Calibration of Partial Safety Factors

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Summary

Based on existing literature an overview about calibration of partial safety factors and loads combination values is presented. The aim is to recommend a standardized basis for calibration of partial safety factors. Such calibrations should be made in order to establish National Application Documents (NADs) and in order to determine partial safety factors and load combination values in the Eurocodes/NADs. The paper includes a specific example formulated to illustrate the described method for code calibration.

1. Introduction

The purpose of the code calibration on the present level of structural design practice is to achieve a uniform reliability level within the given groups of structures considered in the code. However, the code format must be operational (simple) and consequently the load combinations and partial safety factors shall not be too many. Some deviations from the target reliability level are therefore inevitable.

In this paper a method for minimisation of the deviations from the target reliability level is described. The method defined includes a way for setting the target reliability level as a

function of the uncertainty modelling and common codified design practise.

The paper is based on a study performed by the authors for SAKO. SAKO is a Nordic group originally formed to harmonise structural codes in the Nordic countries. Since the development of Eurocodes has been initiated, SAKO has focused on this development. The objective of the study performed for SAKO was to formulated a rational way of determinating partial safety factors in the National Application Documents to the Eurocodes.

2.   Code Calibration Procedure

In Fig. 1 the proposed procedure for code calibration is illustrated. Each of the steps in the procedure is described below.

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│  Optimal structures (Codified design)    │
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┌──────────────────────────────┐        ┌──────────────────────────┐
│        Reliability model      │◄───────│  Uncertainty modelling   │
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┌──────────────────────────────┐                      │
│  Estimation of reliability level │                   │
│    for optimal structures     │                      │
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│      Target reliability level │                      │
└──────────────────────────────┘                      │
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│        Code calibaration      │◄─────────────────────┘
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│       Partial safety factors  │
│      Load combination values  │
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│  Deterministic check calculations │
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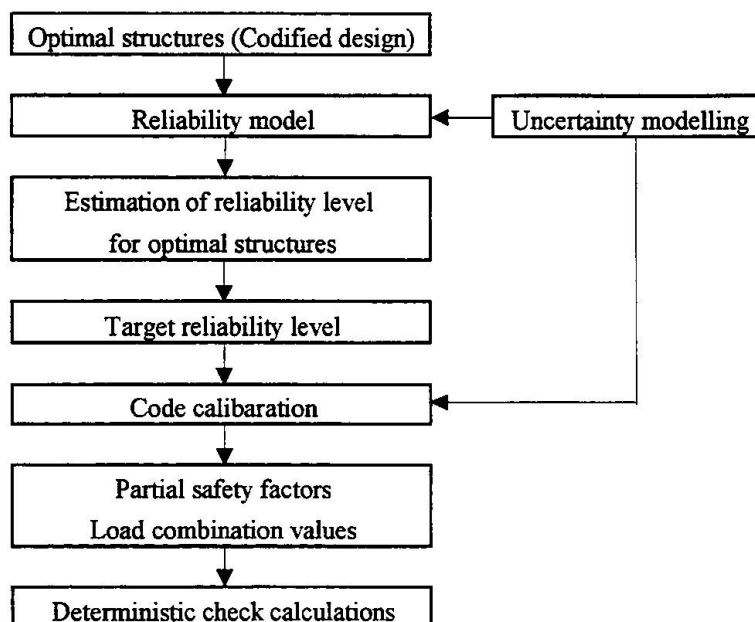
Fig. 1. Code Calibration Procedure

Using the approach in Fig. 1 for the setting of the target reliability level a number of issues has been addressed: 1) The interaction between the target reliability level and the uncertainty modelling has been included; 2) The target reliability level reflects the codified reliability level in each individual country; 3) The codified reliability level in different countries can be compared. This may give a rational basis for discussing the question about optimality of the individual national codes.

Optimal Structures (Code Design) The existing national codes, or at least some parts of the codes, express what the respective countries (or the engineering profession, perhaps) at present consider as being optimal design. Otherwise the national codes should be revised to fit with the prevailing professional anticipation of optimal design and the optimal reliability

level should become revised. Thus, a rational decision rule in connection with choosing target reliability levels can be set up from the postulate that existing codes, when applied to some types of structures, are optimal.

Uncertainty Modelling In connection with codified reliability analysis it is important to keep in mind the direct interaction between the chosen modelling of the uncertainties (choice of distribution, model uncertainties, etc.) and the target reliability index. Thus any possible codified target reliability index must be specified together with codified models for the uncertainties.

