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### **SESSION 3**

#### **EFFECTS OF JOINT ACTION ON FRAMES 2**

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## Seismic Resistance of Semi-rigid Steel Frames

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## Summary

The paper deals with aspects of seismic resistance of steel frames with bolted connections, as an alternative to fully welded and hybrid configurations which have indicated inferior behaviour during recent earthquakes. Experimental and analytical studies undertaken to verify the performance of bolted steel frames in high seismicity areas are briefly outlined. The investigations indicate that, compared with fully welded counterparts, bolted frames may demonstrate favourable seismic behaviour and may be used as a more reliable and cost-effective form for earthquake resistance.

## 1. Introduction

Modern seismic design relies on two fundamental aspects of structural engineering. The first is the realisation of a pre-defined, favourable, failure mode, whilst the second is the provision of deformation capacity sufficient to absorb the earthquake input energy. The former is referred to as 'capacity design' whilst the latter, which is much less developed, is referred to as 'displacement based design'. Modern code provisions, such as Eurocode 8, provide a complete framework for capacity design. However, this is still force-based, with a check on displacement. Recent publications (e.g. Kowalski and Priestley, 1995; Calvi and Kingsley, 1995) give proposals for seismic design based on displacement, with a check on force, for buildings and bridges, respectively. The application of both concepts hinges on a precise knowledge, a priori, of the strength and deformation characteristics of the structure and its constituent components. An alternative to this precision is the provision of a fuse; a structural component which can be rigorously designed to yield prior to the over-stressing of other





components. Both approaches; precision in force and deformation capacity estimates and provision of fuses, lend weight to the use of steel structures in seismic design. This is somewhat balanced by the cost and accessible technology advantage afforded by concrete structures.

Steel moment frames subjected to earthquake loading are traditionally designed with fully welded connections. This is justified by noting that the static stiffness of economical bolted connections is significantly lower than their welded counterparts, hence they are supposed to violate code drift limitations. By imposing drift limits on the inherently flexible bolted connections frame, the cost advantage of eliminating welding is partially or completely lost.

Experimental and analytical work undertaken in recent years (Nander and Astaneh, 1989; Elnashai and Elghazouli, 1994) has highlighted the fallacy of this treatment, which is based on static response. Due to the period elongation and energy dissipation in the connection, bolted frames may indeed displace less than welded structures, since they attract less load and possess higher damping. These studies, amongst others, opened the door to further work on the development of a complete design procedure for such frames. This effort was given an added impetus by the reported failures of welded connections during the Northridge (USA) earthquake of 17 January 1994, followed by further evidence from the Hyogo-ken Nanbu (Japan) earthquake of 17 January 1995. The development of seismic design regulations for bolted connections became not only an issue of economy but also one of safety.

In this paper, a brief history of the effects of earthquakes on steel structures is given alongside comments on the advantages of bolted versus welded connections. This is followed by an expose of experimental investigations undertaken at Imperial College and the University of Tokyo, and supporting analyses. The paper gives a clear indication that many of the drawbacks of steel structures in seismic design are alleviated by bolted partial strength connections.

## **2. Response of Steel Frames in Previous Earthquakes**

For many years, steel structures enjoyed the reputation of being the most suitable form of construction for earthquake resistance. Publications sponsored by the steel industry in the United States (AISI, 1991) found a dearth of cases of steel failure to report, hence concentrated on reinforced concrete damage instead. A review of eleven earthquakes worldwide from 1964 (Alaska) to 1990 (Philippines) indicated that only minor damage is sustained by steel structures, in contrast with the extensive damage and collapse suffered by reinforced concrete structures. In particular the Mexican earthquake of 1985 produced striking statistics (Table 1; Yanev et al, 1991). It is, however, important to note that RC structures are in the overwhelming majority, hence the exposed sample is much larger than for steel. Moreover, land-mark projects are more likely to be in steel, which is better suited to high rise structures. Such projects typically are subject to stringent quality control procedures, in contrast with residential owner-builder structures.

Type of Structure	Extent of Damage	Number
RC Frame	severe	45
	collapse	82
Steel Frame	severe	2
	collapse	10

*Table 1. Damage Comparisons from the Mexico Earthquake, 1985*

Taking into account the distinct characteristics of the exposed building stock, the conclusion drawn in the AISI publication (Yanev et al, 1991) that steel exhibits all the favourable seismic resistance characteristics is not substantiated.

Over the years, isolated cases of steel damage were reported, such as a heavily loaded x-braced warehouse (Miyagi-ken Oki, Japan, 1978) which suffered bracing distress, an eleven storey moment frame with first floor connection failure and the spectacular failure of two of the three structures in Piñeo Suarez complex (Mexico City, 1985). Damage was reported to a four storey braced steel frame in the area affected by the Whittier Narrows earthquake of 1987, with a few buckled bracing members. The damage was blamed though (Yanev et al, 1991) on an attached RC structure which is reported to have caused high torsional forces on the steel frame. With almost complete devastation of the area hit by the Spitak (Armenia) earthquake of 1988, all types of structure, with the exception of steel frames, suffered extensive damage and collapse. In the case of the latter, only a few cases of weld failure were reported. Further confirmation of the high seismic resistance of steel structures was furnished by the Loma Prieta damage assessment (EERI, 1995 amongst others). Only consequential damage to a very small number of steel frame buildings was reported, alongside buckling of a bracing member used to retrofit a reinforced concrete structure in San Francisco (Elnashai et al, 1989).

A few days after the Northridge earthquake of 17 January 1994, reports emerged that several cases of steel beam-column connections have failed in a brittle manner. The extent of this damage unfolded gradually and has now emerged as a most serious concern about the safety of steel frames in seismic areas. Out of a sample of 89 buildings selected as representative of 300 to 400 steel buildings in the area affected by the earthquake (NYA, 1995), 26% of all connections were damaged. If the number of connections inspected (2342) is taken into account, this percentage gives 615 damaged connections in the sample chosen. This further implies that there could be a few thousand failed connections in the area affected by the Northridge earthquake.

Repair of these connections often necessitates evacuation of the building and loss of use, in addition to the expense of repair and recladding. The effect of this earthquake on the reputation of steel structures as the ideal earthquake resistant construction material has been devastating. This prompted the industry, in liaison with Government agencies, professional institutions and academic establishment to create a joint venture (SAC; Structural Engineering Association of California, Applied Technology Council and California Universities Research into Earthquake Engineering) to respond to the questions raised by the extensive damage observed. It is beyond the scope of this paper to discuss possible causes of the observed damage, but it is sufficient to state that laboratory tests have confirmed that connections



designed and manufactured strictly to code requirements and best shop practice failed to provide the necessary levels of ductility.

The standing of steel in seismic design was dealt another blow in the Hyogo-ken Nanbu earthquake of 17 January 1995 (EERI, 1995, Elnashai et al, 1995 amongst many others). Apart from the devastation inflicted on old open lattice column structures, which was neither surveyed nor reported, a very large number of steel frames suffered distress, especially in the beam-column connection. It is noteworthy though that connection configuration in California and in Japan are distinct. Whereas connections in Japan are usually fully welded, hybrid connections are in wide use in California. These comprise welded beam flanges to column flange (intended for moment transfer) and plates welded to the column flange and bolted to the beam web (intended for shear transfer; termed shear tabs). The causes of damage are therefore not strictly related.

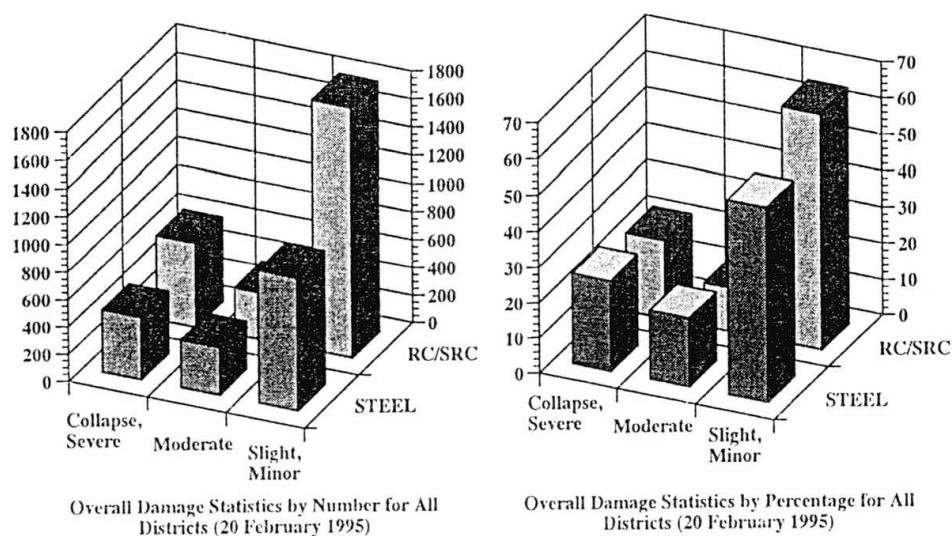


Fig.1. Damage Statistics from the Hyogo-ken Nanbu Earthquake (Elnashai et al, 1995)

In contrast with observations from previous earthquakes (with the exception of Northridge) steel structures suffered as much as reinforced concrete structures, especially when considering the right hand side of Figure 1 above. Moreover, if composite steel/concrete structures (referred to in Japan as SRC) are added to steel and not to concrete, the picture becomes even more bleak.

In the light of the above, it is the writers' opinion that steel structures remain a most suitable solution to earthquake design problems. However, there is no room for complacency in their design, since it has been shown that they are no less vulnerable to earthquake damage than reinforced concrete structures. Moreover, the repeated observation of failure of welded connections lends further weight to the effort dedicated to the development of seismic design rules for bolted connections, as discussed further in this paper.

As a consequence of the damage inflicted on welded and hybrid connections, re-assessment of steel seismic design procedures is underway. The reliability of welded connections, which has been subject to scrutiny in Japan for several years, is being examined fundamentally and new

connection configurations are under development. The half way mark has been reached via the publication of guidance notes on the seismic design of rigid bolted connections (Astanek, 1996). The next step forward is the use of semi-rigid bolted connections, with full or partial strength, which is the subject of the sections below.

In addition to the concerted effort dedicated to improving seismic design regulations for new construction (e.g. SAC Advisory Notes and the Interim Guide), several proposals have been forwarded for the upgrading of existing connections. This may take one of two forms, (i) strengthening of the connection by cover plates or other means or (ii) weakening of the beam by trimming or perforating the flanges.

#### 4. Experimental Investigations

An experimental programme carried out as a joint activity between the Institute of Industrial Science, Tokyo, and Imperial College, London, investigated the feasibility of semi-rigid frames in comparison with rigid alternatives. Fully-welded connections were used for the rigid frames, whereas the semi-rigid frames comprised bolted connections with top and seat angles, and two web cleats. Full details of the models and loading regimes are given elsewhere (Takanashi et al, 1993; Elnashai and Elghazouli, 1994).

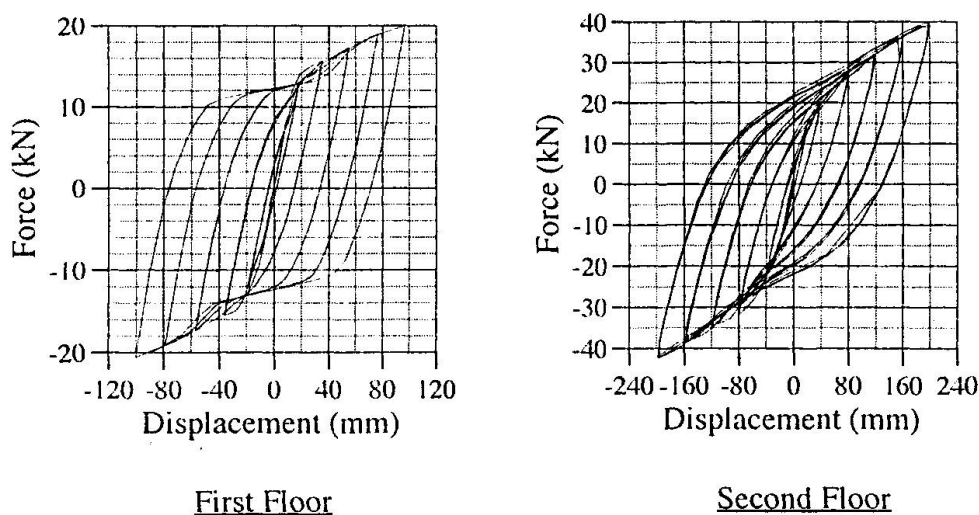


Fig. 2. Hysteretic load-displacement relationships from a cyclic test on a semi-rigid frame

Monotonic, cyclic and earthquake tests were performed on two-storey rigid and semi-rigid steel frames. The monotonic and cyclic tests were carried out under a hybrid displacement/load control procedure whereas the earthquake tests were performed using the pseudo-dynamic technique. In the monotonic and cyclic tests, the semi-rigid connections sustained the expected moment capacity with high rotational ductility and largely stable hysteretic behaviour, as shown in Figure 2.

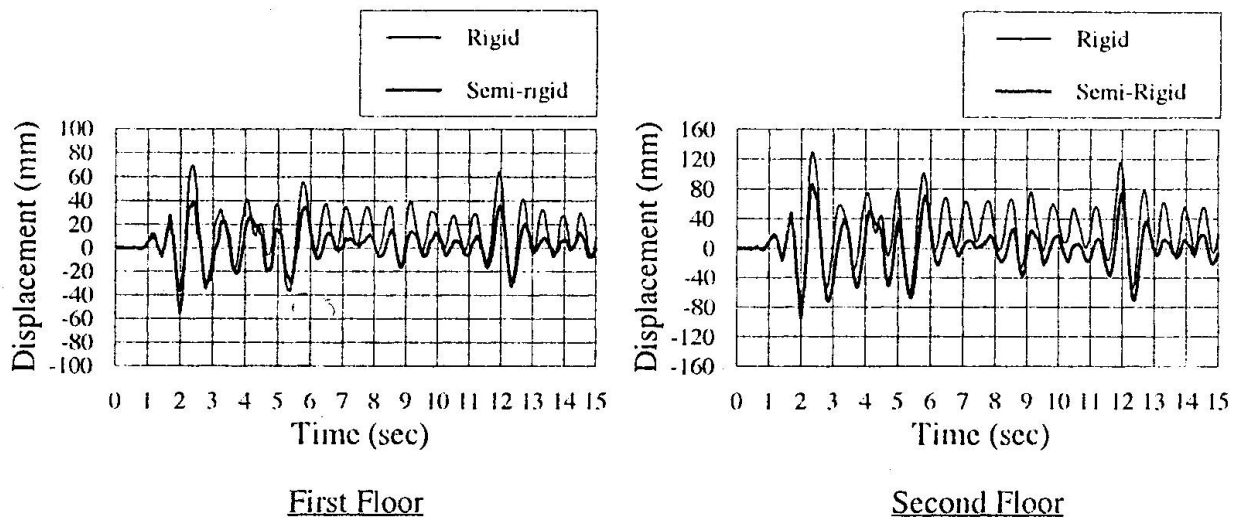


Fig. 3. Displacement response rigid semi-rigid frames under fixed capacity/load ratio

In the pseudo-dynamic tests, the same earthquake records were applied on similar rigid and semi-rigid frames. However, the masses and peak accelerations were appropriately scaled to satisfy a fixed strength-to-load ratio in both frames whilst maintaining a similar fundamental period. As shown in Figure 3, the peak displacement response was considerably lower in the semi-rigid case. This indicates that, despite their relative flexibility semi-rigid frames may, depending on the characteristics of both the structure and the applied load, cause smaller storey deformations as compared to rigid frames.

