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## Endplate and web plate joints: Moment–rotation behaviour

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### Summary

The paper presents short description of methods and computer programs to determine “moment–rotation” characteristics of bolted beam–column joints with endplate and single/double web plate, which have been developed in Scientific Research Institute Promstalkonstruktsiya, Moscow. They are capable to calculate these types of joints taking into account any variation of geometrical and mechanical parameters of beams, columns and connection details. The calculation examples are also given.

### 1. Introduction

Traditionally the joints are considered as rigid or pinned at analysis of steel skeletons. However all joints possess definite bending strength, rigidity and deformativity. Consideration of these properties mutually enables to decrease the total weight of multistory steel structures up to 20%. To describe the behaviour of joints the correlations between the bending moment transmitted by the joint ( $M$ ) and its rotation ( $\phi$ ) are usually used. The works on investigation and determination of  $M$ – $\phi$  characteristics of beam–column joints of different types are being provided now in Scientific Research Institute Promstalkonstruktsiya, Moscow. This paper is devoted to definition of  $M$ – $\phi$  characteristics of joints with endplate and single/double web plate.

### 2. Endplate joints

#### 2.1. General part

The theoretical model of endplate joints behaviour takes into account 9 endplate types, which are shown in *Fig. 1*. Columns can be strengthened by stiffeners. Bolts can be pretension.

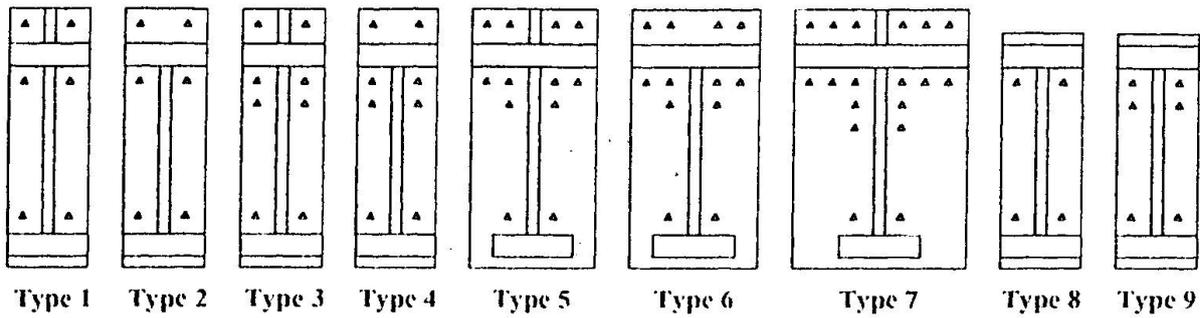


Fig. 1. Endplate types.

Shown in Fig. 2 the "moment-rotation" characteristic is produced with the formula proposed by Chen and Kishi [1]

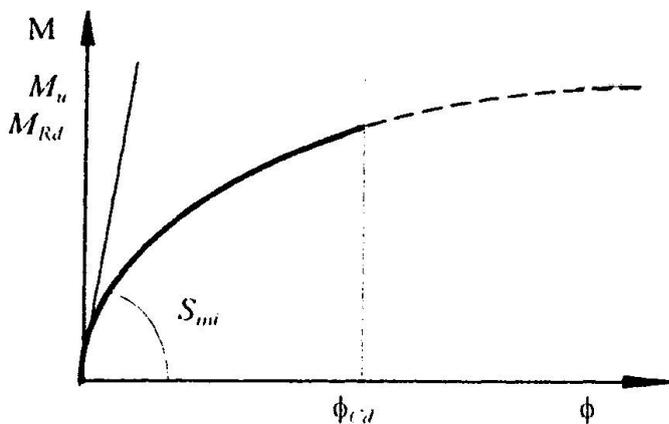


Fig. 2. "Moment-rotation" characteristic.

$$M = \frac{S_{mi} \phi}{\left[ 1 + \left( \frac{S_{mi} \phi}{M_u} \right)^{1.8} \right]^{0.56}} \quad (1)$$

$$\phi_{cd} = \frac{M_u M_{Rd}}{S_{mi} (M_u^{1.8} - M_{Rd}^{1.8})^{0.56}} \quad (2)$$

$S_{mi}$  – initial stiffness

$M_u$  – ultimate moment capacity

$M_{Rd}$  – design moment resistance

$\phi_{cd}$  – design rotation capacity

The partial safety factors are taken equal to 1,0 for the ultimate moment capacity calculation and equal to their real values for design moment resistance calculation.

