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Non-Linear Analysis and Safety Evaluation for Concrete Structures

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Summary

This paper describes the principal features of a reliability based nonlinear finite element method for reinforced concrete beams under static loads. The combination of the theory of structural reliability and the finite element method provides an efficient and comprehensive tool for assessing structural safety. Using the example of a simple statically indeterminate beam, different models of normative safety concepts for nonlinear system analysis are investigated considering the reliability index β for SLS and ULS criteria.

1 Introduction

Modern design-code formulations for civil engineering works are based on reliability methods as described in EC 1 [2]. The main reason for this is the objectivity of an homogeneous level of safety which should be reached on average. This refers to a general probabilistic approach, but it is well known that there are practical difficulties in using probabilistic methods for design. However, it is a useful task to verify or derive adequate safety elements from probabilistic calculations.

A general nonlinear analysis for concrete structures holds for most realistic results under all load levels. Therefore EC 2 [3] allows the application of nonlinear methods, although an adequate safety concept is still discussed within experts. As proposed in EC 2, the simultaneous calculation of the structure with mean values of the material properties for system analysis and design values of the material for the cross-section design leads to unacceptable inconsistencies. Using this background, in the following a reliability based nonlinear finite element method for reinforced concrete beams under static loads will be presented. Some results for SLS and ULS limit state conditions will be outlined by observing the reliability index β.

2 Safety Analysis for Reinforced Concrete Structures

2.1 First Order Reliability Method

The *first order reliability method* (FORM) is an approximate method to calculate failure probabilities for general, non-linear and normal or non-normal distributed problems. The response of a structure is entirely defined by the outcome of a vector $\mathbf{X} = (X_1, X_2, \ldots, X_n)^T$ of basic random variables which may include parameters defining actions, material properties, member sizes etc..

In order to calculate failure probabilities one has to formulate a limit state function g which depends on a set of statistical variables $\mathbf{x} = (x_1, x_2, \ldots, x_n)^T$:

$$g(\mathbf{x}) \geq 0. \tag{1}$$

A limit state function has only two states, a *safe* state and a *failure* state. If $g(\mathbf{x}) \geq 0$, the considered design requirement is fulfilled (safe region), if $g(\mathbf{x}) < 0$, the considered design requirement is failed [1] (failure region). The design condition may be written as $g(\mathbf{x}^\star) = 0$ where $\mathbf{x}^\star$ contains the design values for the particular problem.

The probability of failure P_f is given by the n-fold integral over the failure region of the limit state function in the space of basic variables $\mathbf{X}$, where each point $\mathbf{x}$ is assigned to a joint probabilistic density function $f_{\mathbf{X}}(x_1, x_2, \ldots, x_n) = f_{\mathbf{X}}(\mathbf{x})$.

$$P_f = \int\limits_{\{\mathbf{x}|g(\mathbf{x})<0\}} f_{\mathbf{X}}(\mathbf{x})\,\mathrm{d}\mathbf{x} \tag{2}$$

The evaluation of the integral in Eq. 2 is often impossible because $f_{\mathbf{X}}(\mathbf{x})$ may not be known and the direct evaluation of the integral for general limit-state functions and large n is very extensive. Therefore, the *first order reliability method* provides a consistent and invariant method for deriving the design point $\mathbf{x}^\star$ in the majority of practical design tasks.

The first order reliability method takes advantage of the properties of the so called standard normal space $\mathbf{Y}$. Using the transformation $\mathbf{y} = T(\mathbf{x})$, the limit-state function is then given by $g(\mathbf{x}) = g(T(\mathbf{y})) = h(\mathbf{y})$ and Eq. 2 is rewritten to

$$P_f = \int\limits_{h(\mathbf{y})<0} (2\pi)^{-n/2} \exp\left(-\frac{1}{2}\mathbf{y}^{\mathrm{T}}\mathbf{y}\right) \mathrm{d}\mathbf{y}. \tag{3}$$

with n = the number of random variables. The solution of FORM is schematically drawn in Fig. 1. The limit-state surface will be replaced by its tangent hyper-plane $l(\mathbf{y}) = \nabla h(\mathbf{y}^\star)(\mathbf{y} - \mathbf{y}^\star)$ in the design point $\mathbf{y}^\star$ with $h(\mathbf{y}^\star) = 0$ The design point is characterized by the highest likelihood (maximum of the joint probability density) among all points in the failure set. The distance β between the origin and the design point is

[1] The term " failure" refers to either inadequate safety or serviceability of the structure.

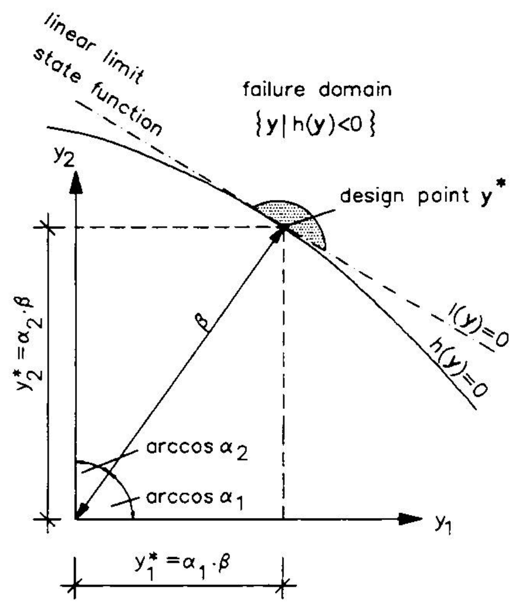

Figure 1: Failure domain and its linear approximation in the standard normal space

a measure of safety because the relation between the failure probability P_f and the so called *reliability index* β is defined by

$$P_f = \Phi(-\beta) \Rightarrow \beta = -\Phi^{-1}(P_f) \tag{4}$$

where Φ is the standard normal probability density function.