## 5. Analytical Studies

Analytical investigations were undertaken to study the effect of different connection types on the frame response. Full results from this investigation together with a detailed description of the analytical models used are given elsewhere (Elnashai and Elghazouli, 1994). Two storey frames similar to those used in the experiments, with beam-to-column connection properties ranging from fully rigid to flexible, were analysed under monotonic and earthquake loads.

The four frames were first analysed under monotonic loading. The monotonic moment-rotation relationships for the connections are shown in Figure 4. The same hybrid displacement-load control used in the experiments was adopted. Displacements were applied at the top floor, whereas the load at the first floor level was kept at half the value of the top-floor restoring force.

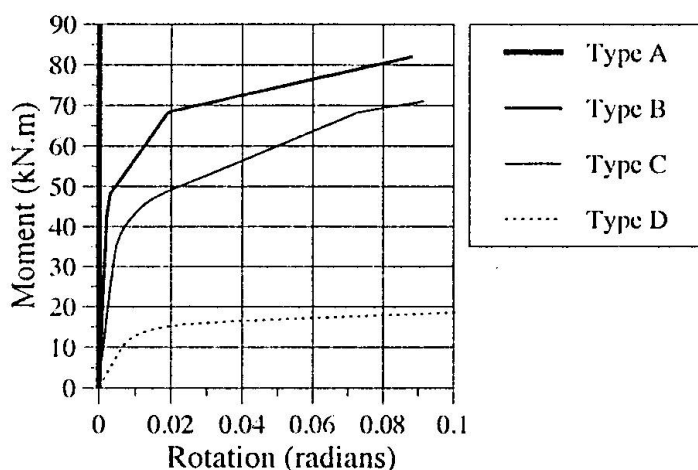


Figure 4. Moment-rotation relationships for different types of connections

Load-displacement relationships for the four frames are shown in Figure 5 at the first and top floor levels, which shows the effect of the connection properties on the frame stiffness and capacity. The reduction in yield and ultimate capacities for semi-rigid and flexible frames is accompanied however by an increase in the deformation at yield.

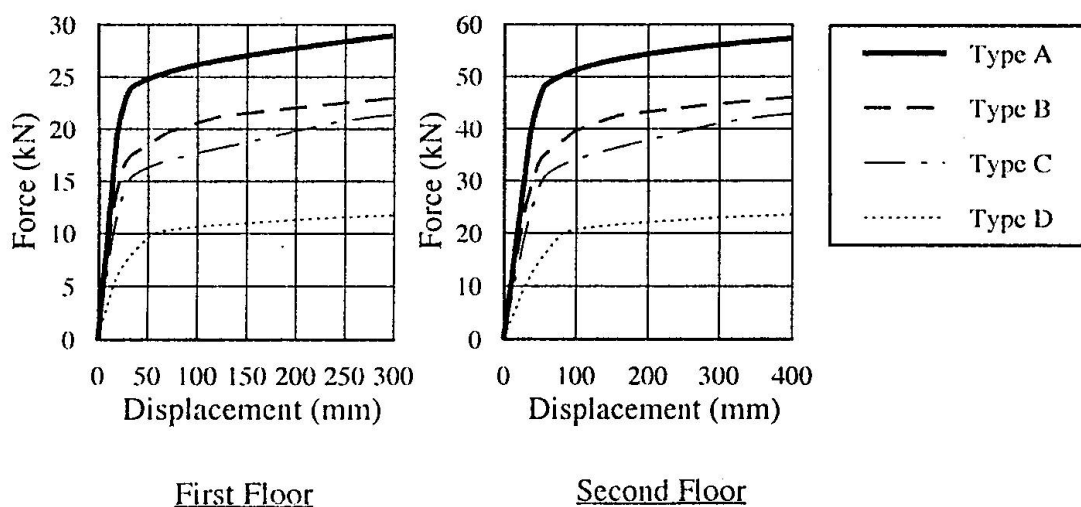


Figure 5. Load displacement relationships for the different frames

The connection properties were also shown to have a significant influence on several important response parameters such as the number, length and location of plastic hinges, and in turn on the overall ductility of the frame. In addition, the influence of the connection stiffness on the dynamic characteristics of the frame was also studied. The four frames were subjected in turn to the full scale El Centro N-S component acceleration time history. Table 2 gives the natural periods of vibration of the frames as well as the peak displacement response at both floor level in each frame. Due to the dynamic characteristics of the frames, the relative response of the four frames showed significant differences in terms of both the response





frequency and the displacement amplitudes. For example, as shown in Table 2, at the first floor level, the largest displacement amplitude of approximately 49.0 mm is observed in the response of the fully rigid frame, Type A, whereas the peak displacement does not exceed 40 mm in any of the other three frames.

Frame Type	T1 (sec)	T2 (sec)	Peak Displ (mm) <i>First Floor</i>	Peak Displ (mm) <i>Second Floor</i>
A	0.596	0.193	48	68
B	0.664	0.201	37	71
C	0.696	0.204	39	77
D	0.736	0.208	36	90

*Table 2. Natural periods and peak displacements for the analysed frames*

The results of the dynamic analyses confirmed the conclusions of the experimental work that, even when all types of frame are subjected to the same ground motion, frames classified as semi-rigid may still exhibit favourable response compared to similar rigid types.

Further analyses were undertaken (Danesh, 1996), with funding from the UK Engineering and Physical Sciences Research Council (EPSRC) to quantify the limits of applicability of semi-rigidly connected frames in areas of medium to high seismicity. Several structural configurations were studied, with different numbers of bays and stories. The main parameters studied were:

- Connection capacity as a percentage of beam capacity
- Column design actions

The objective of studying the former parameter is to give an indication of the minimum connection capacity consistent with seismic integrity, with the consequence of saving in materials and construction costs. The aim from studying the latter parameter is to investigate the possibility of releasing the requirement of column over-strength for capacity design purposes. This would lead to further savings in column sections. A sample of results for two storey two bay and four storey two bay frames is given in Tables 3 and 4.

Frame	Drift %	Beam Rotation	Connection Rotation
Rigid	1.5	0.019	-
22C3R	2.4	0.030	0.021
22C5R	2.8	0.028	0.020
22C7R	2.4	0.026	0.017
22C3C	2.8	0.022	0.029
22C5C	2.3	0.022	0.021
22C7C	2.2	0.024	0.019

*Table 3. Response Parameters for 2 Storey 2 Bay Frames*

In the Tables 3 and 4, C3, C5 and C7 stands for connection strength 30%, 50% and 70% of the beam strength, respectively. Suffix R or C indicate whether the column design actions are based on magnified beam actions (R) or actions from analysis of the frames under earthquake and static loads (C). Maximum drift, beam and connection rotations are obtained by dynamic analysis of the frames using an artificial accelerogram compatible with the EC8 elastic spectrum for soil class B, scaled to a ground acceleration of 0.3g.

Frame	Drift %	Beam Rotation	Connection Rotation
Rigid	1.8	0.020	-
42C3R	2.5	0.017	0.024
42C5R	2.0	0.019	0.017
42C7R	2.0	0.020	0.018
42C3C	2.6	0.016	0.023
42C5C	2.0	0.018	0.017
42C7C	1.9	0.019	0.017

*Table 4. Response Parameters for 4 Storey 2 Bay Frames*

Collectively, the results indicate that semi-rigid frames are indeed a feasible solution even with design accelerations of 0.3g. It is however noted that a connection with 30% of the beam capacity leads to drift limits at odds with code recommendations. It is also indicated that imposing over-strength requirements on columns in the presence of partial strength connections is pointless. Further analysis is underway at Imperial College, but the sample results presented above are quite conclusive in confirming the viability of use of partial strength bolted connections in seismic design.

## 6. Concluding Remarks

The paper discussed the feasibility of frames with bolted connections for seismic resistance as an alternative to welded frames. The alarmingly inferior behaviour of welded connections in recent earthquakes was described. In addition, the potential advantages, in terms of reliability and economy, of using bolted as opposed to welded connections were highlighted. The experimental and analytical studies presented show that semi-rigidly connected frames provide adequate and, in some cases, favourable earthquake-resistant qualities. It was shown that semi-rigid frames do exhibit ductile and stable hysteretic behaviour. Although the stiffness and capacity of semi-rigid frames are lower than similar rigid frames under monotonic and cyclic loading, the response under earthquake loading largely depends on the dynamic characteristics of the both the frames and the input motion. The results demonstrated that bolted frames may displace less than their welded counterparts, contrary to common belief. In general, it was shown that the response of semi-rigidly connected frames may be superior to rigid frames, provided that stable hysteretic behaviour is ensured. Further examination of the effect of connection strength and column design force was carried out. It was shown that provided the connection strength is higher than about 50% of the beam strength, bolted frames satisfy existing seismic code requirements even for high design ground accelerations. Moreover, it is confirmed that capacity design regulations expressed as column over-design factors need not apply. This leads to on the whole further economy in materials.





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## STATIC AND DYNAMIC TESTING OF STRUCTURES WITH SEMI - RIGID CONNECTIONS

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### Summary

The influence of semi-rigid structural connections on the static effects and dynamic characteristics of typic prefabricated construction "MINOMA", developed at the Civil Engineering Faculty in Niš, Yugoslavia, is discussed in this paper, based on the results of experimental testing and theoretical analysis.

Experimental tests has been carried out under static and dynamic load separately. This results are in close agreement with obtained analytical values.

### 1. Introduction

Researches carried out on the Civil Engineering Faculty in Niš, Yugoslavia lasting several Years have led to quite new system of typic reinforced concrete prefabricated structure called "MINOMA".

This construction includes all advantages of prefabricated construction system, but is more economical than similar ones known up to nowadays. As it is designed based on a new concept with semi-rigid connections and prestressing by means of a tie, it requires extensive investigations of long duration, both theoretical and experimental. Precision of some mathematical model parameters can be verified only by testing of considered structure. So, aforementioned structure has been tested under static and dynamic trial load and the results are presented in this paper.

### 2. Basic characteristics of structure type "minoma"

Carring structure is formed of straight precast concrete members wich are fixed in joints by means of either steel or aluminum connectors. The typic frame is supported so that the columns are fixed into prefabricated footings. The main characteristic of "MINOMA" structure is that joint stiffness is a little bit higher concerning the member stiffness.

Columns and beams are prestressed by adhesion, made of appropriate strength clas of concrete, with shaped crossection, reinforced by ribbed bars.



The tie is made of high-quality steel wire according to IMS system ( tendons  $6 \Phi 7$  or  $12 \Phi 5$  mm ). Distance between frames is from 3m up to 6m, while columns length is changeable and depends on hall purposes.

Roof covering can be of durisol, gas-concrete or classic one consisting of concrete, timber or steel purlins.

Up to now, it has been investigated construction types: "MINOMA-1" span 12 m, "MINOMA-2" maximal span 20m and "MINOMA-3" up to maximal span 28 m.

### 3. Testing under the static load

Test under static load are based on Yugoslav code JUS U. M1. 047. Structure tests at the site have been carried out through two phases.

1<sup>st</sup> phase: Test up to elastic limit ( serviceability limit ).

2<sup>nd</sup> phase: Test up to failure ( test of some members and substructures up to failure ).

Static test have been performed through three phases of loading. Load which simulates dead load  $q=253 \text{ daN/m}$  was applied by means of ties which were fixed into reinforced concrete floor and into the contra-beam constructed in the plane of the frame. The tie forces were observed by measuring strips with automatic temperature compensation. Initial force in the system tie was 83 kN.

Also, designation and disposition of measured cross-sections are shown in Fig. 1. and description of the construction is given in [7].

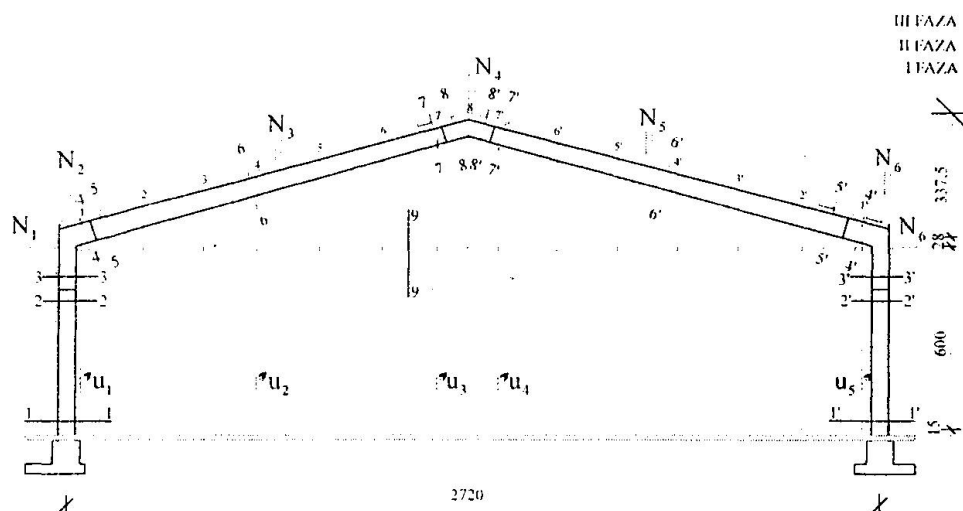


Fig. 1. Designation and distribution of measured cross-sections for static test

Measuring points along the frame were chosen at the sections with maximal static effects in order to be possible to observe them under trial load. Measuring instruments are placed according to principle "encircling" of the section although the cross-section is symmetrical one. Deflections were observed by means of survey instruments as well as by the following measuring instruments:

- measuring strips LY 10/120 and 100/120 Hottinger Baldwin
- deformeters with base  $L=250\text{mm}$ ,  $p=4,0 \times 10^{-6} \text{ mm/mm}$
- dilatometers with base  $L=190\text{mm}$ ,  $p=5,1 \times 10^{-6} \text{ mm/mm}$
- klinometers with scale precision  $p=1 \text{ s}$
- deflectometers with scale precision  $p=0,01 \text{ mm}$

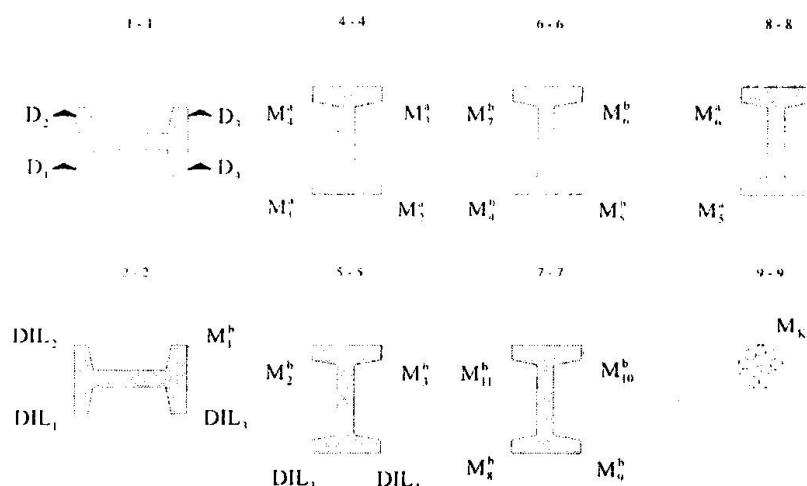


Fig. 2 Designations and distribution of measuring instruments at the cross-sections

### 3.1 Results of static testing

The results about stresses, and deflections based on the measuring data are given in [8]. Bending moment diagram obtained on the base of test data is included in Fig. 4.