The parameters calculation of formulac (1) and (2) is produced for the total loadintroduction "TL" (for points PL and PC in Fig. 3), the column web in shear "s.wc" (for points PS and PC in Fig. 3) and total joint "TJ" (for point PC in Fig. 3).

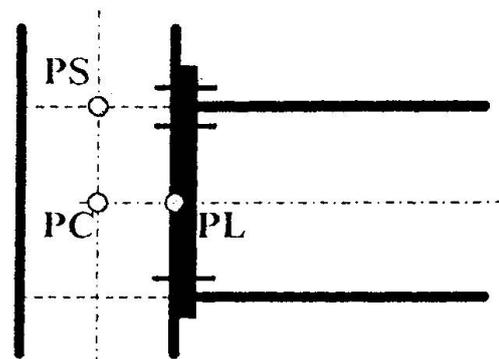


Fig. 3. Points definition according [2].

## 2.2. Total loadintroduction

### 2.2.1 Initial stiffness

$S_{TL,mi,PL}$  is calculated by the computer program "Flora" which is based on the elastic finite element model of endplate joints. The column flange and the endplate represent two independent plates divided to finite elements. The bolts are the internal constraints between the

column flanges and the endplates finite element nodes. Bolt pretension is accepted. The column web and stiffener behaviour is simulated by the constraints in the column flange on its connection line with the column web and stiffener.

$$S_{TL,mi,IV} = \frac{S_{j,b} S_{TL,mi,PL}}{S_{j,b} - S_{TL,mi,PL}} \quad (3)$$

$$S_{j,b} = 2EI_b/h_c \quad (4)$$

### 2.2.2 Strength

Ultimate moment capacity and design moment resistance are defined as minimal bending moment which is necessary to apply to the joint to obtain the limit state in one of its elements. The following limit states for the total joint are considered:

- crushing forces achievement in maximally loaded bolts if moment  $M_{b,u}$  or  $M_{b,Rd}$  acts;
- plastic hinge propagation into the endplate if moment  $M_{ep,u}$  or  $M_{ep,Rd}$  acts;
- plastic hinge propagation into the column flange if moment  $M_{fc,u}$  or  $M_{fc,Rd}$  acts;
- yield stresses achievement in the part of column web subjected to local tension if moment  $M_{t,wc,u}$  or  $M_{t,wc,Rd}$  acts;
- yield stresses achievement in the part of column web subjected to local compression or column web buckling if moment  $M_{c,wc,u}$  or  $M_{c,wc,Rd}$  acts.

$$M_{TL,u,PL} = M_{TL,u,IV} = \min\{M_{b,u}, M_{ep,u}, M_{fc,u}, M_{t,wc,u}, M_{c,wc,u}\} \quad (5)$$

$$M_{TL,Rd,PL} = M_{TL,Rd,IV} = \min\{M_{b,Rd}, M_{ep,Rd}, M_{fc,Rd}, M_{t,wc,Rd}, M_{c,wc,Rd}\} \quad (6)$$

## 2.3. Column web in shear

### 2.3.1 Initial stiffness

$S_{s,wc,mi,PS}$  is calculated by the traditional methods of the theory of elasticity.

$$S_{s,wc,mi,IV} = \frac{S_{j,c} S_{s,wc,mi,PS}}{S_{j,c} - S_{s,wc,mi,PS}} \quad (7)$$

$$S_{j,c} = 4EI_c/h_j \quad \text{-- Internal and End Joints} \quad (8)$$

$$S_{j,c} = 2EI_c/h_j \quad \text{-- T and Knee Joints} \quad (9)$$

### 2.3.2 Strength

Ultimate moment capacity  $M_{s,wc,u,PS} = M_{s,wc,u,PC}$  and design moment resistance  $M_{s,wc,Rd,PS} = M_{s,wc,Rd,PC}$  are defined using the proposals of Innsbruck University [3].

