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STATIC AND DYNAMIC DESIGN OF STRUCTURES WITH SEMI - RIGID CONNECTIONS

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Summary

Constructions with not absolutely rigid connections between members, that allow some relative rotation of member ends, are systems with semi-rigid connections in joints. As such system of connections is very often in constructions, particularly in prefabricated ones, it is of interest to analyze them taking in account flexibility of joint connections.
Because of limited space, here it will be given only expressions for design of semi-rigid connection systems, without deriving.

1. Static design of systems with semi-rigid connections

Design of linear systems, with assumption that member connections are either quite rigid or ideally pinned known as "approximate" slope-deflection method is presented in [1], and in [2] it is given its better assurance up to arbitrary reguired level by introducing the influence of axial forces on deformation. In what follows, semi-rigid connected linear systems 'are considered.

1.1 Design according to first order theory

For semi-rigid connected lynear systems, expressions for bending moments at the ends as well as governing equations of slope-deflection method according to first order theory are derived in [3]. If it is introduced designations $\mu_{ik} = \varphi^*_{ik} / \varphi_i ; \mu_{ki} = \varphi^*_{ki} / \varphi_k$ (where φ_i and φ_k are the angles of rotation of the joint "i" and "k" respectively, and angles φ^*_{ik} and φ^*_{ki} of rotations of end cross-sections of the member "ik") and are named fixing degrees of member "ik" in joints "i" and "k", the expressions for bending moments at the bar ends of such connected members are:

$$M_{ik} = a_{ik}\varphi^*_i + b_{ik}\varphi^*_k - c_{ik}\psi_{ik} + m_{ik}; \qquad M_{ki} = b_{ik}\varphi^*_i + a_{ki}\varphi^*_k - c_{ki}\psi_{ik} + m_{ki} \tag{1}$$

or in terms of $\varphi_i, \varphi_k, \psi_{ik}$, in the shape

$$M^*_{ik} = a^*_{ik}\varphi_i + b^*_{ik}\varphi_k - c^*_{ik}\psi_{ik} + m^*_{ik}; \qquad M^*_{ki} = b^*_{ik}\varphi_i + a^*_{ki}\varphi_k - c^*_{ki}\psi_{ik} + m^*_{ki} \tag{2}$$

Constants $a^*_{ik}, b^*_{ik}, c^*_{ik}$, as well as the initial moments of semi-rigidly connected members can be expressed in terms of corresponding values of rigidly connected members and fixing degree, as it is given in [3], with the form similar to the expressions (8. a) , (8. i), but in this case independent on axial force of the bar.
The final expressions for M^*_{ik} and M^*_{ki} are obtained in the shape:

$$M^*_{ik} = a^*_{ik}\varphi_i + b^*_{ik}\varphi_k - c^*_{ik}\sum_{j=1}^{n}\psi^{(j)}_{ik}\Delta_j + m^*_{ik}; \quad M^*_{ki} = b^*_{ik}\varphi_i + a^*_{ki}\varphi_k - c^*_{ki}\sum_{j=1}^{n}\psi^{(j)}_{ik}\Delta_j + m^*_{ki} \tag{3}$$

where it is denoted
$$m^*_{ik} = m^{*(o)}_{ik} + m^{*(\Delta t)}_{ik} + m^{*(t)}_{ik} + m^{*(c)}_{ik} + m^{*(N)}_{ik} \tag{4}$$

with
$$m^{*(N)}_{ik} = -c^*_{ik}\psi^{(N)}_{ik}; \tag{5}$$

while a_{ik}, b_{ik}, c_{ik} and d_{ig} are the constants of members, $\psi^{(j)}_{ik}, \psi^{(j)}_{ig}$ angles of rotation in the state $\Delta_j = 1$, and Δ_j (j=1,2,...,n) parameters of displacement.
Equations of rotation and equations of displacements are given in [3].
For semi-rigid connected systems it is shown in [7] applicability to matrix analysis, in [8] applicability to force method, as well as reinforced concrete frame design in [8] and [9].

