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Influence of skewness on reliability verification and safety factors

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Summary

The assumptions on the probability distribution of resistance, format of reliability verification and determination of numerical values of partial factors, adopted in Eurocodes, are discussed with regard to the influence of skewness. Error estimates of design reliability conditions and examples of determination of design resistance from tests in the cases of non-conforming skewness are shown. The results are compared with those obtained by a suggested design reliability condition involving an explicit occurrence of the coefficient of skewness of resistance.

1. Introduction

The present Eurocodes are developed as level 1 codes employing the limit state concept in conjunction with a partial factor method [1]. The not exceedance of all relevant limit states is verified comparing the design values of action effects and resistance. Adopting design models, the reliability condition is expressed in terms of the design values of actions, material properties and geometrical data given by their representative values and partial factors. The target level of reliability is achieved adjusting appropriate numerical values to partial factors. Calibration of partial factors is primarily based on comparison to historical and empirical design methods with amendments via a simplification of the first-order reliability method (FORM) [1]. Further development towards a probabilistic justification of numerical values of partial factors and more precise reliability verification format is envisaged.

The application of FORM, utilized in Eurocodes, is the common one - as a level 2 reliability method representing basic random variables and their functions by the first two moments. The representation sets a level of approximation allowing for further simplifications, among others (cf. [2]):

- Assumptions made on probability distributions lead to closed-form or simplified expressions for reliability verification.
- A convenient separation of action effects and resistance in the design reliability condition is adopted.

- Resistance is assumed in a product form with the log-normal distribution of basic random variables and thereby also of the resistance.
- A direct determination of the design resistance from the characteristic value of the product resistance, without explicit determination of design values for individual basic variables, is applied for steel structures (EC 1993) and is often used in connection with design by testing.

However, in application of FORM a more complete probability information can be used. There are good reasons for inclusion of the third moment, i.e. the coefficient of skewness, assuming three-parameter probability distributions of resistance and possibly of some basic random variables. Tichý [3] pointed out that neglect of the third moment may cause considerable errors in determination of the probability of failure. In many practical cases neither basic variables nor resistance itself possess values of the coefficient of skewness approximately equal to three times the coefficient of variation, which is characteristic for the log-normal distribution adopted for resistance in Eurocodes [1]. Long term investigations show that the statistical distributions of strength of higher strength steels and concretes tend to negative skewnesses [4]. This is important for checking the resistance of a compact cross-section which is dominated by the material property. Negative skewnesses were also found on studying strength functions modelling column buckling [5] and post-buckling of plates [6], mainly due to the type of probability distribution of initial deflection.

For utilization of the information on skewness in codification a simple separated form of reliability verification with an explicit occurence of the coefficient of skewness, at least in the fundamental case of reliability margin, is a necessary preliminary. From the by Tichý [3] suggested invariant first-order third-moment method there does not appear to issue a simple (formal) separation of parameters in reliability condition. Recently, for the fundamental case of safety margin the problem has been successfuly treated by Mrázik [7] or in [8] by a FORM-based asymptotic analysis. Let us note, that neither Tichý's method [3], nor Mrázik's approach are FORM oriented. Obviously, the resistance side of reliability condition, while implemented into the procedure for determination of design resistance from tests, directly influences numerical values of partial factors.

A question arises about the determination of the coefficient of skewness of resistance. Since large samples are needed to assess its value, prior knowledge from investigations of model strength functions have to be gained, if necessary. A suitable tool for identification of the model resistance by moments offer an application of the solution of inverse reliability problem [9], based on the first-order reliability index. The procedure was checked against the results obtained by the simple Monte Carlo simulation [9] and non-trivial cases were already treated, cf. [6].

In this contribution, the format of reliability verification is discussed. Especially, error estimates for a design reliability condition adopted in Eurocodes [1] and the one suggested in [8] are shown. In the cases of skewness non-conforming with the assumption of Eurocodes, examples of determination of design resistance from tests as well as the corresponding numerical values of partial factors are presented.

