Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte

Band: 74 (1996)

Rubrik: Parallel session 1A: basis of design

Parallel Session 1A

Basis of Design

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Calibration of Partial Safety Factors

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Summary

Based on existing literature an overview about calibration of partial safety factors and loads combination values is presented. The aim is to recommend a standardized basis for calibration of partial safety factors. Such calibrations should be made in order to establish National Application Documents (NADs) and in order to determine partial safety factors and load combination values in the Eurocodes/NADs. The paper includes a specific example formulated to illustrate the described method for code calibration.

1. Introduction

The purpose of the code calibration on the present level of structural design practice is to achieve a uniform reliability level within the given groups of structures considered in the code. However, the code format must be operational (simple) and consequently the load combinations and partial safety factors shall not be too many. Some deviations from the target reliability level are therefore inevitable.

In this paper a method for minimisation of the deviations from the target reliability level is described. The method defined includes a way for setting the target reliability level as a function of the uncertainty modelling and common codified design practise.

The paper is based on a study performed by the authors for SAKO. SAKO is a Nordic group originally formed to harmonise structural codes in the Nordic countries. Since the development of Eurocodes has been initiated, SAKO has focused on this development. The objective of the study performed for SAKO was to formulated a rational way of determinating partial safety factors in the National Application Documents to the Eurocodes.

2. Code Calibration Procedure

In Fig. 1 the proposed procedure for code calibration is illustrated. Each of the steps in the procedure is described below.

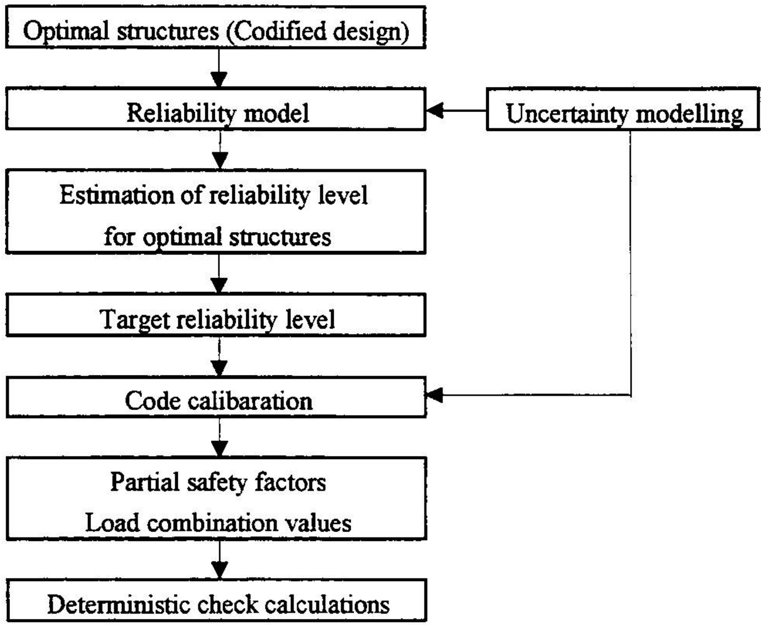

Fig. 1. Code Calibration Procedure

Using the approach in Fig. 1 for the setting of the target reliability level a number of issues has been addressed: 1) The interaction between the target reliability level and the uncertainty modelling has been included; 2) The target reliability level reflects the codified reliability level in each individual country; 3) The codified reliability level in different countries can be compared. This may give a rational basis for discussing the question about optimality of the individual national codes.

Optimal Structures (Code Design) The existing national codes, or at least some parts of the codes, express what the respective countries (or the engineering profession, perhaps) at present consider as being optimal design. Otherwise the national codes should be revised to fit with the prevailing professional anticipation of optimal design and the optimal reliability level should become revised. Thus, a rational decision rule in connection with choosing target reliability levels can be set up from the postulate that existing codes, when applied to some types of structures, are optimal.

Uncertainty Modelling In connection with codified reliability analysis it is important to keep in mind the direct interaction between the chosen modelling of the uncertainties (choice of distribution, model uncertainties, etc.) and the target reliability index. Thus any possible codified target reliability index must be specified together with codified models for the uncertainties.

Reliability Model By means of a reliability model, the reliability level is evaluated in a combination of the limit states specified in the Eurocodes with the probabilistic models for the uncertain elements. Here the reliability evaluation is based on FORM, Ditlevsen and Madsen /1/, and Madsen, Krenk and Lind /4/.

Estimation of Reliability Level for Optimal Structures The basis for the estimation of the reliability level for optimal structures is a set of structures designed to the limit in accordance with the national codes. By analyzing the codified designs by means of a probabilistic model, the reliability indices β for each structure can be calculated. The probabilistic model shall be set up on the basis of the limit states defined in the Eurocodes. By this the national codified designs are evaluated by means of the code format given in the Eurocodes.

Target Reliability Level Since most of the partial safety factors specified in the various national codes have not been based on code calibration calculations and since the code format defined in the Eurocodes may differ from the code format used for the national codes, the calculated values of the reliability level for an individual national code will normally not be constant. However, a representative sample of structural elements designed to the limit on the basis of each individual countries national code can form the basis for choosing the target reliability level, Ditlevsen /6/.

Code Calibration The aim of the code calibration is to achieve a uniform reliability level within the different classes of structures. On the other hand, the code format must be operational (simple) and consequently the load combinations and partial safety factors shall not be too many. Some deviations from the target reliability index are therefore inevitable. The basis of the code calibration is a sample of structural elements designed through the reliability model to the target reliability level. The idea is to find the set of partial safety factors by use of which the structural design gives "the best approximation" to the reliability based designs, Ditlevsen and Madsen /1/.

Partial Safety Factors and Load Combination Values The solution of the code calibration is a set of partial safety factors and load combination values. Together with the limit states defined in the Eurocodes, these factors lead to structural designs that correspond to the target reliability index.

Deterministic Check Calculations (verification of results) Since the solution to the code calibration problem, ie. partial safety factors and load combination values, is obtained as "the best approximation" to the reliability based designs, the reliability level of designs based on the calibrated partial safety factors and load combination values needs to be verified. This verification of the reliability level may also allow an evaluation of the level of safety differentiation in the code. If the reliability level differs significantly within a given class of structures, it might be appropriate to divide the class into a number of subclasses in order to obtain an improvement of the uniformity of the reliability level within the subclasses.

3 Example

Below the code calibration procedure is illustrated by means of a concrete beam subjected to shear forces. The shear capacity is defined by the variable strut method, EC2-1, /5/. For the code calibration only failure in the shear reinforcement is investigated. The area of the shear reinforcement is taken as the design parameter, but otherwise the geometry is fixed.

The total applied shear force is modelled through a linear influence model combining a dead load (G), a short and a long term environmental loads (Q_{EnvS} and Q_{EnvL}), and a long and a short term imposed loads (Q_{long} and Q_{short}).

Optimal structures (Codified design) The codified design of the concrete beam is made in accordance with the partial safety factors outlined in EC1-1 /3/ and EC2-1 /5/ for the Ultimate Limit State, persistent situation, see Table 1.

Variable	Unit	Characteristic value	Fractile value [%]	Partial safety factor	Load combination factor
Yield strength	N/mm^2	475	0.1	1.15	-
G	kN	30	mean	1.0/1.35	-
Q_{EnvS}	kN	20	98	1.5	0.6
Q_{EnvL}	kN	20	98	1.5	0.6
Q_{long}	kN	10	98	1.5	0.7
Q_{Short}	kN	20	98	1.5	0.7

Table 1. Characteristic values and partial safety factors

In order to create a number of codified designs several sets of influence coefficients are simulated by use of Monte Carlo simulation.

Uncertainty modelling In a reliability analysis the uncertain quantities are described by random variables. In the present example the uncertain quantities are the yield strength of the reinforcement, the loads and the model uncertainties.

The yield strength is assumed to be log-normally distributed with mean value 560 N/mm^2 and standard deviation 30 N/mm^2. The dead load is modelled as a normally distributed variable with a coefficient of variation of 0.08. The mean value is equal the characteristic value, ie. 30 kN.

The instantaneous distribution of variable loads are defined as load pulse processes in line with NKB, /2/, by dividing the reference period (1 year) into time intervals of constant length, see Table 2. The yearly extreme value distributions have been obtained from the Poisson pulse occurrence model, Ditlevsen and Madsen /1/.

Variable	Type of distribution	k	λ $[N^{-1}]$	Occurrence probability per interval	Number of intervals per 1 year
Q_{EnvS}	Gamma	0.25	$3.80\ 10^{-4}$	1	730
Q_{EnvL}	Gamma	2.96	$4.94\ 10^{-4}$	0.583	12
Q_{long}	Gamma	8.93	$1.61\ 10^{-4}$	1	1
Q_{Short}	Gamma	2.47	$4.15\ 10^{-4}$	$5.48\ 10^{-3}$	730

Table 2. Distribution of load pulses.

In order to take model uncertainty into account the resistance and loading properties are in the present example multiplied by model uncertainty factors all with a mean value of one. The yield strength is multiplied by a log-normally distributed variable with a coefficient of variation of 0.09, the dead load is multiplied by a normally distributed variable with a coefficient of variation of 0.05 and the variable loads by a normally distributed variable with a coefficient of variation of 0.20.

Estimation of reliability level for optimal structures The basis for the estimation of the reliability level for optimal structures (codified design) is a set of cross-sections designed to the limit in accordance with the code defined by the partial safety factors and the limit state described above.

Turkstra's Rule, Madsen, Krenk and Lind /4/, is applied for the purpose of obtaining combinations of the random load processes. Each combination is in the calibration treated as a separate design case. The combinations together makes a series system for which the failure probability is approximated by the sum of failure probabilities for the combinations.

For a sample of 100 sets of influence coefficients, the reliability indices for the corresponding codified designs are shown in Fig. 2. The estimated mean value and standard deviation for the sample are found as 5.47 and 0.56, respectively.

Target reliability level There is no unique way of setting the target reliability level. For a detailed discussion of the issue reference is made to Ditlevsen /6/, Ditlevsen and Madsen /1/. In the present example the target reliability index is chosen as 5.5, that is close to the mean value.