## 2.4. Total joint

### 2.4.1 Initial stiffness

$$S_{TL,mi,IV} = \frac{S_{TL,mi,IV} \cdot S_{s,wc,mi,IV}}{S_{TL,mi,IV} + S_{s,wc,mi,IV}} \quad (10)$$

### 2.4.2 Strength

$$M_{TL,u,IV} = \min\{M_{TL,u,IV}, M_{s,wc,u,IV}\} \quad (11)$$

$$M_{TL,Rd,IV} = \min\{M_{TL,Rd,IV}, M_{s,wc,Rd,IV}\} \quad (12)$$



## 2.5. Used formulae

All used formulae are included in the documentation of the International Module Bank System, developed at the Institute of Steel and Timber Construction, University of Innsbruck.

## 2.6. Computer program

The computer program is aimed for the determination of "moment-rotation" characteristics of endplate joints and connected to the International Module Bank System as Module No. 3 MOS\_EP [4]. Program output data are:

- values of the initial stiffness', the ultimate moment capacities, the design moment resistances and the coordinates of the relationships "moment-rotation" for total joint and separately for shear panel and total load introduction;
- characteristic points of the beam line for the condition of yield stresses constant level;
- coordinates of a intersection point of "moment-rotation" curve with beam line (support moment and rotation);
- value of the related load  $q_{rel}$  [5] demonstrating how much times the load taken up by the beam with chosen joint exceeds the load taken up by the same beam with pinned supporting (the uniformly distributed load is considered).

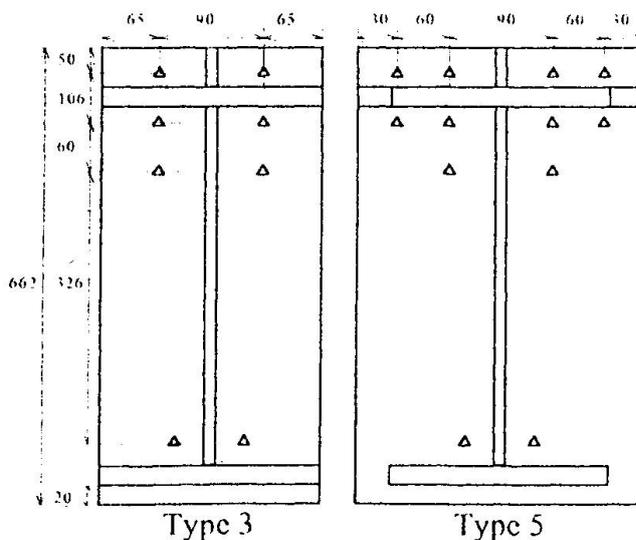
## 2.7. Calculation example

### 2.7.1. Joints characteristics

Ten endplate joints with the following characteristics have been chosen for calculation:

- beam 55B2 ( $h_b=547$  mm,  $b_b=220$  mm,  $t_{fb}=15.5$  mm,  $t_{wb}=10.0$  mm,  $r_b=24$  mm) with the span  $l_b=6$  m and 12 m;
- column 30C1 ( $h_c=296$  mm,  $b_c=300$  mm,  $t_{fc}=13.5$  mm,  $t_{wc}=10.0$  mm,  $r_c=18$  mm);
- steel of beam, column, backing plates and column stiffeners S255 ( $f_y=245$  Mpa,  $\gamma_{M0}=1.025$ );
- endplates steel S390 ( $f_y=390$  MPa,  $\gamma_{M0}=1.025$ );
- bolts: diameter M24, high strength ( $f_u=1100$  MPa,  $\gamma_{MB}=1.43$ ).

