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Reliability analysis of a reinforced concrete column designed according to the Eurocodes

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Summary

Reliability analysis of a built in reinforced concrete column designed according to Eurocodes 1 and 2 is a part of an extended research activity on Eurocode Random Variable Models supervised by JCSS. Presented results indicate that the reliability level of reinforced concrete columns designed according to the present generation Eurocodes may considerably vary depending on actual arrangement of the structure. To harmonise reliability levels provided by the Eurocodes for various structural members further research and calibration is required.

1. Introduction

Reliability analysis of reinforced concrete columns is part of an extensive research activity on Eurocode Random Variable Models supervised by the Joint Committee for Structural Safety JCSS [1]. The whole project covers reliability analysis of different structural members of a model multi-storey frame structure made of concrete or steel. The JCSS aims at providing a standardised set of statistical models for loads and structural properties which would reflect the present state of knowledge. Where necessary, the models should be adjusted in the future. It is expected that these models will be used as a practical design tool in conjunction with a probabilistic design criterion.

In a probabilistic design procedure a decision theoretical approach seems to be the most natural. However, as the models are only partly based on the experimental data, the calculated failure probabilities should not be identified directly with actual failure frequencies. That is why reliability criteria are usually defined through calibration to existing practice. In such a calibration procedure a set of structural elements are designed according to current design practice. For each of these elements the failure probability or reliability index is calculated, using the set of standardised statistical models. The resulting reliability indices may be then used as target reliability for the subsequent probabilistic design procedure. In such a way a combination of mechanical models, statistical models and corresponding target reliability which renders on the average the same design as current practice procedures may be derived.

This contribution presents preliminary results of reliability analysis of a built in reinforced concrete column designed according to newly developing Eurocode 1 [2, 3 and 4] and

Eurocode 2 [5]. The reliability analysis has been carried out using software product COMREL [6] developed by RCP München. It is expected that submitted investigation will contribute to desired calibration and possible future improvement of present generation of Eurocodes.

2. Structural characteristics

A model multi-storey structure considered in the this study is schematically shown in Fig. 1. It is assumed that each plenary frame in the transversal direction of the structure may be considered as unbraced sway frame. These transversal sway frames consist of four columns at a constant distance a_1; in the longitudinal direction of the structure they are located within a constant distance a_2 (see Fig. 1). The columns are considered as fully clamped in booth ends, at the top and at the bottom.

In the following reliability analysis of the edge column of an internal transversal frame having the height L and rectangular cross section $b \times h$ is considered. The cross section dimensions are chosen in such a way that the height h is two times (in one study case three times) the width b, thus $h/b = 2$ or 3. Considering different structural arrangements the total of 12 study cases indicated in Table 1 are analysed.

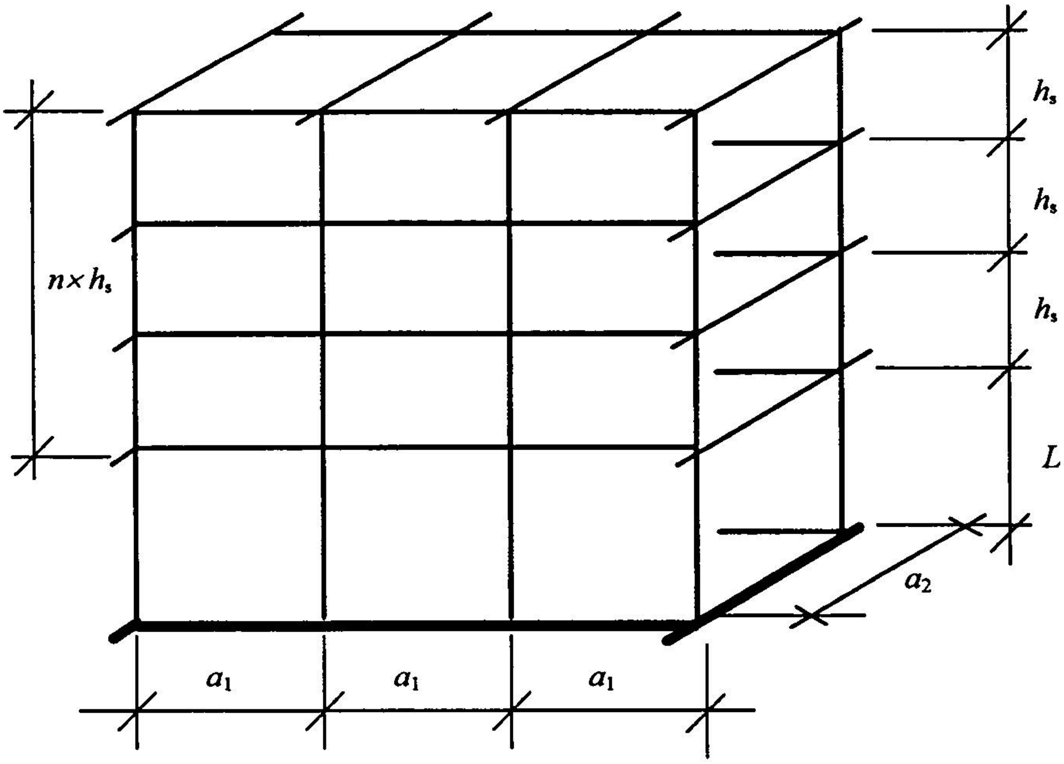

Fig. 1. Transversal frame of a multi-storey structure.