Target failure probabilities are derived by a process of probabilistic calibration to different existing design codes. The failure probabilities should be applicable to a wide range of structural components and provide a reliable and satisfactory performance. Indicative values for the target reliability index β are given in Eurocode 1 [2]. For different safety requirements (intended life time or safety-class I to III), the safety index β is given for the Ultimate-Limit-State (ULS) and the Serviceability-Limit-State[2] (SLS) as shown in Tab. 1. The bold values refer to the formulation of EC 1.

	Safety Class		
	I	II	III
SLS	2.5	**3.0**	3.5
ULS	4.2	**4.7**	5.2

Table 1: Indicative values for the reliability index β (one year)

[2]The values for SLS-conditions are valid, if the limit state function does not contain an inherent safety parameter

2.2 Limit State Functions for Reinforced Concrete Structures

The application of the structural reliability concepts to reinforced concrete structures needs a formulation of the limit state functions. Here, they are given in accordance to the regulation of EC 2 [3] with respect to serviceability and ultimate limit states.

2.2.1 Serviceability Limit State

Steel stresses, which could lead to inelastic deformation of the steel shall be avoided as this will lead to large, permanently open, cracks. So, the limitation of steel stresses under service accounts for adequate durability. Stresses are limited to

$$\sigma_s \leq 0.8 f_{yk} \qquad (\ 1.0 f_{yk} \text{ for imposed deformations}), \tag{5}$$

where f_{yk} is the characteristic yield strength of the steel. σ_s is calculated by assuming a cracked cross-section, if the concrete tensile strength f_{ct} has been exceeded.

Cracking shall be limited to a level that will not impair the proper functioning of the structure or cause its appearance to be unacceptable. An explicit limitation of the crack width may be checked by the following limit state function:

$$w_m \leq w_{lim} \tag{6}$$

where

w_m = mean design crack width, which will be calculated by using the following equation:

$$w_m = \left(50 + 0.25 \cdot k_1 \cdot k_2 \frac{d_s}{\rho_r}\right) \varepsilon_{sm} \quad [\text{mm}]$$

The mean strain ε_{sm} of reinforcement is evaluated by taking tension stiffening effects, shrinkage etc. into account. d_s denotes the average steel diameter, k_1 takes account of the influence of the bond properties, k_2 takes account of the influence of the form of the strain distribution and ρ_r is the effective reinforcement ratio (see EC 2, 4.4 [3]).

w_{lim} = limit of crack width, which will usually be chosen in accordance to exposure classes. For the sake of simplicity, it will here generally be set to 0.3 mm.

2.2.2 Ultimate Limit State

The ultimate bending capacity M_R resp. the rotation capacity Θ_R within critical regions is mainly defined by the ultimate compressive and tensile strength of concrete and reinforcement. Fig. 2 shows the values of ε_{cu} and ε_{su}, which will be assumed as deterministic here. The limit state function for the ultimate bending capacity M_R of a particular system cross-section is given by

$$M_R(\mathbf{x}_{cross\text{-}section}, N_S) \geq M_S(\mathbf{x}). \tag{7}$$

$M_R(\mathbf{x}_{cross\text{-}section}, N_S)$ is the ultimate bending moment, depending on all variables of the cross-section ($\mathbf{x}_{cross\text{-}section}$) and the acting longitudinal force N_S. $M_S(\mathbf{x})$ characterizes the acting bending moment.

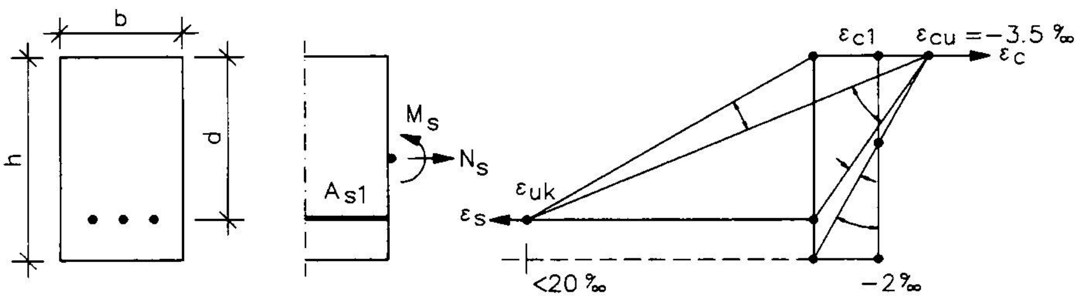

Figure 2: Strain limits for the ULS

2.3 Material Properties

2.3.1 Concrete

The main parameters are the modulus of elasticity E_c, the compressive strength f_c, the compressive strain ε_{c1} at the peak stress and the ultimate compressive strain ε_{cu}. For nonlinear structural analysis, the stress-strain diagram for concrete subjected to uniaxial compression may be written by Eq. 8 and 9. Obviously, it expresses a modification of the parabolic-rectangular stress-strain diagram by adapting the nonlinear branch to the modulus of elasticity E_c.

$$0 \geq \varepsilon_c \geq \varepsilon_{c1} \qquad \sigma_c = -f_c \cdot \left[1 - \left(1 - \frac{\varepsilon_c}{\varepsilon_{c1}}\right)^n\right] \tag{8}$$

$$\varepsilon_{c1} > \varepsilon_c \geq \varepsilon_{cu} \qquad \sigma_c = -f_c \tag{9}$$

By choosing the parameter n in Eq. 8 in accordance to Eq. 10,

$$n = -\frac{\varepsilon_{c1} \cdot E_c}{f_c} \tag{10}$$

one can adapt the stress-strain relation very simple to different situations. The parameter ε_{c1} will here be set to -0.002, ε_{cu} to -0.0035.