Endplate dimension are shown in *Fig. 4*, thickness' of endplate ( $t_{ep}$ ), column stiffeners ( $t_{st}$ ), backing plates ( $t_{bp}$ ) and bolt pretension efforts ( $B_p$ ) are presented in *Table 1*.



*Fig. 4. Endplate dimension.*

No	Type	$t_{ep}$ mm	$t_{st}$ mm	$t_{bp}$ mm	$B_p$ kN
1	3	25	16	16	239
2	3	16	10	10	239
3	3	25	16	16	–
4	3	16	10	10	–
5	3	25	–	–	–
6	5	25	16	16	239
7	5	20	10	10	239
8	5	25	16	16	–
9	5	20	10	10	–
10	5	25	–	–	–

*Table 1. Selected characteristics of endplate joints elements.*

### 2.7.2. Calculation results

The values of initial stiffness, design moment resistances and related loads of all numerical tests are presented in *Table 2*.

N	$S_{IL,ini,PL}$	$S_{s,wc,ini,PS}$	$M_{b,Rd}$	$M_{cp,Rd}$	$M_{jc,Rd}$	$M_{l,wc,Rd}$	$M_{c,wc,Rd}$	$M_{s,wc,Rd}$	$q_{rel}$	$q_{rel}$
	kNm/rad	kNm/rad	kNm	kNm	kNm	kNm	kNm	kNm	$l_b=6m$	$l_b=12m$
1	500000	203698	556	1690	1060	965	846	1526	1,54	1,45
2	377358	168242	538	688	667	743	604	1034	1,66	1,49
3	168067	203698	556	1690	1060	965	846	1526	1,89	1,63
4	148148	168242	538	688	667	743	604	1034	1,78	1,76
5	111111	109147	556	1690	333	374	255	216	1,40	1,32
6	512821	203698	851	1896	1005	965	846	1526	1,70	1,60
7	465116	168242	801	1211	634	743	613	1034	1,97	1,90
8	253165	203698	851	1896	1005	965	846	1526	1,90	1,70
9	229885	168242	801	1211	634	743	613	1034	1,88	1,97
10	116279	109147	851	1896	310	374	255	216	1,40	1,29

Table 2. Endplate joints calculation results.

The corresponding total load introduction “moment–rotation” curves for point PL are shown in *Fig. 5*.

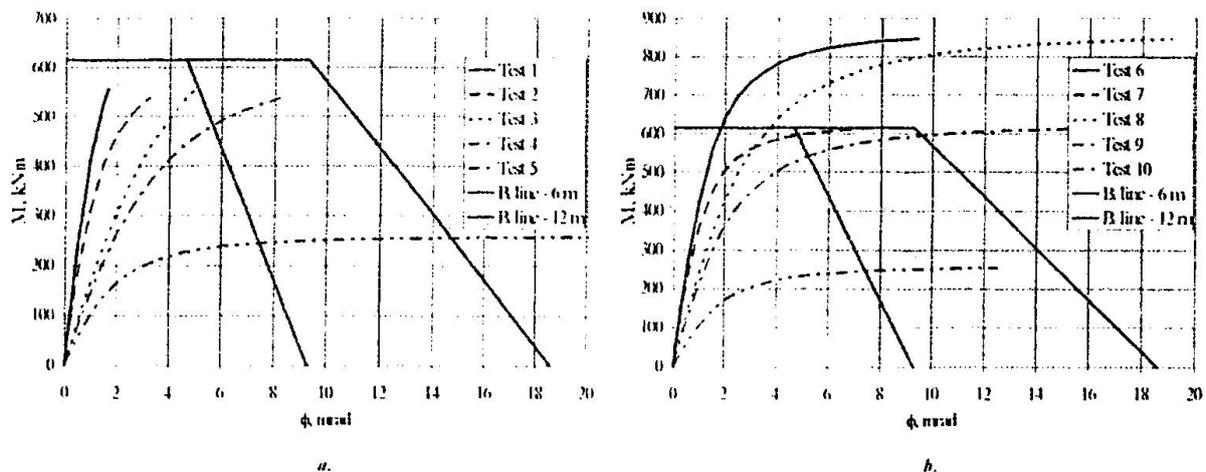


Fig. 5. Total load introduction endplate joints “moment–rotation” curves for point PL: a) tests 1 – 5, b) tests 6 – 10.