Study case	Number of storeys above the column n	Height of the analysed column L [m]	Transversal distance of columns a_1 [m]	Longitudinal distance of columns a_2 [m]	Cross section dimensions: width × height $b \times h$ [m×m]
1	10	6	5	5	0,35 × 0,70
2	10	3	5	5	0,25 × 0,50
3	10	9	5	5	0,35 × 0,70
4	10	12	5	5	0,45 × 0,90
5	10	6	4	5	0,35 × 0,70
6	10	6	7	5	0,35 × 0,70
7	10	6	5	4	0,30 × 0,60
8	10	6	5	7	0,40 × 0,80
9	1	6	5	5	0,25 × 0,50
10	3	6	5	5	0,25 × 0,50
11	20	6	5	5	0,40 × 0,80
12	10	6	5	5	0,25 × 0,75

Table 1. Study cases of a built in column.

Further it is assumed that the story height above the considered column is h_s = 3 m, permanent load is determined assuming reinforced concrete floor of a uniform equivalent thickness of 0.30 m (representing weight due to slab, columns, beams, floor and cladding).

3. Effect of actions

Effects of actions considered in the analysis of built in column consist of the axial force and bending moment, denoted again by N and M with appropriate subscripts. In the design calculation, the axial force and bending moment are represented by the design values N_d and M_d respectively. The maximum design axial force $N_{d,max}$ is given as

$$N_{d,max} = \gamma_G\ N_{W,k} + \gamma_Q \max\ \{N_{imp,k} + \psi_0\ N_{wind,k}\ ;\ N_{wind,k} + \psi_0 N_{imp,k}\ \} \qquad (1)$$

where γ_G = 1,35 is the partial factor for permanent actions, γ_Q = 1,50 is the partial factor for the variable actions, ψ_0 is the factor for combination value, $N_{W,k}$ is the characteristic value of the axial force due to self weight, $N_{imp,k}$ is the characteristic value due to imposed load and $N_{wind,k}$ is the characteristic value due to wind action (positive values are accepted for compressive forces). The minimum design axial force $N_{d,min}$ is given as

$$N_{d,min} = \gamma_G\ N_{W,k} - \gamma_Q\ N_{wind,k} \qquad (2)$$

where γ_G = 1,00 is the partial factor for favourable permanent actions, γ_Q = 1,50 is the partial factor for the variable actions.

Taking into account arrangement of the structure indicated in Fig. 1 the characteristic value due to self weight of n floors and one roof is given as

$$N_{W,k} = (n+1) a_1\, a_2\, t\, \rho_c\ /\ 2 \qquad (3)$$

where ρ_c is the weight of concrete per unit volume considered as 0,024 MN/m^3. $N_{imp,k}$ is the characteristic value of imposed load from n floors given as

$$N_{imp,k} = n\, a_1 a_2\, p_{imp} / 2 \tag{4}$$

Choosing a category B (Public Building) the characteristic value of floor imposed load $p_{imp\,k}$ equals 3 kN/m^2. For $n > 1$ the load reduction according to Eurocode 1 [3] should be included. $N_{wind,k}$ is the wind resulting from a pressure $C_p\, G\, p_{wind\,k}$ on a vertical area equal to $(L + nh_s)\, a_2$; multiplication by the height $(L + nh_s)/2$ gives the overturning moment. This moment is assumed to be balanced by the normal forces in the two outer columns, so:

$$N_{wind,k} = (1/2)(L + nh_s)^2 a_2 C_p G\, p_{wind,k} / (3\, a_1) = 0.271(L + nh_s)^2 a_2 / a_1 \tag{5}$$

where the characteristic value of the wind action is taken for the return period of 50 years as $p_{wind,k} = 0.5$ kN/m^2; further for the gust (exposure) factor the value $G = 2.5$ and for the shape factor the value $C_p = 0.8 + 0.5 = 1.3$ is chosen [4].