The tensile strength of concrete has main influence on the system behavior, e.g. when determining deflections, crack widths or the effective stiffness of a structure. In order to get realistic results, the tension stiffening effects have to be taken into account, especially when performing nonlinear analysis under SLS loading conditions. For the sake of simplicity the stress-strain diagram for concrete in tension may be taken as linear until the tensile strength f_{ct} is reached. In the majority of practical applications within a finite element code, the material behavior of reinforced concrete after exceeding the tensile stress is modeled by a hyperbolic stress-strain relation. See [6] for details.

2.3.2 Reinforcing Steel

The main parameters for reinforcing steel are the tensile strength f_t, the yield stress f_y, the elongation at maximum load ε_u and the modulus of elasticity E_s. The widely used stress-strain diagram which is given by Eq. 11 accounts for these parameters.

$$\begin{aligned} \varepsilon_s &= \frac{\sigma_s}{E_s} + 0.002 \cdot \left(\frac{\sigma_s}{f_y}\right)^m \qquad (11) \\ \text{with } m &= \frac{\ln(\varepsilon_{su}/0.002)}{\ln(f_t/f_y)} \quad \text{and} \quad \varepsilon_{su} = \varepsilon_u - \varepsilon_y. \end{aligned}$$

The exponent m provides an easy way to adapt the relation to different curve characteristics.

2.4 Proposal for Probabilistic Models

Essentially according to proposals made by the *Joint Committee on Structural Safety* (JCSS) [4] and to the recommendation of CEB [5] the statistical characteristics of the governing random variables are taken as listed in Tab. 2. Direct random model

Variable	Type	μ	σ	V[%]	x_k	Frak.
f_c [MPa] $f_{ct} = 0.25 \cdot f_c^{2/3}$ $E_c = 9500 \cdot f_c^{1/3}$	LN	$f_{ck} + 8$	5		f_{ck}	5
f_y [MPa]	LN	$f_{yk} + 60$	30		f_{yk}	5
A_s [cm^2]	N	${}_{nom}A_s$		2.5	${}_{nom}A_s$	50
h, b [mm]	N	${}_{nom}b$	5		${}_{nom}b$	50
c [mm]	N	${}_{nom}c + 5$	5		${}_{nom}c$	50
G [kN/m^2]	N	${}_{nom}G$		10	${}_{nom}G$	50
Q [kN/m^2]	N	$\frac{Q_k}{1.824}$		40	Q_k	98

Table 2: Probability distribution parameters for random variables

uncertainty parameters are generally not applied here.

2.5 Finite-Element-Reliability-Method

In the field of structural engineering, limit state functions or failure criteria are usually not formulated in terms of the basic variables themselves. They are expressed in terms of a response quantity or action effect **S** like stresses, crack widths, deformations etc., that are derived from the basic variables. This derivation $\mathbf{S} = \mathbf{S}(\mathbf{x})$, which is called *mechanical transformation*, is available only in an implicit form, such as the finite element method. This is the principal reason for employing the finite element concept in structural reliability analysis.

The used 3-node/9-degrees of freedom finite beam element for geometrically and physically nonlinear reinforced concrete structures is shown in Fig. 3. Basically the simple beam theory with static loads will be applied. The longitudinal and vertical displacements u and w are interpolated with conventional interpolation functions. This element with its material subroutines as described before is implemented in a standard

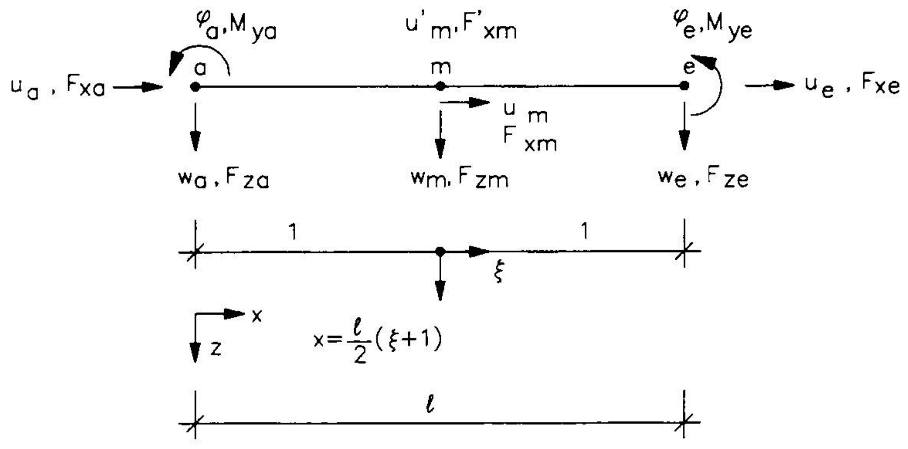

Figure 3: 3-node/9-degrees of freedom beam element

finite element code that was expanded to probabilistic approaches by including the first order reliability theory (FORM). The corresponding flow-chart is drawn in Fig. 4. The conventional solution of a nonlinear structural problem in a finite element displacement problem can be expressed through the equilibrium equation

$$\mathbf{R}(\mathbf{v}) = \mathbf{F}. \tag{12}$$

Thus equilibrium is achieved when the internal resistance forces $\mathbf{R}$, that depend on the displacements $\mathbf{v}$, balance the externally applied nodal forces $\mathbf{F}$. The solution of Eq. 12 may here be calculated by a *Newton-Raphson* iteration scheme with its known advantages. To evaluate the design point, an optimization procedure as known from Rackwitz-Fießler is used.