2. Reliability verification

2.1 Design reliability conditions

Consider a reliability problem given by the fundamental case of safety margin

$$Z = R - S \tag{1}$$

where R denotes resistance and S action effects. For normally distributed R and S, FORM procedure coincides with the well known closed-form solution yielding the design reliability condition

$$\mu_S - \alpha_S\beta_t\sigma_S \le \mu_R - \alpha_R\beta_t\sigma_R \tag{2}$$

where

$$\alpha_R = \frac{\sigma_R/\sigma_S}{\sqrt{1+(\sigma_R/\sigma_S)^2}}\ , \qquad \alpha_S = -\frac{1}{\sqrt{1+(\sigma_R/\sigma_S)^2}} \tag{3}$$

are called the FORM weight factors or sensitivity factors. The preset target value of the reliability index β_t is related to failure probability by

$$P_f = \Phi(-\beta_t) \tag{4}$$

where Φ is the standardized normal distribution function. μ, σ, v, a denote the mean value, standard deviation and coefficients of variation and skewness of a random variable or function indicated in subscript position. Assigning to the weight factors suitable constant values a convenient separation of action effects and resistance is achieved. The empirically-based values

$$\alpha_R = 0{,}8\ , \qquad \alpha_S = -0{,}7 \tag{5}$$

recommended in [2] imply

$$\mu_S + 0{,}7\beta_t\sigma_S \le \mu_R - 0{,}8\beta_t\sigma_R \tag{6}$$

Under the assumption of the log-normal distributions of R and S, another closed-form solution to the reliability problem can be obtained ,cf. [2]. Assigning again to the weight factors the values (5) and assuming that the coefficients of variation of R and S are small a counterpart to the design reliability condition (6) can be found as

$$\mu_S \exp(0{,}7\beta_t v_S) \le \mu_R \exp(-0{,}8\beta_t v_R) \tag{7}$$

In Eurocodes a combination of design values of action effects and resistance, obtained for different assumptions on probability distributions, in reliability verification is admitted [1]. Thus, for self weight usually taken with normal distribution and log-normal resistance, the design reliability condition may read, cf. (6), (7)

$$\mu_S + 0{,}7\beta_t\sigma_S \le \mu_R \exp(-0{,}8\beta_t v_R) \tag{8}$$

In order to gain an insight about the influence of the coefficient of skewness of resistance a_R upon the reliability verification, the case of S normal and R three-parameter log-normal was studied [8]. By an asymptotic FORM-based analysis a design reliability condition with an explicit occurrence of a_R was suggested [8]:

$$\mu_S + 0{,}7\beta_t\sigma_S \le \mu_R - (0{,}8 - 0{,}3a_R)\beta_t\sigma_R \tag{9}$$

2.2 Error estimates

On designing a structural element, the actual reliability measure β_c may differ from the target one. Let us check the design reliability conditions (8) and (9) in an idealized situation. Following [8] we assume that the design is economical, i.e. the equality in the reliability condition is reached, and further, that the, say actual, probability distributions of actions effects and resistance are normal and three-parameter log-normal, respectively. The differences β_c-β_t then issue from:

- Non-conformity of the assumed probability distributions with those used in the derivation of design reliability condition.
- Adopted simplifications.

The value of $\beta_c = |\,\Phi^{-1}(P_f)\,|$ is obtained by the solution of the reliability problem

$$Z = R - S \ge 0 \tag{10}$$

with presumed actual distributions of R,S adjusted to the parameters issuing from the considered economical design. The probability of failure P_f is found by importance sampling technique with sample size n=50.000. For an illustrative presentation of the calculated β_c-β_t, a suitable parametrization of the reliability problem (10) and design reliability conditions under consideration are performed.

Following [8], R and S are standardized to $\tilde{R},\tilde{S}$ and (10) is rearranged to

$$Z^{st} = \beta + \frac{\tilde{R}}{[1+\frac{1}{(\sigma_R/\sigma_S)^2}]^{1/2}} - \frac{\tilde{S}}{[1+(\sigma_R/\sigma_S)^2]^{1/2}} \ge 0 \tag{11}$$

where

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{12}$$

Then it can be shown that for sampling of $\tilde{R},\tilde{S}$ and evaluation of (11) altogether three parameters β, σ_R/σ_S and a_R are needed [8].