Reliability Model By means of a reliability model, the reliability level is evaluated in a combination of the limit states specified in the Eurocodes with the probabilistic models for the uncertain elements. Here the reliability evaluation is based on FORM, Ditlevsen and Madsen /1/, and Madsen, Krenk and Lind /4/.

Estimation of Reliability Level for Optimal Structures The basis for the estimation of the reliability level for optimal structures is a set of structures designed to the limit in accordance with the national codes. By analyzing the codified designs by means of a probabilistic model, the reliability indices β for each structure can be calculated. The probabilistic model shall be set up on the basis of the limit states defined in the Eurocodes. By this the national codified designs are evaluated by means of the code format given in the Eurocodes.

Target Reliability Level Since most of the partial safety factors specified in the various national codes have not been based on code calibration calculations and since the code format defined in the Eurocodes may differ from the code format used for the national codes, the calculated values of the reliability level for an individual national code will normally not be constant. However, a representative sample of structural elements designed to the limit on the basis of each individual countries national code can form the basis for choosing the target reliability level, Ditlevsen /6/.

Code Calibration The aim of the code calibration is to achieve a uniform reliability level within the different classes of structures. On the other hand, the code format must be operational (simple) and consequently the load combinations and partial safety factors shall not be too many. Some deviations from the target reliability index are therefore inevitable. The basis of the code calibration is a sample of structural elements designed through the reliability model to the target reliability level. The idea is to find the set of partial safety factors by use of which the structural design gives "the best approximation" to the reliability based designs, Ditlevsen and Madsen /1/.

Partial Safety Factors and Load Combination Values The solution of the code calibration is a set of partial safety factors and load combination values. Together with the limit states defined in the Eurocodes, these factors lead to structural designs that correspond to the target reliability index.

Deterministic Check Calculations (verification of results) Since the solution to the code calibration problem, ie. partial safety factors and load combination values, is obtained as "the best approximation" to the reliability based designs, the reliability level of designs

based on the calibrated partial safety factors and load combination values needs to be verified. This verification of the reliability level may also allow an evaluation of the level of safety differentiation in the code. If the reliability level differs significantly within a given class of structures, it might be appropriate to divide the class into a number of subclasses in order to obtain an improvement of the uniformity of the reliability level within the subclasses.

3   Example

Below the code calibration procedure is illustrated by means of a concrete beam subjected to shear forces. The shear capacity is defined by the variable strut method, EC2-1, /5/. For the code calibration only failure in the shear reinforcement is investigated. The area of the shear reinforcement is taken as the design parameter, but otherwise the geometry is fixed.

The total applied shear force is modelled through a linear influence model combining a dead load (G), a short and a long term environmental loads $(Q_{EnvS}$ and $Q_{EnvL})$, and a long and a short term imposed loads $(Q_{long}$ and $Q_{short})$.

Optimal structures (Codified design) The codified design of the concrete beam is made in accordance with the partial safety factors outlined in EC1-1 /3/ and EC2-1 /5/ for the Ultimate Limit State, persistent situation, see Table 1.

Variable	Unit	Characteristic value	Fractile value [%]	Partial safety factor	Load combination factor
Yield strength	N/mm^2	475	0.1	1.15	-
G	kN	30	mean	1.0/1.35	-
Q_{EnvS}	kN	20	98	1.5	0.6
Q_{EnvL}	kN	20	98	1.5	0.6
Q_{long}	kN	10	98	1.5	0.7
Q_{Short}	kN	20	98	1.5	0.7

Table 1. Characteristic values and partial safety factors

In order to create a number of codified designs several sets of influence coefficients are simulated by use of Monte Carlo simulation.

Uncertainty modelling In a reliability analysis the uncertain quantities are described by random variables. In the present example the uncertain quantities are the yield strength of the reinforcement, the loads and the model uncertainties.

The yield strength is assumed to be log-normally distributed with mean value 560 N/mm^2 and standard deviation 30 N/mm^2. The dead load is modelled as a normally distributed variable with a coefficient of variation of 0.08. The mean value is equal the characteristic

value, ie. 30 kN.

The instantaneous distribution of variable loads are defined as load pulse processes in line with NKB, /2/, by dividing the reference period (1 year) into time intervals of constant length, see Table 2. The yearly extreme value distributions have been obtained from the Poisson pulse occurrence model, Ditlevsen and Madsen /1/.