The interface between the two codes, namely the reliability analysis and the nonlinear finite element analysis has to account for the spatial variability of the in general correlated random variables $\mathbf{x}$.

3 Cross-Section Reliability Analysis

The main influence parameters for the reliability of concrete structures may be analysed by a cross-section reliability analysis. In the following some results on this topic will be discussed.

Considering a rectangular cross-section, the simple limit state function

$$g(f_c, f_y) = M_R(f_c, f_y) - M_S \tag{13}$$

with only two basic variables (concrete and steel strength), will be investigated in order to get the results of FORM graphically. M_R describes the ultimate resistance moment, M_S the ultimate acting moment which is determined in accordance to EC 2. For the sake of simplicity, M_S is chosen here deterministically. Fig. 5 shows the limit state surface of Eq. 13 for different reinforcement ratios ρ in the standard normal space $\mathbf{Y}$. The boldly drawn arrows in Fig. 5 are equivalent to the reliability index β as the shortest distance

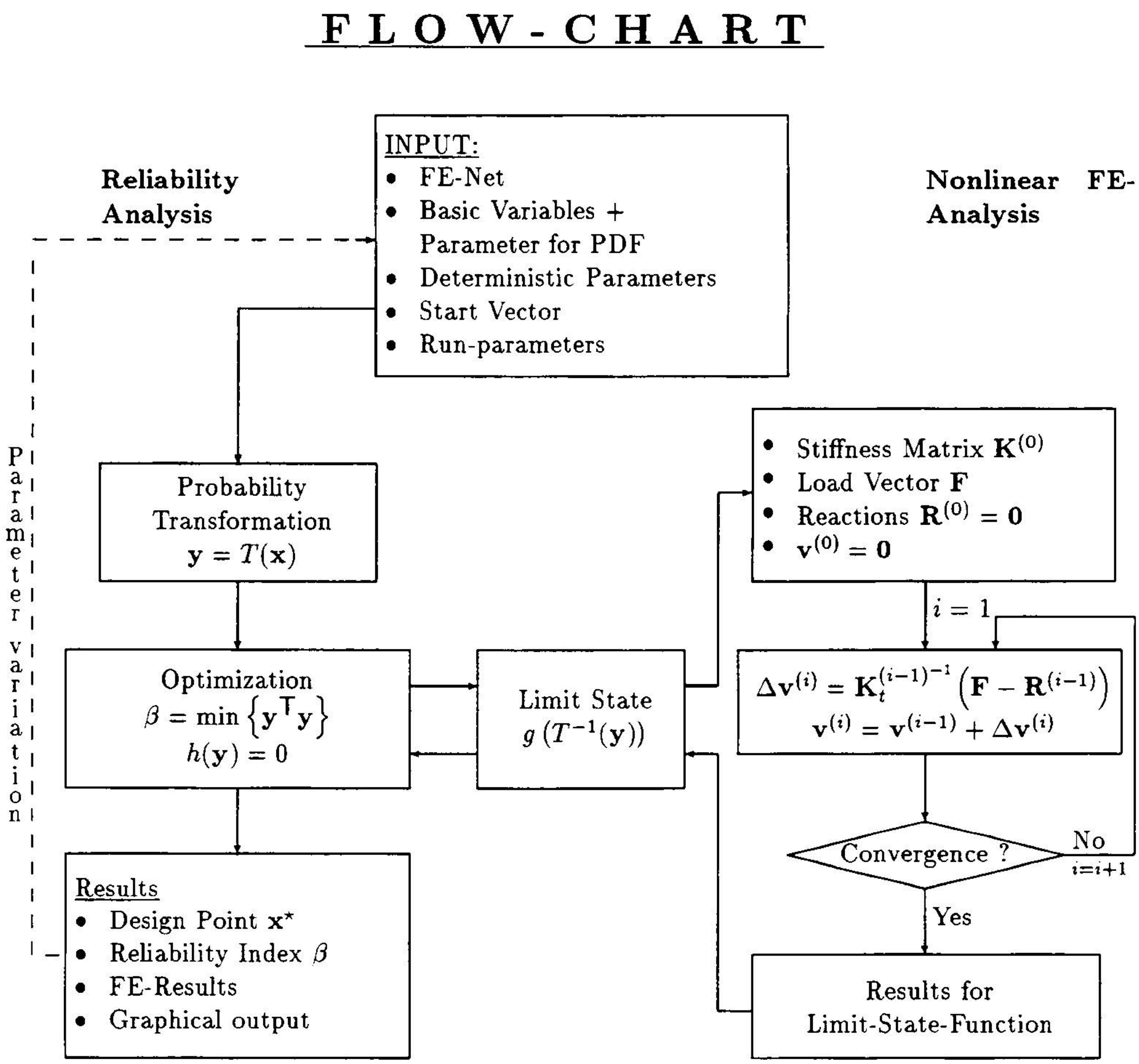

Figure 4: Finite element reliability method

from the origin to the limit state surface. Obviously either the steel strength or the concrete strength has essential influence. The safe region between the origin and the limit state surface is convex limited. That means that the FORM-solution may give unsafe results, but further investigations with statistical simulation methods have shown, that these influence can be neglected [6].