The assumed equality in the design reliability condition sets a relationship between the parameters. Thus, in the case (9) we readily find

$$\beta = \beta_t \frac{0{,}7 + (0{,}8 - 0{,}3a_R)\sigma_R / \sigma_S}{\sqrt{1 + (\sigma_R / \sigma_S)^2}} \qquad (13)$$

Treatment of (8) is not so straightforward. Subsequently we divide (8) by σ_S, introduce the coefficients of variation v_R, v_S and eliminate v_S employing the equality sign. Then we express β by v_R, v_S, σ_R/σ_S and substitute for v_S the obtained expression, which finally yields

$$\beta = \frac{1}{\sqrt{1 + (\sigma_R / \sigma_S)^2}} \{0{,}7\beta_t + \frac{\sigma_R / \sigma_S}{v_R}[1 - \exp(-0{,}8\beta_t v_R)]\} \qquad (14)$$

We see that in this case, besides of σ_R/σ_S, a_R, β_t, moreover the coefficient of variation v_R have to be considered as a parameter.

The error estimates β_c-β_t are calculated for σ_R/σ_S varying from 0,1 to 1,0 ; a_R= 0,5 , 0,25 , 0 , -0,25 , -0,5 ; β_t = 3,8 and v_R= 0,05 , 0,11 , 0,17 . The value of β_t is in Eurocode 1 [1] introduced as reliability level "appropriate for most cases". The choice of v_R is taken after Annex Z of ENV 1993-1-1, where the aforementioned values are attributed, according to test observations, to limit states of excessive yielding or gross deformations, local buckling and overall instability, respectively.

The results of checking the design reliability condition (8) for v_R=0,05 , 0,11 , 0,17 are shown in Figs.1,2,3. We see that with increasing v_R the curves fall deeper in the unsafe side, but the non-uniformity of approximation is smaller.

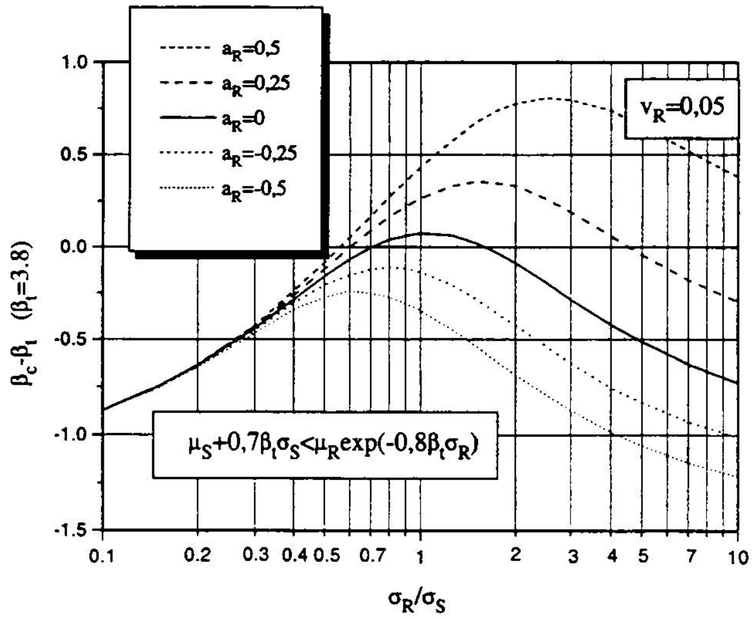

Fig. 1. Error estimates of the reliability verification according to Eurocode 1 - condition (8), β_t =3,8 , v_R=0,05.