The design value M_d of the bending moment M is given as

$$M_d = M_{d0} + N_d(e_a + e_2) = N_d(e_0 + e_a + e_2) \tag{6}$$

where M_{d0} is the first order bending moment, $e_0 = M_{d0} / N_d$ is the first order eccentricity, e_a is the additional eccentricity taking into account geometric imperfections and e_2 is the second order eccentricity taking into account deformations of the column.

It is assumed that the first order moment M_{d0} is caused only by wind action, which is transmitted in each frame section of the width a_2 (see Fig. 1) equally by the four columns fully clamped in and, therefore, the maximum first order bending moment M_{d0} due to wind load about the centroid of a column cross section is determined from the formula

$$M_{d0} = L[\gamma_Q\, C_p\, G\, p_{wind,k}\, (L+nh_s)\, a_2]/8 = 0{,}305\;\; L(L + nh_s)\, a_2 \tag{7}$$

where L denotes the column height.

The eccentricities e_a and e_2 are determined in accordance with Chapter 2 and 4 of Eurocode 2 [5]. The additional eccentricity e_a is given as $e_a = \nu_a\, l_0/2$, where l_0 denotes the effective length of the column considered here by the lowest recommended value 1,12 L (for the case of a column of a sway frame), ν_a inclination from the vertical given by the minimum value 1/200 which is valid for all structures higher than 4 m when the second order effects are taken into account. Thus

$$e_a = 1{,}12\, L/(2 \times 200) = 0{,}0028\, L \tag{8}$$

The second order eccentricity e_2 is dependent on the characteristics of the column cross section and should be generally determined by an iteration process. In accordance with equation (4.69) in [5] the second order eccentricity is given as

$$e_2 = 0{,}1 K_1\, l_0^2\, (1/r) \tag{9}$$

where the coefficient K_1 depends on the slenderness ratio $\lambda = l_0 / i$ (i being radius of gyration) and is given by equations (4.70) and (4.71) in Eurocode 2 [5]. As in the all study cases here $\lambda \geq 35$ the value $K_1 = 1$ is considered. The curvature $1/r$ is given by equation (4.72) in [5] as

$$1/r = 2\, K_2\, \varepsilon_{yd} / (0{,}9\;\; (h - d_1)) \tag{10}$$

where the coefficient K_2 is defined by equation (4.73) in [5] as follows

$$K_2 = (N_{ud} - N_d) / (N_{ud} - N_{bal,d}) \leq 1 \tag{11}$$

where N_{ud} is the design capacity of the cross section, N_d is the design axial force and $N_{bal,d}$ is the force which maximises the ultimate moment of the cross section; in this study for symmetrical reinforcement $N_{bal,d} = 0,5\ \alpha f_{cd} A_c$, where α is a coefficient taking account of long term effects on the compressive strength.

The remaining variables entering equation (10), the design yield strength $\varepsilon_{yd} = f_{yd} / E_s$ and the effective depth of cross section $h - d_1$, are specified bellow (see also Fig. 2). Table 2 and 3 shows the resulting values of the effects of actions for all 12 study cases considered here.

Study case	$N_{d,max}$ [MN]	M_{d0} [MNm]	e_0 [m]	L [m]	e_a [m]	$A_s \times 10^4$ [m^2]	A_s / bh [%]	e_2 [m]	M_d [MNm]
1	2,162	0,329	0,1522	6	0,0168	28,7	1,17	0,0245	0,418
2	2,078	0,151	0,0726	3	0,0084	22,1	1,23	0,0047	0,178
3	2,054	0,535	0,2373	9	0,0252	34,1	1,07	0,0591	0,725
4	2,353	0,768	0,3263	12	0,0336	38,2	0,94	0,1062	1,098
5	1,967	0,329	0,1673	6	0,0168	24,6	1,00	0,0265	0,415
6	2,736	0,329	0,1201	6	0,0168	41,4	1,69	0,0200	0,431
7	1,729	0,263	0,1523	6	0,0168	31,9	1,77	0,0285	0,343
8	3,028	0,461	0.1522	6	0,0168	37,4	1,17	0,0196	0,572
9	0,340	0,082	0,2422	6	0,0168	4,6	0,37	0,0485	0,105
10	0,702	0,137	0,1954	6	0,0168	10,9	0,87	0,0485	0,183
11	4,895	0,603	0,1232	6	0,0168	90,7	2,83	0,0141	0,755
12	2,162	0,329	0,1522	6	0,0168	37,5	2,00	0,0191	0,407

Table 2. Effects of actions for the maximum axial force $N_{d,max}$.

