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Upper bound for combination of action effects

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Summary

The square-wave model of random actions with the Ferry-Borges & Castanheta combination rule is sufficiently exact but too difficult for practical design. The Turkstra rule is simpler but it gives lower bound estimates of action effects A new combination rule is also simple and it gives safe estimates. Combination values of nondominant actions depend on their repetition numbers relative to a specified reference period. The characteristic value of dominant action will be changed if a design working life of the structure is different from the reference period

1. Introduction

1.1 Random variations and time variations

Both permanent loads G and variable actions Q are random. It means that they are variable in population of construction works:

- of similar destination if occupancy loads are concerned,
- in the same climatic zones for wind action, air temperature and insolation, snow or icing.

Characteristic values G_k, Q_k are enhanced by means of load factors γ_G and γ_Q for applications in partial factor design. The load factors cover uncertainties due to random variations of the permanent and variable actions.

Moreover the variable actions Q are variant in time. Combination factors ψ_o reduce characteristic values of simultaneous actions, except the dominant one, because their maxima will not probably occur in the same while. The characteristic values may be also reduced or enhanced if a design period is different from the reference period of the maximal variable actions .

The combination of design action effects S_d is always more than the design value of action effect $\gamma_S S_k$ thanks to geometric summation of the standard deviations according to rules of the first-order second-moment probabilistic theory:

$$S_d = \sum_{j=1}^{m} c_j \gamma_G G_{j.k} + \sum_{i=1}^{n} c_i \gamma_Q \psi_{oi} Q_{i.k} = \sum_{j=1}^{m} c_j \overline{G}_j + \sum_{i=1}^{n} c_i \overline{Q}_i^* + \beta_S (\sum_{j=1}^{m} c_j \sigma_j + \sum_{i=1}^{n} c_i \sigma_i) \quad ; \qquad (1)$$

$$\gamma_S S_k = \sum_{j=1}^{m} c_j \overline{G}_j + \sum_{i=1}^{n} c_i \overline{Q}_i^* + \beta_S \sqrt{\sum_{j=1}^{m} c_j^2 \sigma_j^2 + \sum_{i=1}^{n} c_j^2 \sigma_i^2} \quad ; \qquad (2)$$

where $\overline{G}_j = G_{j.k}$ - mean and characteristic values of permanent loads,
$\overline{Q}_i^*$, $\psi_{oi} Q_{i.k}$ - combination values of variable actions,
σ_j σ_i - standard deviations for j=1, 2, ... m and i = 1, 2,... n,
β_S - a specified load index.

Some authors and codemakers mistake a reduction of S_d to the level $\gamma_S S_k$ with application of combination factors ψ_o. Perhaps additional reduction factors could be introduced to the linear combination of design values (1) in order to make the result S_d of partial factor design closer to the result of probabilistic design $\gamma_S S_k$ (2). Such a reduction factor ξ_j is foreseen for permanent actions only by the draft international standard of ISO: (*DIS2394,* 7.5.1). In addition another ξ factor could be defined for combination values of variable actions or a global ξ for both kinds of actions.The combination factors ψ for variable actions are better not to be amalgamated with ξ factors. The actual value of the global ξ would depend on the number $m+n$ of actions G_j and Q_i as well as proportions among them. The maximum value of the ξ factor occurs when only one action (either permanent or variable) is applied and $\xi = 1$. The minimum will occur when the moments of all $m+n$ particular action effects are equal

$$\xi = \frac{1+\beta_S v_S}{1+\beta_S v_S \sqrt{m+n}} \qquad (3)$$

where $v_S = \sigma_j/\overline{G}_j = \sigma_i/\overline{Q}_i =$ const - coefficients of variation for $j = 1, 2 \ldots m$, $i = 1, 2 \ldots n$.

Further considerations will be limited to combination factors ψ_o applied to ultimate limite states of structures in persistent and transient situations.The subscript o will be omitted.