These values are also calculated according to slope-deflection method by use of STRESS-program and compared with corresponding obtained by testing. Agreement is not satisfying, in addition to everything else, constitutive equations of slope-deflection method according to STRESS-program do not include elasticity of connections of some members in joints.

So, according to expressions of mentioned refined theor derived in [8], based on experimentally determined fixing degrees of connections column-foundation and column-beam, bending moments due to dead load and applied load are calculated for the structure shown in Fig. 3 taking in account the real stiffness of these connections.

Effects of uniformly distributed load with intensity  $q=253 \text{ daN/m'}$  and applied tensile force into the tie  $S=0,80Z=66,40 \text{ kN}$  are calculated separately and after that they are summed. Initial force in tie  $83,0 \text{ kN}$  is reduced 20% because of loss during prestressing of the structure. According to previously given measuring data, fixing degree of connection column-to-foundation is  $\mu_{ik}=0,4$  and of connection column-to-beam  $\mu_{ik}=1,0$ . Bending moments diagrams with analytically obtained values are given in Fig. In the final bending moments diagram, values calculated on the base of test data are given in brackets.

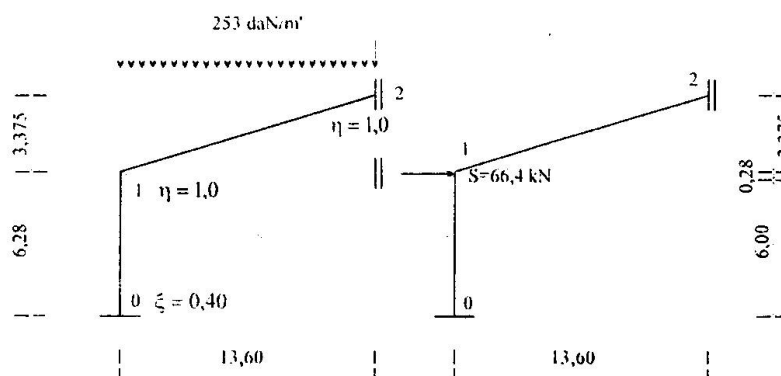


Fig. 3 The structure subjected to applied load and to the tie force

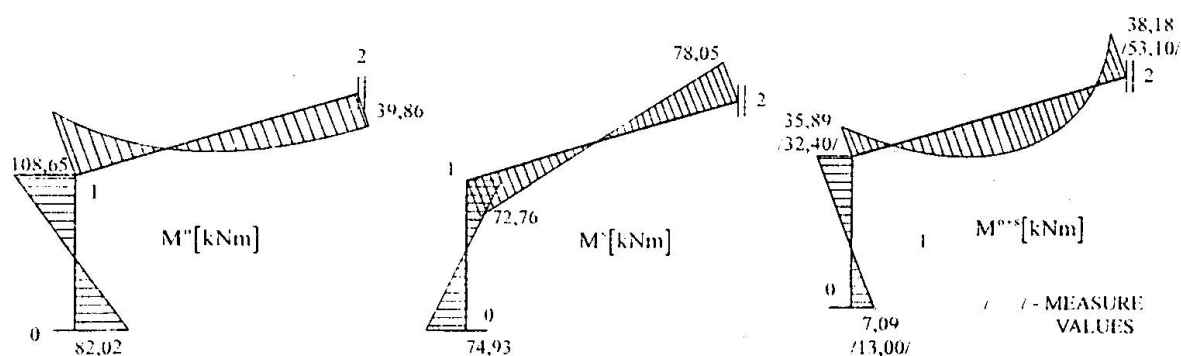


Fig. 4 Bending moment diagrams due to: a) applied load, b) tie force, c) summ under a) and b).

#### 4. Testing under the dynamic load

The purpose of dynamic testing of structures is to define its real dynamic characteristics, i. e. resonant frequencies of basic modes in horizontal and vertical direction, shapes of vibrations at this frequencies, as well as corresponding coefficient of viscous damping. All of these parameters are the base for each mathematical modeling of a structure.

Experimental testing of dynamic characteristics of typic prefabricated structure "MINOMA-3" has been carried out applying the following experiment types:

1-Experiments with forced harmonic excitation

2-Free-vibration experiments

3-Experiments with ambient vibrations

Equipement for dynamic testing of structures in full scale consists of two basic systems that have to be compatibil each to other in the sense of diapason of frequent and amplitude composition. These two systems are: system for vibration excitation ( Mechanical or electrohidraulical and registration system).

In the case of experiments with forced harmonic excitation, it was used the following equipment: Function Generator HP-3310A, Power Amplifier Model-114, Electrodynamic Shaker Model-113, as well as Kistler Accelerometer Model-305A, Kistler Amplifier Model-515, Spectrum Analyser Hp-3582A, Ploter Hp-7045B.

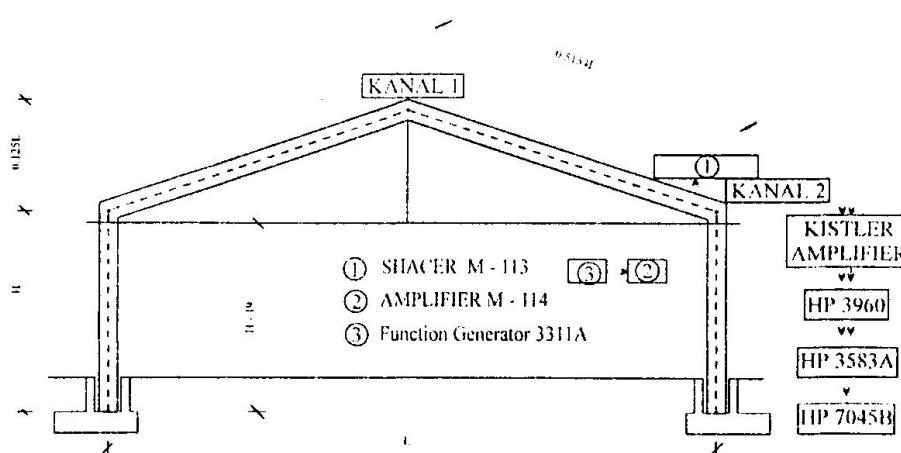


Fig.5 Functional presentation of measuring procedure of forced harmonic vibrations and free vibrations

Free vibrations have been generated in two different ways, so these experiments can be accomplished as:

- experiments with initial displacement,
- experiments when the external harmonic force is switched off.

In the case of experiments with ambient vibrations it was used the following equipment:

Ranger Seismometer Model SS-1, Signal Conditioner Sc-1, Tape recorder HP-3960, Spectrum Analyzer HP-3582A I Ploter HP-7045B.

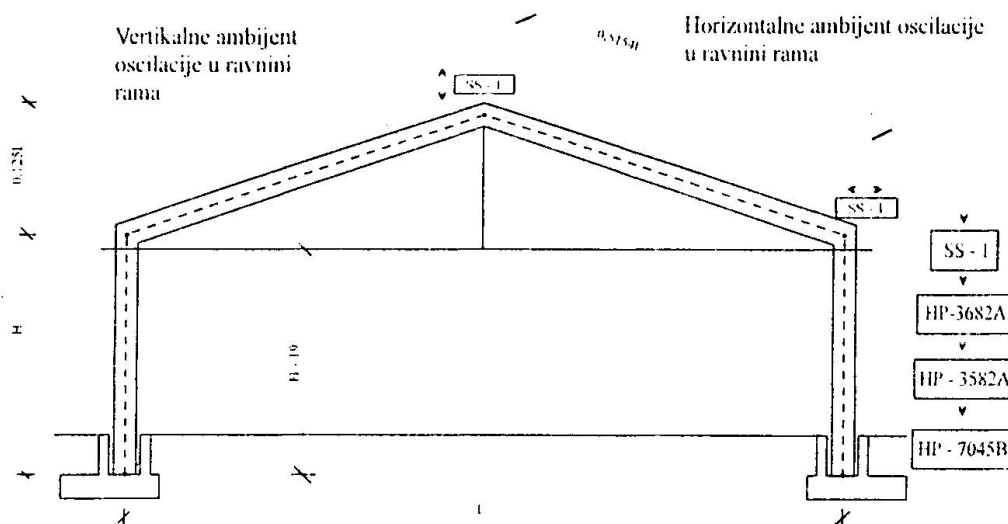


Fig.6 Functional presentation of measuring procedure of ambient vibrations

## 4.1 Results of dynamic testing

In the scope of detailed researches program, listed types of experiments have been performed. The results are given as time records of motion of material points of the structure and their mathematical processing in the form of amplitude spectrum (Furie transformation).

### 4.1.1 Experiment with forced harmonic excitation:

Functional presentation of the measuring procedure, applied equipment for measuring and generating of harmonic excitation are given in Fig. 5.

Obtained mode shape for horizontal direction of frame vibrations in plane is given in Fig. 7.

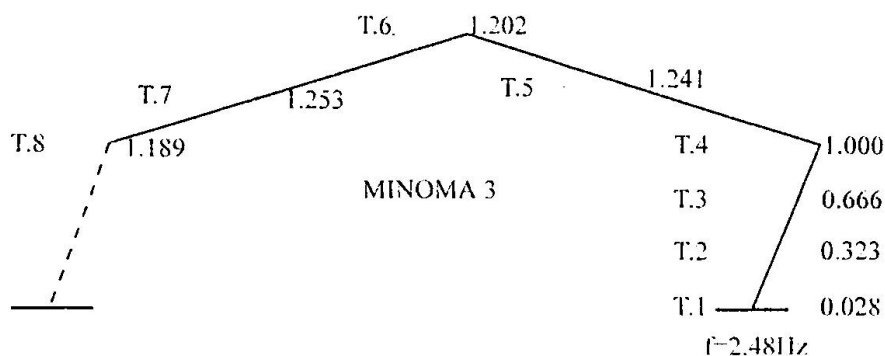


Fig. 7.



Time record of dynamic structure response and amplitude spectrum for horizontal direction is given in Fig. 8.

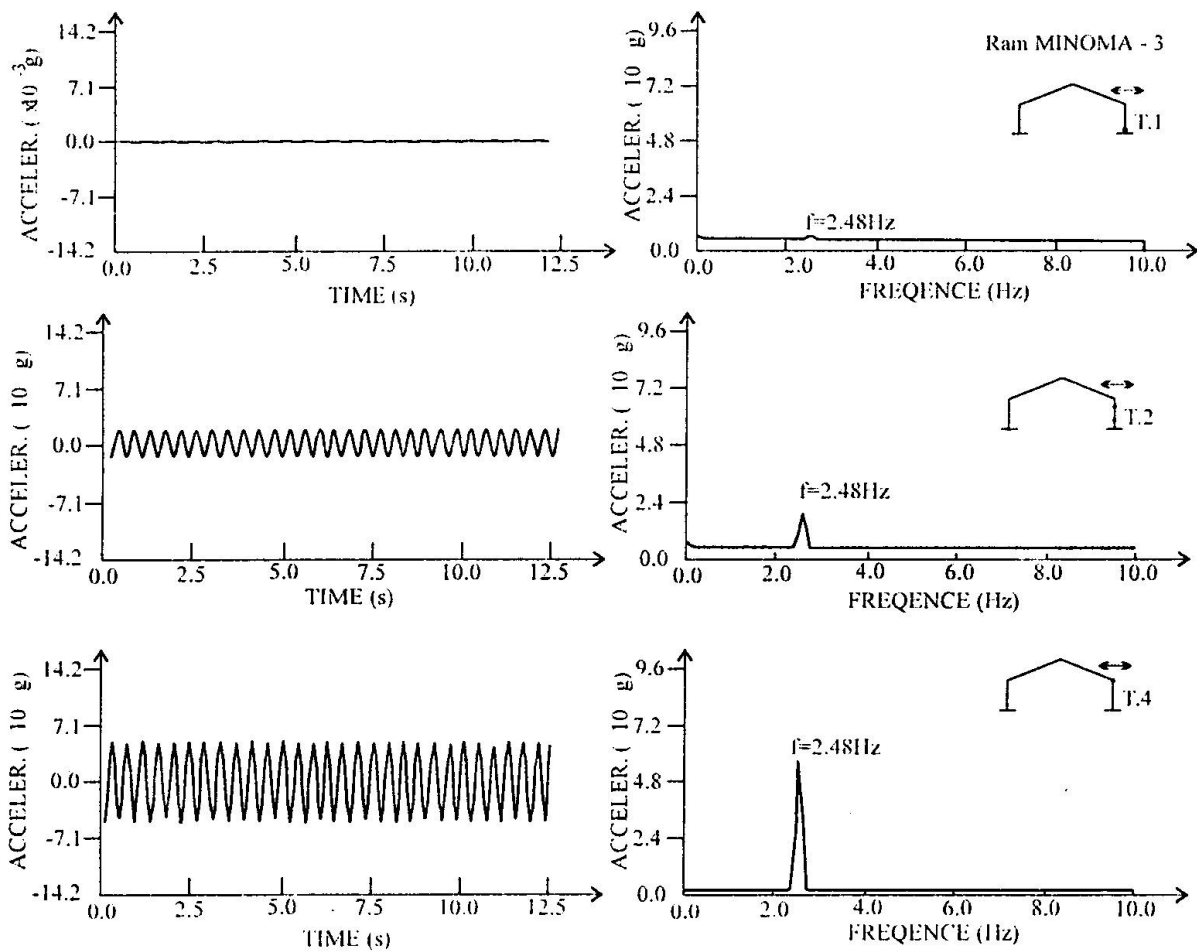


Fig. 8

Time record of harmonic decreasing functions and their amplitude spectra for horizontal vibration direction is shown in Fig. 9. and Fig. 10. This test was performed by sudden switching off the tie previously in the frame ridge.