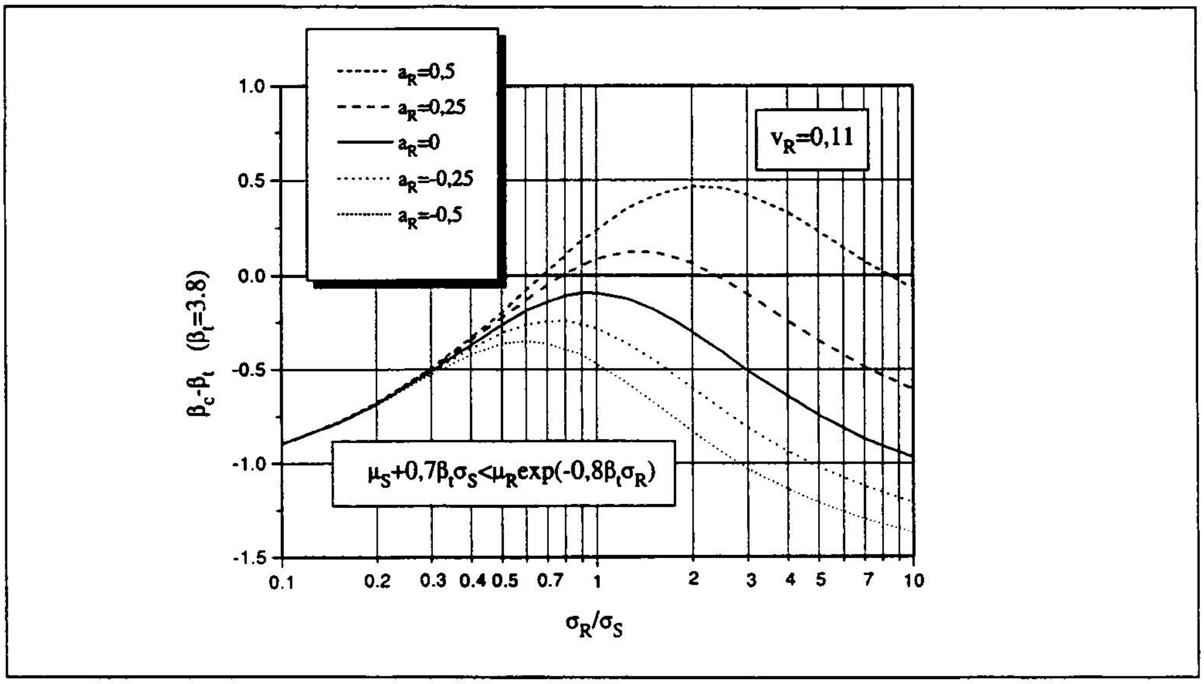

Fig. 2. Error estimates of the reliability verification according to Eurocode 1 - condition (8), β_t =3,8 , v_R=0,11.