2. Design according to second order theory

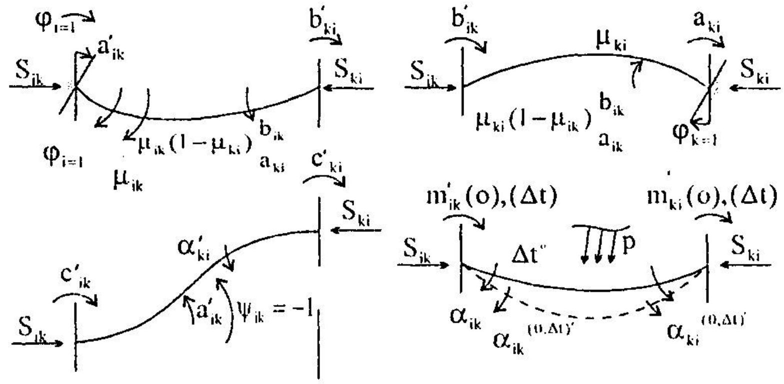
It is assumed fixing degree of a bar ik in nodal point "i" μ_{ik} and "k" μ_{ki} . During deformation, nodal points rotations are φ_i and φ_k, end cross-sections of a bar rotate φ'_i and φ'_k, while bending moments are M'_{ik} and M'_{ki} . In terms of deformation values it can be written:

$$M'_{ik} = a_{ik}\varphi'_i + b_{ik}\varphi'_k - c_{ik}\psi_{ik} + m_{ik}; \qquad M'_{ki} = b_{ik}\varphi'_i + a_{ki}\varphi'_k - c_{ki}\psi_{ik} + m_{ki} \tag{6}$$

or in terms of $\varphi_i, \varphi_k, \psi_{ik}$ in the shape

$$M'_{ik} = a'_{ik}\varphi_i + b'_{ik}\varphi_k - c_{IK}\psi_{ik} + m'_{ik}; \qquad M'_{ki} = b'_{ik}\varphi_i + a'_{ki}\varphi_k - c'_{ki}\psi_{ik} + m'_{ki} \tag{7}$$

Relations between old and new constants as well as initial moments are found on the base of their physical meanings (Fig. 1).



$$\alpha'_{ik} = \mu_{ik} - (1-\mu_{ik})\mu_{ki}\frac{b_{ik}}{a_{ik}} \qquad \alpha'^{(o,\Delta t)}_{ik} = \mu_{ik}\alpha^{(o,\Delta t)}_{ik} - (1-\mu_{ik})\mu_{ki}\frac{b_{ik}}{a_{ik}}\alpha^{(o,\Delta t)}_{ik}$$

$$\alpha'_{ki} = \mu_{ki} - (1-\mu_{ki})\mu_{ik}\frac{b_{ik}}{a_{ki}} \qquad \alpha'^{(o,\Delta t)}_{ki} = \mu_{ki}\alpha^{(o,\Delta t)}_{ki} - (1-\mu_{ik})\mu_{ki}\frac{b_{ik}}{a_{ki}}\alpha^{(o,\Delta t)}_{ki}$$

Fig. 1.

$$a'_{ik} = \mu_{ik}\left[a_{ik} - (1-\mu_{ki})\frac{b_{ik}}{a_{ki}}b_{ik}\right] \quad (8.a) \qquad m'_{ik} = \mu_{ik}\left[m_{ik} - (1-\mu_{ki})\frac{b_{ik}}{a_{ki}}m_{ki}\right] \quad (8.f)$$

$$b'_{ik} = b'_{ki}\mu_{ik}\mu_{ki} \quad (8.b) \qquad m'_{ki} = \mu_{ki}\left[m_{ki} - (1-\mu_{ik})\frac{b_{ik}}{a_{ik}}m_{ik}\right] \quad (8.g)$$

$$c'_{ik} = \mu_{ik}\left[c_{ik} - (1-\mu_{ki})\frac{b_{ik}}{a_{ki}}c_{ki}\right] \quad (8.c) \qquad M'_{ik} = \mu_{ik}\left[M_{ik} - (1-\mu_{ki})\frac{b_{ik}}{a_{ki}}M_{ki}\right] \quad (8.h)$$

$$c'_{ki} = \mu_{ki}\left[c_{ki} - (1-\mu_{ik})\frac{b_{ik}}{a_{ik}}c_{ik}\right] \quad (8.d) \qquad M'_{ki} = \mu_{ki}\left[M_{ki} - (1-\mu_{ik})\frac{b_{ik}}{a_{ik}}M_{ik}\right] \quad (8.i)$$

$$a'_{ki} = \mu_{ki}\left[a_{ki} - (1-\mu_{ik})\frac{b_{ik}}{a_{ik}}b_{ik}\right] \quad (8.e)$$