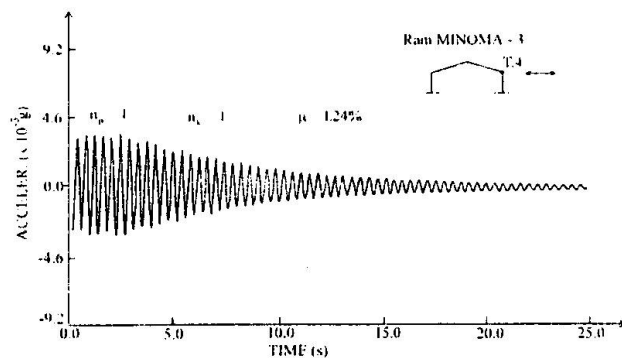


Fig. 9

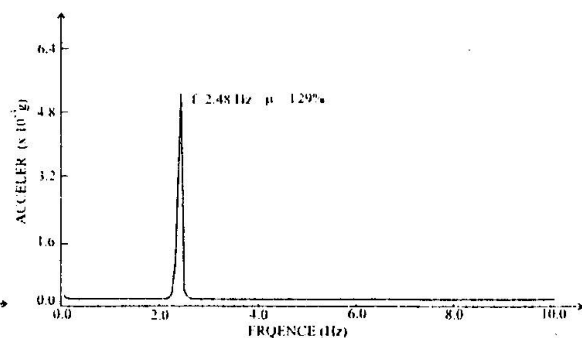


Fig. 10

### 4.1.2 Experiment with ambient vibrations:

Functional presentation is shown in Fig. 6.

In Fig. 11 it is shown time record of ambient vibrations and their amplitude spectrum for horizontal direction and in Fig 12 For vertical direction of vibrations in the plane of frame.

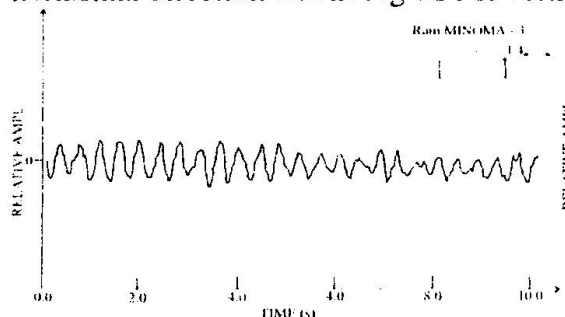


Fig. 11

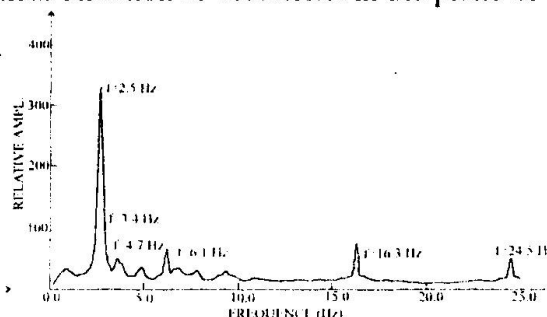


Fig. 12

### 4.2 Calculating of frequencies for horizontal and vertical vibration direction

On the base of experimentally determined fixing degrees of connection column to foundation ( $\mu_{ki} = \eta = 40\%$ ) and connection column to beam ( $\mu_{ik} = \xi = 100\%$ ) bending moments are calculated, as well as circle frequencies for horizontal and vertical vibration direction, taking in account real rigidity of connections.

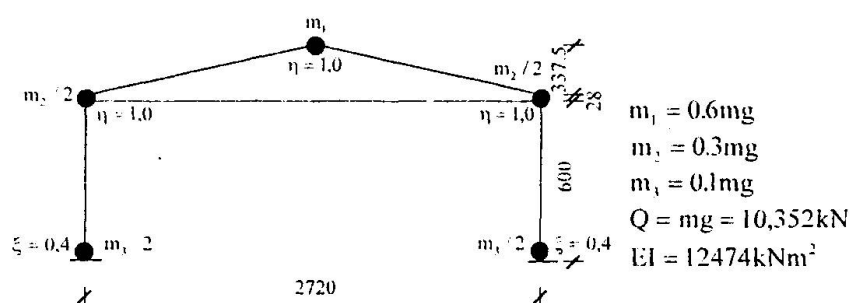


Fig. 13. Dynamically model of frame

Calculated values of circle frequencies are: for horizontal direction  $\omega_h = 16.26 s^{-1} = 2,58 Hz$ , for vertical direction  $\omega_v = 22,56 s^{-1} = 3,59 Hz$ .

## 5. Conclusion

It can be concluded from obtained results that fixing degree should not be neglected, both in static and in dynamic design, and particularly it should be paid attention about that, in analysis of prefabricated constructions. At static and dynamic testing, prefabricated construction "MINOMA" has shown complex behavior whose details are not presented here. Experimental researches and numerical analysis of considered structure have confirmed already known data, but also have pointed out new ones, particularly in the case of prestressing by the tie, as well as in the case of design of structure with semi-rigid connections of members. Calculated values of circle frequencies, based on dynamic model





of the frame (Fig. 13 ), are very closed to measured values for both directions of vibration (  $\omega_{h,1} = 2,58\text{Hz}$  i  $\omega_{v,1} = 3,59\text{Hz}$  ).

Obtained experimental results for dynamic characteristics represent the base for further experimental and analytical researches that would be carried out in order typical prefabricated system "MINOMA" security attest for all types of loading and influences according to available technical regulation. The first that follows will be testing of joints up to failure, as well as defining of design seismic characteristics (design spectra and design earthquake time histories of ground motion). Taking in consideration that in world literature and periodicals data about damping force (expressed in term of viscous damping) are poor for similar typic prefabricated structures, obtained results represents basic values for structural analysis under strong earthquake ground motions and for estimate of global seismic security.

The authors think that it is to be pointed out to the former circumstances in the corresponding regulations and supporting documents (comments or suggestions) because of missing the appropriate technical regulations for design and construction of the structures with members elastical fixed in joints at seismic regions.

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## STATIC AND DYNAMIC DESIGN OF STRUCTURES WITH SEMI - RIGID CONNECTIONS

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### Summary

Constructions with not absolutely rigid connections between members, that allow some relative rotation of member ends, are systems with semi-rigid connections in joints. As such system of connections is very often in constructions, particularly in prefabricated ones, it is of interest to analyze them taking in account flexibility of joint connections.

Because of limited space, here it will be given only expressions for design of semi-rigid connection systems, without deriving.

### 1. Static design of systems with semi-rigid connections

Design of linear systems, with assumption that member connections are either quite rigid or ideally pinned known as "approximate" slope-deflection method is presented in [1], and in [2] it is given its better assurance up to arbitrary required level by introducing the influence of axial forces on deformation. In what follows, semi-rigid connected linear systems 'are considered.

#### 1.1 Design according to first order theory

For semi-rigid connected lynear systems, expressions for bending moments at the ends as well as governing equations of slope-deflection method according to first order theory are derived in [3]. If it is introduced designations  $\mu_{ik} = \varphi_{ik}^* / \varphi_i$ ;  $\mu_{ki} = \varphi_{ki}^* / \varphi_k$  (where  $\varphi_i$  and  $\varphi_k$  are the angles of rotation of the joint "i" and "k" respectively, and angles  $\varphi_{ik}^*$  and  $\varphi_{ki}^*$  of rotations of end cross-sections of the member "ik") and are named fixing degrees of member "ik" in joints "i" and "k", the expressions for bending moments at the bar ends of such connected members are:

$$M_{ik} = a_{ik} \varphi_i^* + b_{ik} \varphi_k^* - c_{ik} \psi_{ik} + m_{ik}; \quad M_{ki} = b_{ik} \varphi_i^* + a_{ki} \varphi_k^* - c_{ki} \psi_{ik} + m_{ki} \quad (1)$$

or in terms of  $\varphi_i, \varphi_k, \psi_{ik}$ , in the shape

$$M_{ik}^* = a_{ik}^* \varphi_i + b_{ik}^* \varphi_k - c_{ik}^* \psi_{ik} + m_{ik}^*; \quad M_{ki}^* = b_{ik}^* \varphi_i + a_{ki}^* \varphi_k - c_{ki}^* \psi_{ik} + m_{ki}^* \quad (2)$$



Constants  $a_{ik}^*$ ,  $b_{ik}^*$ ,  $c_{ik}^*$ , as well as the initial moments of semi-rigidly connected members can be expressed in terms of corresponding values of rigidly connected members and fixing degree, as it is given in [3], with the form similar to the expressions (8. a) , (8. i), but in this case independent on axial force of the bar.

The final expressions for  $M_{ik}^*$  and  $M_{ki}^*$  are obtained in the shape:

$$M_{ik}^* = a_{ik}^* \varphi_i + b_{ik}^* \varphi_k - c_{ik}^* \sum_{j=1}^n \psi_{ik}^{(j)} \Delta_j + m_{ik}^*; \quad M_{ki}^* = b_{ik}^* \varphi_i + a_{ki}^* \varphi_k - c_{ki}^* \sum_{j=1}^n \psi_{ik}^{(j)} \Delta_j + m_{ki}^* \quad (3)$$

$$\text{where it is denoted} \quad m_{ik}^* = m_{ik}^{*(0)} + m_{ik}^{*(\Delta t)} + m_{ik}^{*(1)} + m_{ik}^{*(\infty)} + m_{ik}^{*(N)} \quad (4)$$

$$\text{with} \quad m_{ik}^{*(N)} = -c_{ik}^* \psi_{ik}^{(N)}; \quad (5)$$

while  $a_{ik}$ ,  $b_{ik}$ ,  $c_{ik}$  and  $d_{ig}$  are the constants of members,  $\psi_{ik}^{(j)}$ ,  $\psi_{ig}^{(j)}$  angles of rotation in the state  $\Delta_j = 1$ , and  $\Delta_j$  ( $j=1,2,\dots,n$ ) parameters of displacement.

Equations of rotation and equations of displacements are given in [3].

For semi-rigid connected systems it is shown in [7] applicability to matrix analysis, in [8] applicability to force method, as well as reinforced concrete frame design in [8] and [9].

## 2. Design according to second order theory

It is assumed fixing degree of a bar  $ik$  in nodal point "i"  $\mu_{ik}$  and "k"  $\mu_{ki}$ . During deformation, nodal points rotations are  $\varphi_i$  and  $\varphi_k$ , end cross-sections of a bar rotate  $\varphi'_i$  and  $\varphi'_k$ , while bending moments are  $M'_{ik}$  and  $M'_{ki}$ . In terms of deformation values it can be written:

$$M'_{ik} = a_{ik} \varphi'_i + b_{ik} \varphi'_k - c_{ik} \psi_{ik} + m_{ik}; \quad M'_{ki} = b_{ik} \varphi'_i + a_{ki} \varphi'_k - c_{ki} \psi_{ik} + m_{ki} \quad (6)$$

or in terms of  $\varphi_i$ ,  $\varphi_k$ ,  $\psi_{ik}$  in the shape

$$M'_{ik} = a'_{ik} \varphi_i + b'_{ik} \varphi_k - c'_{ik} \psi_{ik} + m'_{ik}; \quad M'_{ki} = b'_{ik} \varphi_i + a'_{ki} \varphi_k - c'_{ki} \psi_{ik} + m'_{ki} \quad (7)$$

Relations between old and new constants as well as initial moments are found on the base of their physical meanings (Fig. 1).

$$\alpha'_{ik} = \mu_{ik} - (1 - \mu_{ik}) \mu_{ki} \frac{b_{ik}}{a_{ik}} \quad \alpha'_{ik}^{(o, \Delta t)} = \mu_{ik} \alpha_{ik}^{(o, \Delta t)} - (1 - \mu_{ik}) \mu_{ki} \frac{b_{ik}}{a_{ik}} \alpha_{ik}^{(o, \Delta t)}$$

$$\alpha'_{ki} = \mu_{ki} - (1 - \mu_{ki}) \mu_{ik} \frac{b_{ik}}{a_{ki}} \quad \alpha'_{ki}^{(o, \Delta t)} = \mu_{ki} \alpha_{ki}^{(o, \Delta t)} - (1 - \mu_{ki}) \mu_{ik} \frac{b_{ik}}{a_{ki}} \alpha_{ki}^{(o, \Delta t)}$$

Fig. 1.

$$a'_{ik} = \mu_{ik} \left[ a_{ik} - (1 - \mu_{ki}) \frac{b_{ik}}{a_{ki}} b_{ik} \right] \quad (8.a) \quad m'_{ik} = \mu_{ik} \left[ m_{ik} - (1 - \mu_{ki}) \frac{b_{ik}}{a_{ki}} m_{ki} \right] \quad (8.f)$$

$$b'_{ik} = b_{ki} \mu_{ik} \mu_{ki} \quad (8.b) \quad m'_{ki} = \mu_{ki} \left[ m_{ki} - (1 - \mu_{ik}) \frac{b_{ik}}{a_{ik}} m_{ik} \right] \quad (8.g)$$

$$c'_{ik} = \mu_{ik} \left[ c_{ik} - (1 - \mu_{ki}) \frac{b_{ik}}{a_{ki}} c_{ki} \right] \quad (8.c) \quad M'_{ik} = \mu_{ik} \left[ M_{ik} - (1 - \mu_{ki}) \frac{b_{ik}}{a_{ki}} M_{ki} \right] \quad (8.h)$$

$$c'_{ki} = \mu_{ki} \left[ c_{ki} - (1 - \mu_{ik}) \frac{b_{ik}}{a_{ik}} c_{ik} \right] \quad (8.d) \quad M'_{ki} = \mu_{ki} \left[ M_{ki} - (1 - \mu_{ik}) \frac{b_{ik}}{a_{ik}} M_{ik} \right] \quad (8.i)$$

$$a'_{ki} = \mu_{ki} \left[ a_{ki} - (1 - \mu_{ik}) \frac{b_{ik}}{a_{ik}} b_{ik} \right] \quad (8.e)$$

As it is evident, here bars of type “k” and type “g” are considered both as type “k” ( fixed at the both ends ). From the expression (8) it follows for  $\mu_{ik} = \mu_{ki} = 1$  former type “g” ( fixed in “i” and flexible in “k” ) and for  $\mu_{ik} = \mu_{ki} = 0$ , former type “s” is with elastic fixing in joint. At the same way as in (7) are obtained expressions  $M'_{ik}$  and  $M'_{ki}$ :

$$M'_{ik} = a'_{ik} \varphi_i + b'_{ik} \varphi_k - c'_{ik} \sum_{j=1}^n \psi_{ik}^{(j)} \Delta_j + m'_{ik}; \quad M'_{ki} = b'_{ik} \varphi_i + a'_{ki} \varphi_k - c'_{ki} \sum_{j=1}^n \psi_{ik}^{(j)} \Delta_j + m'_{ki} \quad (9)$$

Rotation equations and displacement equations now look like:

$$\sum_k M'_{ik} + M_i = 0 \quad (i=1,2,...,m); \quad (\varphi_i=1) \quad (10.a)$$

$$\sum_{ik} (M'_{ik} + M'_{ki}) \psi_{ik}^{(j)} + R_j(p) + R_j(m^f) = 0 \quad (j=1,2,...,n)(\Delta_j=1) \quad (10.b)$$

where  $R_j(m^f)$  is work of distributed fictive moments.