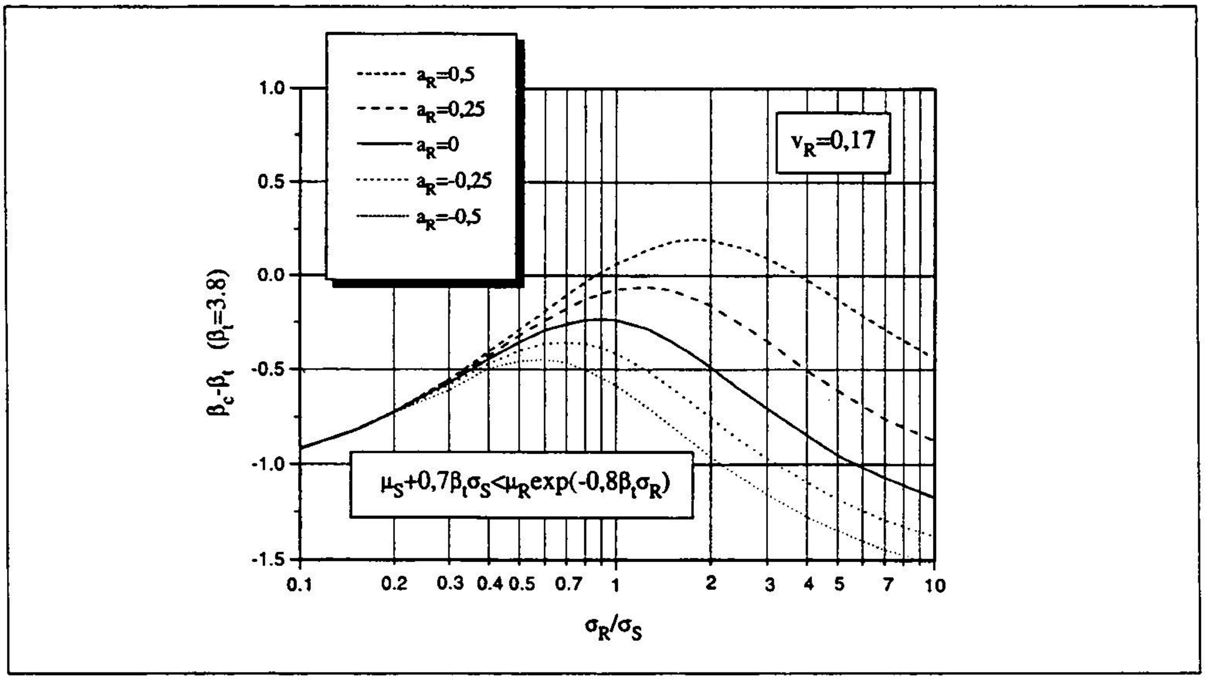

Fig. 3. Error estimates of the reliability verification according to Eurocode 1 - condition (8), β_t =3,8 , v_R=0,17.

Fig. 4 shows the corresponding results for the in [8] suggested design reliability condition (9). The error estimates of (9) were in [8] calculated by an explicit formula issuing from an asymptotic approximation of β_c.

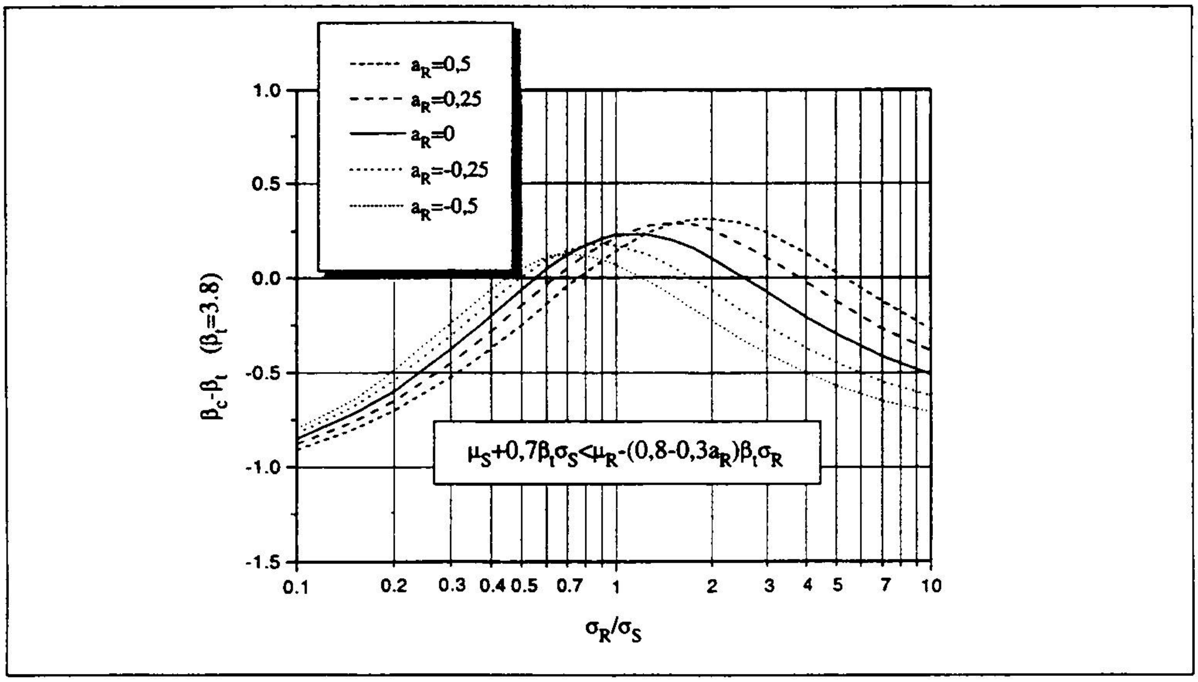

Fig. 4. Error estimates of the suggested reliability verification - condition (9), β_t =3,8.

