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Summary

The object of the paper is the derivation of the damping matrix of timber frames with flexible connections. The basic explicit assumption is that the loss of energy occurs only at the joints and that the modal damping ratios are known either from experiments or from engineering judgement. Damping matrix orthogonality is used to obtain the damping exhibited by each of the joints.

1. Introduction

The performance of timber construction in earthquakes can be very good mainly because of its low mass, flexibility of connections and high damping. Dowrick [1] and Green [2] report that the damping ratio in timber structures can take a very high value, even 15% to 20%. But according to Lazan [3], the wood itself has a material damping ratio less than 1%. It is obvious that a large amount of energy absorption takes place in the secondary structure and in the flexible connections of the primary structure.

The tests performed on some timber portal frames, without secondary structure, by Ceccotti et al. [4], confirm an equivalent damping ratio of about 15%. No doubt, the principal part of the energy absorption takes place just in the flexible connections of the primary structure. These facts prove that the timber structures behave as systems with the discrete dampers which should be properly represented in the model for a correct dynamic response analysis. The other property of flexible connections, their stiffness capacity, is not the subject of the present analysis.

The modal damping ratios for a structure are known either from experiments or from engineering judgement but the contribution of each joint separately is unknown. In the global damping matrix of a timber frame structure in which the flexible connections are the only dampers, the influence coefficients are actually the coefficients associated with the damping forces developed in particular joints. If the procedure of deriving uncoupled equations of motion is followed, which also means the satisfaction of orthogonality condition of the damping matrix, one comes to a set of equations which relate the mode-shapes, the damping coefficients, and the modal damping ratios. From these equations it is possible to compute either viscous damping coefficient for each of the joints or their relative values. It means that the damping uncoupling imposes certain rule on the damping distribution among the flexible joints. This will be the basis for the approach used further in the analysis.

An alternative in handling the damping in structures is to use two methods for the numerical evaluation of orthogonal damping matrices, developed by Wilson and Penzien [5] and also Clough and Penzien [6]. These two methods yield an orthogonal damping matrix which produces specific modal damping ratios, but the damping model is fictitious one, not pretending the stress distribution within the frame to be correct. On the contrary, in the present analysis a particular damping model is observed in which the damping forces are developed internally in specified locations and along specified coordinates. In an earlier attempt, the authors [7] constructed a damping matrix for a model in which the dampers were attached externally to the joints. That was also a fictitious model.

2. Damping distribution analysis

For any structure of N dynamic degrees of freedom, the equations specifying the orthogonality property of the damping matrix have the form

$$C_i = \{\phi_i\}^T [c] \{\phi_i\} = 2 \cdot M_i \cdot \xi_i \cdot \omega_i, \qquad i = 1,2,\dots,N$$

$$\{\phi_i\}^T [c] \{\phi_j\} = 0, \qquad i \neq j \tag{1}$$

where M_i, C_i, ω_i, ξ_i are the generalized mass and damping, the frequency and the damping ratio, all in mode i, respectively.

The number of available equations in (1) is N(N+1)/2. It should be noted that dynamic degrees of freedom are associated with the displacement components in which significant inertia forces are developed.

In structures with flexible connections, the damping forces are developed in dashpots between interconnected members, Fig.1b. For the correct description of damping forces, end rotation of each member connected by a dashpot should be treated as an independent displacement component. The motion is now fully described with displacement components necessary to represent the inertia forces and also the damping forces. In order to perform the dynamic analysis in these displacement coordinates, the mass matrix and the stiffness matrix should be extended to match the order of the displacement vector.

Performing the undamped free vibration analysis in the extended number of displacement components, the frequencies and the mode shapes can be obtained. Having these data evaluated, equations (1) will yield the damping influence coefficients. Without any loss of generality, the idea of damping distribution will be demonstrated first on a two-storey timber frame, Fig.1a. All the data necessary for numerical evaluation and also the displacement components are presented on the figure. The beams are flexibly connected to columns and also there is a flexible connection at the base level. Any flexible connection is regarded as a rotational spring of stiffness s and a rotational dashpot with some viscous damping coefficient c, Fig. 1b. The spring stiffness s depends on the type of joint and fastener, taken the same for all joints in this example, and it is calculated according to DIN 1052 [8].