Equations of slop-deflection method are:

$$A'_{ii} \varphi_i + \sum_k A'_{ik} \varphi_k + \sum_{j=1}^n B'_{ij} \Delta_j + A_{i0} = 0 \quad (i=1,2,...,m) \quad (11.a)$$

$$\sum_{i=1}^m B'_{ji} \varphi_i + \sum_{l=1}^n C'_{jl} \Delta_l + C'_{j0} = 0 \quad (j=1,2,...,n) \quad (11.b)$$

with introduced designations:

$$A'_{ii} = \sum_k a'_{ik} \sum_s e'_{is}; \quad A'_{ik} = b'_{ik}; \quad A'_{i0} = \sum_k m'_{ik} + M'_i \quad (12.a)$$

$$B'_{ij} = - \sum_k c'_{ik} \psi_{ik}^{(j)} = B'_{ji} \quad (12.b)$$

$$C'_{ji} = C'_{ij} = \sum_{ik} (c'_{ik} + c'_{ki}) \psi_{ik}^{(j)} \psi_{ik}^{(i)} \mp EI_c \sum_{ab} \frac{\omega_{ab}^2}{L_{ab}'} \psi_{ab}^{(j)} \psi_{ab}^{(i)} \quad (12.c)$$

$$C'_{j0} = - \sum_{ik} (m'_{ik} + m'_{ki}) \psi_{ik}^{(j)} - R_j(p) \mp EI_c \sum_{ab} \frac{\omega_{ab}^2}{L_{ab}'} \psi_{ab}^{(j)} (\psi_{ab}^{(i)} + \psi_{ab}^{(v)}) \quad (12.d)$$



### 1.2.1 Determining of critical load

Critical load is determined from governing equations (11) when terms  $A_{i0}$  and  $C_{j0}$  are missed.

Equations (11) in matrix form, by use of block matrix, are:

$$\begin{bmatrix} A' & B' \\ B'' & C' \end{bmatrix} \begin{bmatrix} \varphi \\ \Delta \end{bmatrix} = 0 \quad (13) \quad \text{that is} \quad \det \begin{bmatrix} A' & B' \\ B'' & C' \end{bmatrix} = 0 \quad (14)$$

Coefficients of bar as well as initial moments depend on axial forces in members of a girder only at second order theory, while they are constant at first order theory.

## 1.3 Numerical examples

### 1.3.1 Static design

For the purpose of illustration of presented design of structures with semi-rigid connections of members in joints, it is given example of frame in Fig. 2.

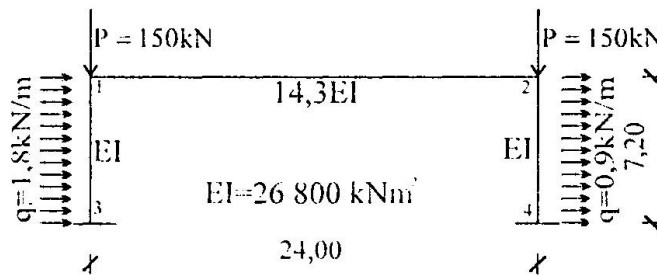


Fig. 2.

This system has three redundant. Unknowns are rotation angles  $\varphi_1$  and  $\varphi_2$ , as well as displacement parameter  $\Delta_1$ . Governing equations of slope-deflection method are given in following shape:

$$\begin{bmatrix} A_{11} & A_{12} & B_{11} \\ A_{21} & A_{22} & B_{21} \\ B'_{11} & B'_{12} & C_{11} \end{bmatrix} \begin{bmatrix} \varphi_1 \\ \varphi_2 \\ \Delta_1 \end{bmatrix} + \begin{bmatrix} A_{10} \\ A_{20} \\ C_{10} \end{bmatrix} = 0 \quad (15)$$

Results are given in Table 1.

### 1.3.2 Determining of critical load and buckling length

Critical load is defined as the smallest value of load parameter at which homogenous problem of linearized second order theory has only one solution different from trivial one (11).

This condition can be expressed in matrix form as (13), that is (14).

For the purpose of determining critical load, it is used equivalent system to that shown in Fig. 2, where intensity of uniformly distributed load is expressed in terms of load parameter P.

Influence of transversal load is neglected in calculation of critical load parameter and buckling length of columns, because its intensity is small ( $q=0,012P$ ), although we are not at the side of security. Having in mind that analysis of geometry imperfection influence on stability of static systems has certain significance, in the case of greater transversal load its influence has to be taken in account.

### 1.3.3 Analysis of influence of fixing degree on bending moments, critical load and buckling length of columns

As it has been done in examples given in 3.1 and 3.2 are given in [6], diagrams of bending moments according to the first and the second order theory, values  $\omega_{cr}$ ,  $P_{cr}$  and buckling length of columns, for different fixing degrees are calculated and presented in Table 1. It is also given comparison of obtained values  $\omega_{cr}$ ,  $P_{cr}$ ,  $l_k$  with those for column rigidly fixed into the foundation and pin jointed to the beam ( $\xi = 0$  and  $\eta = 1$ ).

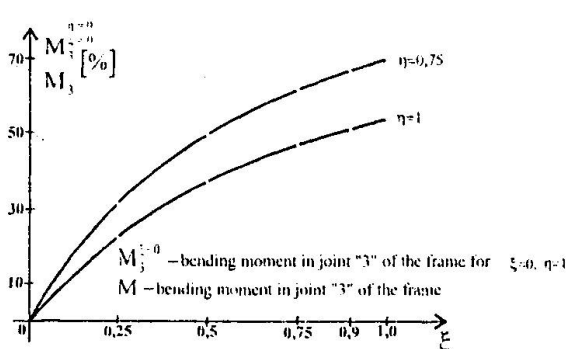
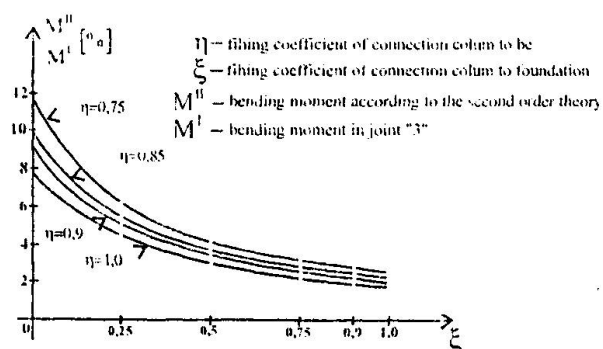
At bending moment diagrams, in parentheses are given values obtained according to the first order theory, and out of parentheses according to the second order theory. Because of limited space it is presented just a small part of obtained results.

Table. 1

fixing coefficient	DIAGRAM M	$\omega_{kr}^*$	$P_{kr}$	$l_k = \beta l_{k0}$
		$\frac{\omega_{kr}^*}{\omega_{k0}^*}$	$\frac{P_{kr}}{P_{k0}}$	$\frac{l_k}{l_{k0}}$
$\xi = 0,25$ $\eta = 0,95$		0,272 1,248	0,074EI 1,558	1,604 $l_{k0}$ 0,802
$\xi = 0,3$ $\eta = 0,75$		0,299 1,371	0,0894EI 2,882	1,459 $l_{k0}$ 0,729

Diagram 1

Diagram 2



At Diagram 1, it is shown difference between bending moment in the foundation of the column (joint "3") calculated according to the first and the second order theory, for different



values of fixing degrees  $\xi$  and  $\eta$ . It is evident that difference is significant for smaller fixing degrees (for  $\xi = 0$  and  $\eta = 0,75$  the difference is 11,6%), while for greater fixing degrees the difference is insignificant (for  $\eta = 1$  and  $\xi = 1$  its 2%). So it can be concluded that in design of systems with semi-rigid connections application of the second order theory is more important.

At Diagram 2, it is shown how the values of bending moment in column-foundation joint "3" are changing for different fixing degrees  $\xi$  and  $\eta$  in comparison with that one for  $\xi = 0$ . It is evident that this change significantly depends on fixing degree  $\xi$  of the joint column to beam, and lower depends on fixing degree of the connection column to foundation  $\eta$ . So, greater attention has to be paid on connection details of column to beam particularly in the case of prefabricated structures.

Similar analysis has been carried out for changing the ratio of critical forces  $P_{kr}/P_{k0}$  (Diagram 3) as well as the ratio of buckling lengths of columns  $l_k/l_{k0}$  (Diagram 4) depending on fixing degrees  $\xi$  and  $\eta$ .

Diagram 3

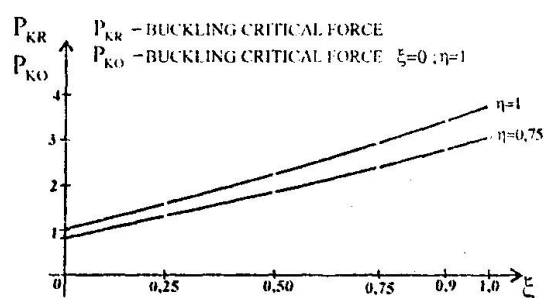
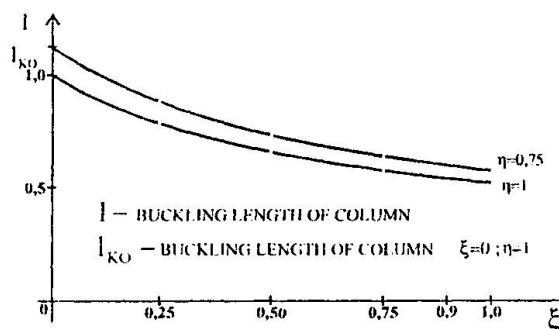


Diagram 4



It can be noticed that for different values of fixing degree  $\eta$ , the ratio  $P_{kr}/P_{k0}$  changes already linear with changing of fixing degree  $\xi$ , that significantly simplifies procedure of designing the parameter of critical load and buckling length of columns.

## 2. Dynamic design

### 2.1 The case of semi-rigid connections

When it is considered systems with flexible connected members, whose masses  $m_i$  are attached by dynamic loading  $P_i(t)$ , in real environment that opposes movement of resistant forces  $P_i(t)$ , and corresponding inertial forces  $I_i(t)$  whose projections on "x" and "y" axis are shown in Fig. 3, taking in account equations (3.99)-(3.115) from [5] can be written equations of forced damped vibrations of system with finite number of degrees of freedom in the shape:

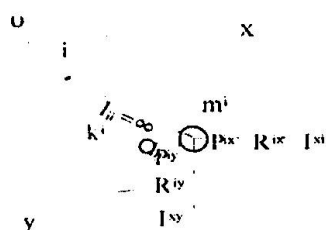


Fig. 3

$$\begin{aligned} R_{ix}(t) &= -\beta_i \dot{u}'_i, R_{iy}(t) = -\beta_i \dot{v}'_i \\ I_{ix}(t) &= -m_i \ddot{u}_i, I_{iy}(t) = -m_i \ddot{v}_i \\ u'_i &= u_i - k_i \varphi_i \sin \alpha_{ii} \\ v'_i &= v_i - k_i \varphi_i \cos \alpha_{ii} \end{aligned} \quad (16)$$

$$\begin{bmatrix} \bar{A} & \bar{B} \\ \bar{B}^T & \bar{C} \end{bmatrix} \begin{Bmatrix} \ddot{\bar{\varphi}} \\ \ddot{\bar{\Delta}} \end{Bmatrix} + \begin{bmatrix} \bar{A} & \bar{B} \\ \bar{B}^T & \bar{C} \end{bmatrix} \begin{Bmatrix} \dot{\bar{\varphi}} \\ \dot{\bar{\Delta}} \end{Bmatrix} + \begin{bmatrix} A^* & B^* \\ B^{*T} & C^* \end{bmatrix} \begin{Bmatrix} \bar{\varphi} \\ \bar{\Delta} \end{Bmatrix} = - \begin{Bmatrix} \bar{A}_0 \\ \bar{C}_0 \end{Bmatrix} \quad (17)$$

Submatrix which appear in (17) are given in [6].

Previously presented is valid for the case where masses are connected by cantilever (length  $k_i$ ) to the joints of girder. The most often case is when masses are put just in joints ( $k_i=0$ ).

Then, matrix  $\bar{C}$ ,  $\bar{C}_0$ ,  $\bar{B}$  and  $\bar{A}_0$  become null matrix, so that from system (17) after m elimination's, can be eliminated all unknowns  $\bar{\varphi}$ , and expression (17) get the shape

$$\begin{bmatrix} \bar{C} \end{bmatrix} \begin{Bmatrix} \ddot{\bar{\Delta}} \end{Bmatrix} + \begin{bmatrix} \bar{C} \end{bmatrix} \begin{Bmatrix} \dot{\bar{\Delta}} \end{Bmatrix} + \begin{bmatrix} C^{**} \end{bmatrix} \begin{Bmatrix} \bar{\Delta} \end{Bmatrix} = - \begin{bmatrix} \bar{C}_0 \end{bmatrix} \quad (18)$$

where matrix  $C^{**} = C^{*(m)}$

(19)

is obtained after m elimination's.

When parameters  $\Delta_i$  are determined from (18), by use of the system

$$\begin{bmatrix} A^* \end{bmatrix} \begin{Bmatrix} \bar{\Phi} \end{Bmatrix} = - \begin{bmatrix} B^* \end{bmatrix} \begin{Bmatrix} \bar{\Delta} \end{Bmatrix} \quad (20)$$

values  $\varphi_i$  ( $i=1,2,\dots,m$ ) can be determined, from (2) end bending moments and after that other internal forces can be calculated.

### 2.1.1 Numerical examples

Proceeding from the new constants of bars, by use of slope-deflection method, for the purpose of illustration, it is worked on an example of a simple frame structure, where fixing degrees are varied from 0 to 1. (Table 2). Because of limited space it is presented just a small part of obtained results.

Table. 2

Structure scheme	$M_d = M_i \times I + M_{st,H}$ Bending moments due to dynamic loading			Dynamic properties and influences caused by perturb. force $P_i(t)$		
	M diagram $H=20 \text{ kN}$ $I=34,783 \text{ kN}$	M diagram $H=10 \text{ kN}$ $I=17,777 \text{ kN}$	M diagram $H=0 \text{ kN}$ $I=0 \text{ kN}$	Circ. freq $\omega$ [s <sup>-1</sup> ]	$I_1$ from $H_1(t) = 0 \sin \theta t$ [kN]	$I_2$ from $H_2(t) = 20 \sin \theta t$ [kN]
$m = 5 \frac{\text{kNs}^2}{\text{m}}$ $EI = 100000 \text{ kNm}^2$ $\theta = 0,8\omega$ 				5,270	17,778	35,556
$\xi = 1$ $q = 10 \text{ kN/m}$ $I = 6,0$ $\eta = 3/4$ 				9,000	17,783	35,356





In the case of dynamic load acting, that can be describe by the function  $P_1(t) = H_1 \sin \theta t$ , circle frequencies and inertial forces are calculated for  $\theta = 0.8\omega$ . On the base of obtained results, it can be concluded that circle frequencies of the structures shown in the Table. 2 are 33% and 71%, respectively, greater than that one of the cantilever column. Inertial forces differs each to other less than 8%, for the considered example, different fixing degrees.