1.2 Pre-standardization of combination factors

International committee about bases for design of structures ISO/TC98 created in 1989 a working group on combination of actions SC2/WG5. This was preceded by a state-of-art report about load combination rules in codified design in ISO member countries (*Mathieu & Murzewski,* 1988). The report has shown that the rules are so different and heterogeneous that their harmonization is not possible.The load combination model of *Ferry-Borges & Castanheta* (1971) was recommended by the Committee as the basis for new unified rules. A special issue of International Journal *"Structural Safety"* devoted to load combinations was edited and combination models and applications have been developed by *Kanda, Murzewski, Nowak, Östlund, Shiraki, Wen* etc.(1993). During years 1989-94 seven drafts of new combination rules were discussed and the last one was submitted as Annex F to the final draft of revised international standard DIS2394: *"General principles on reliability for structures"* (1995). The Annex F after four modifications is a compilation of texts of drafts elaborated by the Working Group, the former edition of the IS2394 and informative documents to Eurocode 1: *"Basis of design and actions on structures"* (1993). The ISO draft standard will be refered further on as *DIS2394* with numbers of paragraphs of the main text or annexes. Similarly the the Eurocode 1. Part 1 will be refered as *EC1-1.*

Both Ferry-Borges & Castanheta model and the Turkstra rule are based on consideration of variations of actions in time. The Ferry-Borges & Castanheta model requires to calculate 2^{n-i} combination cases for each structural element. The *Turkstra* model takes only n cases into account. Combination factors ψ of the Eurocode 1 are associated rather with the *Turkstra* rule. The combination factors ψ of the Eurocode are specific for each variable action and they do not depend on other actions of the combination. It is not so supposed by the draft international standard (*DIS 2394*, F-3.1) .The ISO principles are as follows:

- "One action is chosen as the dominating action and is introduced by means of its characteristic value Q_{1k} .
- A second action is introduced with a reduced combination value $\psi_2 Q_2$, $\psi_2 \leq 1$,
  The combination factor ψ_2 depends on the characteristics of both the dominating action Q_1 and the nondominating action.
- A third action is introduced with a further reduced combination value $\psi_3 Q_3$, $\psi_3 < \psi_2$.
  The value of ψ_3 depends of all three actions. This process is repeated if necessary."

Involving 3 or more actions in one combination factor ψ seems to be too sophisticated. Perhaps 2 actions are sufficient as Ferry-Borges and Castanheta have assumed in their considerations but a practical combination rule should be still simpler as the Turkstra rule is.The problem will be discussed here for linear combinations of action effects. Reduction factors ψ_o for simultaneous actions will be analyzed for persistent and transient loading situations at the ultimate limit states of construction works. The subscript "*o*" will be omitted.

2. Characteristics of variable actions

2.1 Stochastic process of actions

Two moments Q, σ_Q^2 of probability distribution should not be identified with "mean" $Q(t)$ and "variance" $\sigma_Q^2(t)$ determined during an observation time t for one selected construction work. The two moments will be equal one to another if the stochastic process of action is stationary and ergodic. An action process will be stationary if anticipated usage and environmental conditions do not change during the working life period (*Fig.1*). Much more difficult is to prove that the action process is ergodic. If it is even so, the random action $Q(t)$ has to be defined more precisely:

- If maximal values max $Q(t)$ are measured during a total observation period t_0, the mean max $\bar{Q}(t)$ always decreases with increasing t_0 and the variance $\sigma^2_{\max Q}(t)$ can be constant only for "stable" (in reference to maxima) short-term probability distributions of actions Q^*
- If original short-term values $Q^*=Q(t^*)$ are averaged in unit observation periods t^* (e.g. 10 minutes for wind velocities) its variance $\sigma_Q{}^*$ decreases with t^* according to an asymptotic formula $\sim\theta/t^*$ for $t^* \to \infty$ where θ is specific scale of fluctuation.
- If a random action is intermittent, the moments of its probability distribution are different for two cases: when only positive values are measured and when all values are measured. But if two exclusive actions occur periodically one after another, they may be characterized together as a continuous action.