In the following examples, design for ULS-conditions was carried out in accordance to EC 2 for different load combinations N and M. In Fig. 6, the corresponding M-N diagram (interaction diagram) for symmetrically reinforced sections in a nondimensional

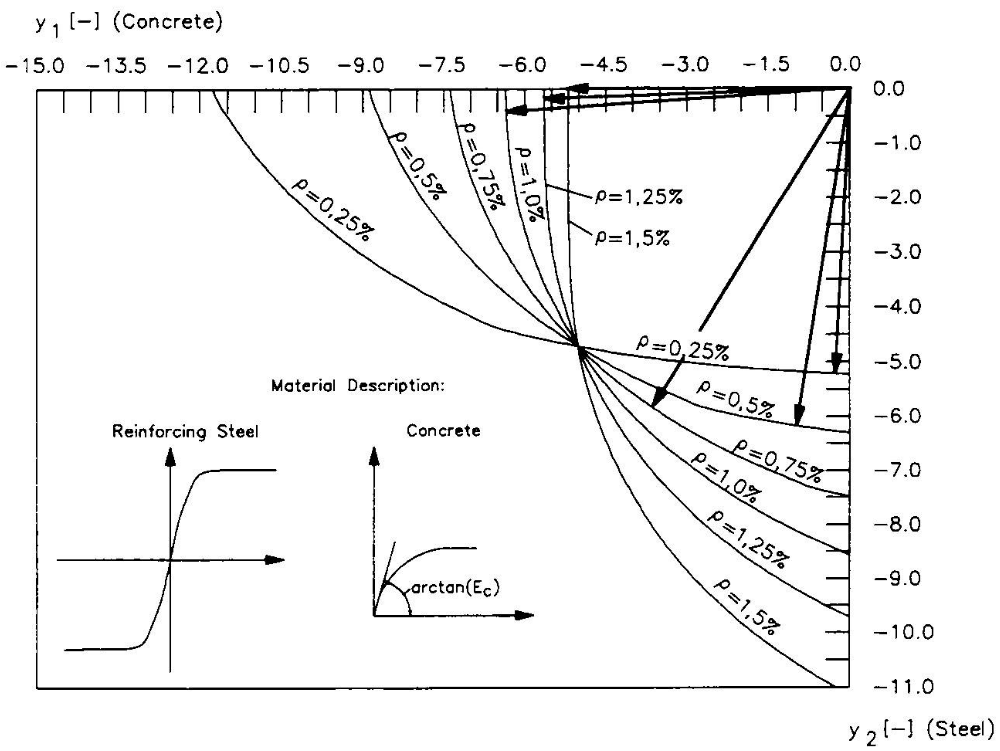

Figure 5: Limit State Surface $M_R(f_c, f_y) - M_S = 0$, Standard Normal Space

form with

$$\mu_n = \frac{\mu_N}{bhf_{cm}} \qquad \mu_m = \frac{\mu_M}{bh^2 f_{cm}} \qquad \omega = \frac{A_s}{bh} \cdot \frac{f_{ym}}{f_{cm}}. \tag{14}$$

is drawn on the left. μ_N and μ_M are determined using mean values of material properties, while the partial load safety factor is assumed as $\gamma_G = 1.35$ (permanent load). If the acting longitudinal force N has a favorable effect, $\gamma_G = 1.0$ is used. ω is the mechanical reinforcement ratio. The applied limit state function with its basic variables is given by $g(\mathbf{x}) = N_R(f_c, f_{y1}, A_{s1}, f_{y2}, A_{s2}, b, h, d_1, d_2, M_S) - N_S = 0$ resp. $g(\mathbf{x}) = M_R(f_c, f_{y1}, A_{s1}, f_{y2}, A_{s2}, b, h, d_1, d_2, N_S) - M_S = 0$. The results of the first order reliability analysis are drawn in Fig. 6 (right) as contour lines for β. The acting forces N_S and M_S, which are determinated as a function of the corresponding design situation are taken as statistically correlated with $\varrho_{NM} = 0.5$ in order to cover an unfavorable case. In general a reliability index of $\beta > 5$ will be achieved. For relative low mechanical reinforcement ratios, values of β can fall below 5 and even down to a value of $\beta = 4.0$. That means, that in general sufficient safety is provided by using the semi-probabilistic design concept for the ULS but caution should be given to low reinforcement ratios.

Fig. 7 shows the according contour lines of β for the steel stress limitation $(0.8 f_{yk})$ as an example for the SLS-conditions ($\varrho_{NM} = 0$). For longitudinal tensile forces ($\mu_n > 0$), a nearly constant reliability index $\beta = 1.6$ is reached while for increasing longitudinal compressive forces ($\mu_n < 0$), the reliability index β increases rapidly. Cross-sections show their minimum of β while mainly loaded by bending moments and low reinforcement ratios. This was also observed for the ULS-conditions.