Code calibration The code calibration is based on the design-value format, which is described in details in Ditlevsen and Madsen /1/.

For the purpose of this example the structures are divided into two classes. In the first class, A, only design cases in which the load combination "No variable load" is the dominating load combination are considered, whereas the second class, B, consists of design cases in which load combinations combining dead load and variable loads are the dominating load combinations.

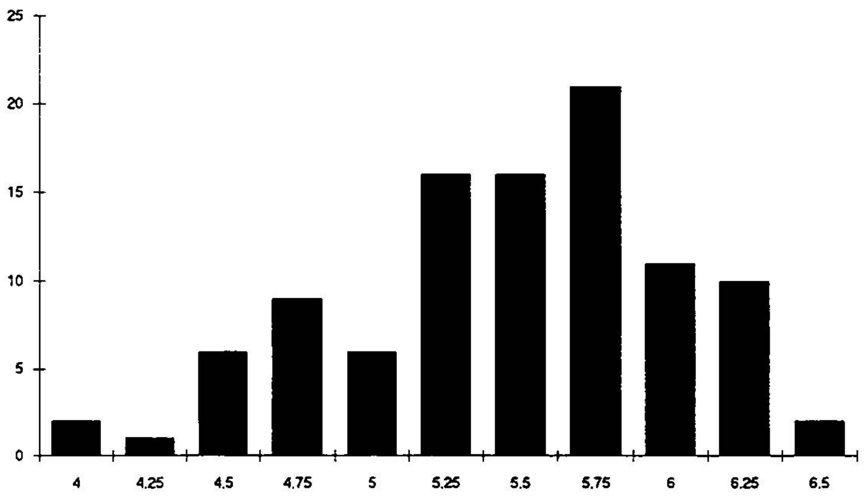

Fig 2. Histogram of reliability indices for codified design.

The reason for making the division of the design cases in these two categories is primarily that there is a relation between the ratio of the partial safety factor for the permanent load and the partial safety factors for the variable loads on the one side and the question of whether the design cases are dominated by permanent load or variable loads on the other side. If the dominance of permanent load is increased, the code calibration procedure will lead to an increase of the partial safety factor for permanent load and a decrease of the partial safety factors for the variable loads and vice versa.

Partial safety factors The results of the code calibration model, partial safety factors on the loads, taking the partial safety factor on reinforcement, γ_R, as 1.15,are given in Table 3.

Load Class	γ_R	γ_G	$\gamma_{Q,EnvS}$	$\gamma_{Q,EnvL}$	$\gamma_{Q,long}$	$\gamma_{Q,short}$	$\psi_{0,EnvS}$	$\psi_{0,EnvL}$	$\psi_{0,long}$	$\psi_{0,short}$
A	1.15	1.54	-	-	-	-	-	-	-	-
B	1.15	1.24/1.15	1.78	2.49	1.40	2.20	0.00	0.43	1.00	0.00

Table 3. Partial safety factors and load combinations factors adjusted to γ_R = 1.15.

Comparing the values in Table 3 with the values given in Table 1 it is seen that the partial safety factors for the variable loads in general are increased whereas the ψ_0 - factors are decreased. This raises a question in relation to the choice of the target reliability index based on a statement about code optimality in the case of specified unreasonably large ψ_0 -

factors. If the factors ψ_0 are specified too large the reliability level obtained by the load combination will increase with the number of variable loads included in the load combination.

As an illustration of this, the shear failure limit state is reconsidered for a situation with only dead load and short term environmental load acting. Taking the partial safety factors for the codified designs as above, the reliability indices for a sample of 100 simulated codified designs has been found with a mean value of 4.64 and the standard deviation is 0.51. It is seen that the use of the (unreasonably) large ψ_0 - factors lead to an increase of the reliability index from 4.64 for the situation with one variable load to 5.47 for the situation with four variable loads.

The large values of the partial safety factors listed in Table 3 are thus a direct consequence of the ψ_0 - factors specified in Table 1. The use of these ψ_0 - factor values implies the large target reliability index of 5.5, which, in turn, by the code optimization is transformed into the large values of the partial safety factors together with a decrease of the values of the ψ_0 - factors.

With the reservation for the coupling between the probabilistic model and the target reliability, the analysis indicates that if the target reliability level is required to approximatively 4.7, the Eurocode ψ_0 - factors appear to be too large rather than the partial safety factors appear to be too small.

From Table 3 it is further seen that there is a direct relation between the ratio of the partial safety factor for the permanent load and the partial safety factors for the variable loads on the one side, and on the other side the question of to what extent the design cases are dominated by permanent load or variable loads. In class A - dead load alone - it is seen that the partial safety factor for dead load is must larger (20%) than in the combination in which the dead load combined with the variable loads, class (B).

The value of 1.54 for the dead load in class A is due to the large target reliability index. However, if only class A is considered, the reliability index for the codified designs based on Table 1 and Table 2 is 4.48. This means that even if the target level is decreased to 4.7, as recommended in NKB /2/ and EC 1-1 /3/, a partial safety factor for the dead load of 1.35 in the situation with dead load as the only acting load is somewhat too small. It is noted that the coefficient of variation of 0.08 on the dead load may be slightly conservative, and thus the value of 1.35 for a target index of 4.7 may be appropriate for the situation with the dead load as the only load applied.

With respect to the value of the partial safety factor for the dead load in class B, it is seen that the Eurocode value of 1.35 is somewhat too large. In the present example it is approximately 9% too large. If the target index is lowered from 5.5 to 4.7 it will be even more than 9% too large.

It appears to be recommendable to introduce different partial safety factors for the dead load depending on to what extent the design case is dominated by the dead load (or the permanent load in general). Further, it appears that the ψ_0 - factors stated in the Eurocode are somewhat too large, especially in the case of several variable loads.

Deterministic check calculations The reliability level of the same sample of design cases as used as the basis for the code optimization has been evaluated. The results have shown a mean value of the reliability indices of 5.70, and the standard deviation is 0.39. This implies that the standard deviation is decreased from 0.56 to 0.39 by the code calibration process. However, the mean value of the reliability indices for the codified designs based on the partial safety factors and load combination values found in the code optimization is seen to be larger than the target reliability index, $\beta_t = 5.5$. Where the reliability indices of structural elements in class A in mean equals the target reliability index, the reliability indices in class B in the mean are larger that the target reliability index.

The key problem is that the most likely failure points for the different design cases may be situated in different direction in the space of random variables. In the design cases with different dominating loads the influence of other loads may differ substantial. Further, the extent to which a design case is dominated by the dead load or the permanent load may have a great influence on the ratio between the partial safety factor for dead load and the partial safety factors for the variable loads.

This may call for further separation of the design cases in class B, a separation which can be made dependent on the degree of dominance of the dead load. Alternatively and without introducing additional load combinations, a change of the division line between class A and class B may be considered. This last approach has been used on the Øresund Link Bridge, where the dead load and traffic loads are combined through two combinations - one in which the partial safety factor on the dead load is high and the factor on traffic load low - and one in which the partial safety factor on the dead load is low and the factor on traffic load high, /7/.

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Reliability analysis of a reinforced concrete column designed according to the Eurocodes

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Summary

Reliability analysis of a built in reinforced concrete column designed according to Eurocodes 1 and 2 is a part of an extended research activity on Eurocode Random Variable Models supervised by JCSS. Presented results indicate that the reliability level of reinforced concrete columns designed according to the present generation Eurocodes may considerably vary depending on actual arrangement of the structure. To harmonise reliability levels provided by the Eurocodes for various structural members further research and calibration is required.

1. Introduction

Reliability analysis of reinforced concrete columns is part of an extensive research activity on Eurocode Random Variable Models supervised by the Joint Committee for Structural Safety JCSS [1]. The whole project covers reliability analysis of different structural members of a model multi-storey frame structure made of concrete or steel. The JCSS aims at providing a standardised set of statistical models for loads and structural properties which would reflect the present state of knowledge. Where necessary, the models should be adjusted in the future. It is expected that these models will be used as a practical design tool in conjunction with a probabilistic design criterion.

In a probabilistic design procedure a decision theoretical approach seems to be the most natural. However, as the models are only partly based on the experimental data, the calculated failure probabilities should not be identified directly with actual failure frequencies. That is why reliability criteria are usually defined through calibration to existing practice. In such a calibration procedure a set of structural elements are designed according to current design practice. For each of these elements the failure probability or reliability index is calculated, using the set of standardised statistical models. The resulting reliability indices may be then used as target reliability for the subsequent probabilistic design procedure. In such a way a combination of mechanical models, statistical models and corresponding target reliability which renders on the average the same design as current practice procedures may be derived.

This contribution presents preliminary results of reliability analysis of a built in reinforced concrete column designed according to newly developing Eurocode 1 [2, 3 and 4] and

Eurocode 2 [5]. The reliability analysis has been carried out using software product COMREL [6] developed by RCP München. It is expected that submitted investigation will contribute to desired calibration and possible future improvement of present generation of Eurocodes.

2. Structural characteristics

A model multi-storey structure considered in the this study is schematically shown in Fig. 1. It is assumed that each plenary frame in the transversal direction of the structure may be considered as unbraced sway frame. These transversal sway frames consist of four columns at a constant distance a_1; in the longitudinal direction of the structure they are located within a constant distance a_2 (see Fig. 1). The columns are considered as fully clamped in booth ends, at the top and at the bottom.

In the following reliability analysis of the edge column of an internal transversal frame having the height L and rectangular cross section $b \times h$ is considered. The cross section dimensions are chosen in such a way that the height h is two times (in one study case three times) the width b, thus $h/b = 2$ or 3. Considering different structural arrangements the total of 12 study cases indicated in Table 1 are analysed.