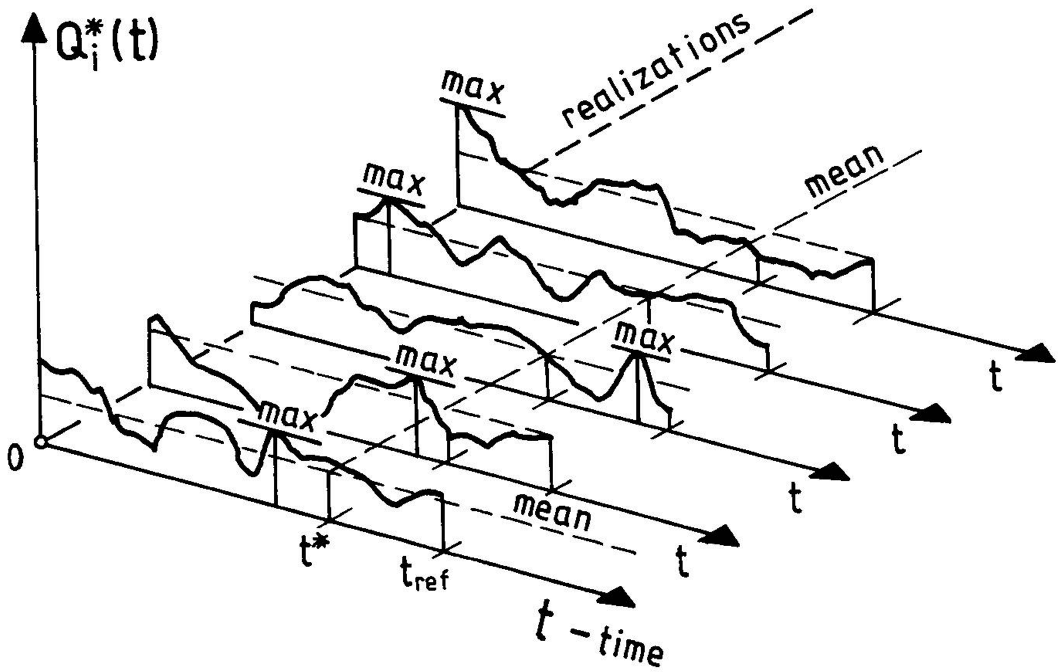

Fig. 1 Realizations of a continuous stationary and ergodic stochastic process

Characteristic values of maximal actions Q_k will be comparable if a constant reference period t_{ref} is selected for any kind of variable action and any country. A design period t_d is not necessarily equal to the reference period t_{ref}. The design period is identified with intended working life specified for construction works (*EC1-1*, Table 2.1; *DIS2394*, Table 2.1) which are classified as:

- temporary for 1-5 years,
- short life for 25 years,
- ordinary for 50 years,
- long life for 100 years.

Now the reference period t_{ref} is determined by codemakers of particular load standards. It is 50 years for wind action (*EC2-4*), the same for snow (*EC2-3*) although 1 year only is recommended by the Eurocode (*EC1-1*, 4.2.8). The reference period t_{ref}=50 years is better because:

- it is equal to the design period t_d for ordinary buildings and it is equal or close to conventional characteristic values of national standard specifications,
- asymptotic distribution functions of extreme values can be taken for 50 or more years with a much better accuracy than it would follow from the relation

$$F(Q \mid t_d) = [F^*(Q \mid t_o)]^r \tag{4}$$

where $F^*(Q \mid t_o)$ - the CDF of short term (e.g. one-year or "point-in-time") random variables
$r = t_d/t_o$ - repetition number of the short-term values during the design period t_d.