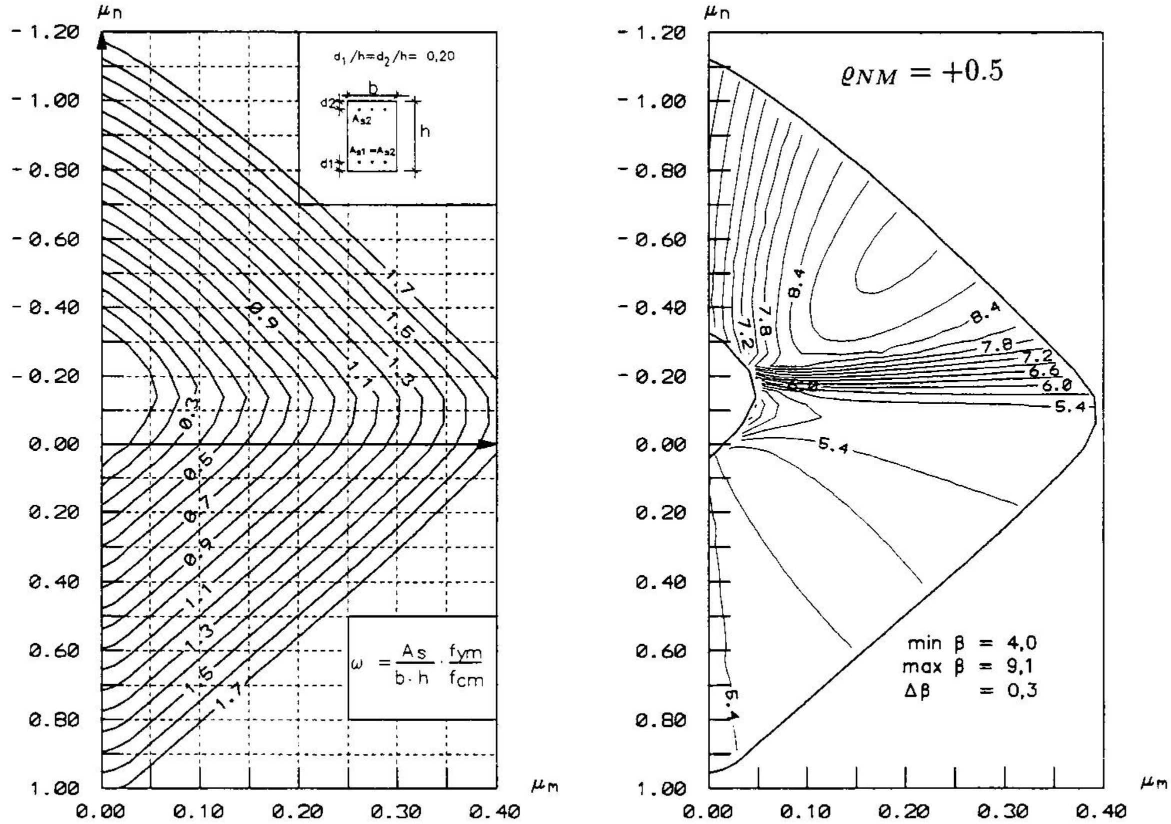

Figure 6: Interaction diagram and contour lines of β [T=1 year] for a symmetrically reinforced concrete cross-section

In Fig. 8 the safety index β is drawn for the limit state of cracking as a function of the reinforcement ratio $\rho = A_s/bh$. Here, the section is loaded only with a bending moment, considering different ratios of permanent (G) and variable (Q) loads. The deterministic design was carried out for SLS and ULS conditions.

Summarizing the results of the cross-section reliability analysis, it is evident that the reliability requirements of the cross-section design for reinforced concrete structures are fulfilled. It should be pointed out that a linear relation between the applied acting loads and the internal forces has been assumed. The extent to which these results are true for statically indeterminate structures with non-linear behavior has still to be investigated.

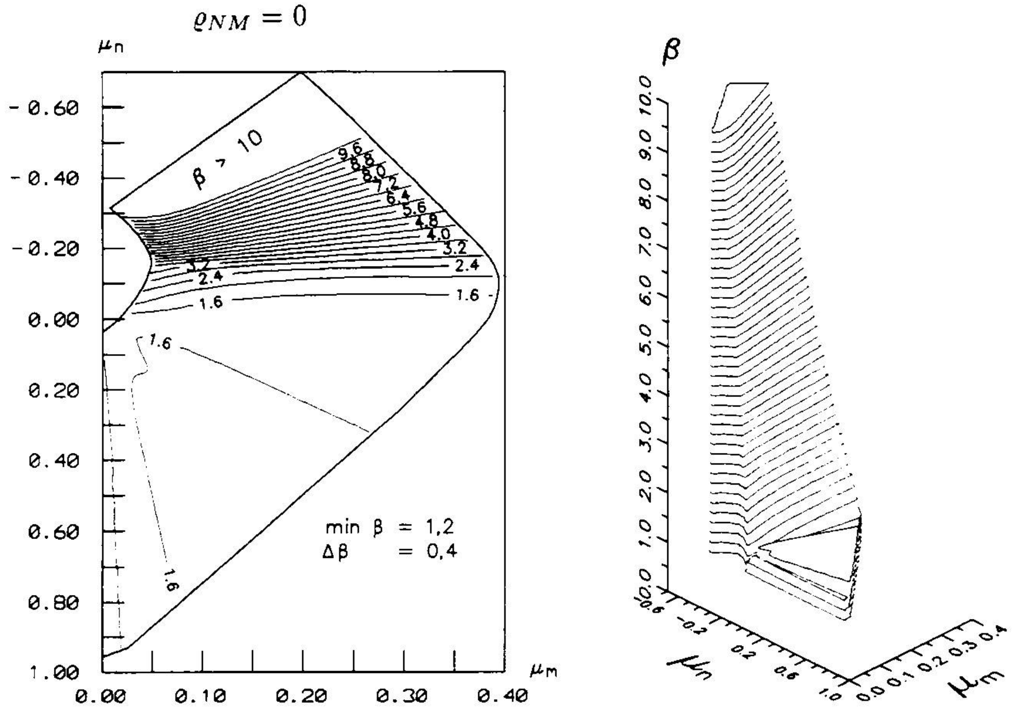

Figure 7: Contour lines and isoparametric plot of the reliability index β [T=1 year] for the limit state of steel stress limitation ($\sigma_s = 0.8 f_{yk}$)

