related to the external work of the uniform load acting on the jth beam of the kth storey, given by:

$$D_{v,jk} = \frac{L_j\, x_{jk}}{2} \tag{9}$$

when the jth beam of the kth storey participate to the collapse mechanism and $D_{v,jk} = 0$ in the opposite case;
- $F^T = \{ F_1, F_2, \ldots\ldots\ldots, F_k, \ldots\ldots\ldots, F_{n_s} \}$ is the vector of the design horizontal forces, where F_k is the horizontal force applied to the kth storey;
- $h^T = \{ h_1, h_2, \ldots\ldots\ldots, h_k, \ldots\ldots\ldots, h_{n_s} \}$ is the vector of the storey heights, where h_k is the height of the kth storey;
- s is the shape vector of the storey horizontal virtual displacements ($du = s\, d\theta$, where $d\theta$ is the virtual rotation of the plastic hinges of the columns involved in the mechanism;

- $V^T = \{ V_1, V_2, \ldots\ldots\ldots\ldots, V_k, \ldots\ldots\ldots, V_{n_s} \}$ is the vector of the storey vertical loads, where V_k is the total vertical load acting at the kth storey given by:

$$V_k = \sum_{j=1}^{n_b} q_{jk} L_j \qquad (10)$$

- B is a matrix of order $n_b \times n_s$ accounting for the location of the plastic hinges within the beams, the element B_{jk} of B is defined as:

$$B_{jk} = \frac{\overline{m}_{l,jk} + \overline{m}_{r,jk}}{2} M_{b,jk} \quad \text{for} \quad x_{jk} = 0 \qquad (11)$$

and:

$$B_{jk} = \frac{1 + \overline{m}_{r,jk}}{2} M_{b,jk} \quad \text{for} \quad x_{jk} > 0 \qquad (12)$$

- C is the matrix of order $n_c \times n_s$ whose elements C_{ik} are equal to the column plastic moments (i.e. $C_{ik} = M_{c,ik}$);
- R_b is the matrix (order $n_b \times n_s$) of the coefficients $R_{b,jk}$;
- R_c is the matrix (order $n_c \times n_s$) of the coefficients $R_{c,ik}$;
- D_v is the matrix (order $n_b \times n_s$) of the coefficients $D_{v,jk}$;
- $M^T_{c,k} = \{ M_{c,1k}, M_{c,2k}, \ldots\ldots, M_{c,ik}, \ldots\ldots, M_{c,n_c k} \}$ is the vector of the plastic moments of the columns of the kth storey, reduced due to the influence of the axial force;
- q is the matrix (order $n_b \times n_s$) of the uniform loads acting on the beams.

3.2 Mechanism equilibrium curves

As already pointed out, the collapse mechanisms of moment resisting frames under seismic horizontal forces can be considered belonging to three main typologies (Fig.1). The collapse mechanism of the global type is a particular case of type 2 mechanism.
The linearized mechanism equilibrium curve can be always expressed as:

$$\alpha_c = \alpha - \gamma\delta \qquad (13)$$

where α is the kinematically admissible multiplier of horizontal forces and γ is the slope of the mechanism equilibrium curve.

Concerning the evaluation of the kinematically admissible multiplier of horizontal forces corresponding to the generic mechanism, it is easy to recognize that, for a virtual rotation $d\theta$ of the plastic hinges of the columns involved in the mechanism, the internal work can be expressed as:

$$W_i = [\, tr(C^T R_c) + 2\, tr(B^T R_b) \,]\, d\theta \qquad (14)$$

where tr denotes the trace of the matrix.

The external work due to the horizontal forces and to the uniform load acting on the beams can be written as:

$$W_e = [\alpha F^T s + tr(q^T D_v)\,]\, d\theta \qquad (15)$$

Therefore the application of the virtual work principle provides the kinematically admissible multiplier as:

$$\alpha = \frac{[\, tr(C^T R_c) + 2\, tr(B^T R_b) - tr(q^T D_v) \,]}{F^T s} \qquad (16)$$

In order to compute the slope of the mechanism equilibrium curve, it is necessary to evaluate the second order work due to vertical loads. With reference to Fig.4, it can be observed that the horizontal displacement of the kth storey involved in the generic mechanism is given by $u_k = r_k \sin\theta$, where r_k is the distance of the kth storey from the center of rotation C and θ the angle of rotation.