As it is evident, here bars of type "k" and type "g" are considered both as type "k" (fixed at the both ends). From the expression (8) it follows for $\mu_{ik} = \mu_{ki} = 1$ former type "g" (fixed in "i" and flexible in "k") and for $\mu_{ik} = \mu_{ki} = 0$, former type "s" is with elastic fixing in joint. At the same way as in (7) are obtained expressions M'_{ik} and M'_{ki}:

$$M'_{ik} = a'_{ik}\varphi_i + b'_{ik}\varphi_k - c'_{ik}\sum_{j=1}^{n}\psi_{ik}^{(j)}\Delta_j + m'_{ik}; \quad M'_{ki} = b'_{ik}\varphi_i + a'_{ki}\varphi_k - c'_{ki}\sum_{j=1}^{n}\psi_{ik}^{(j)}\Delta_j + m'_{ki} \quad (9)$$

Rotation equations and displacement equations now look like:

$$\sum_k M'_{ik} + M_i = 0 \quad (i=1,2,...,m); \quad (\varphi_i = 1) \quad (10.a)$$

$$\sum_{ik}(M'_{ik} + M'_{ki})\psi_{ik}^{(j)} + R_j(p) + R_j(m^f) = 0 \quad (j=1,2,...,n)(\Delta_j = 1) \quad (10.b)$$

where $R_j(m^f)$ is work of distributed fictive moments.
Equations of slop-deflection method are:

$$A'_{ii}\varphi_i + \sum_k A'_{ik}\varphi_k + \sum_{j=1}^{n} B'_{ij}\Delta_j + A_{i0} = 0 \quad (i=1,2,...,m) \quad (11.a)$$

$$\sum_{i=1}^{m} B''_{ji}\varphi_i + \sum_{l=1}^{n} C'_{jl}\Delta_l + C'_{j0} = 0 \quad (j=1,2,...,n) \quad (11.b)$$

with introduced designations:

$$A'_{ii} = \sum_k a'_{ik}\sum_s e'_{is}; \quad A'_{ik} = b'_{ik}; \quad A'_{i0} = \sum_k m'_{ik} + M'_i \quad (12.a)$$

$$B'_{ij} = -\sum_k c'_{ik}\psi^j_{ik} = B''_{ji} \quad (12.b)$$

$$C'_{jl} = C'_{lj} = \sum_{ik}(c'_{ik} + c'_{ki})\psi_{ik}^{(j)}\psi_{ik}^{(l)} \mp EI_c\sum_{ab}\frac{\omega^2_{ab}}{L'_{ab}}\psi_{ab}^{(j)}\psi_{ab}^{(l)} \quad (12.c)$$

$$C'_{j0} = -\sum_{ik}(m'_{ik} + m'_{ki})\psi_{ik}^{(j)} - R_j(p) \mp EI_c\sum_{ab}\frac{\omega^2_{ab}}{L'_{ab}}\psi_{ab}^{(j)}(\psi_{ab}^{(t)} + \psi_{ab}^{(c)}) \quad (12.d)$$

1.2.1 Determining of critical load

Critical load is determined from governing equations (11) when terms A_{i0} and C_{j0} are missed.
Equations (11) in matrix form, by use of block matrix, are:

$$\begin{bmatrix} A' & B' \\ B'' & C' \end{bmatrix}\begin{bmatrix} \varphi \\ \Delta \end{bmatrix} = 0 \qquad (13) \qquad \text{that is} \quad \det\begin{bmatrix} A' & B' \\ B'' & C' \end{bmatrix} = 0 \qquad (14)$$

Coefficients of bar as well as initial moments depend on axial forces in members of a girder only at second order theory, while they are constant at first order theory.

1. 3 Numerical examples

1. 3. 1 Static design

For the purpose of illustration of presented design of structures with semi-rigid connections of members in joints, it is given example of frame in Fig. 2.

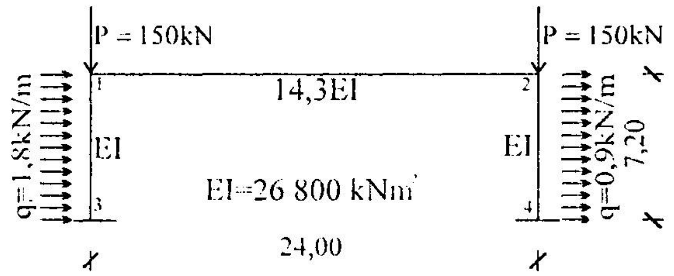


Fig. 2.