At the Table T.2, fixing degrees of joint column to foundation is denoted by  $\mu_{k1} = \xi$ , and column to beam  $\mu_{k2} = \eta$ . Bending moment diagrams, in the case of dynamic loading are calculated for uniformly distributed loading  $q=10$  kN/m and for horizontal force  $H=10$  kN,  $H=20$  kN and  $H=0$ . For different fixing degrees and  $\eta$ , it is evident from given diagrams that changes of bending moments are significant.

### 3. Conclusion

On the basis of carried out analyze, it can be concluded that fixing degree of the connections would be taken in account, in static and dynamic design, so as in design of stability of constructions. Special attention has to be poyed on analysis of prefabricated constructions where relatively low fixing degree can be favorable for distribution of bending moments. So this circumstance is to be used in designing, because accompanied measures are easy to realize. At the otter side, not enough insured but assumed rigid connections can cause unfavorable consequences.

Depending on physic-mechanical characteristics of used materials and behavior of joint connections, i.e flexibility of the system during force acting, it is often necessary to calculate effects according to the second order theory.

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## Flexible connections as the discrete dampers

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### Summary

The object of the paper is the derivation of the damping matrix of timber frames with flexible connections. The basic explicit assumption is that the loss of energy occurs only at the joints and that the modal damping ratios are known either from experiments or from engineering judgement. Damping matrix orthogonality is used to obtain the damping exhibited by each of the joints.

### 1. Introduction

The performance of timber construction in earthquakes can be very good mainly because of its low mass, flexibility of connections and high damping. Dowrick [1] and Green [2] report that the damping ratio in timber structures can take a very high value, even 15% to 20%. But according to Lazan [3], the wood itself has a material damping ratio less than 1%. It is obvious that a large amount of energy absorption takes place in the secondary structure and in the flexible connections of the primary structure.

The tests performed on some timber portal frames, without secondary structure, by Ceccotti et al. [4], confirm an equivalent damping ratio of about 15%. No doubt, the principal part of the energy absorption takes place just in the flexible connections of the primary structure. These facts prove that the timber structures behave as systems with the discrete dampers which should be properly represented in the model for a correct dynamic response analysis. The other property of flexible connections, their stiffness capacity, is not the subject of the present analysis.

The modal damping ratios for a structure are known either from experiments or from engineering judgement but the contribution of each joint separately is unknown. In the global damping matrix of a timber frame structure in which the flexible connections are the only dampers, the influence coefficients are actually the coefficients associated with the damping forces developed in particular joints. If the procedure of deriving uncoupled equations of motion is followed, which also means the satisfaction of orthogonality condition of the damping matrix, one comes to a set of equations which relate the mode-shapes, the damping coefficients, and the modal damping ratios. From



these equations it is possible to compute either viscous damping coefficient for each of the joints or their relative values. It means that the damping uncoupling imposes certain rule on the damping distribution among the flexible joints. This will be the basis for the approach used further in the analysis.

An alternative in handling the damping in structures is to use two methods for the numerical evaluation of orthogonal damping matrices, developed by Wilson and Penzien [5] and also Clough and Penzien [6]. These two methods yield an orthogonal damping matrix which produces specific modal damping ratios, but the damping model is fictitious one, not pretending the stress distribution within the frame to be correct. On the contrary, in the present analysis a particular damping model is observed in which the damping forces are developed internally in specified locations and along specified coordinates. In an earlier attempt, the authors [7] constructed a damping matrix for a model in which the dampers were attached externally to the joints. That was also a fictitious model.

## 2. Damping distribution analysis

For any structure of  $N$  dynamic degrees of freedom, the equations specifying the orthogonality property of the damping matrix have the form

$$\begin{aligned} C_i &= \{\phi_i\}^T [c] \{\phi_i\} = 2 \cdot M_i \cdot \xi_i \cdot \omega_i, & i = 1, 2, \dots, N \\ \{\phi_i\}^T [c] \{\phi_j\} &= 0, & i \neq j \end{aligned} \quad (1)$$

where  $M_i$ ,  $C_i$ ,  $\omega_i$ ,  $\xi_i$  are the generalized mass and damping, the frequency and the damping ratio, all in mode  $i$ , respectively.

The number of available equations in (1) is  $N(N+1)/2$ . It should be noted that dynamic degrees of freedom are associated with the displacement components in which significant inertia forces are developed.

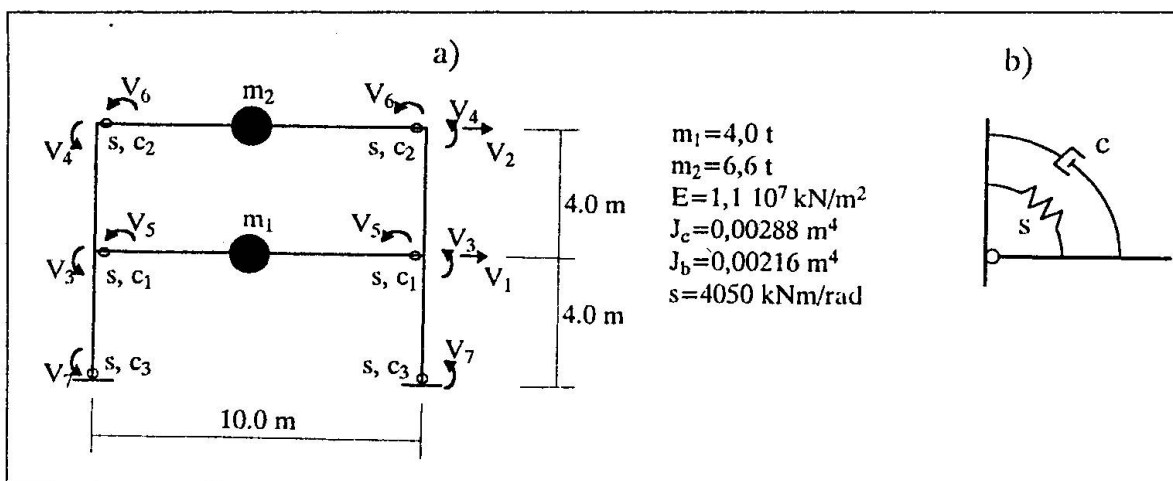
In structures with flexible connections, the damping forces are developed in dashpots between interconnected members, Fig. 1b. For the correct description of damping forces, end rotation of each member connected by a dashpot should be treated as an independent displacement component. The motion is now fully described with displacement components necessary to represent the inertia forces and also the damping forces. In order to perform the dynamic analysis in these displacement coordinates, the mass matrix and the stiffness matrix should be extended to match the order of the displacement vector.

Performing the undamped free vibration analysis in the extended number of displacement components, the frequencies and the mode shapes can be obtained. Having these data evaluated, equations (1) will yield the damping influence coefficients. Without any loss of generality, the idea of damping distribution will be demonstrated first on a two-storey timber frame, Fig.1a. All the data necessary for numerical evaluation and also the displacement components are presented on the figure. The beams are flexibly connected to columns and also there is a flexible connection at the base level. Any flexible connection is regarded as a rotational spring of stiffness  $s$  and a rotational dashpot with some viscous damping coefficient  $c$ , Fig. 1b. The spring stiffness  $s$  depends on the type of joint and fastener, taken the same for all joints in this example, and it is calculated according to DIN 1052 [8].

The frame performs lateral vibrations, antisymmetric in character. Because of dashpots, end rotations of flexibly connected members must be taken as independent coordinates. Observing antisymmetry, the displacement vector is of order seven which gives the following damping matrix

$$[c] = \begin{bmatrix} 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & 0 \\ 0 & 0 & c_1 & 0 & -c_1 & 0 & 0 \\ 0 & 0 & 0 & c_2 & 0 & -c_2 & 0 \\ 0 & 0 & -c_1 & 0 & c_1 & 0 & 0 \\ 0 & 0 & 0 & -c_2 & 0 & c_2 & 0 \\ 0 & 0 & 0 & 0 & 0 & 0 & c_3 \end{bmatrix} \quad (2)$$

where  $c_1$ ,  $c_2$ , and  $c_3$  are the damping coefficients for the dashpots on the beams and at the base, Fig. 1a.



*Fig. 1. Dynamic model of a two-storey laminated timber portal frame*



The only nonzero coefficients in the mass matrix are  $m_{11}=m_1$  and  $m_{22}=m_2$ . The undamped free vibration frequencies, mode-shapes, and generalized masses are computed and presented here as

$$\begin{aligned}\{\omega\}^T &= [6,287 \quad 47,207] \text{ sec}^{-1} \\ \{\phi_1\}^T &= [1,0 \quad 1,994 \quad -0,2496 \quad -0,2255 \quad -0,05522 \quad -0,05 \quad -0,22148] \\ \{\phi_2\}^T &= [1,0 \quad -0,3043 \quad 0,0351 \quad 0,42887 \quad 0,00776 \quad 0,095 \quad -0,34805] \\ M_1 &= 30,24 \text{ t}, \quad M_2 = 4,611 \text{ t}\end{aligned} \quad (3)$$

Equations (1) which express generalized damping and the orthogonality property of the damping matrix, have the following developed form

$$\begin{aligned}c_1(\phi_{31} - \phi_{51})^2 + c_2(\phi_{41} - \phi_{61})^2 + c_3\phi_{71}^2 &= 2M_1 \cdot \xi_1 \cdot \omega_1 \\ c_1(\phi_{32} - \phi_{52})^2 + c_2(\phi_{42} - \phi_{62})^2 + c_3\phi_{72}^2 &= 2M_2 \cdot \xi_2 \cdot \omega_2 \\ c_1(\phi_{31} - \phi_{51})(\phi_{32} - \phi_{52}) + c_2(\phi_{41} - \phi_{61})(\phi_{42} - \phi_{62}) + c_3\phi_{71}\phi_{72} &= 0\end{aligned} \quad (4)$$

where  $\xi_1$  and  $\xi_2$  are the modal damping ratios for the first and the second mode, respectively. If the modal damping ratios are assigned some specific values, for example  $\xi_1 = 0,15$  and  $\xi_2 = 0,05$ , equations (4) result in  $c_1 = 1304,82$ ,  $c_2 = 48,77$  and  $c_3 = 126,76$ . This way the damping distribution among the flexible connections is achieved and also the damping matrix (2) is evaluated.

A different situation may arise if the joint on the base level, Fig. 1a, is the fixed joint. In that case, the number of unknown damping coefficients is less than the number of available equations in (1) and for that reason only one of the modal damping ratios can be assigned while the other should result from the equations. The free vibration analysis, with a stiffness matrix of order six, gives the following frequencies and the mode-shapes

$$\begin{aligned}\{\omega\}^T &= [9,86 \quad 62,67] \text{ sec}^{-1} \\ \{\phi_1\}^T &= [1,0 \quad 2,8816 \quad -0,4058 \quad -0,4572 \quad -0,0898 \quad -0,101146] \\ \{\phi_2\}^T &= [1,0 \quad -0,2103 \quad -0,0681 \quad 0,4437 \quad -0,01506 \quad 0,09816] \\ M_1 &= 58,8 \text{ t}, \quad M_2 = 4,29 \text{ t}\end{aligned} \quad (5)$$

Using these data, and also adopting  $\xi_1 = 0,15$ , the solution procedure will yield:  $c_1 = 1484,91$ ,  $c_2 = 202,303$ , and also  $\xi_2 = 0,052$ .

It should be noticed that the position of zero and nonzero elements in the damping matrix (2) strictly follows the displacement components in which the damping forces are actually developed. Wilson and Penzien [5] do not observe this fact since the methods proposed by them are conceptually different from the present approach. Using their method, the one based on Caughey series [9] for which a proportional damping matrix is a special case, an orthogonal damping matrix is evaluated for the illustrative purposes. For damping ratios as above,  $\xi_1 = 0,15$ ,  $\xi_2 = 0,05$ , and also using the stiffness matrix with the extended number of coordinates, for the frame with fixed joints at the base, one comes to a proportional damping matrix of the form

$$[c] = a_0[m] + a_1[k] \quad (6)$$

$$= \begin{bmatrix} 32,024 & -10,264 & 0 & -20,528 & 0 & 0 \\ -10,264 & 29,232 & 20,528 & 20,528 & 0 & 0 \\ 0 & 10,264 & 58,242 & 13,686 & -3,5 & 0 \\ -10,264 & 10,264 & 13,686 & 30,871 & 0 & -3,5 \\ 0 & 0 & -3,5 & 0 & 15,816 & 0 \\ 0 & 0 & 0 & -3,5 & 0 & 15,816 \end{bmatrix}$$

where  $a_0$  and  $a_1$  are the coefficients related to frequencies and damping ratios and  $[m]$  and  $[k]$  are the mass and stiffness matrices, respectively. For any conclusion, this matrix should be compared with the damping matrix in (2).

Lateral vibrations of one-storey frame, Fig.2, are described with four coordinates. The damping in the joints is described with the following matrix

$$[c] = \begin{bmatrix} 0 & 0 & 0 & 0 \\ 0 & c_1 & -c_1 & 0 \\ 0 & -c_1 & c_1 & 0 \\ 0 & 0 & 0 & c_2 \end{bmatrix} \quad (7)$$

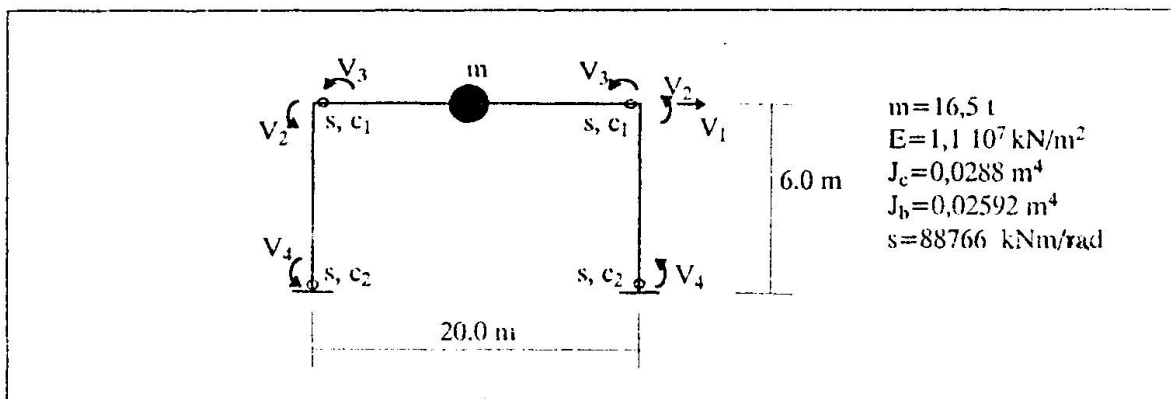


Fig. 2. Dynamic model of a one-storey laminated timber portal frame



The undamped free vibration analysis gives

$$\omega = 18,834 \text{ sec}^{-1} \quad (8)$$

$$\{\phi_1\}^T = [1,0 \quad -0,15725 \quad -0,08008 \quad -0,12066]$$

$$M = 16,5 \text{ t}$$

The term mode-shape here is unusual for a system of one dynamic degree of freedom and the confusion may arise. But it should be understood as the deformed shape in the extended number of displacement components.