Study case	$N_{d,max}$ [kN]	M_{d0} [MNm]	e_0 [m]	L [m]	e_a [m]	$A_s \times 10^4$ [m^2]	A_s / bh [%]	e_2 [m]	M_d [MNm]
1	0,464	0,329	0,7100	6	0,0168	17,9	0,73	0,0346	0,353
2	0,548	0,151	0,2755	3	0,0084	4,0	0,22	0,0101	0,161
3	0,372	0,535	1,4374	9	0,0252	31,4	0,98	0,0682	0,589
4	0,273	0,768	2,8125	12	0,0336	44,2	1,09	0,1078	0,806
5	0,134	0,329	2,4649	6	0,0168	24,0	0,98	0,0346	0,336
6	1,001	0,329	0,3289	6	0,0168	12,9	0,53	0,0346	0,381
7	0,372	0,263	0,7077	6	0,0168	18,6	1,03	0,0404	0,285
8	0,650	0,461	0.7093	6	0,0168	20,0	0,63	0,0303	0,491
9	0,147	0,082	0,5596	6	0,0168	6,8	0,54	0,0485	0,092
10	0,269	0,137	0,5106	6	0,0168	11,6	0,93	0,0485	0,155
11	0,120	0,603	5,0273	6	0,0168	40,5	1,27	0,0303	0,609
12	0,464	0,329	0,7100	6	0,0168	16,6	0,89	0,0323	0,352

Table 3. Effects of actions for the minimum axial force $N_{d,min}$.

4. Material characteristics

The following materials characteristics for concrete and reinforcing steel are considered in the deterministic design of reinforced concrete columns. Concrete class C 20/25 having the characteristics

$$f_{ck} = 20 \text{ MPa}, \gamma_c = 1{,}5, f_{cd} = 13{,}33 \text{ MPa}, \alpha = 0{,}85 \quad (12)$$

is considered here. It should be noted that the coefficient α equal to one is considered in some countries. Reinforcing steel S 500 having the strength values

$$f_{yk} = 500 \text{ MPa}, \gamma_s = 1{,}15, f_{yd} = 435 \text{ MPa} \quad (13)$$

is considered. Assuming further the modulus of elasticity E_s = 200 GPa, the design yield strain ε_{yd} = 2,17 ‰ corresponds to the yield strength f_{yd} given above.

5. Deterministic design

The following simplifications are accepted for design of column cross sections (see figure 2):

- symmetrical reinforcement ($A_{s1} = A_{s2} = A_s / 2$) is considered only,
- the square shape of the column cross section having dimensions h and b rounded to 5×10^{-2} m are chosen such that h/b = 2 (in the last study case h/b = 3).
- distance of reinforcing bars from the edge is chosen as $d_{1(2)} = 0.1\ h$.

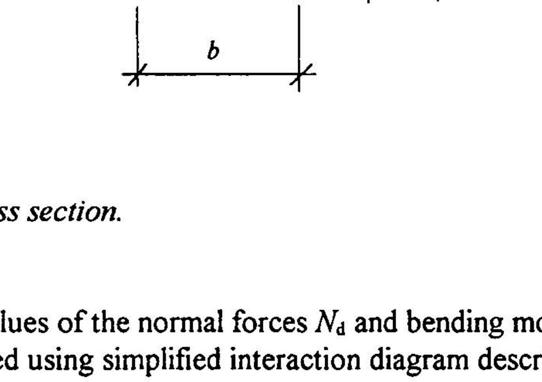

Fig. 2. Column cross section.

For given design values of the normal forces N_d and bending moments M_d, the column cross sections are designed using simplified interaction diagram described by the following formula:
for $N_d < \alpha\, b\, h f_{cd} / 2$

$$[A_s f_{yd} (h - 2d_1) + h N_d (1 - N_d / (\alpha\, b\, h f_{cd})] / 2 - M_d > 0 \quad (14)$$

for $N_d > \alpha\, b\, h f_{cd} / 2$

$$K_2 [A_s f_{yd} (h - 2d_1) / 2 + \alpha\, b\, h^2 f_{cd} / 8] - M_d > 0 \quad (15)$$

$$K_2 = (N_{ud} - N_d) / (N_{ud} - N_{bal,d}) \tag{16}$$

$$N_{ud} = \alpha\ b\,h f_{cd} + A_s f_{yd} \tag{17}$$

$$N_{bal,d} = \alpha\ b\,h f_{cd} / 2 \tag{18}$$

These relationships approximate well interaction diagrams derived from appropriate rules of Eurocode 2 [5] and, because of their simplicity, shall be used in the following reliability analysis. Moreover, detail analysis show that in common cases the ultimate bending moment given by these relationships is mostly on the safe side and differs insignificantly (by less than few percent) from that obtained by more accurate procedure based on Eurocode 2 [5]. The total reinforcement area A_s should satisfy the conditions of clause 5.4 in [5]:

$$0{,}15\ |N_d|\ / f_{yd} < A_s,\quad 0{,}003\ b\,h < A_s < 0{,}08\ b\,h \tag{19}$$

which specifies the minimum and maximum reinforcement ratio.