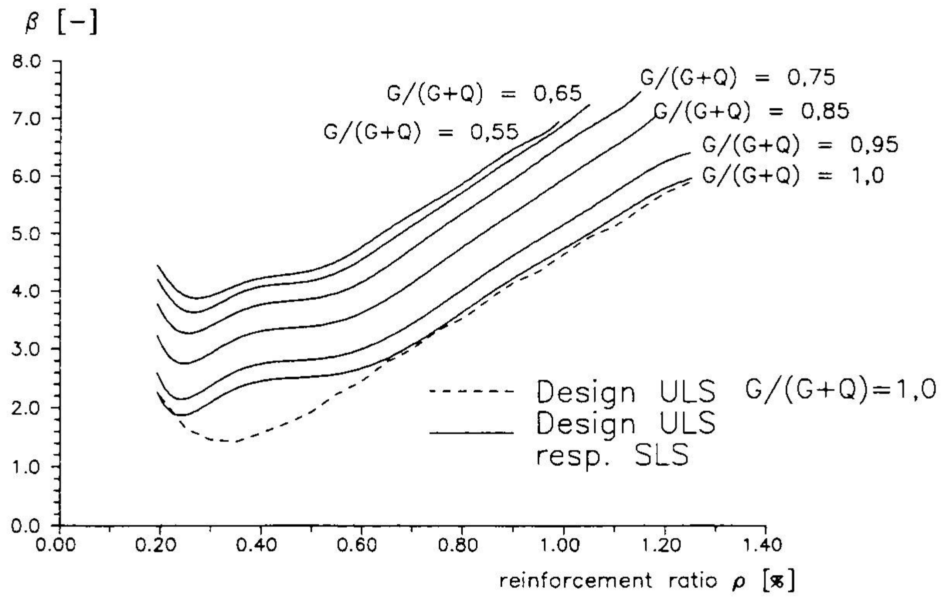

Figure 8: Reliability index β [T=1 year] for different ratios $G_k/(G_k + Q_k)$ for the limit state of cracking ($w_{lim} = 0.3$ mm)

4 Safety Analysis of an Indeterminate Reinforced Concrete Beam

Referring to EC 2 [3], system analysis may be performed by using non-linear methods. Useful safety concepts for nonlinear analysis are still discussed [1].The following example will investigate different proposals in order to compare the reliability in critical regions by using the Finite Element Reliability Method. Fig. 9 shows a statical system which may be taken as a symmetrical half of a two span reinforced concrete beam. The beam

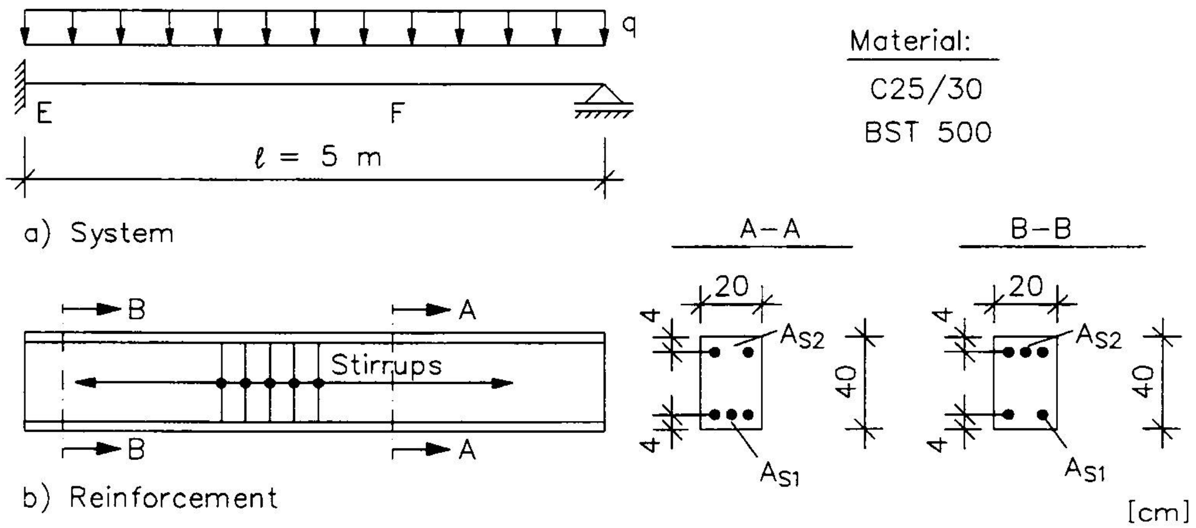

Figure 9: Reinforced concrete beam

is equally loaded with a permanent load q to cover the most unfavorable case. Using the linear elastic model for system analysis, design is carried out deterministically by SLS-criteria only (crack width = 0.3 mm, steel stress = $0.8 f_{yk}$). The design bearing capacity $q_{T,d}$ for the ULS is determinated by non-linear analysis, using different safety concepts for the material properties. In the following, to determine the bearing capacity, the investigated models with different safety concepts are discussed:

PLA plastic analysis with design values of material properties within critical zones. The assumption of design properties in the critical zones (plastic hinges) which are determinated by using the partial safety factors for steel ($\gamma_s = 1.15$) and concrete ($\gamma_c = 1.5$) covers the material uncertainty.

NLD non-linear analysis with design values of material properties for the whole structure. To determine the bearing capacity, the applied load q will be increased until the rotational capabilities[3] or instability of the system is reached, respectively. The assumption of design values for material covers the uncertainty.

NLMG non-linear analysis with mean values of material properties and gamma. The bearing capacity q_T is defined by reaching the rotational capabilities or instability of the system, respectively. The material uncertainty will be considered by

[3]The rotational capabilities are here defined by $\varepsilon_{cu} = -3.5\,‰$ and $\varepsilon_u = 20\,‰$ $(f_t/f_y = 1.0)$.

applying a partial safety factor $\gamma_R = 1.25$ to q_T. That means, it has to be shown that

$$\gamma_F \cdot q \leq \frac{q_T}{\gamma_R}. \tag{15}$$

where γ_F summarizes the load safety factor. Eq. 15 may lead to a global safety factor concept because γ_R can also be used on the left side. However, it is much more reasonable and in the consequence of the well established partial safety concept to apply γ_R to q_T.