Variable	Type of distribution	k	λ [N^{-1}]	Occurrence probability per interval	Number of intervals per 1 year
Q_{EnvS}	Gamma	0.25	3.80 10^{-4}	1	730
Q_{EnvL}	Gamma	2.96	4.94 10^{-4}	0.583	12
Q_{long}	Gamma	8.93	1.61 10^{-4}	1	1
Q_{Short}	Gamma	2.47	4.15 10^{-4}	5.48 10^{-3}	730

Table 2. Distribution of load pulses.

In order to take model uncertainty into account the resistance and loading properties are in the present example multiplied by model uncertainty factors all with a mean value of one. The yield strength is multiplied by a log-normally distributed variable with a coefficient of variation of 0.09, the dead load is multiplied by a normally distributed variable with a coefficient of variation of 0.05 and the variable loads by a normally distributed variable with a coefficient of variation of 0.20.

Estimation of reliability level for optimal structures The basis for the estimation of the reliability level for optimal structures (codified design) is a set of cross-sections designed to the limit in accordance with the code defined by the partial safety factors and the limit state described above.

Turkstra's Rule, Madsen, Krenk and Lind /4/, is applied for the purpose of obtaining combinations of the random load processes. Each combination is in the calibration treated as a separate design case. The combinations together makes a series system for which the failure probability is approximated by the sum of failure probabilities for the combinations.

For a sample of 100 sets of influence coefficients, the reliability indices for the corresponding codified designs are shown in Fig. 2. The estimated mean value and standard deviation for the sample are found as 5.47 and 0.56, respectively.

Target reliability level There is no unique way of setting the target reliability level. For a detailed discussion of the issue reference is made to Ditlevsen /6/, Ditlevsen and Madsen /1/. In the present example the target reliability index is chosen as 5.5, that is close to the mean value.

Code calibration The code calibration is based on the design-value format, which is described in details in Ditlevsen and Madsen /1/.

For the purpose of this example the structures are divided into two classes. In the first class, A, only design cases in which the load combination "No variable load" is the dominating load combination are considered, whereas the second class, B, consists of design cases in which load combinations combining dead load and variable loads are the dominating load combinations.
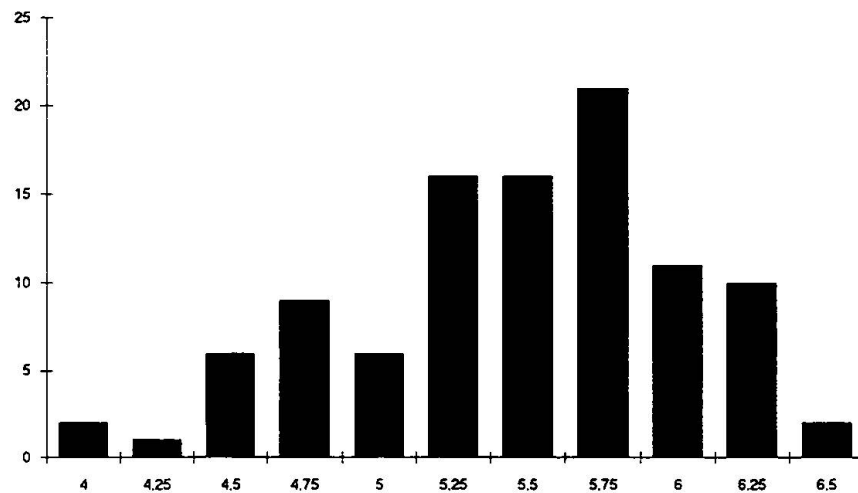

Fig 2. Histogram of reliability indices for codified design.

The reason for making the division of the design cases in these two categories is primarily that there is a relation between the ratio of the partial safety factor for the permanent load and the partial safety factors for the variable loads on the one side and the question of whether the design cases are dominated by permanent load or variable loads on the other side. If the dominance of permanent load is increased, the code calibration procedure will lead to an increase of the partial safety factor for permanent load and a decrease of the partial safety factors for the variable loads and vice versa.

Partial safety factors The results of the code calibration model, partial safety factors on the loads, taking the partial safety factor on reinforcement, γ_R, as 1.15,are given in Table 3.

Load Class	γ_R	γ_G	$\gamma_{Q,EnvS}$	$\gamma_{Q,EnvL}$	$\gamma_{Q,long}$	$\gamma_{Q,short}$	$\Psi_{0,EnvS}$	$\Psi_{0,EnvL}$	$\Psi_{0,long}$	$\Psi_{0,short}$
A	1.15	1.54	-	-	-	-	-	-	-	-
B	1.15	1.24/1.15	1.78	2.49	1.40	2.20	0.00	0.43	1.00	0.00

Table 3. Partial safety factors and load combinations factors adjusted to $\gamma_R = 1.15$.