Using relationships (14) to (18.), material properties given by equations (12) and (13) and the design values of effects of actions described by equations (1) to (11), the resulting reinforcement areas A_s and rations A_s / bh shown in Table 2 and 3 have been obtained for the maximum axial force $N_{d,max}$ and the minimum axial forces $N_{d,min}$ respectively. Note that the reinforcement areas A_s given in Table 2 and 3 satisfy the conditions (19) required by Eurocode 2 [5]. Theoretical values of reinforcement area A_s rounded upward to the last digit indicated in Table 2 and 3, which do not correspond to any specific bar size, shall be considered in the following reliability analysis.

It follows from Tables 2 and 3 that in the study cases 4, 9 and 10 the greater reinforcement areas follow from the design situation corresponding to the minimum axial force $N_{d,min}$; this reinforcement should be used. However, to show the effect of the design procedure considering the maximum axial force $N_{d,max}$ only, both reinforcement areas (the greater due to the minimum axial force and smaller due to the maximum axial force) are considered in the following reliability analysis of the study cases 4, 9 and 10.

6. Limit state function

In the time variant reliability analysis the actual axial force N is considered as a simple sum of actual axial forces due to all the considered actions:

$$N = N_W + N_{imp} + N_{wind} \tag{20}$$

where N_W is the axial force due to self weight, N_{imp} is the axial force due to imposed load and N_{wind} is the axial force due to wind action (positive values are again accepted for compressive forces). Thus, the time variant reliability analysis presented here concerns only the permanent design situation with the maximum axial force (corresponding to $N_{d,max}$ given by (1)).

The bending moment M is given by equation (6) used in the design calculation in which actual values are applied instead of the design values and a new additional eccentricity e_a are considered, thus

$$M = M_0 + N(e_a + e_2) = N(e_0 + e_a + e_2) \tag{21}$$

where the first order eccentricity $e_0 = M_0 / N$, where M_0 is given as

$$M_0 = L[C_p\, G\, p_{wind}\, (L+nh_s)\, a_2]/8 \tag{22}$$

The additional eccentricity e_a is given in terms of the initial sway ζ as

$$e_a = \zeta L/2 \tag{23}$$

where ζ is given in Table 4. The second order eccentricity e_2 is given by modified equations (9) in which $l_0 = L$ (the minimum value $l_0 = 1{,}12\, L$ required by Eurocode 2 [5] is neglected in the reliability analysis), thus

$$e_2 = 0{,}1\, K_1 L^2 (1/r) \tag{24}$$

where $K_1 = 1$ and r is given by equation (10), in which, again, actual values of basic variables shall be used instead of the design values.

The limit state function g may be expressed as the difference of resistance bending moment and the actual bending moment about the centroid.

$$g = \xi_R M_R - \xi_E M \tag{25}$$

Two coefficients of model uncertainties ξ_R and ξ_E are considered as random variables to cover imprecision and incompleteness of the relevant theoretical models. Taking into account (15) to (18) the limit state function (25) becomes
for $N < \alpha\, b\, h\, f_c / 2$

$$\xi_R\, [A_s f_y\, (h - 2d_1) + h\, N\, (1 - N / (\alpha\, b\, h f_c))] / 2 - \xi_E M > 0 \tag{26}$$

for $N > \alpha\, b\, h\, f_c / 2$

$$\xi_R\, \kappa\, [A_s f_y\, (h - 2d_1) / 2 + \alpha\, b\, h^2 f_c / 8] - \xi_E M > 0 \tag{27}$$

$$\kappa = (N_u - N) / (N_u - N_{bal}) \tag{28}$$

$$N_u = \alpha\, b\, h f_c + A_s f_y \tag{29}$$

$$N_{bal} = \alpha\, b\, h f_c / 2 \tag{30}$$

The limit state function given by equations (26) to (30) is applied in the reliability analysis of the column in conjunction with appropriate probabilistic models for basic random variables described bellow.

7. Statistical properties of basic variables

Basic variables applied in the reliability analysis are listed in Table 4. Note that the initial overall sway ζ_0 (which is not used in the design - see note (1) below Table 4) is applied now in the reliability analysis of the column. Some of the basic variables are assumed to be deterministic values - denoted "DET" (A_s, E_s, a_1, a_2, L, and n), the others are considered as random variables having the normal distribution - "N", lognormal distribution - "LN", Gumbel distribution - "GUM" and Gamma distribution - "GAM". Statistical properties of the random variables are further described by the moment characteristics, the mean and standard deviation, partly taken from CIB Reports [7] and [8].