This system has three redundant. Unknowns are rotation angles φ_1 and φ_2, as well as displacement parameter Δ_1. Governing equations of slope-deflection method are given in following shape:

$$\begin{bmatrix} A_{11} & A_{12} & B_{11} \\ A_{21} & A_{22} & B_{21} \\ B'_{11} & B'_{12} & C_{11} \end{bmatrix}\begin{Bmatrix} \varphi_1 \\ \varphi_2 \\ \Delta_1 \end{Bmatrix} + \begin{Bmatrix} A_{10} \\ A_{20} \\ C_{10} \end{Bmatrix} = 0 \qquad (15)$$

Results are given in Table 1.

1. 3. 2 Determining of critical load and buckling length

Critical load is defined as the smallest value of load parameter at which homogenous problem of linearized second order theory has only one solution different from trivial one (11).
This condition can be expressed in matrix form as (13), that is (14).
For the purpose of determining critical load, it is used equivalent system to that shown in Fig. 2, where intensity of uniformly distributed load is expressed in terms of load parameter P.

Influence of transversal load is neglected in calculation of critical load parameter and buckling length of columns, because its intensity is small (q=0,012P), although we are not at the side of security. Having in mind that analysis of geometry imperfection influence on stability of static systems has certain significance, in the case of greater transversal load its influence has to be taken in account.

1. 3. 3 Analysis of influence of fixing degree on bending moments, critical load and buckling length of columns

As it has been done in examples given in 3.1 and 3.2 are given in [6], diagrams of bending moments according to the first and the second order theory, values ω_{cr}, P_{cr} and buckling length of columns, for different fixing degrees are calculated and presented in Table 1. It is also given comparison of obtained values ω_{cr}, P_{cr}, l_k with those for column rigidly fixed into the foundation and pin jointed to the beam ($\xi = 0$ and $\eta=1$).
At bending moment diagrams, in parentheses are given values obtained according to the first order theory, and out of parentheses according to the second order theory. Becouse of limited space it is presented just a small part of obtained results.

Table. 1

fixing coefficient	DIAGRAM M	ω^*_{kr} / $\omega^*_{kr}/\omega^*_{k0}$	P_{kr} / P_{kr}/P_{k0}	$l_k = \beta l_{k0}$ / l_k/l_{k0}
$\xi = 0,25$ $\eta = 0,95$		0,272 1,248	0,074EI 1,558	$1,604 l_{k0}$ 0,802
$\xi = 0,5$ $\eta = 0,75$		0,299 1,371	0,0894EI 2,882	$1,459 l_{k0}$ 0,729

Diagram 1

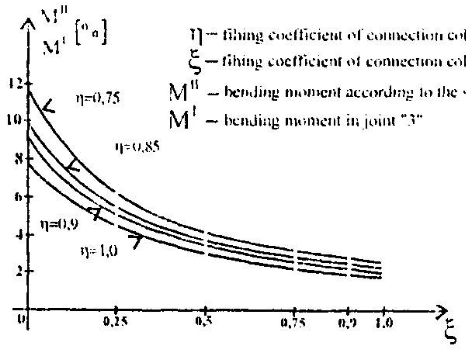
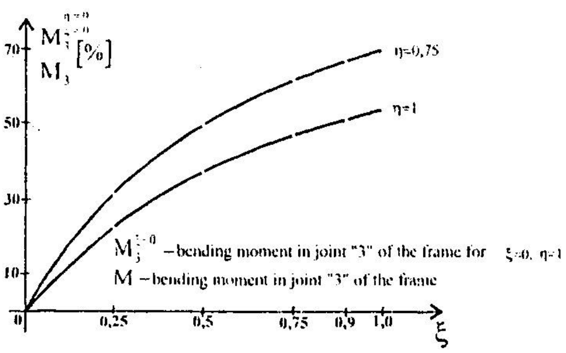


Diagram 2



At Diagram 1, it is shown difference between bending moment in the foundation of the column (joint "3") calculated according to the first and the second order theory, for different values of fixing degrees ξ and η. It is evident that difference is significant for smaller fixing degrees (for $\xi = 0$ and $\eta = 0,75$ the difference is 11,6%), while for greater fixing degrees the difference is insignificant (for $\eta = 1$ and $\xi = 1$ its 2%). So it can be concluded that in design of systems with semi-rigid connections application of the second order theory is more important.