There are objections relative to equation (4). It requires that the extreme values Q^* be independent in not always well defined unit observation intervals t_0 and it happens that:

- the occupancy loads and other actions are autocorrelated for time intervals which may be longer than the short term periods t_0,
- There are many distribution functions F^* proposed for particular short-term actions and statistical tests do not give precise solutions (*Sedlacek*, 1992).

The situation is different in the case of extreme values which happen in a longer time period e.g. t_{ref}= 50 years. There are 3 types and only 3 asymptotic distributions of extreme values: the Gumbel (I), the Fréchet (II) and the Weibull (III). No empirical tests are necessary to verify this theorem of *R.A.Fisher and L.H.Tippett* (from *Gumbel*, 1954). The central parameter $\hat{Q}$ of any extreme value distribution has been called characteristic value in mathematical statistics. The characteristic maximum $\hat{Q}$ will be equal to the codified characteristic value Q_k (*EC1-1*, 1.5.3.14) if the prescribed probability of not been exceeded is exactly e^{-1}=0,368...
The probability that it will be exceeded once and only once during t_{ref} is the same . The upcrossing events are rare and the *Poisson* law may be applied. So the characteristic value $\hat{Q}$ will be exceeded on average once during the reference period of the Poisson sequence of events.

2.2 The Gumbel probability distribution of extreme actions

Preference should be given to the type I distribution for maximal actions during the reference period

$$F(Q) = \exp(-\exp\frac{\hat{Q}-Q}{u}) \tag{5}$$

where Q - characteristic maximum in the sense of mathematical statistics,
u - the Gumbel deviation - a parameter characterizing dispersion..

- The characteristic maximum $\hat{Q}$ will be equal to the mode $\tilde{Q}$, i.e. the most probable value during the reference period, for the Gumbel probability distribution,

$$f Q) = dF(Q)/dQ = \max \rightarrow df(Q)/dQ=0 \rightarrow Q=\tilde{Q}=\hat{Q} \rightarrow F(\tilde{Q}) = e^{-1}. \tag{6}$$

- The characteristic maximum Q_t of the Gumbel distribution may be predicted for a period t longer than 50 years so that only the model maximum increases (*Fig*.2)

$$\tilde{Q}_t = \tilde{Q} + u\ \ln(t/50), \qquad u_t = u = \text{const.} \tag{7}$$

- The first and second moments of the Gumbel probability distribution are related to its parameters in a simple way:

$$\bar{Q} = \tilde{Q} + u\,C, \qquad \sigma^2 = u^2\,\pi^2/6 \qquad \text{with C=0,5772... the Euler number.} \tag{8}$$

The normal coefficient of variation v and the Gumbel one υ are related as follows

$$v = (\upsilon\,\pi/\sqrt{6})/(1 + C\,\upsilon) = \upsilon/(0,780 + 0,450\,\upsilon)\ . \tag{9}$$