Category of basic var.	Symbol	Name of basic variable	Distrib type	Dimen.	Mean	Standard deviation
Material properties	α	reduction factor	N	-	0,85	0,085
	A_s	reinforcement area	DET	m^2	nom	0
	f_c	concrete strength	LN	Mpa	30	5
	f_y	yield strength	LN	Mpa	560	30
	E	modulus of elasticity	DET	GPa	200	0
Geometric data	a_1	column distance in plane	DET	m	nom	0
	a_2	perpend. dist. of column	DET	m	nom	0
	b	width of cross section	N	m	nom	0,005
	$d_{1(2)}$	distance of bars from edge	N	m	0.1h+0.00	0,005
	h	height of cross section	N	m	nom	0,005
	L	height of column	DET	m	nom	0
	n	number of floors	DET	-	nom	0
	ζ	initial overall sway$^{(1)}$	N	rad	0	$0,0015^{(1)}$
Model uncertainty	ξ_E	uncertainty of load	N	-	1,0	0,1
	ξ_R	uncertainty of column	N	-	1,1	0,11
Actions	ρ	weight of reinf. concrete	N	MNm^{-2}	0,0240	0,00192
	C_p	shape coefficient	LN	-	1,0	0,15
	G	gust factor	GUM	-	2,5	0,25
	p_{wind}	wind pressure	GUM	MNm^{-2}	0,00035	$0,00006^{(2)}$
	p_{impl}	imposed long term load	GAM	MNm^{-2}	0,0006	ean × $\nu^{(3)}$
	p_{imps}	imposed short term load	GAM	MNm^{-2}	0,0002	ean × $\nu^{(4)}$

Notes:
(1) The initial overall sway ζ is used to calculate the additional eccentricity e_a of the built in column according to equation (23).
(2) The mean and standard deviation correspond to the distribution of one year maximum.
(3) The mean and standard deviation correspond to the distribution of 7 years maximum; v^2=(0,16+8/(a_1 a_2))(1/n+ρ (1-1/n)) (see CIB report [8]), where the coefficient of correlation of the long term loads in two floors is considered as ρ = 0.5 (see also table 5).
(4) The mean and standard deviation correspond to the distribution of the 12 hours (one day) maximum, v^2= 50/(a_1 a_2) (see also table 5).

Table 4. Statistical properties of basic variables for built in column.

Study case	$A_s \times 10^4$ [m^2]	a_1 [m]	a_2 [m]	n	$\sigma_{p,impl}$ [MN/m^2]	$\sigma_{p,imps}$ [MN/m^2]
1	24,3	5	5	10	0,00031	0,00028
2	28,2	5	5	10	0,00031	0,00028
3	46,4	5	5	10	0,00031	0,00028
4	28,5	5	5	10	0,00031	0,00028
5	23,2	4	5	10	0,00033	0,00032
6	30,1	7	5	10	0,00028	0,00024
7	26,1	5	4	10	0,00033	0,00032
8	31,1	5	7	10	0,00028	0,00024
9	5,3	5	5	1	0,00042	0,00028
10	9,4	5	5	3	0,00034	0,00028
11	73,8	5	5	20	0,00030	0,00028
12	29,8	5	5	10	0,00031	0,00028

Table 5. Standard deviation $\sigma_{p,impl}$ and $\sigma_{p,imps}$ of the imposed loads.

8. Reliability analysis

Time variant reliability analysis is based on the Borges - Castanheta model for wind action, long term and short term imposed loads indicated in Fig. 3 (see also [1]). Program COMREL-JP [6] have been applied for time variant reliability analysis (jump process) of the columns assuming life time of 50 years and the probabilistic models given in Table 4 and 5.

The wind load is modelled as a sequence of independent rectangular pulses, each pulls having a duration of approximately 1 day. The statistical properties of the pulls intensity is tuned in such a way that the maximum pressure in a year has a distribution specified in Table 4. The long term imposed load is defined for the interval of 7 years. It is assumed to be changed simultaneously on all floors of a building. The short term load is present during one interval of 1 day in each year; the simultaneous occurrence of short term imposed loads on more than 1 floor at the same time may be neglected; so an independent short term single floor load imposed on the column occurs n times a year, n being the number of floors. Note that long term loads are considered as being correlated over various floors.