At Diagram 2, it is shown how the values of bending moment in column-foundation joint "3" are changing for different fixing degrees ξ and η in comparison with that one for $\xi = 0$. It is evident that this chanqe significantly depends on fixing degree ξ of the joint column to beam, and lower depends on fixing degree of the connection column to foundation η. So, greater attention has to be payed on connection details of column to beam particulary in the case of prefabricated structures.

Similar analysis has been carried out for changing the ratio of critical forces P_{kt} / P_{k0} (Diagram 3) as well as the ratio of buckling lengths of columns l_k / l_{k0} (Diagram 4) depending on fixing degrees ξ and η.

Diagram 3

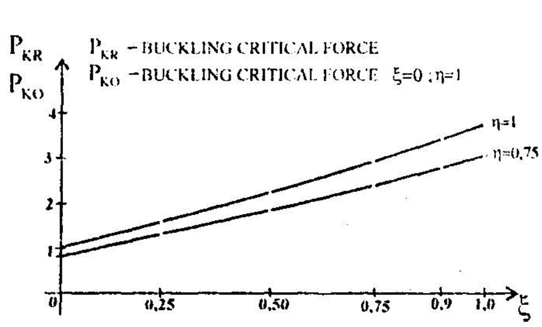
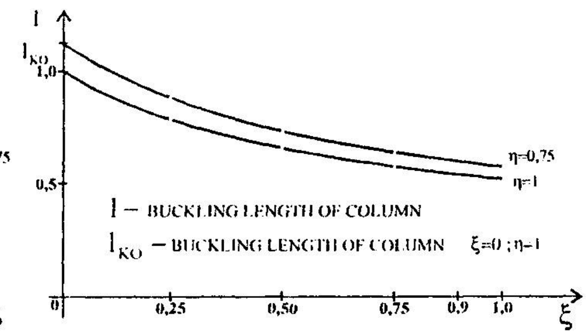


Diagram 4



It can be noticed that for different values of fixing degree η, the ratio P_{kt} / P_{k0} chanqes already linear with changing of fixing degree ξ, that significantly simplifies procedure of designing the parameter of critical load and buckling length of columns.

2. Dynamic design

2. 1 The case of semi-rigid connections

When it is considered systems with flexibil connected members, whose masses m_i are attached by dynamic loading $P_i(t)$, in real environment that opposes movement of resistant forces Pi(t), and corresponding inertial forces Ii(t) whose projections on "x" and "y" axis are shown in Fig. 3, taking in account equations (3.99)-(3.115) from [5] can be written equations of forced damped vibrations of system with finite number of degrees of freedom in the shape:

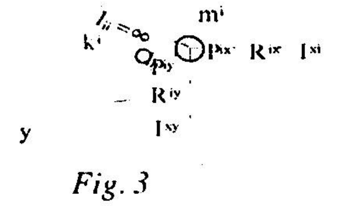


Fig. 3

$$
\begin{aligned}
& R_{ix}(t) = -\beta_i \dot{u}' R_{iy}(t) = -\beta_i \dot{v}' \\
& I_{ix}(t) = -m_i \ddot{u}_i I_{iy}(t) = -m_i \ddot{v}_i' \\
& \dot{u}_i = u_i - k_i \varphi_i \sin \alpha_{ii} \\
& \dot{v}_i = v_i - k_i \varphi_i \cos \alpha_{ii}
\end{aligned}
\qquad (16)
$$

$$\begin{bmatrix} \bar{\bar{A}} & \bar{\bar{B}} \\ \bar{\bar{B}}^T & \bar{\bar{C}} \end{bmatrix} \begin{Bmatrix} \ddot{\vec{\varphi}} \\ \ddot{\vec{\Delta}} \end{Bmatrix} + \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{B}^T & \bar{C} \end{bmatrix} \begin{Bmatrix} \dot{\vec{\varphi}} \\ \dot{\vec{\Delta}} \end{Bmatrix} + \begin{bmatrix} A^* & B^* \\ B^{*T} & C^* \end{bmatrix} \begin{Bmatrix} \vec{\varphi} \\ \vec{\Delta} \end{Bmatrix} = -\begin{Bmatrix} \vec{A}_0 \\ \vec{C}_0 \end{Bmatrix} \quad (17)$$

Submatrix wich appear in (17) are given in [6].