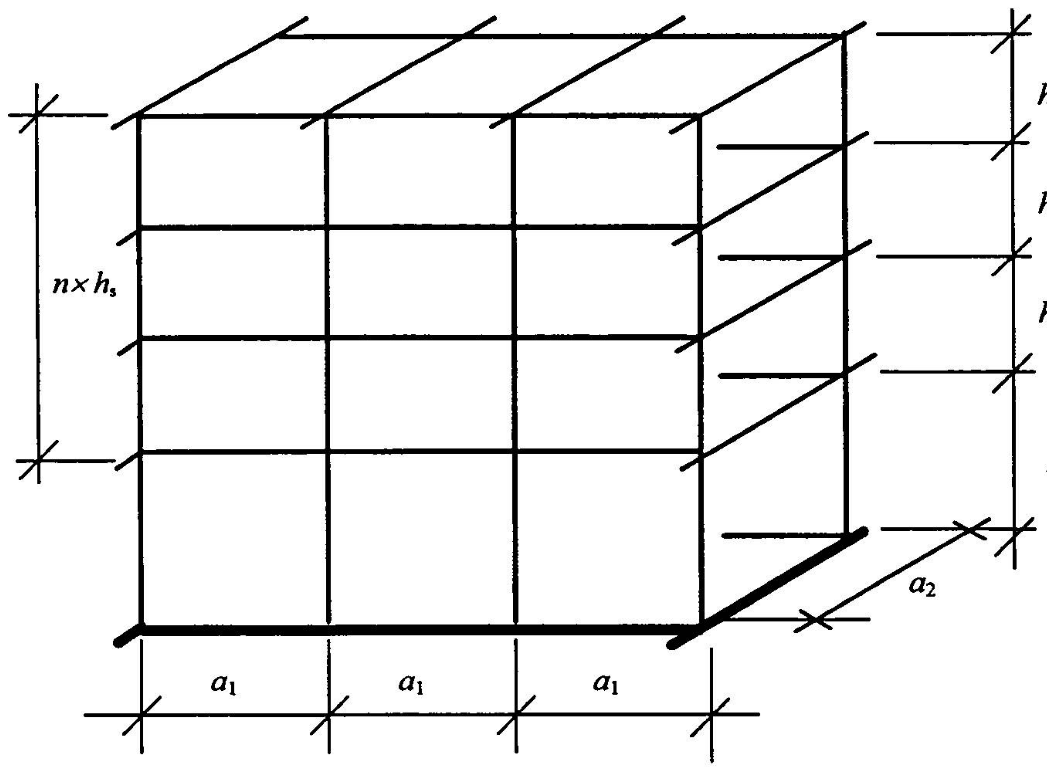

Fig. 1. Transversal frame of a multi-storey structure.

Study case	Number of storeys above the column n	Height of the analysed column L [m]	Transversal distance of columns a_1 [m]	Longitudinal distance of columns a_2 [m]	Cross section dimensions: width × height $b \times h$ [m×m]
1	10	6	5	5	0,35 × 0,70
2	10	3	5	5	0,25 × 0,50
3	10	9	5	5	0,35 × 0,70
4	10	12	5	5	0,45 × 0,90
5	10	6	4	5	0,35 × 0,70
6	10	6	7	5	0,35 × 0,70
7	10	6	5	4	0,30 × 0,60
8	10	6	5	7	0,40 × 0,80
9	1	6	5	5	0,25 × 0,50
10	3	6	5	5	0,25 × 0,50
11	20	6	5	5	0,40 × 0,80
12	10	6	5	5	0,25 × 0,75

Table 1. Study cases of a built in column.

Further it is assumed that the story height above the considered column is $h_s = 3$ m, permanent load is determined assuming reinforced concrete floor of a uniform equivalent thickness of 0.30 m (representing weight due to slab, columns, beams, floor and cladding).

3. Effect of actions

Effects of actions considered in the analysis of built in column consist of the axial force and bending moment, denoted again by N and M with appropriate subscripts. In the design calculation, the axial force and bending moment are represented by the design values N_d and M_d respectively. The maximum design axial force $N_{d,max}$ is given as

$$N_{d,max} = \gamma_G\ N_{W,k} + \gamma_Q \max \{N_{imp,k} + \psi_0\, N_{wind,k}\ ;\ N_{wind,k} + \psi_0 N_{imp,k}\} \tag{1}$$

where $\gamma_G = 1{,}35$ is the partial factor for permanent actions, $\gamma_Q = 1{,}50$ is the partial factor for the variable actions, ψ_0 is the factor for combination value, $N_{W,k}$ is the characteristic value of the axial force due to self weight, $N_{imp,k}$ is the characteristic value due to imposed load and $N_{wind,k}$ is the characteristic value due to wind action (positive values are accepted for compressive forces). The minimum design axial force $N_{d,min}$ is given as

$$N_{d,min} = \gamma_G\ N_{W,k} - \gamma_Q\, N_{wind,k} \tag{2}$$

where $\gamma_G = 1{,}00$ is the partial factor for favourable permanent actions, $\gamma_Q = 1{,}50$ is the partial factor for the variable actions.

Taking into account arrangement of the structure indicated in Fig. 1 the characteristic value due to self weight of n floors and one roof is given as

$$N_{W,k} = (n+1) a_1 a_2 t\, \rho_c / 2 \tag{3}$$

where ρ_c is the weight of concrete per unit volume considered as 0,024 MN/m^3. $N_{imp,k}$ is the characteristic value of imposed load from n floors given as

$$N_{\text{imp,k}} = n\, a_1 a_2\, p_{\text{imp}} / 2 \tag{4}$$

Choosing a category B (Public Building) the characteristic value of floor imposed load $p_{\text{imp k}}$ equals 3 kN/m^2. For $n > 1$ the load reduction according to Eurocode 1 [3] should be included. $N_{\text{wind,k}}$ is the wind resulting from a pressure $C_p\, G\, p_{\text{wind k}}$ on a vertical area equal to $(L + nh_s)\, a_2$; multiplication by the height $(L + nh_s)/2$ gives the overturning moment. This moment is assumed to be balanced by the normal forces in the two outer columns, so:

$$N_{\text{wind, k}} = (1/2)(L + nh_s)^2 a_2 C_p G\, p_{\text{wind ,k}} / (3\, a_1) = 0.271(L + nh_s)^2 a_2 / a_1 \tag{5}$$

where the characteristic value of the wind action is taken for the return period of 50 years as $p_{\text{wind,k}} = 0.5$ kN/m^2; further for the gust (exposure) factor the value $G = 2.5$ and for the shape factor the value $C_p = 0.8 + 0.5 = 1.3$ is chosen [4].

The design value M_d of the bending moment M is given as

$$M_d = M_{d0} + N_d(\, e_a + e_2) = N_d(\, e_0 + e_a + e_2) \tag{6}$$

where M_{d0} is the first order bending moment, $e_0 = M_{d0} / N_d$ is the first order eccentricity, e_a is the additional eccentricity taking into account geometric imperfections and e_2 is the second order eccentricity taking into account deformations of the column.

It is assumed that the first order moment M_{d0} is caused only by wind action, which is transmitted in each frame section of the width a_2 (see Fig.1) equally by the four columns fully clamped in and, therefore, the maximum first order bending moment M_{d0} due to wind load about the centroid of a column cross section is determined from the formula

$$M_{d0} = L[\gamma_Q\, C_p\, G\, p_{\text{wind,k}}\, (L{+}nh_s)\, a_2]/8 = 0{,}305\;\; L(L + nh_s)\, a_2 \tag{7}$$

where L denotes the column height.

The eccentricities e_a and e_2 are determined in accordance with Chapter 2 and 4 of Eurocode 2 [5]. The additional eccentricity e_a is given as $e_a = \nu_a\, l_0/2$, where l_0 denotes the effective length of the column considered here by the lowest recommended value 1,12 L (for the case of a column of a sway frame), ν_a inclination from the vertical given by the minimum value 1/200 which is valid for all structures higher than 4 m when the second order effects are taken into account. Thus

$$e_a = 1{,}12\, L /(2 \times 200) = 0{,}0028\, L \tag{8}$$

The second order eccentricity e_2 is dependent on the characteristics of the column cross section and should be generally determined by an iteration process. In accordance with equation (4.69) in [5] the second order eccentricity is given as

$$e_2 = 0{,}1 K_1\, l_0^{\,2}\, (1/\, r) \tag{9}$$

where the coefficient K_1 depends on the slenderness ratio $\lambda = l_0 / i$ (i being radius of gyration) and is given by equations (4.70) and (4.71) in Eurocode 2 [5]. As in the all study cases here $\lambda \geq 35$ the value $K_1 = 1$ is considered. The curvature $1/r$ is given by equation (4.72) in [5] as

$$1/r = 2\, K_2\, \varepsilon_{yd} / (0{,}9\;\; (h - d_1)) \tag{10}$$

where the coefficient K_2 is defined by equation (4.73) in [5] as follows

$$K_2 = (N_{ud} - N_d) / (\, N_{ud} - N_{\text{bal,d}}) \;\leq 1 \tag{11}$$

where N_{ud} is the design capacity of the cross section, N_d is the design axial force and $N_{bal,d}$ is the force which maximises the ultimate moment of the cross section; in this study for symmetrical reinforcement $N_{bal,d}$ = 0,5 $\alpha f_{cd} A_c$, where α is a coefficient taking account of long term effects on the compressive strength.

The remaining variables entering equation (10), the design yield strength $\varepsilon_{yd} = f_{yd} / E_s$ and the effective depth of cross section h - d_1, are specified bellow (see also Fig. 2). Table 2 and 3 shows the resulting values of the effects of actions for all 12 study cases considered here.

Study case	$N_{d,max}$ [MN]	M_{d0} [MNm]	e_0 [m]	L [m]	e_a [m]	$A_s \times 10^4$ [m^2]	A_s/bh [%]	e_2 [m]	M_d [MNm]
1	2,162	0,329	0,1522	6	0,0168	28,7	1,17	0,0245	0,418
2	2,078	0,151	0,0726	3	0,0084	22,1	1,23	0,0047	0,178
3	2,054	0,535	0,2373	9	0,0252	34,1	1,07	0,0591	0,725
4	2,353	0,768	0,3263	12	0,0336	38,2	0,94	0,1062	1,098
5	1,967	0,329	0,1673	6	0,0168	24,6	1,00	0,0265	0,415
6	2,736	0,329	0,1201	6	0,0168	41,4	1,69	0,0200	0,431
7	1,729	0,263	0,1523	6	0,0168	31,9	1,77	0,0285	0,343
8	3,028	0,461	0.1522	6	0,0168	37,4	1,17	0,0196	0,572
9	0,340	0,082	0,2422	6	0,0168	4,6	0,37	0,0485	0,105
10	0,702	0,137	0,1954	6	0,0168	10,9	0,87	0,0485	0,183
11	4,895	0,603	0,1232	6	0,0168	90,7	2,83	0,0141	0,755
12	2,162	0,329	0,1522	6	0,0168	37,5	2,00	0,0191	0,407

Table 2. Effects of actions for the maximum axial force $N_{d,max}$.