The top sway displacement is given by $\delta = H_o \sin\theta$, where H_o is the sum of the interstorey heights of the storeys involved by the generic mechanism.

The relationship between vertical and horizontal virtual displacements is given by $dv_k = du_k \sin\theta$. It shows that, as the ratio dv_k/du_k is independent of the considered storey, vertical and horizontal virtual displacement vectors have the same shape. In fact, the virtual horizontal displacements are given by $du_k = r_k\, d\theta$, where r_k defines the shape of the virtual horizontal displacement vector, while the virtual vertical displacements are given by $dv_k = (\delta / H_o)\, r_k\, d\theta$ and, therefore, they have the same shape r_k of the horizontal ones. It can be concluded that:

$$dv = \frac{\delta}{H_o} s\, d\theta \tag{17}$$

As a consequence, the second order work due to vertical loads is given by:

$$W_v = V^T s \frac{\delta}{H_o} d\theta \tag{18}$$

Therefore, the slope of the mechanism equilibrium curve is given by:

$$\gamma = \frac{V^T s \dfrac{1}{H_o}}{F^T s} \tag{19}$$

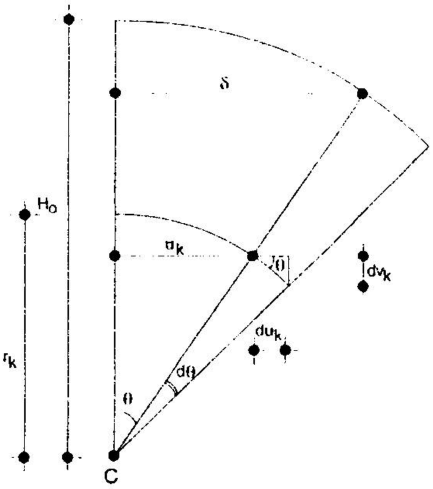

Fig.4 - Vertical displacements

The following notation will be used to denote the parameters of the equilibrium curve of the considered mechanisms:

- $\alpha^{(g)}$ and $\gamma^{(g)}$ are, respectively, the kinematically admissible multiplier of the horizontal forces (rigid-plastic theory) and the slope of the softening branch of the α–δ curve, corresponding to the global type mechanism;
- $\alpha_{i_m}^{(t)}$ and $\gamma_{i_m}^{(t)}$ have the same meaning of the previous symbols, but they are referred to the i_mth mechanism of the tth typology (t=1,2,3).

The expressions of the above parameters will be furtherly developed in order to evidence the contribution of the columns to the internal work.

3.3 Global type mechanism

In the case of global type mechanism (Fig.1), the shape vector of the horizontal displacements is given by $s^{(g)} = h$. In addition, as all storeys participate to the collapse mechanism, all beams are involved. This is taken into account through the matrix $R_b^{(g)}$ related to the rotation of the plastic hinges and the matrix $D_v^{(g)}$ related to the beam vertical displacements. $R_b^{(g)}$ is the value of R_b and $D_v^{(g)}$ is the value of D_v for the specific case of global mechanism.

Therefore, the kinematically admissible multiplier is given by:

$$\alpha^{(g)} = \frac{M_{c1}^T I + 2\, tr(B^T R_b^{(g)}) - tr(q^T D_v^{(g)})}{F^T s^{(g)}} \tag{20}$$

where I is the unit vector of order n_c. In addition, taking into account that $H_o = h_{n_s}$, because all storeys are involved in the collapse mechanism, the slope $\gamma^{(g)}$ of the mechanism equilibrium curve is obtained from equation (19) for $s = s^{(g)}$ and $H_o = h_{n_s}$.