The frame performs lateral vibrations, antisymmetric in character. Because of dashpots, end rotations of flexibly connected members must be taken as independent coordinates. Observing antisymmetry, the displacement vector is of order seven which gives the following damping matrix

$$[c] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_1 & 0 & -c_1 & 0 & 0 \\ 0 & 0 & 0 & c_2 & 0 & -c_2 & 0 \\ 0 & 0 & -c_1 & 0 & c_1 & 0 & 0 \\ 0 & 0 & 0 & -c_2 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_3 \end{bmatrix} \qquad (2)$$

where c_1, c_2, and c_3 are the damping coefficients for the dashpots on the beams and at the base, Fig.1a.

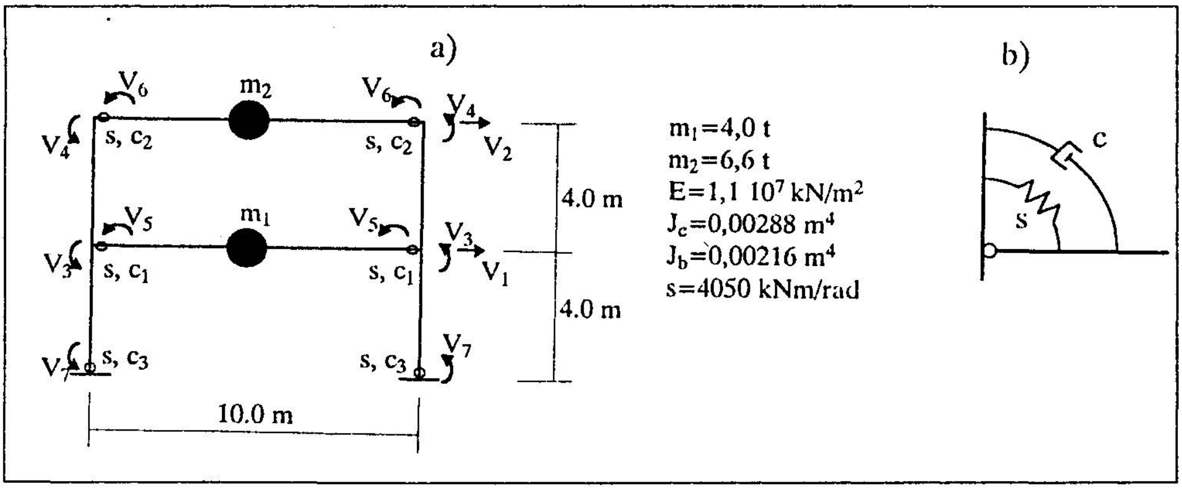

Fig. 1. Dynamic model of a two-storey laminated timber portal frame

The only nonzero coefficients in the mass matrix are $m_{11}=m_1$ and $m_{22}=m_2$. The undamped free vibration frequencies, mode-shapes, and generalized masses are computed and presented here as

$$\{\omega\}^T = \begin{bmatrix} 6,287 & 47,207 \end{bmatrix} \sec^{-1}$$

$$\{\phi_1\}^T = \begin{bmatrix} 1,0 & 1,994 & -0,2496 & -0,2255 & -0,05522 & -0,05 & -0,22148 \end{bmatrix} \qquad (3)$$

$$\{\phi_2\}^T = \begin{bmatrix} 1,0 & -0,3043 & 0,0351 & 0,42887 & 0,00776 & 0,095 & -0,34805 \end{bmatrix}$$