Study case	$N_{d,max}$ [kN]	M_{d0} [MNm]	e_0 [m]	L [m]	e_a [m]	$A_s \times 10^4$ [m^2]	A_s/bh [%]	e_2 [m]	M_d [MNm]
1	0,464	0,329	0,7100	6	0,0168	17,9	0,73	0,0346	0,353
2	0,548	0,151	0,2755	3	0,0084	4,0	0,22	0,0101	0,161
3	0,372	0,535	1,4374	9	0,0252	31,4	0,98	0,0682	0,589
4	0,273	0,768	2,8125	12	0,0336	44,2	1,09	0,1078	0,806
5	0,134	0,329	2,4649	6	0,0168	24,0	0,98	0,0346	0,336
6	1,001	0,329	0,3289	6	0,0168	12,9	0,53	0,0346	0,381
7	0,372	0,263	0,7077	6	0,0168	18,6	1,03	0,0404	0,285
8	0,650	0,461	0.7093	6	0,0168	20,0	0,63	0,0303	0,491
9	0,147	0,082	0,5596	6	0,0168	6,8	0,54	0,0485	0,092
10	0,269	0,137	0,5106	6	0,0168	11,6	0,93	0,0485	0,155
11	0,120	0,603	5,0273	6	0,0168	40,5	1,27	0,0303	0,609
12	0,464	0,329	0,7100	6	0,0168	16,6	0,89	0,0323	0,352

Table 3. Effects of actions for the minimum axial force $N_{d,min}$.

4. Material characteristics

The following materials characteristics for concrete and reinforcing steel are considered in the deterministic design of reinforced concrete columns. Concrete class C 20/25 having the characteristics

$$f_{ck} = 20 \text{ MPa}, \gamma_c = 1{,}5, f_{cd} = 13{,}33 \text{ MPa}, \alpha = 0{,}85 \quad (12)$$

is considered here. It should be noted that the coefficient α equal to one is considered in some countries. Reinforcing steel S 500 having the strength values

$$f_{yk} = 500 \text{ MPa}, \gamma_s = 1{,}15, f_{yd} = 435 \text{ MPa} \quad (13)$$

is considered. Assuming further the modulus of elasticity E_s = 200 GPa, the design yield strain ε_{yd} = 2,17 ‰ corresponds to the yield strength f_{yd} given above.

5. Deterministic design

The following simplifications are accepted for design of column cross sections (see figure 2):
- symmetrical reinforcement ($A_{s1} = A_{s2} = A_s / 2$) is considered only,
- the square shape of the column cross section having dimensions h and b rounded to 5×10^{-2} m are chosen such that h/b = 2 (in the last study case h/b = 3).
- distance of reinforcing bars from the edge is chosen as $d_{1(2)} = 0.1\ h$.

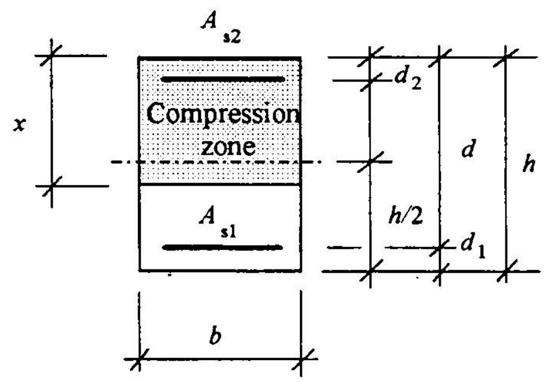

Fig. 2. Column cross section.

For given design values of the normal forces N_d and bending moments M_d, the column cross sections are designed using simplified interaction diagram described by the following formula:
for $N_d < \alpha\, b\, h f_{cd} / 2$

$$[A_s f_{yd}\, (h - 2d_1) + h\, N_d\, (1 - N_d / (\alpha\, b\, h f_{cd})] / 2\ -\ M_d > 0 \quad (14)$$

for $N_d > \alpha\, b\, h f_{cd} / 2$

$$K_2\, [A_s f_{yd}\, (h - 2d_1) / 2 + \alpha\, b\, h^2 f_{cd} / 8]\ -\ M_d > 0 \quad (15)$$

$$K_2 = (N_{ud} - N_d) / (N_{ud} - N_{bal,d}) \tag{16}$$

$$N_{ud} = \alpha \; b\,h\,f_{cd} + A_s f_{yd} \tag{17}$$

$$N_{bal,d} = \alpha \; b\,h\,f_{cd} / 2 \tag{18}$$

These relationships approximate well interaction diagrams derived from appropriate rules of Eurocode 2 [5] and, because of their simplicity, shall be used in the following reliability analysis. Moreover, detail analysis show that in common cases the ultimate bending moment given by these relationships is mostly on the safe side and differs insignificantly (by less than few percent) from that obtained by more accurate procedure based on Eurocode 2 [5]. The total reinforcement area A_s should satisfy the conditions of clause 5.4 in [5]:

$$0{,}15\,|N_d| \,/ f_{yd} < A_s, \quad 0{,}003\; b\,h < A_s < 0{,}08\; b\,h \tag{19}$$

which specifies the minimum and maximum reinforcement ratio.

Using relationships (14) to (18.), material properties given by equations (12) and (13) and the design values of effects of actions described by equations (1) to (11), the resulting reinforcement areas A_s and rations A_s / bh shown in Table 2 and 3 have been obtained for the maximum axial force $N_{d,max}$ and the minimum axial forces $N_{d,min}$ respectively. Note that the reinforcement areas A_s given in Table 2 and 3 satisfy the conditions (19) required by Eurocode 2 [5]. Theoretical values of reinforcement area A_s rounded upward to the last digit indicated in Table 2 and 3, which do not correspond to any specific bar size, shall be considered in the following reliability analysis.

It follows from Tables 2 and 3 that in the study cases 4, 9 and 10 the greater reinforcement areas follow from the design situation corresponding to the minimum axial force $N_{d,min}$; this reinforcement should be used. However, to show the effect of the design procedure considering the maximum axial force $N_{d,max}$ only, both reinforcement areas (the greater due to the minimum axial force and smaller due to the maximum axial force) are considered in the following reliability analysis of the study cases 4, 9 and 10.

6. Limit state function

In the time variant reliability analysis the actual axial force N is considered as a simple sum of actual axial forces due to all the considered actions:

$$N = N_W + N_{imp} + N_{wind} \tag{20}$$

where N_W is the axial force due to self weight, N_{imp} is the axial force due to imposed load and N_{wind} is the axial force due to wind action (positive values are again accepted for compressive forces). Thus, the time variant reliability analysis presented here concerns only the permanent design situation with the maximum axial force (corresponding to $N_{d,max}$ given by (1)).

The bending moment M is given by equation (6) used in the design calculation in which actual values are applied instead of the design values and a new additional eccentricity e_a are considered, thus

$$M = M_0 + N(\,e_a + e_2) = N(\,e_0 + e_a + e_2) \tag{21}$$

where the first order eccentricity $e_0 = M_0 / N$, where M_0 is given as

$$M_0 = L[C_p\, G\, p_{wind}\, (L+nh_s)\, a_2]/8 \tag{22}$$

The additional eccentricity e_a is given in terms of the initial sway ζ as

$$e_a = \zeta L/2 \tag{23}$$

where ζ is given in Table 4. The second order eccentricity e_2 is given by modified equations (9) in which $l_0 = L$ (the minimum value $l_0 = 1{,}12\, L$ required by Eurocode 2 [5] is neglected in the reliability analysis), thus

$$e_2 = 0{,}1\, K_1 L^2 (1/r) \tag{24}$$

where $K_1 = 1$ and r is given by equation (10), in which, again, actual values of basic variables shall be used instead of the design values.

The limit state function g may be expressed as the difference of resistance bending moment and the actual bending moment about the centroid.

$$g = \xi_R M_R - \xi_E M \tag{25}$$

Two coefficients of model uncertainties ξ_R and ξ_E are considered as random variables to cover imprecision and incompleteness of the relevant theoretical models. Taking into account (15) to (18) the limit state function (25) becomes
for $N < \alpha\, b\, h\, f_c / 2$

$$\xi_R\, [A_s f_y\, (h - 2d_1) + h\, N\, (1 - N / (\alpha\, b\, h f_c))] / 2 - \xi_E M > 0 \tag{26}$$

for $N > \alpha\, b\, h\, f_c / 2$

$$\xi_R\, \kappa\, [A_s f_y\, (h - 2d_1) / 2 + \alpha\, b\, h^2 f_c / 8] - \xi_E M > 0 \tag{27}$$

$$\kappa = (N_u - N) / (N_u - N_{bal}) \tag{28}$$

$$N_u = \alpha\, b\, h f_c + A_s f_y \tag{29}$$

$$N_{bal} = \alpha\, b\, h f_c / 2 \tag{30}$$

The limit state function given by equations (26) to (30) is applied in the reliability analysis of the column in conjunction with appropriate probabilistic models for basic random variables described bellow.

7. Statistical properties of basic variables

Basic variables applied in the reliability analysis are listed in Table 4. Note that the initial overall sway ζ_0 (which is not used in the design - see note (1) below Table 4) is applied now in the reliability analysis of the column. Some of the basic variables are assumed to be deterministic values - denoted "DET" (A_s, E_s, a_1, a_2, L, and n), the others are considered as random variables having the normal distribution - "N", lognormal distribution - "LN", Gumbel distribution - "GUM" and Gamma distribution - "GAM". Statistical properties of the random variables are further described by the moment characteristics, the mean and standard deviation, partly taken from CIB Reports [7] and [8].