The expanded form of generalized damping is

$$c_1(\phi_{21} - \phi_{31}) + c_1\phi_{31}(-\phi_{21} + \phi_{31}) + c_2\phi_{41}^2 = 2M \cdot \xi \cdot \omega \quad (9)$$

For a given damping ratio, equation (9) relates  $c_1$  and  $c_2$ . If they are equal, and for  $\xi_1 = 0,15$ , the equation results in  $c_1 = c_2 = 4544,6$ .

The same frame with fixed joints at the base is the only case with no distribution of damping since the beam to column joints take all of the dissipated energy. For the three displacement components, the damping matrix takes the form

$$[c] = \begin{bmatrix} 0 & 0 & 0 \\ 0 & c & -c \\ 0 & -c & c \end{bmatrix} \quad (10)$$

With the data from the undamped free vibration analysis

$$\omega = 28,406 \text{ sec}^{-1}$$

$$\{\phi_1\}^T = [1,0 \quad -0,20725 \quad -0,1055] \quad (11)$$

$$M = 16,5 \text{ t}$$

and with the use of the expanded form of generalized damping

$$c[\phi_{21}(\phi_{21} - \phi_{31}) + \phi_{31}(-\phi_{21} + \phi_{31})] = 2 \cdot M \cdot \xi \cdot \omega \quad (12)$$

one obtains, for  $\xi = 0,15$ , that  $c$  is equal to 13581,46.

From the damping matrices in previous examples, and also directly from a dashpot, it can be seen that the damping force developed in a particular joint is proportional to the rate of change of relative rotation in that joint. But, in a dynamic analysis, that fact would not reduce the total number of independent coordinates.

A totally different problem, which is not the subject of this paper, would be if some arbitrary distribution of damping is adopted. In that case a set of uncoupled equations of motion could be obtained by the use of damped mode-shapes. But, it requires someone to deal with the complex eigenproblem and to solve it as in Hurty and Rubinstein [10].

### 3. Conclusion

A substantial part of energy absorption occurs in the flexible connections of some structures, especially the timber structures since they belong to the group of prefabricated systems. To deal with the mechanics of different types of connections and fasteners, one should know not only their stiffness property but also their damping capacity.

The damping model of a structure adopted here observes the fact that the joints behave as the discrete dampers. Based on such model, the damping matrix with unknown damping coefficients is constructed. The uncoupling procedure results in a system of equations from which the numerical evaluation of damping coefficients is possible. The resulting damping matrix in explicit form then may be used, for example, in the dynamic response analysis of some nonlinear systems.

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## Nonlinear Seismic Analysis of a Semi-Rigid Frame Building

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### Summary

Full-scale pseudodynamic tests of a 4-story RC frame building in Ispra demonstrated semi-rigid behaviour of beam-column joints due to the slippage of the steel bars. In the paper the mathematical modelling of the structure is discussed. The results of nonlinear dynamic analyses are compared with experimental results and with the results obtained by a simplified nonlinear seismic analysis.

### 1. Introduction

A series of pseudodynamic tests on a full-scale four-story reinforced concrete building designed according to Eurocodes 8 and 2 was carried out in the European Laboratory for Structural Assessment (ELSA) of the Joint Research Centre of the European Commission in Ispra. The test results indicate "that the slippage of the steel bars in the joints dominates the behaviour of the structure leading to a very pronounced pinching of the story shear-drift diagrams, consequently reducing the potential dissipation capacity of the structure" [1]. Due to this effect, the behaviour of beam-column joints can be described as semi-rigid. In order to numerically simulate the seismic structural response of such a structure appropriate mathematical modelling is required. In the paper, a relatively simple mathematical model of the building structure is presented. The main results of nonlinear dynamic analysis are compared with experimental results and with the results obtained by a simplified nonlinear seismic analysis.

### 2. Description of the structure and of the tests

The general layout of the structure is shown in Fig. 1. The materials used for the test structure are normal-weight concrete C25/30 as specified by Eurocode 2, and B500 Temcore rebars and welded meshes. The design was made in accordance to Eurocodes 2 and 8, assuming effective peak ground acceleration 0.3 g, soil type B, importance factor 1, ductility class H and behaviour factor  $q=5$ . The detailed data on the reinforcement are given in [2]. The lumped



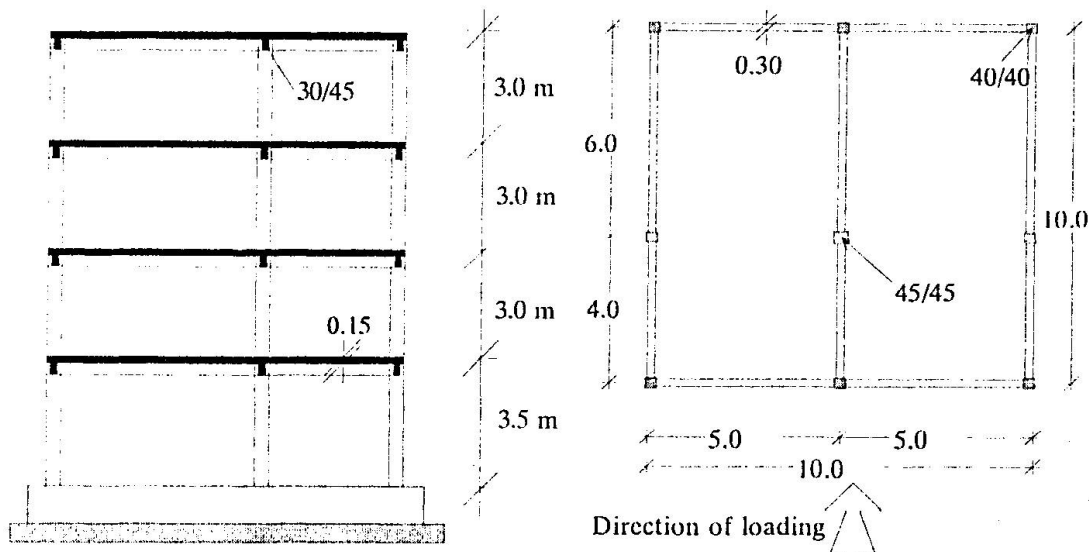


Fig. 1. Layout of the test structure

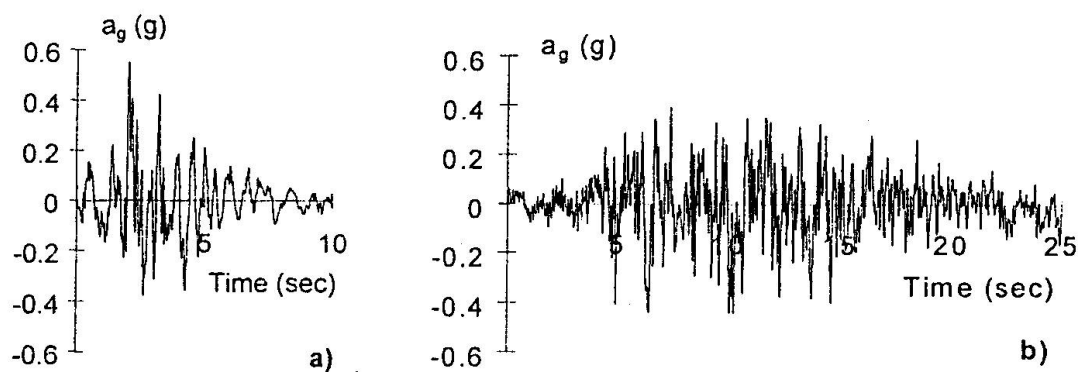


Fig. 2. Accelerograms used in the study

(a) Friuli based accelerogram used in the high-level pseudodynamic test

(b) Montenegro based accelerogram

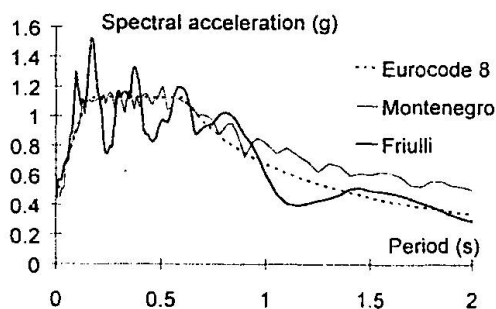


Fig. 3. Elastic spectra for 5 percent damping

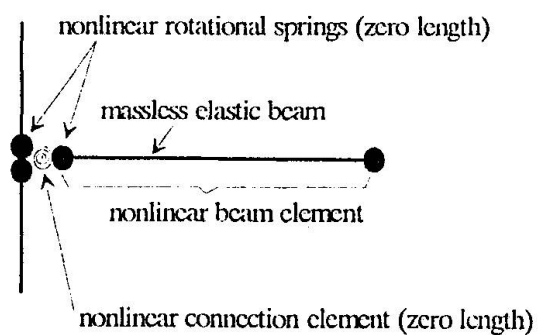


Fig. 4. Arrangement of elements

masses amount to 86.9, 85.9, 85.9 and 83 tons in stories 1 to 4, respectively. An artificial accelerogram (Fig. 2a) was generated to fit the elastic response spectrum given by Eurocode 8 for soil profile B and 5% damping (Fig. 3). The accelerogram is based on an actual record from the 1976 Friuli earthquake. The duration is 10 seconds. A low-level and a high-level pseudodynamic tests were performed. In the first test, the accelerogram was scaled to 40% of design intensity, i. e. to  $0.4 \times 0.3 = 0.12$  g nominal ground acceleration. In the second test, the design intensity was increased by a factor of 1.5. The nominal peak ground acceleration was thus 0.45 g. More detailed data on the test can be found in [2] and [3].

### 3. Mathematical modelling

All analyses were performed with a modified version of the DRAIN-2DX. A new element was implemented in the program and used for modelling of all beams and columns: a perfectly elastic, massless beam element with two nonlinear rotational springs at the two ends. The moment-rotation relationship for each spring was defined by a trilinear envelope (representing the three typical parts of the moment-rotation relationships of RC sections: up to cracking, between cracking and yielding, after yielding) and by Takeda's hysteretic rules. Asymmetric backbone curves were used for beams, the behaviour of which when subjected to positive and negative moments is different. In addition to beam elements, simple rotational connection elements, defined in the DRAIN-2DX as type 04 elements with "inelastic unloading with gap", were used to model the influence of the slippage of steel bars in the joints. They were placed between beams and joints. The rigid connection was assumed between columns and joints. The arrangement of elements is shown in Fig. 4.

The moment-rotation relationships in rotational springs representing nonlinear behaviour of beams and columns were calculated based on moment-curvature relationships and assuming the antisymmetrical moment distribution along the length of the element. The initial stiffness was calculated using experimentally obtained moduli of elasticity  $E$  (from 28.5 to 35.3 kN/mm<sup>2</sup>). The effective widths of the slab contributing to the beam of 1.5 m and 0.9 m were adopted for the internal and external frames, respectively [1]. Cracking in beams was determined using the concrete tensile strength, which amounted to 10 percent of cylindrical strength, and axial force due to dead load. In columns, the strength at cracking was reduced to one half of the computed value. In such a way the influence of varying axial force due to horizontal loading was approximately taken into account. Analytical procedures for the determination of post-yield stiffness are quite unreliable. Based on several calculations with different assumptions the following values were chosen for post-yield stiffness expressed in percentage of the secant stiffness to the yield point: columns 15%, beams 2% and 20% for positive and negative moments, respectively. It is well known that the region in which slab reinforcement yields progressively spreads with increasing beam rotation. When calculating the ultimate moment and curvature, in the case in a negative bending moment (with the slab in tension), this phenomenon can be approximately taken into account by assuming a greater effective width. As a result, a relatively high post-yield stiffness is obtained.

In the Takeda's hysteretic model the unloading stiffness is controlled by the parameter  $\alpha$ . This parameter proved, in the case of the investigated building, to exhibit a quite important effect on the time-history of the structural response. Best correlation with the experimental results was obtained by using the maximum possible value, i. e.  $\alpha = 1.0$ . As a consequence of this



assumption, the unloading stiffness after yielding is relatively small and little energy is dissipated in inelastic cycles after yielding.

The initial stiffness of connection elements was determined according to Fillippou et al [4]. The semi-rigidity of joints introduced by the connection elements has increased the resulting beam end rotations in elastic region for about 50 percent. The yield moments of connection elements were assumed to be the same as the (positive and negative) yield moments of the corresponding beams. The post-yield stiffness was arbitrarily chosen as 5 percent of the initial stiffness. The hysteretic behaviour is shown in Fig. 5.

The influence of the viscous damping is, due to the low hysteretic energy dissipation capacity, important even in the inelastic region. Only mass proportional damping was used. The best correlation with the experimental results was obtained by using one percent of critical viscous damping, determined using the initial first mode period.

The analysis was performed in sequence. First, the low-level excitation was applied. It was followed by the high level excitation. For comparison, the structural model without the connection elements (i. e. with rigid joints, R - model) was analysed. In addition to the ground acceleration time-history, used in the tests, an additional accelerogram was generated (Fig. 2b) It fits the same spectrum (Fig. 3), but it is based on the Petrovac record from the 1979 Montenegro earthquake. Its duration is 25 seconds. In addition to dynamic time-history analyses, a simplified nonlinear analysis called N2 method [5] was performed. It consists of a static analysis of the MDOF model under monotonically increasing horizontal loading (push-over analysis) and a spectral analysis of the equivalent SDOF model. The relative values of horizontal loads amounted to 0.528, 0.934, 0.926, and 1.0, from the first to the top story, respectively. In order to obtain the results, which are comparable to the results of time-history analyses, the Eurocode 8 design spectrum (normalised to 0.45 g effective peak ground acceleration) for one percent damping was used.

#### 4. Results

The most important results are shown in Figs. 6 - 13. If not stated otherwise, the results correspond to the mathematical model with connection elements (SR - model) and to the time-history analysis with the high-level Friuli based accelerogram. It can be seen that the developed mathematical model yields results which correlate quite well with the test results (with some exception of the top story). As far as time- histories are concerned, such a good correlation would not have been possible without fitting of some parameters. However, the maximum response values proved again to be relatively insensitive to moderate variations of structural properties. As an exception, the computed maximum rotations of beams in the upper part of the structure are much larger than the measured rotations.