Comparing the values in Table 3 with the values given in Table 1 it is seen that the partial safety factors for the variable loads in general are increased whereas the ψ_0 - factors are decreased. This raises a question in relation to the choice of the target reliability index based on a statement about code optimality in the case of specified unreasonably large ψ_0 -

factors. If the factors ψ_0 are specified too large the reliability level obtained by the load combination will increase with the number of variable loads included in the load combination.

As an illustration of this, the shear failure limit state is reconsidered for a situation with only dead load and short term environmental load acting. Taking the partial safety factors for the codified designs as above, the reliability indices for a sample of 100 simulated codified designs has been found with a mean value of 4.64 and the standard deviation is 0.51. It is seen that the use of the (unreasonably) large ψ_0 - factors lead to an increase of the reliability index from 4.64 for the situation with one variable load to 5.47 for the situation with four variable loads.

The large values of the partial safety factors listed in Table 3 are thus a direct consequence of the ψ_0 - factors specified in Table 1. The use of these ψ_0 - factor values implies the large target reliability index of 5.5, which, in turn, by the code optimization is transformed into the large values of the partial safety factors together with a decrease of the values of the ψ_0 - factors.

With the reservation for the coupling between the probabilistic model and the target reliability, the analysis indicates that if the target reliability level is required to approximatively 4.7, the Eurocode ψ_0 - factors appear to be too large rather than the partial safety factors appear to be too small.

From Table 3 it is further seen that there is a direct relation between the ratio of the partial safety factor for the permanent load and the partial safety factors for the variable loads on the one side, and on the other side the question of to what extent the design cases are dominated by permanent load or variable loads. In class A - dead load alone - it is seen that the partial safety factor for dead load is must larger (20%) than in the combination in which the dead load combined with the variable loads, class (B).

The value of 1.54 for the dead load in class A is due to the large target reliability index. However, if only class A is considered, the reliability index for the codified designs based on Table 1 and Table 2 is 4.48. This means that even if the target level is decreased to 4.7, as recommended in NKB /2/ and EC 1-1 /3/, a partial safety factor for the dead load of 1.35 in the situation with dead load as the only acting load is somewhat too small. It is noted that the coefficient of variation of 0.08 on the dead load may be slightly conservative, and thus the value of 1.35 for a target index of 4.7 may be appropriate for the situation with the dead load as the only load applied.

With respect to the value of the partial safety factor for the dead load in class B, it is seen that the Eurocode value of 1.35 is somewhat too large. In the present example it is approximately 9% too large. If the target index is lowered from 5.5 to 4.7 it will be even more than 9% too large.

It appears to be recommendable to introduce different partial safety factors for the dead load depending on to what extent the design case is dominated by the dead load (or the permanent load in general). Further, it appears that the ψ_0 - factors stated in the Eurocode are somewhat too large, especially in the case of several variable loads.

Deterministic check calculations The reliability level of the same sample of design cases as used as the basis for the code optimization has been evaluated. The results have shown a mean value of the reliability indices of 5.70, and the standard deviation is 0.39. This implies that the standard deviation is decreased from 0.56 to 0.39 by the code calibration process. However, the mean value of the reliability indices for the codified designs based on the partial safety factors and load combination values found in the code optimization is seen to be larger than the target reliability index, $\beta_t = 5.5$. Where the reliability indices of structural elements in class A in mean equals the target reliability index, the reliability indices in class B in the mean are larger that the target reliability index.

The key problem is that the most likely failure points for the different design cases may be situated in different direction in the space of random variables. In the design cases with different dominating loads the influence of other loads may differ substantial. Further, the extent to which a design case is dominated by the dead load or the permanent load may have a great influence on the ratio between the partial safety factor for dead load and the partial safety factors for the variable loads.

This may call for further separation of the design cases in class B, a separation which can be made dependent on the degree of dominance of the dead load. Alternatively and without introducing additional load combinations, a change of the division line between class A and class B may be considered. This last approach has been used on the Øresund Link Bridge, where the dead load and traffic loads are combined through two combinations - one in which the partial safety factor on the dead load is high and the factor on traffic load low - and one in which the partial safety factor on the dead load is low and the factor on traffic load high, /7/.

References

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