Category of basic var.	Symbol	Name of basic variable	Distrib type	Dimen.	Mean	Standard deviation
Material properties	α	reduction factor	N	-	0,85	0,085
	A_s	reinforcement area	DET	m^2	nom	0
	f_c	concrete strength	LN	Mpa	30	5
	f_y	yield strength	LN	Mpa	560	30
	E	modulus of elasticity	DET	GPa	200	0
Geometric data	a_1	column distance in plane	DET	m	nom	0
	a_2	perpend. dist. of column	DET	m	nom	0
	b	width of cross section	N	m	nom	0,005
	$d_{1(2)}$	distance of bars from edge	N	m	0.1h+0.00	0,005
	h	height of cross section	N	m	nom	0,005
	L	height of column	DET	m	nom	0
	n	number of floors	DET	-	nom	0
	ζ	initial overall sway$^{(1)}$	N	rad	0	$0{,}0015^{(1)}$
Model uncertainty	ξ_E	uncertainty of load	N	-	1,0	0,1
	ξ_R	uncertainty of column	N	-	1,1	0,11
Actions	ρ	weight of reinf. concrete	N	MNm^{-2}	0,0240	0,00192
	C_p	shape coefficient	LN	-	1,0	0,15
	G	gust factor	GUM	-	2,5	0,25
	p_{wind}	wind pressure	GUM	MNm^{-2}	0,00035	$0{,}00006^{(2)}$
	p_{impl}	imposed long term load	GAM	MNm^{-2}	0,0006	ean $\times\ v^{(3)}$
	p_{imps}	imposed short term load	GAM	MNm^{-2}	0,0002	ean $\times\ v^{(4)}$

Notes:
(1) The initial overall sway ζ is used to calculate the additional eccentricity e_a of the built in column according to equation (23).
(2) The mean and standard deviation correspond to the distribution of one year maximum.
(3) The mean and standard deviation correspond to the distribution of 7 years maximum; $v^2=(0{,}16+8/(a_1\ a_2))(1/n+\rho\ (1-1/n))$ (see CIB report [8]), where the coefficient of correlation of the long term loads in two floors is considered as $\rho = 0.5$ (see also table 5).
(4) The mean and standard deviation correspond to the distribution of the 12 hours (one day) maximum, $v^2= 50/(a_1\ a_2)$ (see also table 5).

Table 4. Statistical properties of basic variables for built in column.

Study case	$A_s \times 10^4$ [m²]	a_1 [m]	a_2 [m]	n	$\sigma_{p,\text{impl}}$ [MN/m²]	$\sigma_{p,\text{imps}}$ [MN/m²]
1	24,3	5	5	10	0,00031	0,00028
2	28,2	5	5	10	0,00031	0,00028
3	46,4	5	5	10	0,00031	0,00028
4	28,5	5	5	10	0,00031	0,00028
5	23,2	4	5	10	0,00033	0,00032
6	30,1	7	5	10	0,00028	0,00024
7	26,1	5	4	10	0,00033	0,00032
8	31,1	5	7	10	0,00028	0,00024
9	5,3	5	5	1	0,00042	0,00028
10	9,4	5	5	3	0,00034	0,00028
11	73,8	5	5	20	0,00030	0,00028
12	29,8	5	5	10	0,00031	0,00028

Table 5. Standard deviation $\sigma_{p,impl}$ and $\sigma_{p,imps}$ of the imposed loads.

8. Reliability analysis

Time variant reliability analysis is based on the Borges - Castanheta model for wind action, long term and short term imposed loads indicated in Fig. 3 (see also [1]). Program COMREL-JP [6] have been applied for time variant reliability analysis (jump process) of the columns assuming life time of 50 years and the probabilistic models given in Table 4 and 5.

The wind load is modelled as a sequence of independent rectangular pulses, each pulls having a duration of approximately 1 day. The statistical properties of the pulls intensity is tuned in such a way that the maximum pressure in a year has a distribution specified in Table 4. The long term imposed load is defined for the interval of 7 years. It is assumed to be changed simultaneously on all floors of a building. The short term load is present during one interval of 1 day in each year; the simultaneous occurrence of short term imposed loads on more than 1 floor at the same time may be neglected; so an independent short term single floor load imposed on the column occurs n times a year, n being the number of floors. Note that long term loads are considered as being correlated over various floors.

In the first type of the time variant analysis the short term action was assumed to be absent, $p_{\text{imps}} = 0$, and only wind action p_{wind} and long term imposed load p_{impl}, were considered as time dependent ergodic and stationary random variables. As the statistical properties of the wind action p_{wind} given in Table 4 refer to the distribution of one year maximum values and properties of the long term imposed load p_{impl} refer to 7 years maximum, the "jump rates" (number of jumps within one year) $\lambda_{p,\text{wind}}$ and $\lambda_{p,\text{impl}}$ of the rectangular wave renewal jump process were considered as follows:

$$\lambda_{p,\text{wind}} = 1{,}0/\text{year}\ ;\ \lambda_{p,\text{impl}} = 0{,}143/\text{year} \tag{31}$$

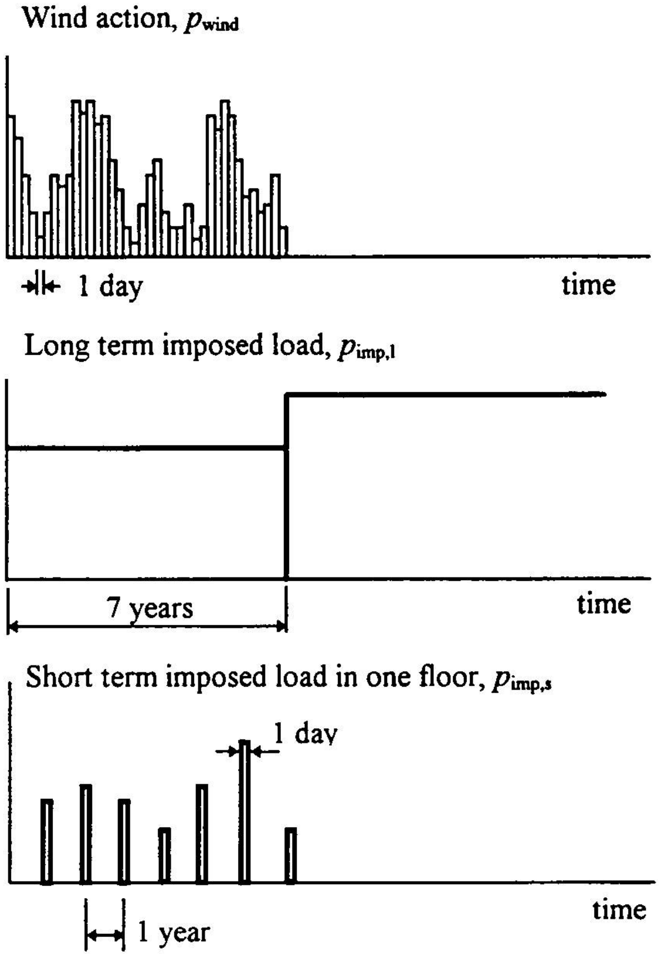

Fig. 3. Models of actions for time variant reliability analysis.

The second type of the time variant analysis concerns the period of time when the short term imposed load p_{imps} is present. As already mentioned above it is assumed that in each floor the short time imposed load may independently occur once a year. Thus, in every year there is n days, where n is the number of floors, when the short time load is active. The total number of 'active' days during the assumed life time of 50 years is therefore 50 n. This period is considered now as the total time of the time variant reliability analysis. One day is considered now as a unit of time. Jump rate of the short term imposed load p_{imps} is thus $\lambda_{p,imps}$ = 1,0/day.

Taking into account properties of the Gumbel distribution, statistical properties of the wind action p_{wind} were adjusted to one day period as follows

$$\mu_{day} = \mu_{year} - 0{,}78\ \sigma_{year} \ln(365) = 0{,}00035\text{-}0{,}00028\text{=}0{,}00007\ \text{MN/m}^2\ ,\ \sigma_{day} = \sigma_{year} \qquad (32)$$

Jump rate of the wind action p_{wind} is thus $\lambda_{p,wind}$ = 1,0/day.

Statistical parameters of the long term imposed load p_{impl} given in Table 4 for 7 years correspond now to the period of *7n* "active" days (one year is "compressed" to *n* "active days"). Appropriate jump rate $\lambda_{p,impl}$ (number of jumps within one active day) is therefore

$$\lambda_{p,impl} = 1/\,(7\,n)\,/\,\text{day} \tag{33}$$

Using the FORM methods of probability integration [6], resulting values of the reliability index β_1 and β_2 of the first and second type of reliability analysis respectively for the 12 study cases are given in Table 6.

Study case	Reinforcement area	Reinforcement ratio	Cross section dimensions	Column height	Time variant analysis, short term load not present	Time variant analysis, short term load present
	$A_s \times 10^4$ [m^2]	A_s/bh [%]	$b \times h$ [m]	L [m]	β_1	β_2
1	28,7	1,17	0,35×0,70	6	5,6	6,1
2	22,1	1,23	0,25×0,50	3	4,7	5,3
3	34,1	1,07	0,35×0,70	9	4,0	4,6
4(1)	44,2 (38,2)	1,09 (0,94)	0,45×0,90	12	4,5 (4,2)	5,1 (4,8)
5	24,0	1,00	0,35×0,70	6	5,3	5,8
6	41,4	1,69	0,35×0,70	6	6,1	6,5
7	31,9	1,77	0,30×0,60	6	5,5	6,0
8	37,4	1,17	0,40×0,80	6	5,7	6,2
9(1)	6,8 (4,6)	0,54 (0,37)	0,25×0,50	6	3,7 (2,9)	4,9 (4,2)
10(1)	11,6 (10,9)	0,93 (0,87)	0,25×0,50	6	3,9 (3,8)	4,8 (4,7)
11	90,7	2,83	0,40×0,80	6	5,6	6,0
12	37,5	2,00	0,25×0,75	6	5,6	6,2

Note: (1) In the study cases 4, 9 and 10 the reinforcement area is designed considering the minimum axial force $N_{d,min}$ due to permanent load and wind action only (imposed load being absent); values given in brackets ( ) correspond to the design considering the maximum axial force $N_{d,max}$.