3.4 Type-1 mechanisms

With reference to the i_mth mechanism of type-1 (Fig.1), the shape vector of the horizontal displacements can be written as:

$$s_{i_m}^{(1)T} = \{h_1, h_2, h_3, \ldots\ldots, h_{i_m}, h_{i_m}, h_{i_m}\} \tag{21}$$

where the first element equal to h_{i_m} corresponds to the i_mth component.
The kinematically admissible multiplier corresponding to the i_mth mechanism of type-1 is given by:

$$\alpha_{i_m}^{(1)} = \frac{M_{c1}^T I + 2\, tr(B^T R_{b,i_m}^{(1)}) + M_{ci_m}^T I - tr(q^T D_{v,i_m}^{(1)})}{F^T s_{i_m}^{(1)}} \tag{22}$$

where $R_{b,i_m}^{(1)}$ is the value of R_b for the i_mth mechanism of this type and $D_{v,i_m}^{(1)}$ is the value of D_v for the i_mth mechanism of type-1.
In addition, only the first i_m storeys participate to the collapse mechanism, so that $H_o = h_{i_m}$. As a consequence, the slope $\gamma_{i_m}^{(1)}$ of the mechanism equilibrium curve is still computed through equation (19), but assuming $s = s_{i_m}^{(1)}$ and $H_o = h_{i_m}$.

3.5 Type-2 mechanisms

With reference to the i_mth mechanism of type-2 (Fig.1), the shape vector of the horizontal displacements can be written as:

$$s_{i_m}^{(2)T} = \{0, 0, 0, \ldots\ldots, 0, h_{i_m} - h_{i_m-1}, h_{i_m+1} - h_{i_m-1}, \ldots\ldots, h_{n_s} - h_{i_m-1}\} \tag{23}$$

where the first non-zero element is the ith one.
The kinematically admissible multiplier corresponding to the i_mth mechanism of the type-2 is given by:

$$\alpha_{i_m}^{(2)} = \frac{M_{ci_m}^T I + 2\, tr(B^T R_{b,i_m}^{(2)}) - tr(q^T D_{v,i_m}^{(2)})}{F^T s_{i_m}^{(2)}} \tag{24}$$

where $R_{b,i_m}^{(2)}$ is the value of R_b for the i_mth mechanism of type-2 and $D_{v,i_m}^{(2)}$ is the corresponding value of the matrix D_v.
In addition, the i_mth storey and those above it participate to the mechanism. Therefore, the slope of the mechanism equilibrium curve is obtained from equation (19) with $H_o = h_{n_s} - h_{i_m-1}$ and $s = s_{i_m}^{(2)}$.

3.6 Type-3 mechanisms

Finally, with reference to the i_mth mechanism of type-3 (Fig.1), the shape vector of the horizontal displacements can be written as:

$$s_{i_m}^{(3)T} = \{ 0, 0, .. 0, 1, 1, 1, \ldots, 1 \} (h_{i_m} - h_{i_m-1}) \tag{25}$$

where the first term different from zero is the i_mth one.
Moreover, both the matrix $R_{b,i_m}^{(3)}$ and the matrix $D_{v,i_m}^{(3)}$ are null matrix, because in this mechanism there is not any beam participating to the collapse mechanism. Therefore, the kinematically admissible multiplier of the i_mth mechanism of type-3 is given by:

$$\alpha_{i_m}^{(3)} = \frac{2 M_{ci_m}^T I}{F^T s_{i_m}^{(3)}} \tag{26}$$

which accounts for the fact that the columns of the i_mth storey are yielded at both ends.
As the i_mth storey only is involved in the mechanism $H_o = h_{i_m} - h_{i_m-1}$, and the corresponding slope $\gamma_{i_m}^{(3)}$ of the mechanism equilibrium curve can be obtained by substituting this value in equation (19) where also $s = s_{i_m}^{(3)}$ has to be assumed.

4. Failure mode control

4.1 Design conditions

In order to design frames failing in global mode, the cross-sections of columns have to be dimensioned so that, according to the upper bound theorem, the kinematically admissible horizontal force multiplier corresponding to the global type mechanism is the minimum among all kinematically admissible multipliers.

This condition is sufficient to assure the desired collapse mechanism provided that the structural material behaves as rigid-plastic so that the horizontal displacements are equal to zero up to the complete development of the collapse mechanism. On the contrary, the actual behaviour is elasto-plastic with significant displacements before the complete development of the collapse mechanism. These displacements give rise to second order effects which cannot be neglected in the design process, particularly in the case of semirigid frames.