In the first type of the time variant analysis the short term action was assumed to be absent, $p_{imps} = 0$, and only wind action p_{wind} and long term imposed load p_{impl}, were considered as time dependent ergodic and stationary random variables. As the statistical properties of the wind action p_{wind} given in Table 4 refer to the distribution of one year maximum values and properties of the long term imposed load p_{impl} refer to 7 years maximum, the "jump rates" (number of jumps within one year) $\lambda_{p,wind}$ and $\lambda_{p,impl}$ of the rectangular wave renewal jump process were considered as follows:

$$\lambda_{p,wind} = 1{,}0/\text{year} \; ; \; \lambda_{p,impl} = 0{,}143/\text{year} \qquad (31)$$

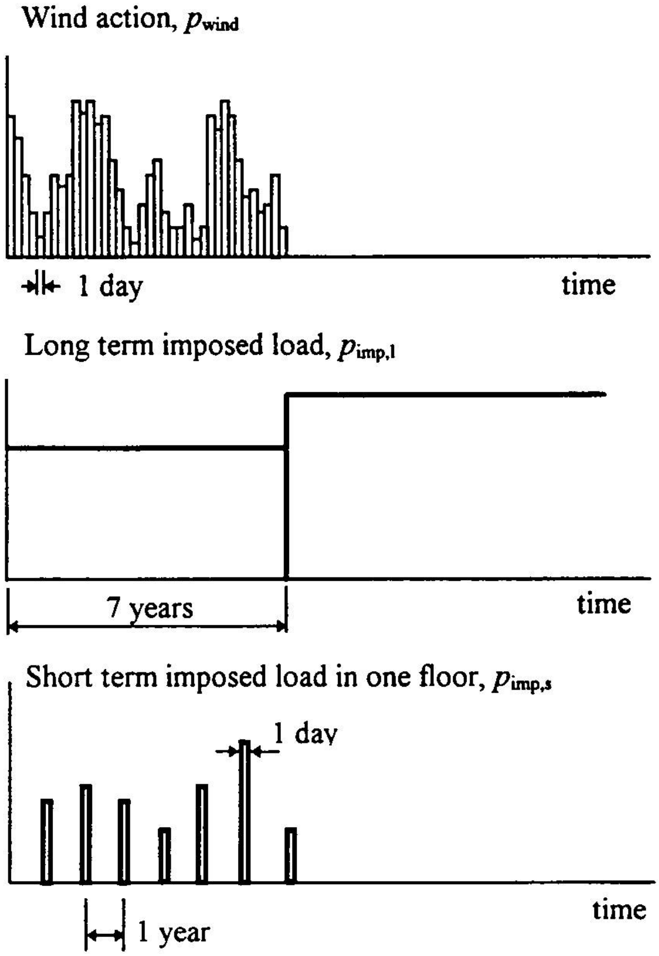

Fig. 3. Models of actions for time variant reliability analysis.

The second type of the time variant analysis concerns the period of time when the short term imposed load p_{imps} is present. As already mentioned above it is assumed that in each floor the short time imposed load may independently occur once a year. Thus, in every year there is n days, where n is the number of floors, when the short time load is active. The total number of 'active' days during the assumed life time of 50 years is therefore 50 n. This period is considered now as the total time of the time variant reliability analysis. One day is considered now as a unit of time. Jump rate of the short term imposed load p_{imps} is thus $\lambda_{p,imps}$ = 1,0/day.

Taking into account properties of the Gumbel distribution, statistical properties of the wind action p_{wind} were adjusted to one day period as follows

$$\mu_{day} = \mu_{year} - 0,78\ \sigma_{year} \ln(365) = 0,00035\text{-}0,00028\text{=}0,00007\ \text{MN/m}^2\ ,\ \sigma_{day} = \sigma_{year} \qquad (32)$$

Jump rate of the wind action p_{wind} is thus $\lambda_{p,wind}$ = 1,0/day.

Statistical parameters of the long term imposed load p_{impl} given in Table 4 for 7 years correspond now to the period of *7n* "active" days (one year is "compressed" to *n* "active days"). Appropriate jump rate $\lambda_{p,impl}$ (number of jumps within one active day) is therefore

$$\lambda_{p,impl} = 1/\,(7\,n)\ /\ \text{day} \tag{33}$$

Using the FORM methods of probability integration [6], resulting values of the reliability index β_1 and β_2 of the first and second type of reliability analysis respectively for the 12 study cases are given in Table 6.

Study case	Reinforcement area	Reinforcement ratio	Cross section dimensions	Column height	Time variant analysis, short term load not present	Time variant analysis, short term load present
	$A_s \times 10^4$ [m^2]	A_s/bh [%]	$b \times h$ [m]	L [m]	β_1	β_2
1	28,7	1,17	0,35×0,70	6	5,6	6,1
2	22,1	1,23	0,25×0,50	3	4,7	5,3
3	34,1	1,07	0,35×0,70	9	4,0	4,6
4(1)	44,2 (38,2)	1,09 (0,94)	0,45×0,90	12	4,5 (4,2)	5,1 (4,8)
5	24,0	1,00	0,35×0,70	6	5,3	5,8
6	41,4	1,69	0,35×0,70	6	6,1	6,5
7	31,9	1,77	0,30×0,60	6	5,5	6,0
8	37,4	1,17	0,40×0,80	6	5,7	6,2
9(1)	6,8 (4,6)	0,54 (0,37)	0,25×0,50	6	3,7 (2,9)	4,9 (4,2)
10(1)	11,6 (10,9)	0,93 (0,87)	0,25×0,50	6	3,9 (3,8)	4,8 (4,7)
11	90,7	2,83	0,40×0,80	6	5,6	6,0
12	37,5	2,00	0,25×0,75	6	5,6	6,2