NLM non-linear analysis with mean values of material properties. To determine the bearing capacity, the applied load q will be increased until the rotational capabilities or instability of the system is reached, respectively. This is an analysis without any safety elements for the material. The results should point to the influence on the material uncertainty in general.

The probabilistic model used here was discussed in chapter 2.4. The random variables are taken as perfectly correlated (random field) over the system, between single variables (e.g. f_{y1} and f_{y2}) no correlation is assumed. The vector x of basic variables is given by

$$\mathbf{x} = (f_c, h, b, f_{yi}, A_{si}, d_i, q), \qquad i = 2E, 1F$$

The marked points F and E in Fig. 9 refer to the critical regions which will be observed. The point F is not fixed but depends on the actual stiffness of the system. First order reliability analyses are carried out for each design situation. The results have in accordance to the random variable description a reference period of one year. Fig. 10

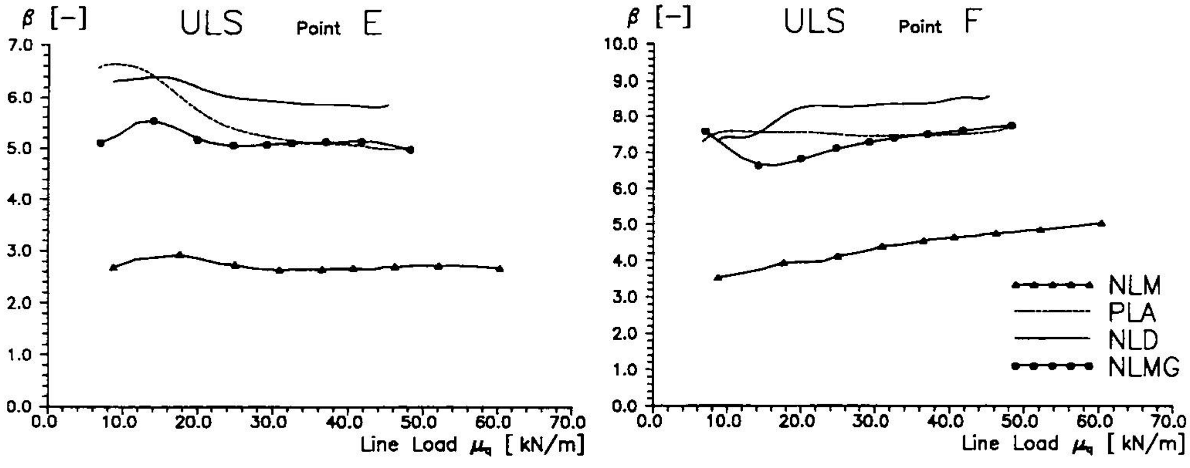

Figure 10: Reliability index β [T=1 year] for the ULS at points E and F

and 11 show the reliability index β, independently calculated for the critical system points E and F. Obviously, the assumption of NLM leads to a very low reliability for ULS which finally points to the requirement of material safety elements. The results for PLA, NLD and NLMG show that mainly independent of the applied safety model, a reasonable safety margin for the ULS is reachable while the values of γ may still be justified. However, the deterministic design using SLS requirements in conjunction with the applied material model for the ULS shows a sufficient reliability for the SLS.

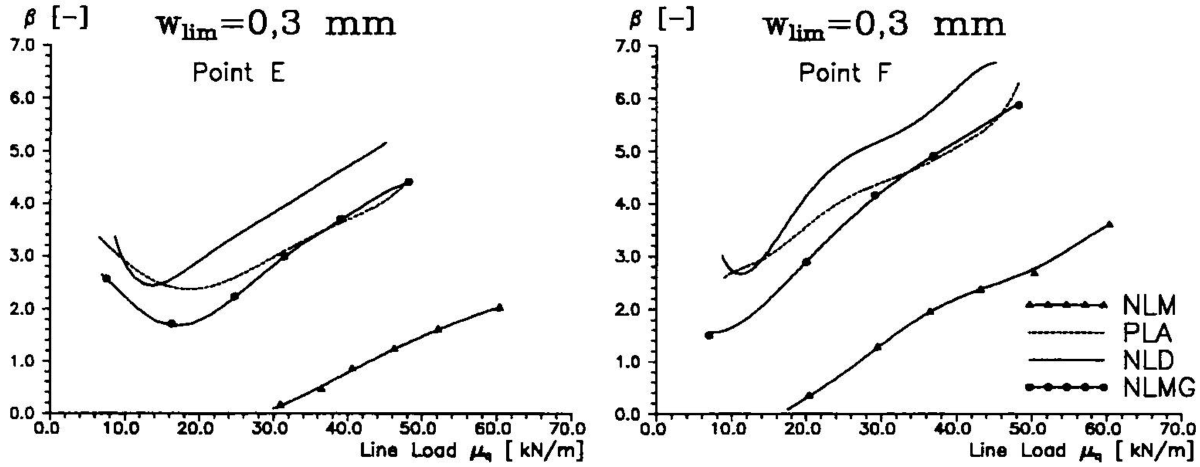

Figure 11: Reliability index β [T=1 year] for the SLS (cracking) at points E and F

5 Conclusions

The first order reliability method has been proven as a general tool to determine structural safety. An application of such safety assessment to reinforced concrete structures allows a comparative point of view to different deterministic design rules. As far as some investigated examples for nonlinear analysis of reinforced concrete structures shows, it should be noticed, that a homogeneous level of safety for the ULS may be reached with different (partial) safety concepts for design. Further investigations on this topic are still in progress. Namely the influence of M-N interaction and random field effects will be observed. In the future topics such as shear and prestressing should be made assessable to safety analysis of reinforced concrete, especially in conjunction with non-linear analysis for beams and plates.

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