Table 6. Reliability indices β_1, and β_2 of time variant analysis for built in column.

It follows from Table 6 that obtained values of the reliability indices are within a broad ranges from 3,7 (2,9 when the 'the maximum axial force design' is considered only) to 6,5. Such a broad range for reliability indices has been, however, reported also in previous probabilistic analyses (see for example [9]). Values of the reliability index β_1 are within a range from 3,7 (2,9) up to 6,1, values of β_2 within a range from 4,6 (4,2) up to 6,5. In the study cases 9 the reliability index β_1 = 3,7 (2,9) is less than recommended value 3,8 [1], relatively low value of β_1 are obtained also for the study cases 3, 4 and 10 (see Table 6). In all these cases the reinforcement ratio is relatively low (around or less than 1%), though still above the required minimum 0,3 %. In the study case 9 and 10 there may be also an unfavourable effect of relatively small cross section dimensions (0,25 × 0,50 m). Higher and perhaps uneconomical values of the reliabilty indices (around 6) seem to correspond to relatively great reinforcement ratios (study cases 7, 11 and 12).

The resulting reliability index β for the column is given by a combination of both reliability indices β_1, and β_2 that are given in Table 6. As a simple approximation the minimum of both values β_1, and β_2 may be considered as the resulting reliability index β. It follows from Table 6 that in all the study cases considered here $\beta_1 < \beta_2$; thus the first design situation with the short term imposed load being absent seems to be decisive.

10. Conclusions

Results of the reliability analysis of 12 study cases of reinforced concrete column show considerable differences in the reliability level of the column in different structural arrangements. Considering 50 years life time, wind action and long term imposed load as time variant actions (short time imposed load being absent) obtained values of the reliability index β varies within a broad range from 2,9 up to 6,1. Generally higher values of β (from 4,2 to 6,5) correspond to the reliability of columns during those days when short term imposed load is present.

It appears that the reliability level of reinforced concrete columns designed according to Eurocodes may be in some cases insufficient in other cases, depending on actual structural arrangements, it may become uneconomical. To harmonise reliability levels obtained for various structural members further research on random variable models using available experimental data and calibration of present generation of Eurocodes to existing structures is urgently needed.

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Reliability analysis of a reinforced concrete column designed according to the Eurocodes

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Abstract

Reliability analysis of reinforced concrete columns is a part of an extended research activity on Eurocode Random Variable Models supervised by the Joint Committee for Structural Safety. Submitted analysis concerns reliability of a built in reinforced concrete column designed according to Eurocodes 1 and 2. Reliability of a column of the first floor of a multi-storey frame structure is analysed using software product COMREL developed by RCP Műnchen. Preliminary results of the analysis are presented for the total of 12 study cases corresponding to different structural arrangements.

The design effects of actions are determined in accordance with Eurocode 1 considering the permanent load due to self weight and variable load due to wind, long term and short term imposed load. The column cross sections are designed using a simplified interaction diagram for axial force and bending moment and material properties specified in Eurocode 2. Dimensions b and h of rectangular cross sections rounded to 5 10^{-2} m are chosen such that h/b = 2 (in one study case h/b = 3). Symmetrical reinforcement having the theoretical area A_s rounded upward to 10^{-5} m^2, which do not necessarily correspond to any specific bar size, is considered in the reliability analysis.

Using the FORM method of probability integration results of time variant reliability analysis of columns for long term and short term actions are submitted for the all 12 study cases. Considering 50 years life time, wind action and long term imposed load as time variant actions (short time imposed load being absent) obtained values of the reliability index β varies within a broad range from 2,9 up to 6,1. Generally higher values of β (from 4,2 to 6,5) correspond to the reliability of columns during those days when short term imposed load is also present.

It appears that the reliability level of reinforced concrete columns designed according to Eurocodes may be in some cases insufficient in other cases, depending on actual structural arrangements, it may become uneconomical. To harmonise reliability levels provided for various structural members further research of random variable models using available experimental data and calibration of present generation of Eurocodes to existing structures is urgently needed.

Non-Linear Analysis and Safety Evaluation for Concrete Structures

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Summary

This paper describes the principal features of a reliability based nonlinear finite element method for reinforced concrete beams under static loads. The combination of the theory of structural reliability and the finite element method provides an efficient and comprehensive tool for assessing structural safety. Using the example of a simple statically indeterminate beam, different models of normative safety concepts for nonlinear system analysis are investigated considering the reliability index β for SLS and ULS criteria.

1 Introduction

Modern design-code formulations for civil engineering works are based on reliability methods as described in EC 1 [2]. The main reason for this is the objectivity of an homogeneous level of safety which should be reached on average. This refers to a general probabilistic approach, but it is well known that there are practical difficulties in using probabilistic methods for design. However, it is a useful task to verify or derive adequate safety elements from probabilistic calculations.

A general nonlinear analysis for concrete structures holds for most realistic results under all load levels. Therefore EC 2 [3] allows the application of nonlinear methods, although an adequate safety concept is still discussed within experts. As proposed in EC 2, the simultaneous calculation of the structure with mean values of the material properties for system analysis and design values of the material for the cross-section design leads to unacceptable inconsistencies. Using this background, in the following a reliability based nonlinear finite element method for reinforced concrete beams under static loads will be presented. Some results for SLS and ULS limit state conditions will be outlined by observing the reliability index β.

2 Safety Analysis for Reinforced Concrete Structures

2.1 First Order Reliability Method

The *first order reliability method* (FORM) is an approximate method to calculate failure probabilities for general, non-linear and normal or non-normal distributed problems. The response of a structure is entirely defined by the outcome of a vector $\mathbf{X} = (X_1, X_2, \dots, X_n)^T$ of basic random variables which may include parameters defining actions, material properties, member sizes etc..

In order to calculate failure probabilities one has to formulate a limit state function g which depends on a set of statistical variables $\mathbf{x} = (x_1, x_2, \dots, x_n)^T$:

$$g(\mathbf{x}) \geq 0. \tag{1}$$

A limit state function has only two states, a *safe* state and a *failure* state. If $g(\mathbf{x}) \geq 0$, the considered design requirement is fulfilled (safe region), if $g(\mathbf{x}) < 0$, the considered design requirement is failed [1] (failure region). The design condition may be written as $g(\mathbf{x}^\star) = 0$ where $\mathbf{x}^\star$ contains the design values for the particular problem.

The probability of failure P_f is given by the n-fold integral over the failure region of the limit state function in the space of basic variables $\mathbf{X}$, where each point $\mathbf{x}$ is assigned to a joint probabilistic density function $f_{\mathbf{X}}(x_1, x_2, \dots, x_n) = f_{\mathbf{X}}(\mathbf{x})$.

$$P_f = \int\limits_{\{\mathbf{x}|g(\mathbf{x})<0\}} f_{\mathbf{X}}(\mathbf{x})\,\mathrm{d}\mathbf{x} \tag{2}$$

The evaluation of the integral in Eq. 2 is often impossible because $f_{\mathbf{X}}(\mathbf{x})$ may not be known and the direct evaluation of the integral for general limit-state functions and large n is very extensive. Therefore, the *first order reliability method* provides a consistent and invariant method for deriving the design point $\mathbf{x}^\star$ in the majority of practical design tasks.

The first order reliability method takes advantage of the properties of the so called standard normal space $\mathbf{Y}$. Using the transformation $\mathbf{y} = T(\mathbf{x})$, the limit-state function is then given by $g(\mathbf{x}) = g(T(\mathbf{y})) = h(\mathbf{y})$ and Eq. 2 is rewritten to

$$P_f = \int\limits_{h(\mathbf{y})<0} (2\pi)^{-n/2} \exp\left(-\frac{1}{2}\mathbf{y}^{\mathrm{T}}\mathbf{y}\right) \mathrm{d}\mathbf{y}. \tag{3}$$

with n = the number of random variables. The solution of FORM is schematically drawn in Fig. 1. The limit-state surface will be replaced by its tangent hyper-plane $l(\mathbf{y}) = \nabla h(\mathbf{y}^\star)(\mathbf{y} - \mathbf{y}^\star)$ in the design point $\mathbf{y}^\star$ with $h(\mathbf{y}^\star) = 0$ The design point is characterized by the highest likelihood (maximum of the joint probability density) among all points in the failure set. The distance β between the origin and the design point is

[1]The term " failure" refers to either inadequate safety or serviceability of the structure.

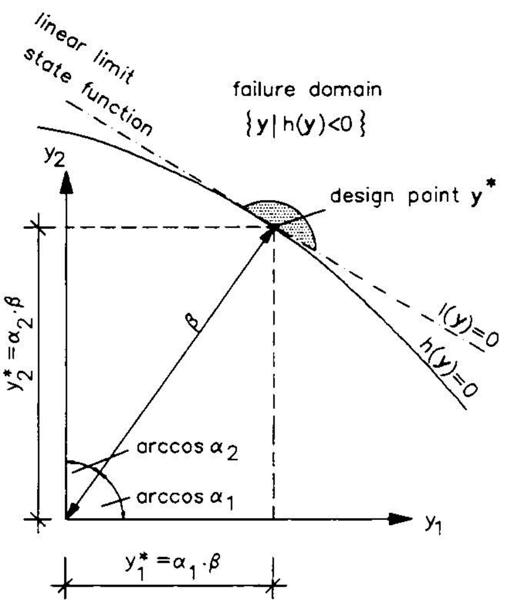

Figure 1: Failure domain and its linear approximation in the standard normal space

a measure of safety because the relation between the failure probability P_f and the so called *reliability index* β is defined by

$$P_f = \Phi(-\beta) \Rightarrow \beta = -\Phi^{-1}(P_f) \tag{4}$$

where Φ is the standard normal probability density function.

Target failure probabilities are derived by a process of probabilistic calibration to different existing design codes. The failure probabilities should be applicable to a wide range of structural components and provide a reliable and satisfactory performance. Indicative values for the target reliability index β are given in Eurocode 1 [2]. For different safety requirements (intended life time or safety-class I to III), the safety index β is given for the Ultimate-Limit-State (ULS) and the Serviceability-Limit-State[2] (SLS) as shown in Tab. 1. The bold values refer to the formulation of EC 1.