3. Partial factors

In Eurocodes basic variables are introduced by their representative values usually defined as:

- characteristic values with a prescribed or intended probability of beeing exceeded
- nominal values

The design values are introduced indirectly, by the representative values and a set of partial factors and load combination factors.

One of the aforementioned simplifications admits a direct determination of the design resistance by testing expressing it by the characteristic value r_k - the 5% fractile of a product resistance and partial factor γ_R as

$$r_d = r_k / \gamma_R \tag{15}$$

From the viewpoint of practical utilization, it is preferable to relate the design value of resistance to the value r_n of strength function obtained for nominal values of parameters. Then the partial factor γ_R^* is defined

$$\gamma_R^* = \frac{r_n}{r_d} \tag{16}$$

On studying the numerical values of partial factors, we resort again to idealized situation. The procedure for determination of design resistance from tests is applied to a strength function identical to a basic random variable considering a perfect correlation between the design model and experiments. In this connection we may immagine about the study of the yield strength of steel specimen. The statistical characteristics are supposed to be evaluated by an almost infinite number of tests and thereby the statistical uncertainty can be neglected. S is assumed with normal probability distribution and R implied by tests as three-parameter log-normal. Then the right-hand sides of the considered design reliability conditions (8) or (9) represent the design resistance and the assumed probability distributions imply the characteristic values of resistance, thus yielding γ_R by (15).

For the reliability verification (8) with the log-normal distribution of resistance according to Eurocode 1 the corresponding partial factor denoted γ_R^{EC} is

$$\gamma_R^{EC} = \frac{\mu_R \exp(-1{,}645 v_R)}{\mu_R \exp(-0{,}8\beta_t v_R)} = \exp((0{,}8\beta_t - 1{,}645)v_R) \qquad (17)$$

Thus, for given β_t, it depends only on v_R. Some numerical values of γ_R^{EC} are for β_t =3,8 shown in Table 1.

γ_R^{EC} - (17)			
v_R =	0,05	0,11	0,17
γ_R^{EC} =	1,072	1,150	1,268

Table 1. Partial factor γ_R^{EC} according to Eurocode 1 (8).

Considering the suggested condition (9), the normal distribution $N(\mu_R,\sigma_R)$ can be attributed to the resistance. The related partial factor denoted γ_R^{aR} is

$$\gamma_R^{aR} = \frac{\mu_R - 1{,}645\sigma_R}{\mu_R - (0{,}8 - 0{,}3a_R)\beta_t\sigma_R} = \frac{1 - 1{,}645 v_R}{1 - (0{,}8 - 0{,}3a_R)\beta_t v_R} \qquad (18)$$

Naturally, in addition the coefficient of skewness a_R has appeared. Examples of evaluations of γ_R^{aR} are for β_t =3,8 shown in Table 2. We see that unusually high values of partial factors were obtained for v_R=0,17 and small and negative skewnesses. Due to different r_k values, we do not intend to compare the partial factors γ_R^{EC} and γ_R^{aR}.

γ_R^{aR} - (18)					
a_R =	0,5	0,25	0	-0,25	-0,5
v_R = 0,05	1,047	1,064	1,082	1,101	1,120
= 0,11	1,100	1,153	1,200	1,252	1,308
= 0,17	1,242	1,355	1,491	1,657	1,865

Table 2. Partial factor γ_R^{aR} (18) corresponding to the suggested reliability verification (9).

A meaningful comparison offer the values of partial factor γ_R^* (16) determined in correspondance with (8) or (9) (distinguished by superscript EC and aR, respectively). Obviously, the ratio

$$\frac{\gamma_R^{*EC}}{\gamma_R^{*aR}} = \frac{1-(0{,}8-0{,}3a_R)\beta_t v_R}{\exp(-0{,}8\beta_t v_R)} \tag{19}$$

equates to the reversed ratio of design resistances, thus, estimating a relative exploitation of a structural element when designed according to (8) or (9). Numerical results are shown in Table 3.