In the case of the model without the connection elements (R - model), the displacement time-history for the low-level test, where the structural behaviour is essentially elastic, does not correlate well with the observed time-history. However, in the case of the high-level excitation, not only the maximum values but also the complete time-history agree quite well with the measured results. It seems that the time-history response of the tested structure is

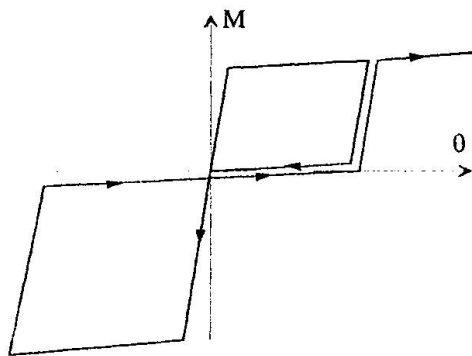


Fig. 5. Moment - rotation relationship for connection element

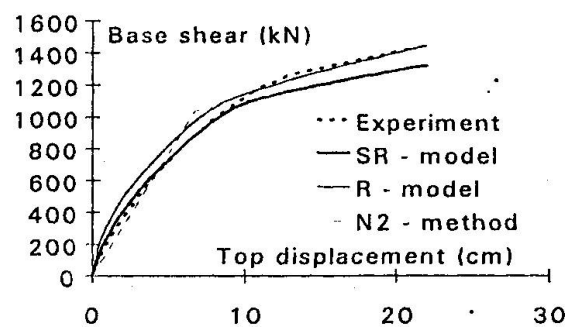


Fig. 6. Top displacement - base shear relationships

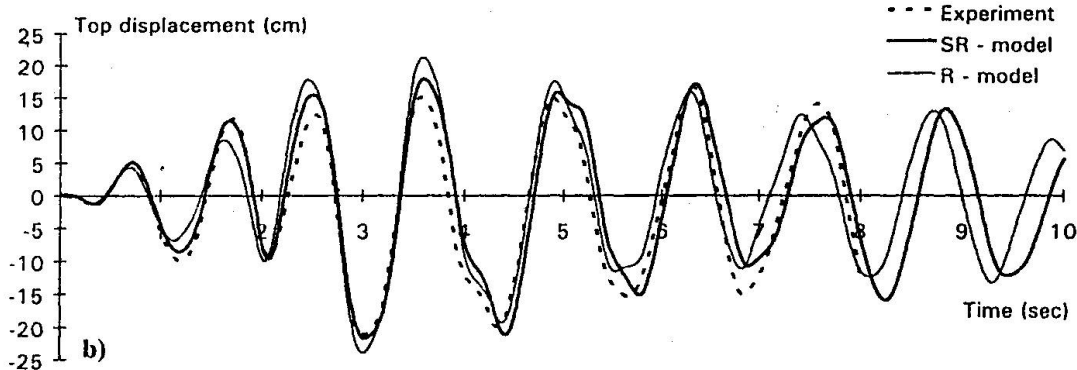
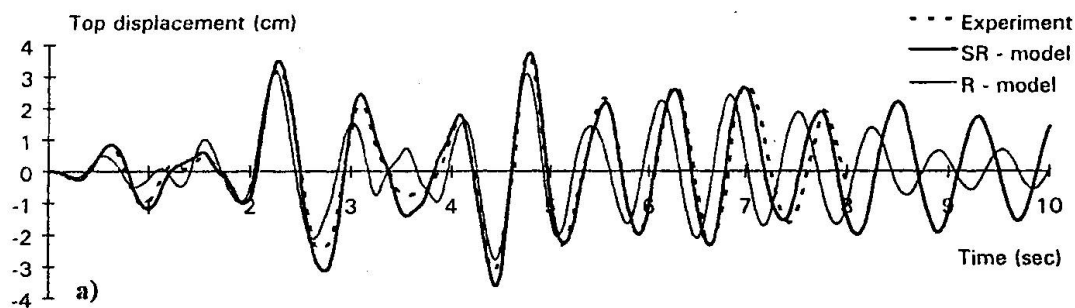


Fig. 7. Time - history of top displacement: (a) low-level excitation  
(b) high-level excitation

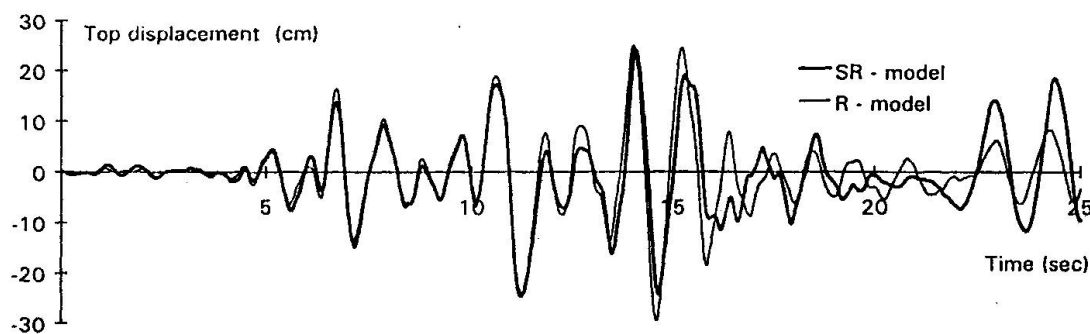


Fig. 8. Time - history of top displacement for the Montenegro based accelerogram

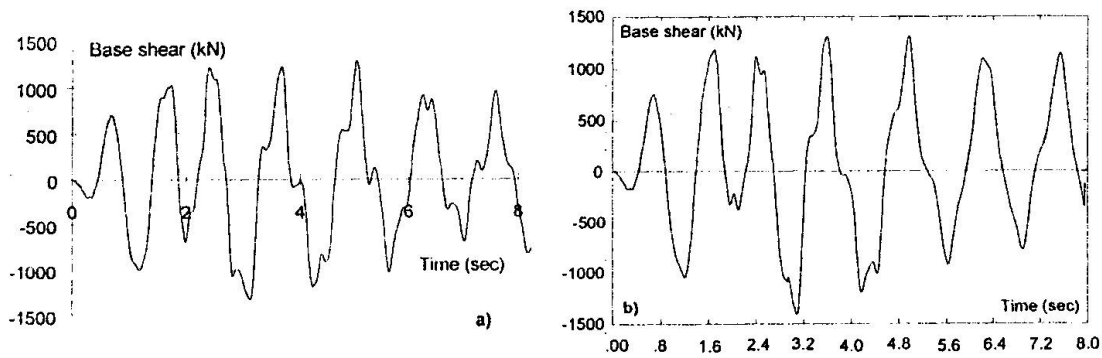


Fig. 9. Computed (a) and experimentally obtained (b) time - histories of base shear

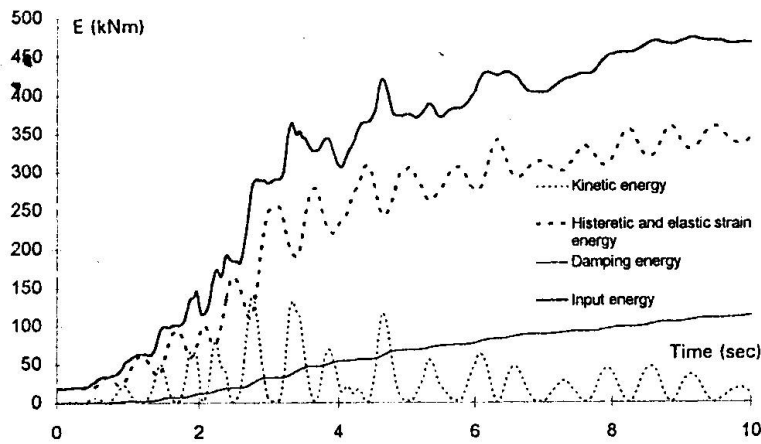


Fig. 10. Computed time-histories of energies

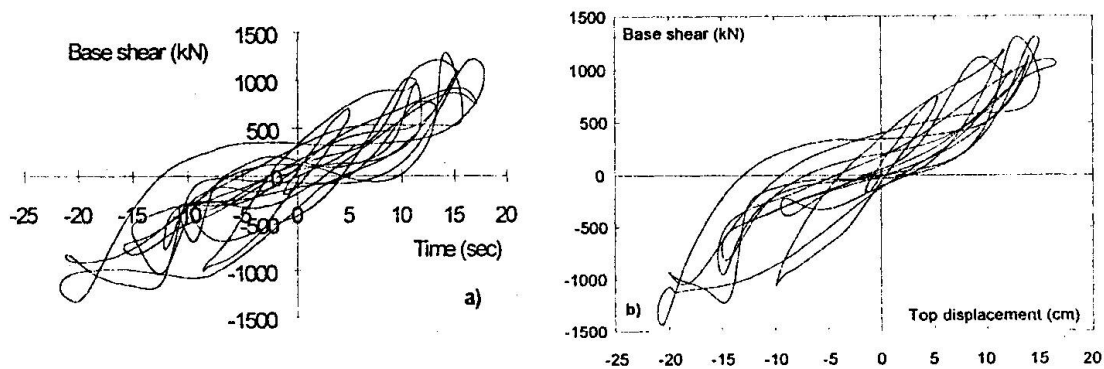


Fig. 11. Computed (a) and experimentally obtained (b) base shear - top displacement relationships

highly influenced by the low energy dissipation capacity during non-peak inelastic cycles. This feature was incorporated in both structural models (with and without connection elements) by using the unloading stiffness parameter  $\alpha = 1$ . The connection elements influence the stiffness in the elastic range and the hysteretic behaviour in the inelastic range. It seems that the second

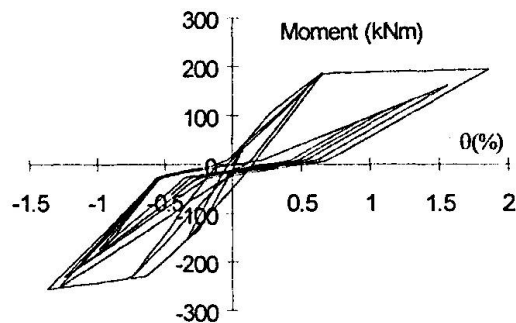
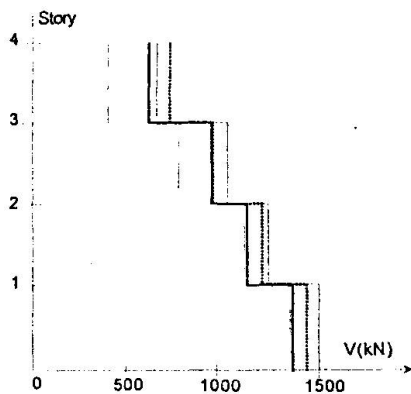
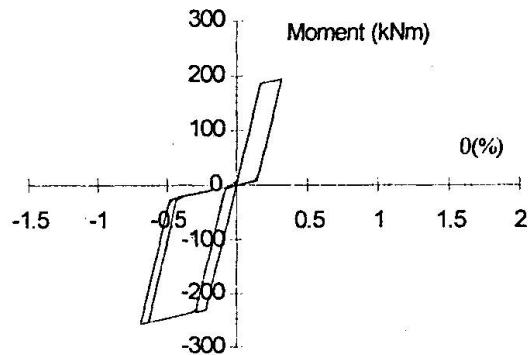
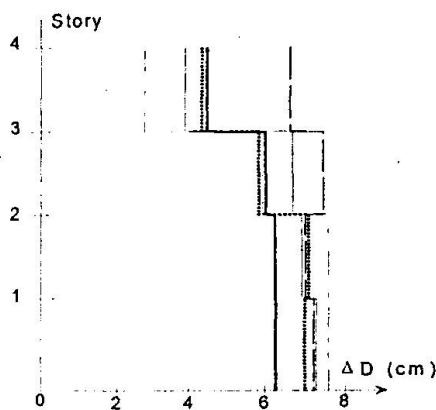
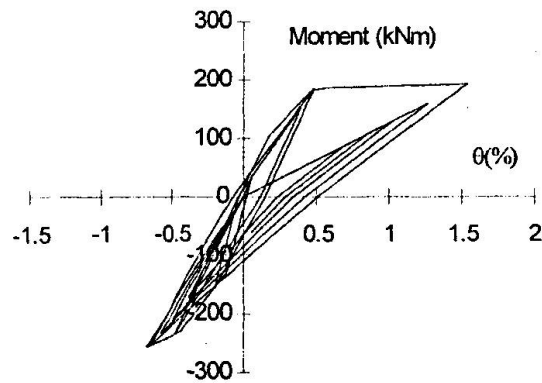
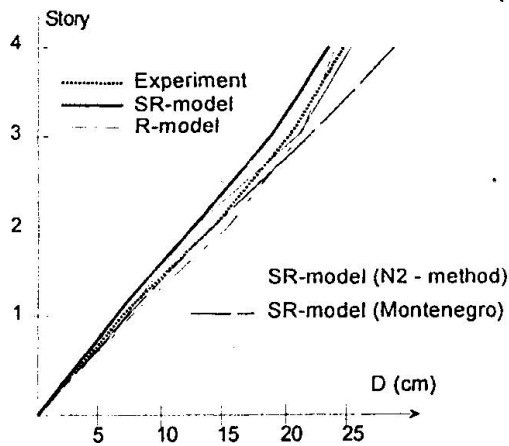


Fig. 12. Vertical distribution of maximum displacements, story drifts and shear forces

Fig. 13. Moment-rotation relationships for a typical beam, connection element, and combined beam and connection element

effect has only a minor influence on the overall structural behaviour. The major difference between the both models subjected to the high-level excitation can be observed in the computed rotation ductilities which are larger in the case of the model without connection elements. The maximum computed rotation ductility factor in the SR - model amounts to 3 and 4.4 for columns and beams, respectively. The corresponding values in the R - model amount to 3.5 and 6.2.





The structural response to the second (Montenegro based) accelerogram differs to some extent from the corresponding results for the first (Friuli based) accelerogram, although both of them are supposed to fit the Eurocode 8 design spectrum (Fig. 3). The differences in response are a consequence of only a small part of uncertainties connected with the input ground motion which characteristics can be only roughly predicted. Having in mind this fact, one can appreciate a relatively simple method, like the N2 method, which is able to produce the main results, determining seismic demand, with similar accuracy as a more sophisticated nonlinear dynamic analysis. In the case of the investigated building are a small exception underestimated response quantities in the upper story. The N2 method has been designed for analysis of structures vibrating mainly in the fundamental mode. Higher mode effects can be taken into account only if a correction procedure is applied.

## 5. Conclusions

It has been shown that the semi-rigid behaviour of a reinforced concrete frame, which is caused by the slippage of the steel bars in joints, can be adequately simulated by a combination of conventional nonlinear elements, i. e. beam elements with concentrated plasticity, and connection elements. While some fitting of parameters is necessary for obtaining close correlation of computed and measured response time-history, are the most important global response characteristics, like maximum displacements, story drifts and story shears, relatively insensitive to the details of mathematical models (having in mind the uncertainties involved in seismic design). Moreover, they can be predicted with a reasonable accuracy by a relatively simple nonlinear procedure which includes a nonlinear static analysis of the MDOF system and a spectral analysis of an equivalent SDOF system.

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