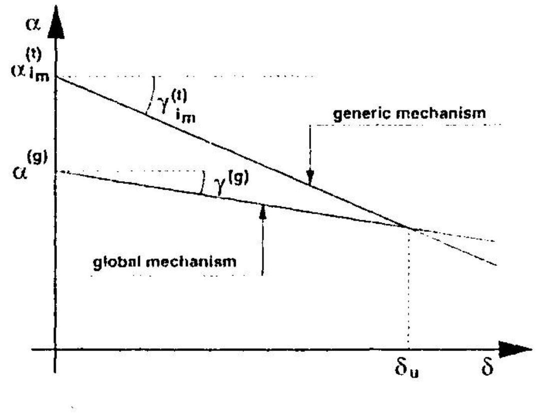

Fig.5 - Design requirements

From the practical point of view, the influence of second order effects can be taken into account by imposing that the mechanism equilibrium curve corresponding to the global mechanism has to lie below those corresponding to all other mechanisms. However, the fulfilment of this requirement is necessary only up to a selected ultimate displacement δ_u which has to be compatible with the plastic rotation capacity of members and/or connections ($\delta_u = \theta_{p_u} h_{n_s}$) (Fig.5).

Therefore, the following design conditions have to be imposed:

$$\alpha^{(g)} - \gamma^{(g)}\ \delta_u \leq \alpha^{(t)}_{i_m} - \gamma^{(t)}_{i_m}\ \delta_u \qquad i_m = 1,2,3,.....,n_s \qquad t = 1,2,3 \tag{27}$$

This means that there are $3n_s$ design conditions to be satisfied in the case of a frame having n_s storeys. These conditions, which derive directly from the extension of the upper bound theorem to the mechanism equilibrium curves, will be integrated by conditions related to technological limitations.

4.2 Conditions to avoid undesired mechanisms

As an example, the method for deriving the design conditions to be satisfied to avoid any undesired collapse mechanism will be presented with reference to type-1 mechanisms only. The extension to the other collapse mechanism typologies can be developed in analogous way. Even though considering full-strength connections only, the complete series of design conditions (i.e. including those for type-2 and type-3 mechanisms) are given in references [2,3].

The n_s conditions given by relationship (27) for $t=1$ can be conveniently expressed by introducing the following parameters:

$$\mu^{(g)} = 2\ tr(B^T R_b^{(g)}) \qquad v^{(g)} = \frac{1}{h_{n_s}} V^T s^{(g)} \qquad \tau^{(g)} = tr(q^T D_v^{(g)}) \tag{28}$$

With reference to the global mechanism, the parameter $\mu^{(g)}$ represents the internal work developed by the beams and/or the connections, the parameter $\tau^{(g)}$ is the external work due to the uniform loads acting on the beams, while the parameter $v^{(g)}$ is related to the second order work due to vertical loads. These parameters are known quantities, because it is intended that a preliminary design of beams and connections has been carried out according to the procedure suggested in [5,6] while the values of vertical loads are data of the design problem.

In addition, it is useful to introduce the following non-dimensional functions of the mechanism index i_m:

$$\xi_{i_m} = \frac{2\ tr(B^T R^{(1)}_{b_{i_m}})}{2\ tr(B^T R_b^{(g)})} = \frac{2\ tr(B^T R^{(1)}_{b_{i_m}})}{\mu^{(g)}} \qquad \lambda_{i_m} = \frac{F^T s^{(1)}_{i_m}}{F^T s^{(g)}} \tag{29}$$

$$\zeta^{(1)}_{i_m} = \frac{tr(q^T D^{(1)}_{v_{i_m}})}{tr(q^T D^{(g)}_{v})} = \frac{tr(q^T D^{(1)}_{v_{i_m}})}{\tau^{(g)}} \tag{30}$$

The function ξ_{i_m} represents the ratio between the internal work which the beams and/or the connections develop in the i_mth mechanism of type-1 and that developed in the global mechanism. The function λ_{i_m} represents the ratio between the external work which the horizontal forces develop in the i_mth mechanism of type-1 and that developed in the global mechanism. Finally, the function ζ_{i_m} represents tha ratio between the external work which the uniform vertical loads develop in the i_mth mechanism of type-1 and that developed in the global mechanism. All these functions are known, because the plastic moments of beams ($M_{b,jk}$) and of connections ($\overline{m}_{l,jk}$ and $\overline{m}_{r,jk}$) are known. In fact, the beam sections are designed to resist vertical loads. In addition, both the horizontal forces F_k and the uniform loads q_{jk} are assigned.