$\gamma_R^{*EC} / \gamma_R^{*aR}$ - (19)					
$a_R =$	0,5	0,25	0	-0,25	-0,5
$v_R = 0{,}05$	1,020	1,004	0,987	0,971	0,954
$= 0{,}11$	1,018	0,974	0,930	0,886	0,842
$= 0{,}17$	0,973	0,891	0,810	0,726	0,648

Table 3. Ratio (19) *of* γ_R^* *values calculated according to Eurocode 1* (8), *and the suggested reliability verification* (9).

4. Conclusions

The influence of skewness of resistance upon reliability verification and partial factors has been studied.

- The error estimates of reliability verification (8) according to Eurocode 1, expressed in terms of the difference between the actual and target reliability indices, show a high non-uniformity of approximation with respect to the skewness, Figs. 1,2,3.
- Checking of the suggested design reliability condition (9), with an explicit occurrence of the coefficient of skewness, shows that the scatter can be diminished to the level obtained for the case of normally distributed action effects and resistance, Fig.4, cf.[8].

The procedure for the determination of design resistance from tests has been applied in an idealized situation, employing the original assumption of the log-normal distribution of resistance adopted in Eurocode 1 [1] and the suggested normal distribution expressing the influence of skewness.

- An assumption on probability distribution to some extent predetermines the partial factors, Tables 1,2.
- The conjunction of extremely high coefficient of variation with small and negative skewnesses leads to high - unrealistically appearing partial factors, Table 2.
- The results presented in Table 3 show that generally the approach of Eurocode 1 may lead to optimistic assessments - smaller values of partial factors of resistance.

As stated in Eurocode 1, p.65 [1] "the same level of formal reliability can be obtained in many different ways". Thereby any improvement should be considered within an overall safety format of a code. Let us mention some points of possible further development in the spirit of this contribution:

- combinations of various types of probability distribution in reliability verification involving the influence of skewness
- prior knowledge of the coefficient of skewness for classes of structural elements obtained from realistic models by e.g. the approach of [9]
- implementation of the prior knowledge of statistical characteristics and assumed probability distributions into the procedure for the determination of design resistance from tests.

To cope succesfully with the outlined problems a broader cooperation on the topic is necessary.

References

[1] Eurocode 1: *Basis of design and actions on structures, Part 1: Basis of design*, Brussels, CEN, 1994.

[2] P. THOFT-CHRISTENSEN and M.J. BAKER, *Structural ReliabilityTheory and Its Applications*, Springer-Verlag, Berlin, Heidelberg, New York, 1982.

[3] M. TICHÝ, *First-order third-moment reliability method*, Struct. Safety, 16(1994), 4, 189-200.

[4] A. MRÁZIK, M. KRIZMA and Z. SADOVSKÝ, *Triparametric distribution and design or characteristic values*, in: ICOSSAR'93, Innsbruck 9-13 August 1993, Eds. Schueller, G. I., Shinozuka, M.and Yao, J. T. P., Vol.2 (1994), 1309-1315.

[5] Z. SADOVSKÝ and D. PÁLEŠ, *Buckling Resistance of Axially Compressed Members and European Standards Unification*, in: The IX-th Int. Conf. Metal Structures,Kraków, Poland 26-30 June 1995, Vol.II, 283-292.

[6] Z. SADOVSKÝ, *Stochastic postbuckling strength of compressed plates*, in: Int. Colloq. Stability of Steel Structures, Budapest, 21-23.9.1995, Vol.II, 313-320.

[7] A. MRÁZIK, *Separation of random variables in reliability condition with asymmetric distribution of probability*, (reviewed).

[8] Z. SADOVSKÝ, *Asymptotic analysis of safety margin with normal action effects and log-normal resistance*, (submitted to Structural Safety).

[9] Z. SADOVSKÝ and D. PÁLEŠ, *Efficient solution of inverse reliability problem*, (reviewed).