$M_1=30,24$ t, $\qquad M_2=4,611$ t

Equations (1) which express generalized damping and the orthogonality property of the damping matrix, have the following developed form

$$c_1(\phi_{31}-\phi_{51})^2+c_2(\phi_{41}-\phi_{61})^2+c_3\phi_{71}^2 = 2M_1\cdot\xi_1\cdot\omega_1$$

$$c_1(\phi_{32}-\phi_{52})^2+c_2(\phi_{42}-\phi_{62})^2+c_3\phi_{72}^2 = 2M_2\cdot\xi_2\cdot\omega_2 \qquad (4)$$

$$c_1(\phi_{31}-\phi_{51})(\phi_{32}-\phi_{52})+c_2(\phi_{41}-\phi_{61})(\phi_{42}-\phi_{62})+c_3\phi_{71}\phi_{72}=0$$

where ξ_1 and ξ_2 are the modal damping ratios for the first and the second mode, respectively. If the modal damping ratios are assigned some specific values, for example $\xi_1=0,15$ and $\xi_2=0,05$, equations (4) result in $c_1=1304,82$, $c_2=48,77$ and $c_3=126,76$. This way the damping distribution among the flexible connections is achieved and also the damping matrix (2) is evaluated.

A different situation may arise if the joint on the base level, Fig.1a, is the fixed joint. In that case, the number of unknown damping coefficients is less than the number of available equations in (1) and for that reason only one of the modal damping ratios can be assigned while the other should result from the equations. The free vibration analysis, with a stiffness matrix of order six, gives the following frequencies and the mode-shapes

$$\{\omega\}^T = \begin{bmatrix} 9,86 & 62,67 \end{bmatrix} \sec^{-1}$$

$$\{\phi_1\}^T = \begin{bmatrix} 1,0 & 2,8816 & -0,4058 & -0,4572 & -0,0898 & -0,101146 \end{bmatrix} \qquad (5)$$

$$\{\phi_2\}^T = \begin{bmatrix} 1,0 & -0,2103 & -0,0681 & 0,4437 & -0,01506 & 0,09816 \end{bmatrix}$$

$M_1=58,8$ t, $\qquad M_2=4,29$ t

Using these data, and also adopting $\xi_1=0,15$, the solution procedure will yield: $c_1=1484,91$, $c_2=202,303$, and also $\xi_2=0,052$.

It should be noticed that the position of zero and nonzero elements in the damping matrix (2) strictly follows the displacement components in which the damping forces are actually developed. Wilson and Penzien [5] do not observe this fact since the methods proposed by them are conceptually different from the present approach. Using their method, the one based on Caughey series [9] for which a proportional damping matrix is a special case, an orthogonal damping matrix is evaluated for the illustrative purposes. For damping ratios as above, $\xi_1 = 0{,}15$, $\xi_2 = 0{,}05$, and also using the stiffness matrix with the extended number of coordinates, for the frame with fixed joints at the base, one comes to a proportional damping matrix of the form

$$[c] = a_0[m] + a_1[k] \tag{6}$$

$$= \begin{bmatrix} 32{,}024 & -10{,}264 & 0 & -20{,}528 & 0 & 0 \\ -10{,}264 & 29{,}232 & 20{,}528 & 20{,}528 & 0 & 0 \\ 0 & 10{,}264 & 58{,}242 & 13{,}686 & -3{,}5 & 0 \\ -10{,}264 & 10{,}264 & 13{,}686 & 30{,}871 & 0 & -3{,}5 \\ 0 & 0 & -3{,}5 & 0 & 15{,}816 & 0 \\ 0 & 0 & 0 & -3{,}5 & 0 & 15{,}816 \end{bmatrix}$$

where a_0 and a_1 are the coefficients related to frequencies and damping ratios and [m] and [k] are the mass and stiffness matrices, respectively. For any conclusion, this matrix should be compared with the damping matrix in (2).

Lateral vibrations of one-storey frame, Fig.2, are described with four coordinates. The damping in the joints is described with the following matrix

$$[c] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & c_1 & -c_1 & 0 \\ 0 & -c_1 & c_1 & 0 \\ 0 & 0 & 0 & c_2 \end{bmatrix} \tag{7}$$