Moreover, in order to account for the influence of second order effects, an additional function related to the slopes of the mechanism equilibrium curves has to be defined:

$$\Delta^{(1)}_{i_m} = \frac{F^T s^{(g)}}{F^T s^{(1)}_{i_m}} \frac{\frac{1}{h_{i_m}} V^T s^{(1)}_{i_m}}{\frac{1}{h_{n_s}} V^T s^{(g)}} = \frac{1}{\lambda_{i_m}} \frac{\frac{1}{h_{i_m}} V^T s^{(1)}_{i_m}}{\nu^{(g)}} \tag{31}$$

The parameter $\Delta^{(1)}_{i_m}$ represents the ratio between the slope of the equilibrium curve of the i_mth mechanism of type-1 and that of the global mechanism.

In addition, it is convenient to introduce the following parameter:

$$\rho^{(1)}_{i_m} = \frac{M^T_{c.i_m} I}{M^T_{c1} I} = \frac{\sum_{i=1}^{n_c} M_{c.ii_m}}{\sum_{i=1}^{n_c} M_{c.i1}} \tag{32}$$

which is the ratio between the sum of the reduced plastic moments of the i_mth storey columns and the same sum corresponding to the first storey columns.

By means of the above parameters, the i_mth condition to be satisfied to avoid type-1 collapse mechanisms can be written in the following form:

$$\rho^{(1)}_{i_m} \geq \frac{\left(1 - \frac{1}{\lambda_{i_m}}\right)\sum_{i=1}^{n_c} M_{c.i1} + \left(1 - \frac{\xi_{i_m}}{\lambda_{i_m}}\right)\mu^{(g)} + \nu^{(g)}\left(\Delta^{(1)}_{i_m} - 1\right)\delta + \tau^{(g)}\left(\frac{\zeta^{(1)}_{i_m}}{\lambda_{i_m}} - 1\right)}{\frac{1}{\lambda_{i_m}} \sum_{i=1}^{n_c} M_{c.i1}} \tag{33}$$

which has to be applied for $i_m=1,2,3,......n_s$.

The design conditions to be satisfied to avoid type-2 and type-3 collapse mechanisms are obtained in similar way, leading to other two series of parameters ($\rho^{(2)}_{i_m}$ and $\rho^{(3)}_{i_m}$). These parameters are still defined as the ratio between the sum of the reduced plastic moments of the columns of the i_mth storey and the same sum corresponding to the first storey, but they provide the values of this ratio to avoid type-2 and type-3 mechanisms [2,3].

Obviously, as these design conditions have to be contemporaneously satisfied for each storey, the ratios ρ_{i_m} ($\rho_{i_m} = M_{ci_m} I / M_{c1} I$) has to satisfy the following relationship:

$$\rho_{i_m} = \max\left\{ \rho^{(1)}_{i_m}, \rho^{(2)}_{i_m}, \rho^{(3)}_{i_m} \right\} \tag{34}$$

In addition, as the section of columns can only decrease along the height of the frame, the values of ρ_{i_m} (with $i_m=1,2,.....,n_s$) obtained by means of the conditions derived through the application of the upper bound theorem have to be modified in order to satisfy the following technological limitation:

$$\rho_1 \geq \rho_2 \geq \rho_3 \geq \ldots\ldots \geq \rho_{n_s} \tag{35}$$

4.3 Evaluation of the axial load in the columns at the collapse state

If the sum of the reduced plastic moments of columns of the first storey is specified, then the previously explained design conditions allow the definition, through the ratios ρ_k ($k=1,2,\ldots,n_s$), of the same sum corresponding to the kth storey, which guarantees that failure does not occur according to mechanisms belonging to the three examined typologies. In order to define the plastic section modulus of the columns, the evaluation of the axial load in the columns at the collapse state is required.

The evaluation of the column axial forces can be performed taking into account that, at the collapse state, the shear forces transmitted by the beams are given by:

$$S = \frac{q\,L}{2} \pm \frac{2\,(\overline{m}_r + \overline{m}_l)\,M_b}{L} \tag{36}$$

provided that the limit value of the uniform vertical load is not exceeded (i.e. equation (5) is not satisfied) and by:

$$S = \frac{q\,L}{2} \pm \frac{M_o + \overline{m}_r\,M_b}{L} \tag{37}$$

where M_o is provided by equation (4), in the opposite case.