	Safety Class		
	I	II	III
SLS	2.5	**3.0**	3.5
ULS	4.2	**4.7**	5.2

Table 1: Indicative values for the reliability index β (one year)

[2]The values for SLS-conditions are valid, if the limit state function does not contain an inherent safety parameter

2.2 Limit State Functions for Reinforced Concrete Structures

The application of the structural reliability concepts to reinforced concrete structures needs a formulation of the limit state functions. Here, they are given in accordance to the regulation of EC 2 [3] with respect to serviceability and ultimate limit states.

2.2.1 Serviceability Limit State

Steel stresses, which could lead to inelastic deformation of the steel shall be avoided as this will lead to large, permanently open, cracks. So, the limitation of steel stresses under service accounts for adequate durability. Stresses are limited to

$$\sigma_s \leq 0.8 f_{yk} \qquad (\ 1.0 f_{yk} \text{ for imposed deformations}), \tag{5}$$

where f_{yk} is the characteristic yield strength of the steel. σ_s is calculated by assuming a cracked cross-section, if the concrete tensile strength f_{ct} has been exceeded.

Cracking shall be limited to a level that will not impair the proper functioning of the structure or cause its appearance to be unacceptable. An explicit limitation of the crack width may be checked by the following limit state function:

$$w_m \leq w_{lim} \tag{6}$$

where

w_m = mean design crack width, which will be calculated by using the following equation:

$$w_m = \left(50 + 0.25 \cdot k_1 \cdot k_2 \frac{d_s}{\rho_r}\right) \varepsilon_{sm} \quad [\text{mm}]$$

The mean strain ε_{sm} of reinforcement is evaluated by taking tension stiffening effects, shrinkage etc. into account. d_s denotes the average steel diameter, k_1 takes account of the influence of the bond properties, k_2 takes account of the influence of the form of the strain distribution and ρ_r is the effective reinforcement ratio (see EC 2, 4.4 [3]).

w_{lim} = limit of crack width, which will usually be chosen in accordance to exposure classes. For the sake of simplicity, it will here generally be set to 0.3 mm.

2.2.2 Ultimate Limit State

The ultimate bending capacity M_R resp. the rotation capacity Θ_R within critical regions is mainly defined by the ultimate compressive and tensile strength of concrete and reinforcement. Fig. 2 shows the values of ε_{cu} and ε_{su}, which will be assumed as deterministic here. The limit state function for the ultimate bending capacity M_R of a particular system cross-section is given by

$$M_R(\mathbf{x}_{cross\text{-}section}, N_S) \geq M_S(\mathbf{x}). \tag{7}$$

$M_R(\mathbf{x}_{cross\text{-}section}, N_S)$ is the ultimate bending moment, depending on all variables of the cross-section ($\mathbf{x}_{cross\text{-}section}$) and the acting longitudinal force N_S. $M_S(\mathbf{x})$ characterizes the acting bending moment.

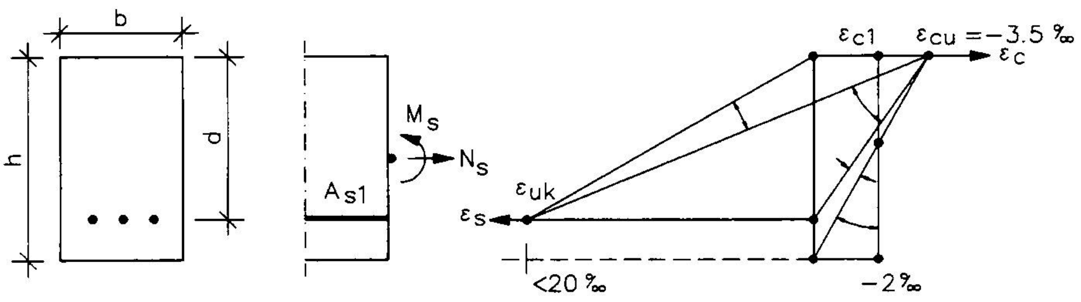

Figure 2: Strain limits for the ULS

2.3 Material Properties

2.3.1 Concrete

The main parameters are the modulus of elasticity E_c, the compressive strength f_c, the compressive strain ε_{c1} at the peak stress and the ultimate compressive strain ε_{cu}. For nonlinear structural analysis, the stress-strain diagram for concrete subjected to uniaxial compression may be written by Eq. 8 and 9. Obviously, it expresses a modification of the parabolic-rectangular stress-strain diagram by adapting the nonlinear branch to the modulus of elasticity E_c.

$$0 \geq \varepsilon_c \geq \varepsilon_{c1} \qquad \sigma_c = -f_c \cdot \left[1 - \left(1 - \frac{\varepsilon_c}{\varepsilon_{c1}}\right)^n\right] \tag{8}$$

$$\varepsilon_{c1} > \varepsilon_c \geq \varepsilon_{cu} \qquad \sigma_c = -f_c \tag{9}$$

By choosing the parameter n in Eq. 8 in accordance to Eq. 10,

$$n = -\frac{\varepsilon_{c1} \cdot E_c}{f_c} \tag{10}$$

one can adapt the stress-strain relation very simple to different situations. The parameter ε_{c1} will here be set to -0.002, ε_{cu} to -0.0035.

The tensile strength of concrete has main influence on the system behavior, e.g. when determining deflections, crack widths or the effective stiffness of a structure. In order to get realistic results, the tension stiffening effects have to be taken into account, especially when performing nonlinear analysis under SLS loading conditions. For the sake of simplicity the stress-strain diagram for concrete in tension may be taken as linear until the tensile strength f_{ct} is reached. In the majority of practical applications within a finite element code, the material behavior of reinforced concrete after exceeding the tensile stress is modeled by a hyperbolic stress-strain relation. See [6] for details.

2.3.2 Reinforcing Steel

The main parameters for reinforcing steel are the tensile strength f_t, the yield stress f_y, the elongation at maximum load ε_u and the modulus of elasticity E_s. The widely used stress-strain diagram which is given by Eq. 11 accounts for these parameters.

$$\varepsilon_s = \frac{\sigma_s}{E_s} + 0.002 \cdot \left(\frac{\sigma_s}{f_y}\right)^m \tag{11}$$
$$\text{with } m = \frac{\ln(\varepsilon_{su}/0.002)}{\ln(f_t/f_y)} \quad \text{and} \quad \varepsilon_{su} = \varepsilon_u - \varepsilon_y.$$

The exponent m provides an easy way to adapt the relation to different curve characteristics.

2.4 Proposal for Probabilistic Models

Essentially according to proposals made by the *Joint Committee on Structural Safety* (JCSS) [4] and to the recommendation of CEB [5] the statistical characteristics of the governing random variables are taken as listed in Tab. 2. Direct random model

Variable	Type	μ	σ	$V[\%]$	x_k	Frak.
f_c [MPa] $f_{ct} = 0.25 \cdot f_c^{2/3}$ $E_c = 9500 \cdot f_c^{1/3}$	LN	$f_{ck} + 8$	5		f_{ck}	5
f_y [MPa]	LN	$f_{yk} + 60$	30		f_{yk}	5
A_s [cm^2]	N	$_{nom}A_s$		2.5	$_{nom}A_s$	50
h, b [mm]	N	$_{nom}b$	5		$_{nom}b$	50
c [mm]	N	$_{nom}c + 5$	5		$_{nom}c$	50
G [kN/m^2]	N	$_{nom}G$		10	$_{nom}G$	50
Q [kN/m^2]	N	$\frac{Q_k}{1.824}$		40	Q_k	98

Table 2: Probability distribution parameters for random variables

uncertainty parameters are generally not applied here.

2.5 Finite-Element-Reliability-Method

In the field of structural engineering, limit state functions or failure criteria are usually not formulated in terms of the basic variables themselves. They are expressed in terms of a response quantity or action effect $\mathbf{S}$ like stresses, crack widths, deformations etc., that are derived from the basic variables. This derivation $\mathbf{S} = \mathbf{S}(\mathbf{x})$, which is called *mechanical transformation*, is available only in an implicit form, such as the finite element method. This is the principal reason for employing the finite element concept in structural reliability analysis.

The used 3-node/9-degrees of freedom finite beam element for geometrically and physically nonlinear reinforced concrete structures is shown in Fig. 3. Basically the simple beam theory with static loads will be applied. The longitudinal and vertical displacements u and w are interpolated with conventional interpolation functions. This element with its material subroutines as described before is implemented in a standard

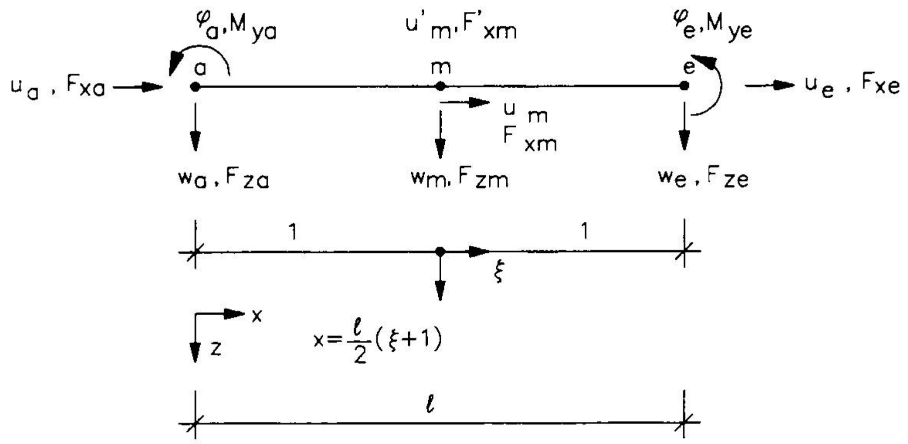

Figure 3: 3-node/9-degrees of freedom beam element

finite element code that was expanded to probabilistic approaches by including the first order reliability theory (FORM). The corresponding flow-chart is drawn in Fig. 4. The conventional solution of a nonlinear structural problem in a finite element displacement problem can be expressed through the equilibrium equation

$$\mathbf{R}(\mathbf{v}) = \mathbf{F}. \tag{12}$$

Thus equilibrium is achieved when the internal resistance forces $\mathbf{R}$, that depend on the displacements $\mathbf{v}$, balance the externally applied nodal forces $\mathbf{F}$. The solution of Eq. 12 may here be calculated by a *Newton-Raphson* iteration scheme with its known advantages. To evaluate the design point, an optimization procedure as known from Rackwitz-Fießler is used.