Present examples demonstrate the efficiency of endplate joints with four vertical rows and pretension bolts.

## 3. Single/double web plate joints

### 3.1. Description of algorithm and computer program

The theoretical algorithm to determine the “moment–rotation” characteristics is developed on the base the investigation of real behaviour of joints. The main consumption into deformativity



of joints is made by the deformations of mutual rotation between connected elements. These deformations take place because of the bolts bearing and bending, the connected elements bearing, the beam slippage relatively to the plate when the tolerances between the bolts and the holes are available.

The determination of "moment-rotations" characteristics which characterize mutual rotation of connected elements is obtained by the computer program SDVIG. The main principles of the analytical model which is used in the program are:

- the displacements  $\delta_i$  of the beam web points relatively to the plate are proportionally to their distance to the rotation center  $r_i$ ;
- the respective forces  $N_i$  taken up by the bolts and the connected elements are obtained with the help of real non-linear curves "force-displacement" defined from the tests of single-bolts connections subjected to shear;
- moment  $M$  taken up by the joint is defined as sum of moments  $M_i$ , taken up by each couple of bolts symmetrically arranged relatively to rotation center:  $M = \sum M_i = \sum N_i r_i$ ;
- specified successive steps for gradual increase of rotation it is executed the calculation of the according moments values.

It is necessary to point out that the presented model contains principally new approach to determine the forces distribution between the bolts in the connection comparatively to the traditional approaches. These traditional methods consider that the forces taken up by the bolts is distributed between them proportionally to distance from their rotation center. In proposed method the bolt forces are defined depending upon the real displacements of the connected elements in the places where the bolts are installed.

Program SDVIG input data are: Russian and European beam profiles; plate thickness; steel of beam and plate; number of plates; number of bolt rows; number of bolts per row; dimension of connection; bolts grade - high strength, 10.9, 8.8 or 5.8; bolts diameter - M24 or M20; holes diameter; beam length; maximum rotation and number of steps. Output data of the program SDVIG are the same one of the program MOS\_EP.

### 3.2. Calculation example

Six single web plate joints with the following characteristics have been chosen for calculation:

- beam 80B1 ( $h_b=791$  mm,  $h_p=280$  mm,  $t_{pb}=17.0$  mm,  $t_{wb}=13.5$  mm,  $r_b=26$  mm) with the span  $l_b=6$  m and 9 m;
- beam steel S345-3 ( $f_y=320$  MPa,  $\gamma_{M0}=1.025$ );
- web plate ( $t_p = 20$  mm, 18 mm, 16 mm, 14 mm, 12 mm, 10 mm);
- web plate steel S255 ( $f_y=245$  MPa,  $\gamma_{M0}=1.025$ );
- bolts: diameter M24, high strength ( $f_u=1100$  MPa,  $\gamma_{MB}=1.43$ );
- holes diameter 27 mm.

This calculation example shows that the changing of the web plate thickness can provide the best combination of joint strength and deformativity which give an opportunity for the structure to take up the maximal load. Fig. 6 shows "moment-rotation" curves of the joint with different plate thickness'. The values of relative loads  $q_{rel}$  for the beam with the spans 6 m and 12 m are presented in Table 3.