Note: (1) In the study cases 4, 9 and 10 the reinforcement area is designed considering the minimum axial force $N_{d,min}$ due to permanent load and wind action only (imposed load being absent); values given in brackets ( ) correspond to the design considering the maximum axial force $N_{d,max}$.

Table 6. Reliability indices β_1, and β_2 of time variant analysis for built in column.

It follows from Table 6 that obtained values of the reliability indices are within a broad ranges from 3,7 (2,9 when the 'the maximum axial force design' is considered only) to 6,5. Such a broad range for reliability indices has been, however, reported also in previous probabilistic analyses (see for example [9]). Values of the reliability index β_1 are within a range from 3,7 (2,9) up to 6,1, values of β_2 within a range from 4,6 (4,2) up to 6,5. In the study cases 9 the reliability index β_1 = 3,7 (2,9) is less than recommended value 3,8 [1], relatively low value of β_1 are obtained also for the study cases 3, 4 and 10 (see Table 6). In all these cases the reinforcement ratio is relatively low (around or less than 1%), though still above the required minimum 0,3 %. In the study case 9 and 10 there may be also an unfavourable effect of relatively small cross section dimensions (0,25 × 0,50 m). Higher and perhaps uneconomical values of the reliabilty indices (around 6) seem to correspond to relatively great reinforcement ratios (study cases 7, 11 and 12).

The resulting reliability index β for the column is given by a combination of both reliability indices β_1, and β_2 that are given in Table 6. As a simple approximation the minimum of both values β_1, and β_2 may be considered as the resulting reliability index β. It follows from Table 6 that in all the study cases considered here $\beta_1 < \beta_2$; thus the first design situation with the short term imposed load being absent seems to be decisive.

10. Conclusions

Results of the reliability analysis of 12 study cases of reinforced concrete column show considerable differences in the reliability level of the column in different structural arrangements. Considering 50 years life time, wind action and long term imposed load as time variant actions (short time imposed load being absent) obtained values of the reliability index β varies within a broad range from 2,9 up to 6,1. Generally higher values of β (from 4,2 to 6,5) correspond to the reliability of columns during those days when short term imposed load is present.

It appears that the reliability level of reinforced concrete columns designed according to Eurocodes may be in some cases insufficient in other cases, depending on actual structural arrangements, it may become uneconomical. To harmonise reliability levels obtained for various structural members further research on random variable models using available experimental data and calibration of present generation of Eurocodes to existing structures is urgently needed.

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Reliability analysis of a reinforced concrete column designed according to the Eurocodes

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Abstract

Reliability analysis of reinforced concrete columns is a part of an extended research activity on Eurocode Random Variable Models supervised by the Joint Committee for Structural Safety. Submitted analysis concerns reliability of a built in reinforced concrete column designed according to Eurocodes 1 and 2. Reliability of a column of the first floor of a multi-storey frame structure is analysed using software product COMREL developed by RCP Műnchen. Preliminary results of the analysis are presented for the total of 12 study cases corresponding to different structural arrangements.

The design effects of actions are determined in accordance with Eurocode 1 considering the permanent load due to self weight and variable load due to wind, long term and short term imposed load. The column cross sections are designed using a simplified interaction diagram for axial force and bending moment and material properties specified in Eurocode 2. Dimensions b and h of rectangular cross sections rounded to 5 10^{-2} m are chosen such that $h/b = 2$ (in one study case $h/b = 3$). Symmetrical reinforcement having the theoretical area A_s rounded upward to 10^{-5} m^2, which do not necessarily correspond to any specific bar size, is considered in the reliability analysis.

Using the FORM method of probability integration results of time variant reliability analysis of columns for long term and short term actions are submitted for the all 12 study cases. Considering 50 years life time, wind action and long term imposed load as time variant actions (short time imposed load being absent) obtained values of the reliability index β varies within a broad range from 2,9 up to 6,1. Generally higher values of β (from 4,2 to 6,5) correspond to the reliability of columns during those days when short term imposed load is also present.

It appears that the reliability level of reinforced concrete columns designed according to Eurocodes may be in some cases insufficient in other cases, depending on actual structural arrangements, it may become uneconomical. To harmonise reliability levels provided for various structural members further research of random variable models using available experimental data and calibration of present generation of Eurocodes to existing structures is urgently needed.