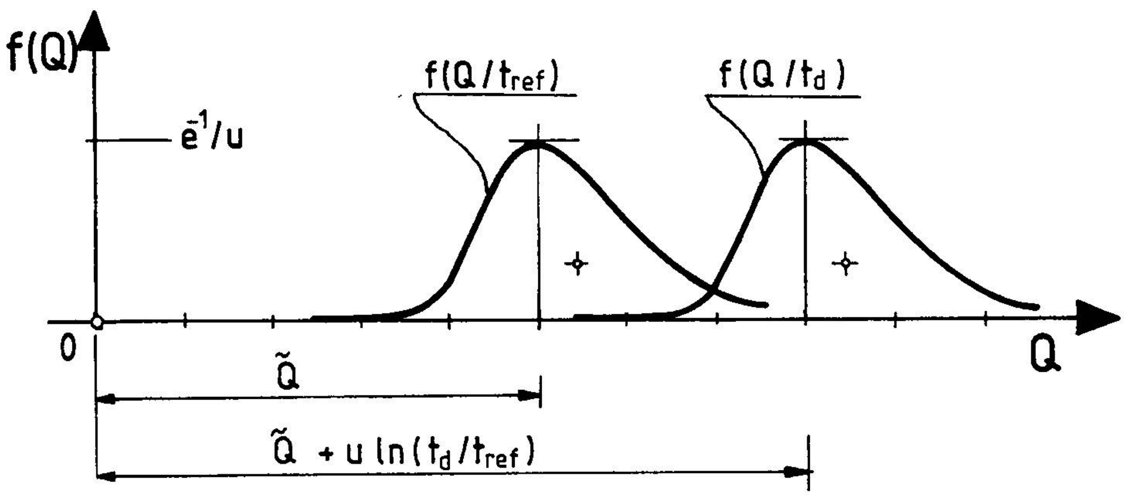

Fig.2. Modal values Q of extreme actions for the reference and design periods

If t<50 years, equation (8) is not necessarily exact. Short-term probability functions $F^*(Q)$can be quite different than their asymptotic distribution. A concept which enables to simplify the load model is to define a basic time interval θ and relative repetition number $r=t_{ref}/\theta$ so that the characteristic values be equal when estimated in two ways

$$\tilde{Q} - u \ln r = Q^* \rightarrow r = \exp\frac{\tilde{Q}-\hat{Q}^*}{u} \tag{10}$$

where $\hat{Q}^* = F^{*-1}(e^{-1})$ - inverse function to the CDF of short-term action from equation (4).

Thanks to the concept of basic time interval θ no extensive statistical investigations are necessary for probability functions of actions during 5-years, 1-year etc. Only the characteristic value $\hat{Q}^*$ is needed.

3. Combination rules for variable actions

3.1 Square-wave model of actions

It is assumed that random values of the same variable action Q_i are independent in any two basic time intervals θ_i, θ_j. That is the essential feature of the square-wave model of random action process. The equations (4), (5), (6), (7), (8), (9) will be actual if the Gumbel probability distribution is accepted for the variable actions and their combinations. Explanations and applications will be easier with this assumption however Ferry-Borges & Castanheta and Turkstra have considered their combination rules in more general formats.

Special numbering order of variable actions is important. Actions Q_1, Q_2, Q_3... Q_n are ordered in sequence of their repetition numbers $r_1 < r_2 < r_3 < ... \; r_n$. according to the Ferry-Borges & Castanheta rule. There are other numbering rules, e.g. an action which gives the highest effect has number 1 and so on according to permutation rule recommended by some national standards, e.g. the Polish standard *PN-82/B-02000*. The numbering order is not important for applications of the Turkstra rule.

One variable action Q_c, c=1, 2, 3,... , is taken as dominant for each combination case. Its characteristic value will not be reduced (i.e. ψ_c=1) unless the design period t_d is different from the reference period t_{ref}. But nondominant actions Q_i are reduced with combination factors ψ_i<1, $i \neq c$, and they do not depend on the design period t_d. They depend on either the reference period t_{ref} or a basic interval θ_j of another variable action Q_j not necessarily the preceding one. The international draft standard does not give exact advice for this point.

There is no difference between the Ferry-Borges & Castanheta, the Turkstra and the new combination rule in the case of two variable actions only. The differences can be shown when at least three simultaneous variable actions Q_1, Q_2, Q_3 occur.