Both in equation (36) and in equation (37), for positive horizontal forces (from left towards right), the sign plus is referred to the right end of the beam and the sign minus is referred to the left end of the beam.

The sum of these shear forces transmitted by the beams at each storey, above the considered one, provides the axial forces in the columns of the considered storey.

4.4 Design algorithm

It has been pointed out that the upper bound theorem allows the assessment of a condition for avoiding each undesired collapse mechanism. As three different collapse mechanism typologies have been considered, there are $3n_s$ design conditions to be satisfied. These design conditions have to be integrated by the technological condition (35). The above mentioned relationships can be used to design frames failing in global mode and, therefore, having a mechanism equilibrium curve given by equation (13), with the kinematically admissible multiplier of horizontal forces given by equation (20) and the slope given by relationship (19) (with $H_o = h_{n_s}$ and $s = s^{(g)}$). The fulfilment of the above design requirements is a linear programming problem which can be solved through the algorithm already described in [2,3].

5. Design procedure

The main difficulty in the elastic design of semirigid frames is due the fact that the internal actions, which the members and the joints have to withstand, depend on the joint rotational stiffness whose value, in turn, affects the flexural resistance that the joints are able to provide [5,6]. This difficulty can be overcome provided that a plastic design approach, such as that previously described, is adopted. Notwithstanding, some iterations can be required as soon as serviceability requirements are also considered. In fact, the fulfilment of a given limit concerning the interstorey drift or the top sway displacement can lead to the need to increase the joint rotational stiffness and/or the beam sizes. In such a case, the increase of the plastic internal work due to the beams (the joint flexural resistance increases as the joint rotational stiffness increases) can undermine the expected failure mode, so that the plastic design for failure mode control has to be repeated starting from the new beam-to-column joint details and/or the new beam sizes. As a consequence, the complete design procedure can be based on the following steps, where the plastic method of design for failure mode control, previously described, has to be intended as a «subroutine» only of the proposed design method:

a) perform a preliminary design of beams (i.e. $M_{b,jk}$), connections and columns to withstand vertical loads only. This step can be accomplished through the method described in [5,6]. According to Eurocode 3 [7], the combination of actions $1.35\ G_k + 1.5\ Q_k$ has to be considered for the ultimate limit state and the combination $G_k + Q_k$ for the serviceability limit state;
b) compute the preliminary values of the joint flexural resistance ($\overline{m}_{r,jk}$ and $\overline{m}_{l,jk}$) through the component method [8];
c) design the column sections to assure a collapse mechanism of global type (i.e. through the plastic design method described in the previous section), starting from the preliminary values of $M_{b,jk}$, $\overline{m}_{r,jk}$ and $\overline{m}_{l,jk}$. According to Eurocode 8 [9], the vertical loads to be considered in this step are those corresponding to the load combination $G_k + \psi_2\ Q_k$ while the seismic horizontal forces have to be computed accounting for the presence of all gravity loads appearing in the combination $G_k + \sum\ \psi_{E,i}\ Q_{ki}$;
d) modify, if necessary, the structural detail of beam-to-column joints to keep constant the $\overline{m}$ values. In fact, as the previous step leads generally to column sections greater than those obtained from preliminary design (step a), the joint flexural resistance could increase (this depends on the weakest joint component) undermining the expected collapse mechanism;
e) compute the joint rotational stiffness through the component method;
f) check the beams, the joints, the interstorey drifts and the top sway displacement for the loading condition $\sum G_{ki} + \gamma_I\ A_{Ed} + \sum \psi_{2i}\ Q_{ki}$ [9]. If anyone of the above checks is not satisfied, modify the beam sizes or the joint structural detail (increasing $\overline{m}$ and the joint rotational stiffness) and return to step c.

6. Conclusions

A new method to design semirigid frames failing in global mode has been presented in this paper. The method is based on the extension of the kinematic theorem of plastic collapse to the concept of mechanism equilibrium curve. This allows to include into the design process the influence of second order effects, which play a very important role in the seismic design of steel frames, particularly in the case of semirigid frames.

In addition, a complete design procedure including the fulfilment of the serviceability requirements has been proposed.

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