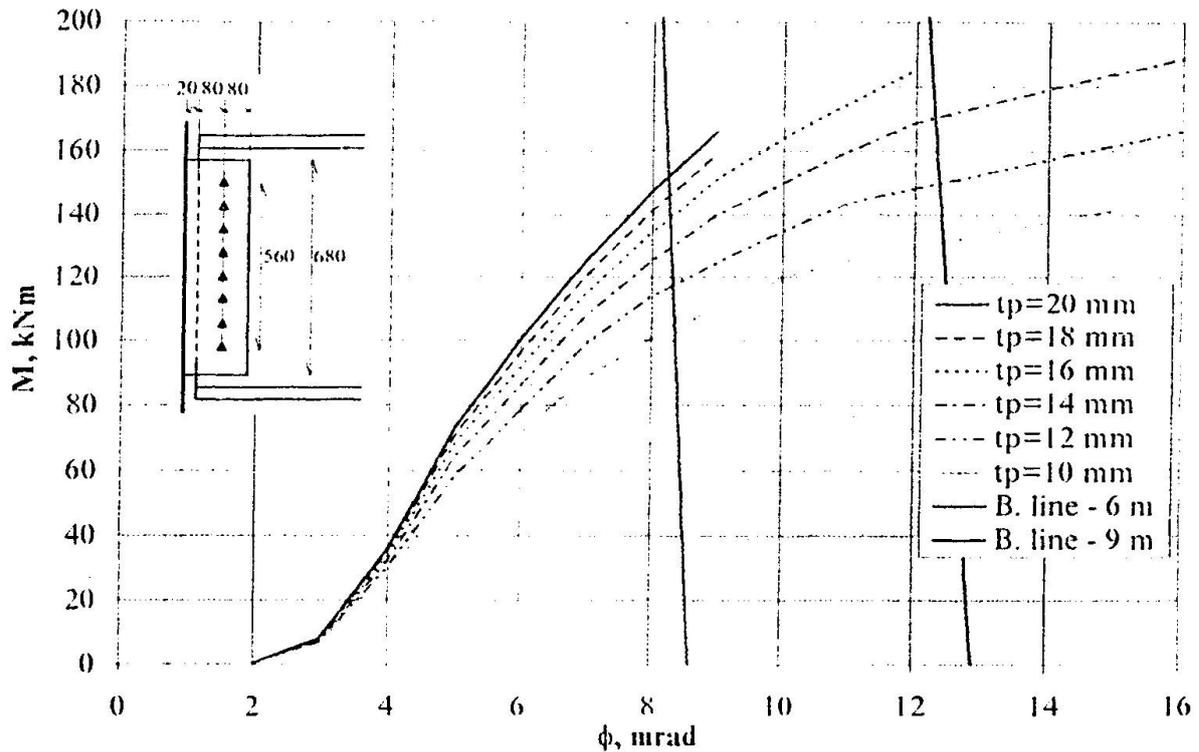


Fig. 6. "Moment-rotation" curves of single web plate joint.

Beam span, m	Plate thickness, mm					
	20	18	16	14	12	10
6	1,084	1,081	1,077	1,072	1,065	1,058
9	0,833	0,827	1,081	1,094	1,083	1,073

Table 3. Relative loads values.

Fig. 6 and Table 3 analysis has got clear that the web plate of 20 mm thickness is the most effective ( $q_{rel}=1,084$ ) for the structure with 6 m beam span. However its use at the beam with the span 9 m is wrong because the joint gets up no enough flexible in this case. Value of relative load is  $q_{rel}=0,833$  or the permitted load applied to the beam with real supports is 16% less than to the same one with pinned connections. Decreasing the plate thickness or increasing the flexibility of the joint demonstrates that the web plate of 14 mm thickness is the most useful for the structure with 9 m beam span ( $q_{rel}=1,094$ ).

#### 4. Comparison with the test results

There is a good correspondence between the calculation methods of endplate and single/double web plate joints and the experimental results. The Russian tests, concerning the mentioned joints, are included to the International Data Bank SERICON [6] (tests 110.001 – 110.009, 111.001 – 111.014).



## 5. Conclusion

In this manner the calculation methods and computer programs to determine “moment–rotation” characteristics of T–stub and top and seat angle joints have been developed. “Moment–rotation” curves of the joints have been used for the global analysis of the building steel skeletons. The total steel weight was decreased up to 15–20% relatively to the results obtained by the traditional frame design.

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