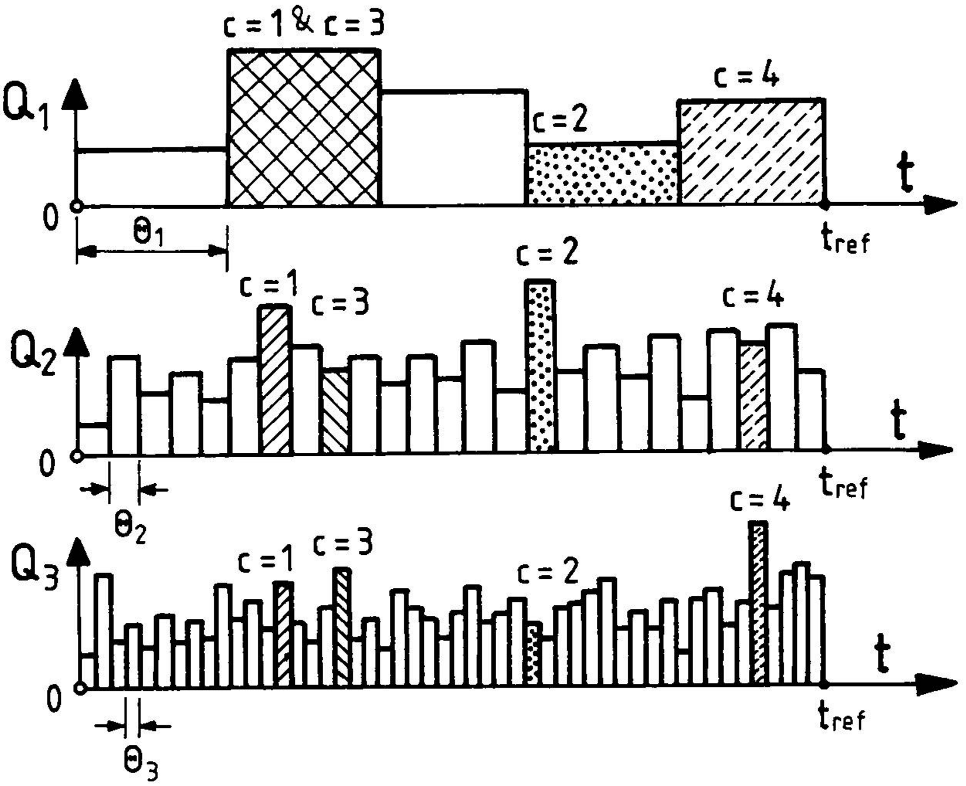

Fig.3. Three variable actions with different basic time intervals

3.2 The Ferry-Borges & Castanheta combination rule

The combnation rule is such that after the dominant action has been chosen, another variable action is selected, not necessarily the next as a sub-dominant one. It is selected from actions with shorter basic intervals. Then again a sub-sub-dominant action may be selected etc. if there are more variable actions in the combination. An extension (*Murzewski*, 1983) of the original Ferry-Borges & Castanheta combination model consists in numbering not only actions: i=1, 2, 3,... n but also their combinations: c = 1, 2, 3,... 2^{n-1} in such a way that periodic order of the combinations is revealed. A current number m=1, 2, 3, ... helps to indicate the column where dominant action can be found from the matrix of combination factors $[\psi_{ic}]$

$$
\begin{aligned}
&\psi_{ic} = 1 + u_i \ln(t_d/t_{ref}) && \text{for } c = 2^i\,(m\text{-}1/2) \\
&\psi_{ic} = 1 - u_i \ln(t_{ref}/\theta_i) && \text{for } c < 2^i\,(m\text{-}1/2) \\
&\psi_{ic} = 1 - u_i \ln(\theta_j/\theta_i) && \text{for } c > 2^i\,(m\text{-}1/2) \text{ and } j>i \\
&\psi_{ic} = 1 - u_i \ln(t_{ref}/\theta_i) && \text{for } c < 2^i\,(m\text{-}1/2) \text{ and } j<i
\end{aligned}
\tag{11}
$$

where $\upsilon_i = u_i/\tilde{Q}_i$ - the Gumbel coefficient of variation.

There are 2^{n-1} combinations to check for each structural element in the case of the Ferry-Borges & Castanheta rule. It is perhaps too many for practical design. However still more combinations (if $n>2$) are required to be checked for each structural element, namely $n!$, in the case of the permutation rule. But only n combinations are necessary with the Turkstra rule.