Previously presented is valued for the case where masses are connected by cantilever (length k_i) to the joints of girder. The most often case is when masses are put just in joints (k_j=0). Then, matrix $\bar{\bar{C}}$, $\bar{C}$,$\bar{\bar{B}}$, $\bar{B}$ and $\vec{A}_0$ become null matrix, so that from system (17) after m elimination's, can be eliminated all unknowns φ, and expression (17) get the shape

$$\left[\bar{\bar{C}}\right]\left\{\ddot{\vec{\Delta}}\right\} + \left[\bar{C}\right]\left\{\dot{\vec{\Delta}}\right\} + \left[C^{**}\right]\left\{\vec{\Delta}\right\} = -\left[\vec{C}_0\right] \quad (18)$$

where matrix $C^{**} = C^{*(m)}$ (19)

is obtained after m elimination's.

When parameters Δ_j are determined from (18), by use of the system

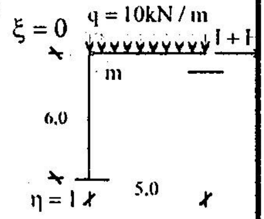
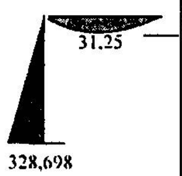
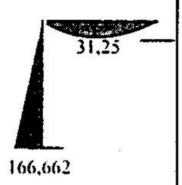
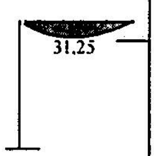
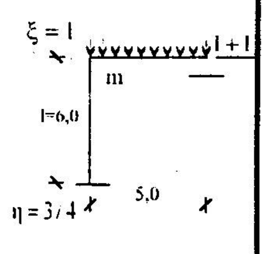
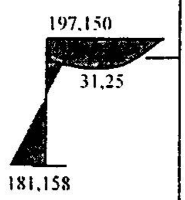
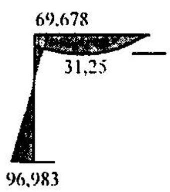
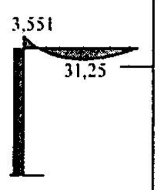
$$\left[A^*\right]\left\{\vec{\Phi}\right\} = -\left[B^*\right]\left\{\vec{\Delta}\right\} \quad (20)$$

values φ_i (i=1,2,...,m) can be determined, from (2) end bending moments and after that other internal forces can by calculated.

2. 1. 1 Numerical examples

Proceeding from the new constants of bars, by use of slope-deflection method, for the purpose of illustration, it is worked on an example of a simple frame structure, where fixing degrees are varied from 0 to 1. (Table 2). Becouse of limited space it is presented just a small part of obtained results.

Table. 2

Structure scheme	$M_d = M_i xI + M_{st,H}$ Bending moments due to dynamic loading			Dynamic properties and influences caused by perturb. force P_i (t)		
$m = 5\frac{kNs2}{m}$ EI=100000 kNm^2 $\theta = 0,8\omega$	M diagram H=20 kN I=34,783 kN	M diagram H=10 kN I=17,777 kN	M diagram H=0 kN I=0 kN	Circ. freq ω $[s^{-1}]$	I_1 from $H_1(t) =$ $0 \sin \theta t$ [kN]	I_2 from $H_2(t) =$ $20 \sin \theta t$ [kN]
				5,270	17,778	35,556
				9,000	17,783	35,356

In the case of dynamic load acting, that can be describe by the function $P_i(t) = H_i \sin \theta t$, circle frequencies and inertial forces are calculated for $\theta = 0.8\omega$. On the base of obtained results, it can be concluded that circle frequencies of the structures shown in the Table. 2 are 33% and 71%, respectively, greater than that one of the cantilever column. Inertial forces differs each to other less than 8%, for the considered example, different fixing degrees.
At the Table T.2. fixing degrees of joint column to foundation is denoted by $\mu_{ik} = \xi$, and column to beam $\mu_{ki} = \eta$. Bending moment diagrams, in the case of dynamic loading are calculated for uniformly distributed loading q=10 kN/m and for horizontal force H=10 kN, H=20 kN and H=0. For different fixing degrees and η, it is evident from given diagrams that changes of bending moments are significant.

3. Conclusion

On the basis of carried out analyze, it can be concluded that fixing degree of the connections would be taken in account, in static and dynamic design, so as in design of stability of constructions. Special attention has to be poyed on analysis of prefabricated constructions where relatively low fixing degree can be favorable for distribution of bending moments. So this circumstance is to be used in designing, because accompanied measures are easy to realize. At the otter side, not enough insured but assumed rigid connections can cause unfavorable consequences.
Depending on physic-mechanical characteristics of used materials and behavior of joint connections, i.e flexibility of the system during force acting, it is often necessary to calculate effects according to the second order theory.

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