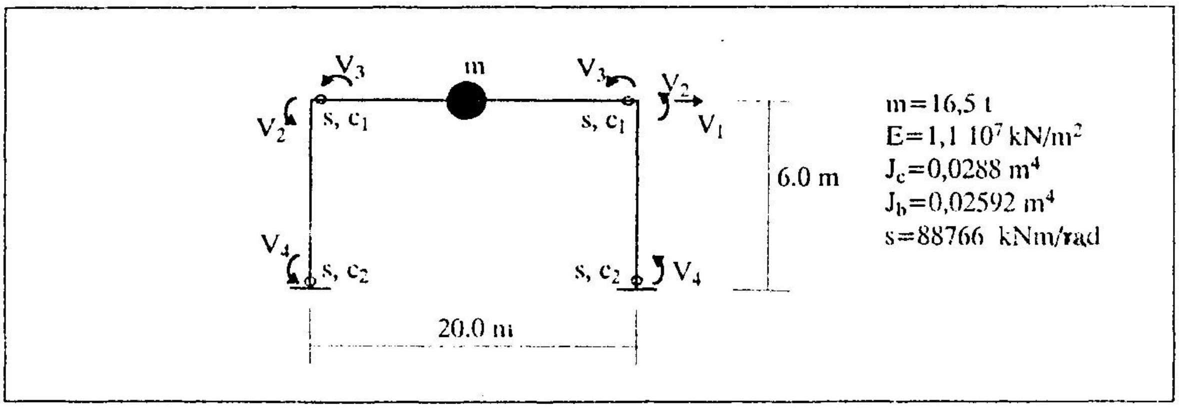

Fig. 2. Dynamic model of a one-storey laminated timber portal frame

The undamped free vibration analysis gives

$$\omega = 18{,}834\,\mathrm{sec}^{-1} \tag{8}$$

$$\{\phi_1\}^T = \begin{bmatrix} 1{,}0 & -0{,}15725 & -0{,}08008 & -0{,}12066 \end{bmatrix}$$

$$\mathrm{M} = 16{,}5\ \mathrm{t}$$

The term mode-shape here is unusual for a system of one dynamic degree of freedom and the confusion may arise. But it should be understood as the deformed shape in the extended number of displacement components.

The expanded form of generalized damping is

$$c_1(\phi_{21} - \phi_{31}) + c_1\phi_{31}(-\phi_{21} + \phi_{31}) + c_2\phi_{41}^2 = 2M\cdot\xi\cdot\omega \tag{9}$$

For a given damping ratio, equation (9) relates c_1 and c_2 . If they are equal, and for ξ_1=0,15, the equation results in c_1=c_2=4544,6.

The same frame with fixed joints at the base is the only case with no distribution of damping since the beam to column joints take all of the dissipated energy. For the three displacement components, the damping matrix takes the form

$$[c] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & c & -c \\ 0 & -c & c \end{bmatrix} \tag{10}$$

With the data from the undamped free vibration analysis

$$\omega = 28{,}406\ \ \mathrm{sec}^{-1}$$

$$\{\phi_1\}^T = \begin{bmatrix} 1{,}0 & -0{,}20725 & -0{,}1055 \end{bmatrix} \tag{11}$$

$$M = 16{,}5\ t$$

and with the use of the expanded form of generalized damping

$$c\left[\phi_{21}(\phi_{21} - \phi_{31}) + \phi_{31}(-\phi_{21} + \phi_{31})\right] = 2\cdot M\cdot\xi\cdot\omega \tag{12}$$

one obtains, for ξ = 0,15, that c is equal to 13581,46.

From the damping matrices in previous examples, and also directly from a dashpot, it can be seen that the damping force developed in a particular joint is proportional to the rate of change of relative rotation in that joint. But, in a dynamic analysis, that fact would not reduce the total number of independent coordinates.

A totally different problem, which is not the subject of this paper, would be if some arbitrary distribution of damping is adopted. In that case a set of uncoupled equations of motion could be obtained by the use of damped mode-shapes. But, it requires someone to deal with the complex eigenproblem and to solve it as in Hurty and Rubinstein [10].

3. Conclusion

A substantial part of energy absorption occurs in the flexible connections of some structures, especially the timber structures since they belong to the group of prefabricated systems. To deal with the mechanics of different types of connections and fasteners, one should know not only their stiffness property but also their damping capacity.

The damping model of a structure adopted here observes the fact that the joints behave as the discrete dampers. Based on such model, the damping matrix with unknown damping coefficients is constructed. The uncoupling procedure results in a system of equations from which the numerical evaluation of damping coefficients is possible. The resulting damping matrix in explicit form then may be used, for example, in the dynamic response analysis of some nonlinear systems.

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