3.3 The Turkstra combination rule

The concept of Turkstra is that all nondominant actions are taken in their instantaneous values. If the square-wave model (*Fig.3*) and the Gumbel probability distribution are assumed, the values $\psi_{ic}Q_i$, $i \neq c$, are determined for their basic time intervals θ_i.
The combination factors ψ_{ic} are as follows for dominant and nondominant actions:

$$
\begin{aligned}
&\psi_{ic} = 1 + \upsilon_i \ln(t_d/t_{ref}) && \text{for } c = i \\
&\psi_{ic} = 1 - \upsilon_i \ln(t_{ref}/\theta_i) && \text{for } c \neq i
\end{aligned}
\tag{12}
$$

The Turkstra combination factors ψ_{ic} for some nondominant actions are lower than corresponding factors according to the Ferry-Borges & Castanheta rule.
Thus the Turkstra rule will underestimate the action effects.

3.4 New combination rule

A new rule for combination of actions provides also only n different combinations of actions as the Turkstra rule does but it gives safe upper bound estimates of action effects. The concept of the new combination rule is such that maxima of nondominant actions, $\psi_{ic}\tilde{Q}_i$ for $i \neq c$ are determined during the basic interval θ_c of dominant action if this time is longer than the basic interval θ_i of the action Q_i,

$$
\begin{aligned}
&\psi_{ic} = 1 + \upsilon_i \ln(t_d/t_{ref}) && \text{for } c=i\,, \\
&\psi_{ic} = 1 - \upsilon_i \ln(t_{ref}/\theta_i) && \text{for } c>i\,, \\
&\psi_{ic} = 1 - \upsilon_i \ln(\theta_j/\theta_i) && \text{for } c<i\,.
\end{aligned}
\tag{13}
$$

The new combination factors for some nondominant actions are higher than corresponding factors according to the Ferry-Borges and Castanheta rule. That is why it gives always a safe upper bound of the load effect.

3.5 Numerical example

Combination factors ψ_{ic} are calculated and shown in Tables 1, 2, 3 for three variable actions: Q_1 - occupancy load, Q_2 - snow in winter or temperature increase in summer, Q_3 -wind. Snow and elevated temperature are exclusive events with durations of no more than half a year that is why they are taken as one variable action with two variants. It is a new concept how to treat intermittent actions with long periods of absence.

The Gumbel coefficients of variation of the actions are equal: $\upsilon_1=\upsilon_2=\upsilon_3=0{,}160$;
they correspond to the normal coefficients of variation (9): $v_1=v_2=v_3=0{,}160\ \pi/\sqrt{6}=0{,}188$;
The coefficients are equal because there are equal load factors: $\gamma_S=$ 1,50 (*EC1-1*, Table 9.2).
If also the load index is accepted (*EC1-1*, Table A.2 and A3.2) $\beta_S=0{,}7{\cdot}3{,}8=2{,}66$,
the value $v=0{,}188$ agrees with the Eurocode load factor: $\gamma_S=1+2{,}66\cdot0{,}188=1{,}50$.

The design period is equal to the reference period: $t_d=t_{ref}=$ 50 years
:
and the basic intervals of the variable actions are :

$$\begin{cases} \theta_1= 5 \text{ years for occupancy load,} \\ \theta_2= 1 \text{ year for snow/temperature,} \\ \theta_3= 1 \text{ week for wind.} \end{cases}$$

The new ψ_i values are more likely than ψ_1=0,7 and $\psi_2=\psi_3$=0,6 which would follow from the Turkstra and the Eurocode combination factors (*EC1-1*, Table 9.3): θ_1=2,32 and $\theta_2=\theta_3$=0,83.

i \ *c*	1	2	3	4
1	1	0,775	1	0,775
2	0,843	1	0,618	0,618
3	0,614	0,614	0,544	1

Table 1. Combination factor matrix according to Ferry- Borges & Castanheta

i \ *c*	1	2	3
1	1	0,775	0,775
2	0,618	1	0,618
3	0,235	0,235	1

Table 2. Combination factor matrix according to Turkstra

i \ *c*	1	2	3
1	1	0,775	0,775
2	0,843	1	0,618
3	0,544	0,614	1

Table 3. Combination factor matrix according to the new rule

4. Conclusions

4.1 One reference period t_{ref} for all variable actions and a well defined characteric value Q_k are necessary to make reasonable comparison, unification or differentiation of numerical values. The value t_{ref}= 50 years should be mentioned as a standard in Eurocode 1. It is better than t_{ref}= 1 year for reasons explained in sub-chapter 2.1.

4.2 The codified characteristic value Q_k should be equal to the characteristic extreme value Q in the reference period t_{ref} as it is defined in mathematical statistics: a fractile with intended probability of not been exceeded: e^{-1} = 0,3678... instead of the recommended value 0,98 (*EC1-1*, 4.2.8). So defined characteristic value $Q_k = Q$ may be easily changed if the design period t_d differs from the reference period t_{ref} .

$$\psi_d Q_k = [1 + \upsilon \ln(t_d / t_{ref})] \tilde{Q} \tag{14}$$

Equations (8) and (9) relate the modal value $Q_k = \tilde{Q}$ and the Gumbel coefficient of variation $\upsilon = u/\tilde{Q}$ with the normal parameters: $\bar{Q}$ and v

4.3 A value $\gamma_Q \psi_d Q_k$ may be introduced to ultimate limit states design with the load factor γ_Q.

$$\gamma_Q = 1 + (C + \beta_S \pi/\sqrt{6})\, \upsilon \qquad \text{with } C = 0{,}5772... \tag{15}$$

The product $\gamma_Q \psi_d$ gives a little different value than the exact design value Q_d according to probabilistic theory

$$Q_d = Q_k \{1 + [C + \beta_S \pi/\sqrt{6} + \ln(t_d / t_{ref})\, \upsilon]\} \tag{16}$$

4.4 The new combination rule (13) gives safe estimates for combination values of variable actions. They are upper bounds for the Ferry-Borges & Castanheta combination values. The new combination rule requires n trials to evaluate the maximum action effect for each structural element, so many as the Turkstra rule does but less than 2^{n-1} according to the Ferry-Borges & Castanheta. The exemplary combination factors ψ_{ic} (*Table* 3) have been determined for likely basic intervals θ.

4.5 A joint effect of independent permanent and variable actions is reduced thanks to geometrical summation of standard variations. No general rule can be found how to take advantage of that in partial factor design except perhaps a simple rule given for the case of a permanent load combined with one variable load (*Murzewski*, 1993). No reduction factor is used in the design (like ξ from *DIS2394*, 7.5.1) i.e. the upper bound value ξ= 1 is used.

4.6 Uncoupled reliability-based format may solve the above problem and simplify the design. Separate load and resistance indices β_S ,β_R can be calibrated in two ways:

- conventional way (*EC1-1*, A-3) such that constant split indices β_S β_R are specified for each safety class of construction works with the same propoportion β_S / β_R = const. The joint reliability index β may be variable for each design case,

$$\beta = a_S \beta_S + \alpha_R \beta_R \tag{17}$$

The sensitivity factors α_S, α_R depend on proportions of standard deviations σ_S/σ_R or coefficients of variation v_S/v_R ;

- optimal way such that the β_S and β_R values depend on the safety class and the coefficients of variation v_S and v_R. of the action effect or resistance, respectively. The separate indices β_S and β_R may be derived from minimum failure probability taken as the objective function of the optimization procedure (*Murzewski*, 1989, 1994,1995b).

The commonly known approach to probabilistic design (*Rshanitsin*, 1978; *Madsen, Krenk & Lind*, 1986; *Thoft-Christensen & Murotsu*, 1986) is based on maximum failure frequency as the objective function. The split indices β_S, β_R and design values S_d R_d are coupled in result of such calibration method, i.e. β_S depends on v_S and v_R and vice-versa - β_R depends on both v_S and v_R .

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