The interface between the two codes, namely the reliability analysis and the nonlinear finite element analysis has to account for the spatial variability of the in general correlated random variables $\mathbf{x}$.

3 Cross-Section Reliability Analysis

The main influence parameters for the reliability of concrete structures may be analysed by a cross-section reliability analysis. In the following some results on this topic will be discussed.

Considering a rectangular cross-section, the simple limit state function

$$g(f_c, f_y) = M_R(f_c, f_y) - M_S \tag{13}$$

with only two basic variables (concrete and steel strength), will be investigated in order to get the results of FORM graphically. M_R describes the ultimate resistance moment, M_S the ultimate acting moment which is determined in accordance to EC 2. For the sake of simplicity, M_S is chosen here deterministically. Fig. 5 shows the limit state surface of Eq. 13 for different reinforcement ratios ρ in the standard normal space $\mathbf{Y}$. The boldly drawn arrows in Fig. 5 are equivalent to the reliability index β as the shortest distance

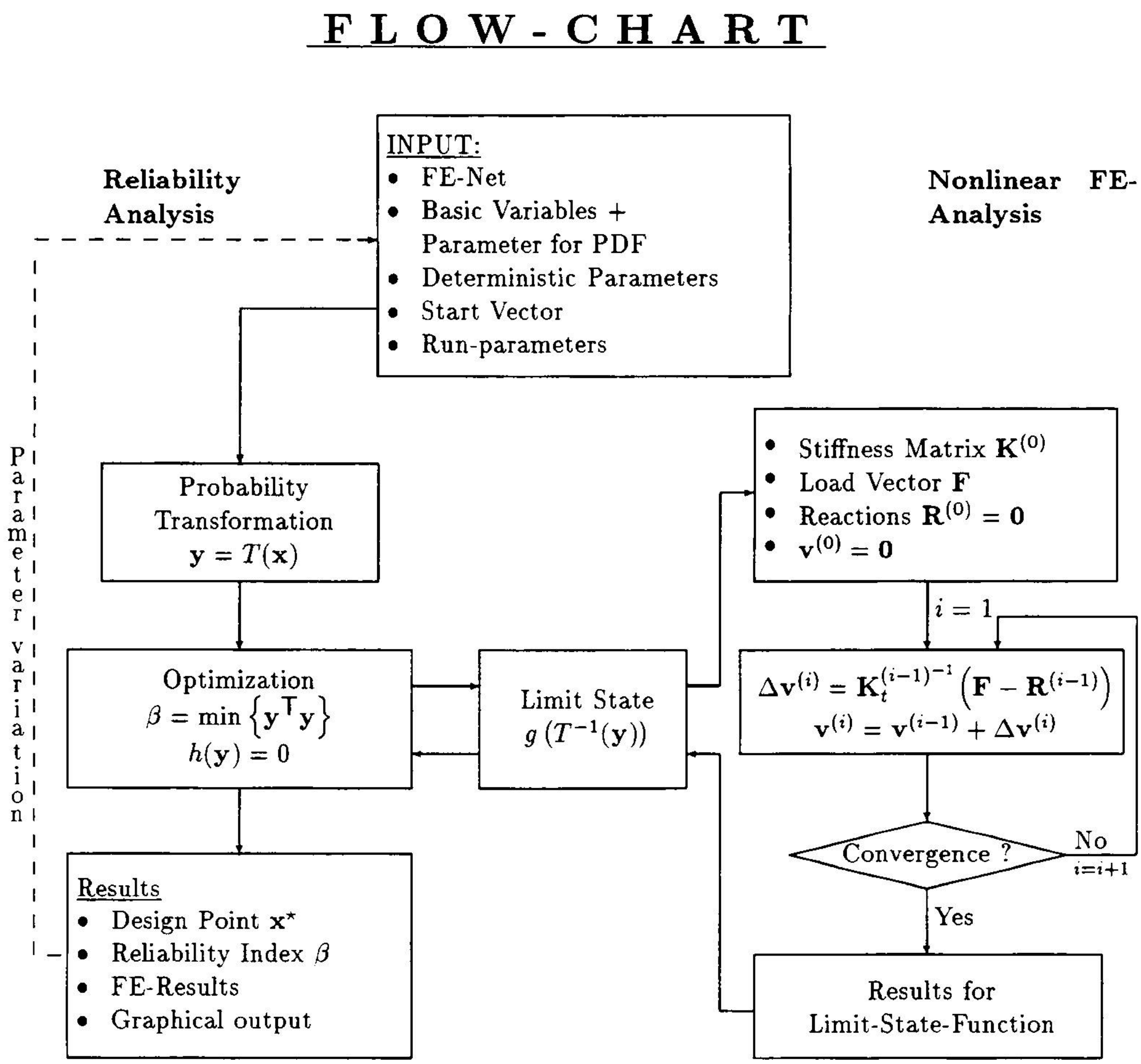

Figure 4: Finite element reliability method

from the origin to the limit state surface. Obviously either the steel strength or the concrete strength has essential influence. The safe region between the origin and the limit state surface is convex limited. That means that the FORM-solution may give unsafe results, but further investigations with statistical simulation methods have shown, that these influence can be neglected [6].

In the following examples, design for ULS-conditions was carried out in accordance to EC 2 for different load combinations N and M. In Fig. 6, the corresponding M-N diagram (interaction diagram) for symmetrically reinforced sections in a nondimensional

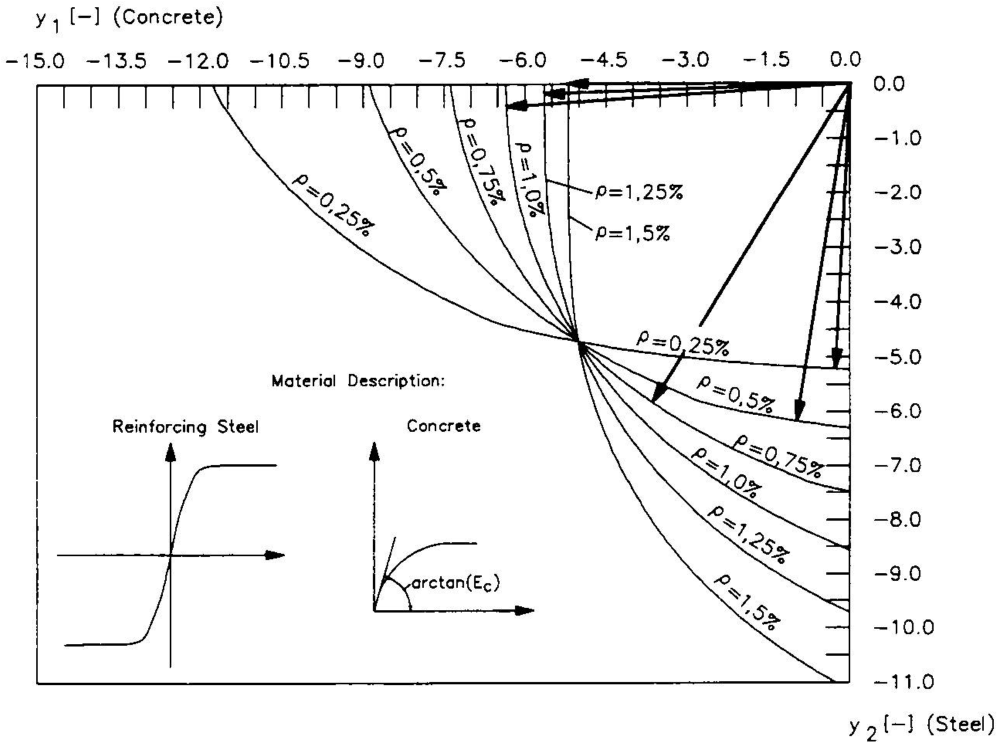

Figure 5: Limit State Surface $M_R(f_c, f_y) - M_S = 0$, Standard Normal Space

form with

$$\mu_n = \frac{\mu_N}{bhf_{cm}} \qquad \mu_m = \frac{\mu_M}{bh^2 f_{cm}} \qquad \omega = \frac{A_s}{bh} \cdot \frac{f_{ym}}{f_{cm}}. \tag{14}$$

is drawn on the left. μ_N and μ_M are determined using mean values of material properties, while the partial load safety factor is assumed as $\gamma_G = 1.35$ (permanent load). If the acting longitudinal force N has a favorable effect, $\gamma_G = 1.0$ is used. ω is the mechanical reinforcement ratio. The applied limit state function with its basic variables is given by $g(\mathbf{x}) = N_R(f_c, f_{y1}, A_{s1}, f_{y2}, A_{s2}, b, h, d_1, d_2, M_S) - N_S = 0$ resp. $g(\mathbf{x}) = M_R(f_c, f_{y1}, A_{s1}, f_{y2}, A_{s2}, b, h, d_1, d_2, N_S) - M_S = 0$. The results of the first order reliability analysis are drawn in Fig. 6 (right) as contour lines for β. The acting forces N_S and M_S, which are determinated as a function of the corresponding design situation are taken as statistically correlated with $\varrho_{NM} = 0.5$ in order to cover an unfavorable case. In general a reliability index of $\beta > 5$ will be achieved. For relative low mechanical reinforcement ratios, values of β can fall below 5 and even down to a value of $\beta = 4.0$. That means, that in general sufficient safety is provided by using the semi-probabilistic design concept for the ULS but caution should be given to low reinforcement ratios.

Fig. 7 shows the according contour lines of β for the steel stress limitation ($0.8 f_{yk}$) as an example for the SLS-conditions ($\varrho_{NM} = 0$). For longitudinal tensile forces ($\mu_n > 0$), a nearly constant reliability index $\beta = 1.6$ is reached while for increasing longitudinal compressive forces ($\mu_n < 0$), the reliability index β increases rapidly. Cross-sections show their minimum of β while mainly loaded by bending moments and low reinforcement ratios. This was also observed for the ULS-conditions.

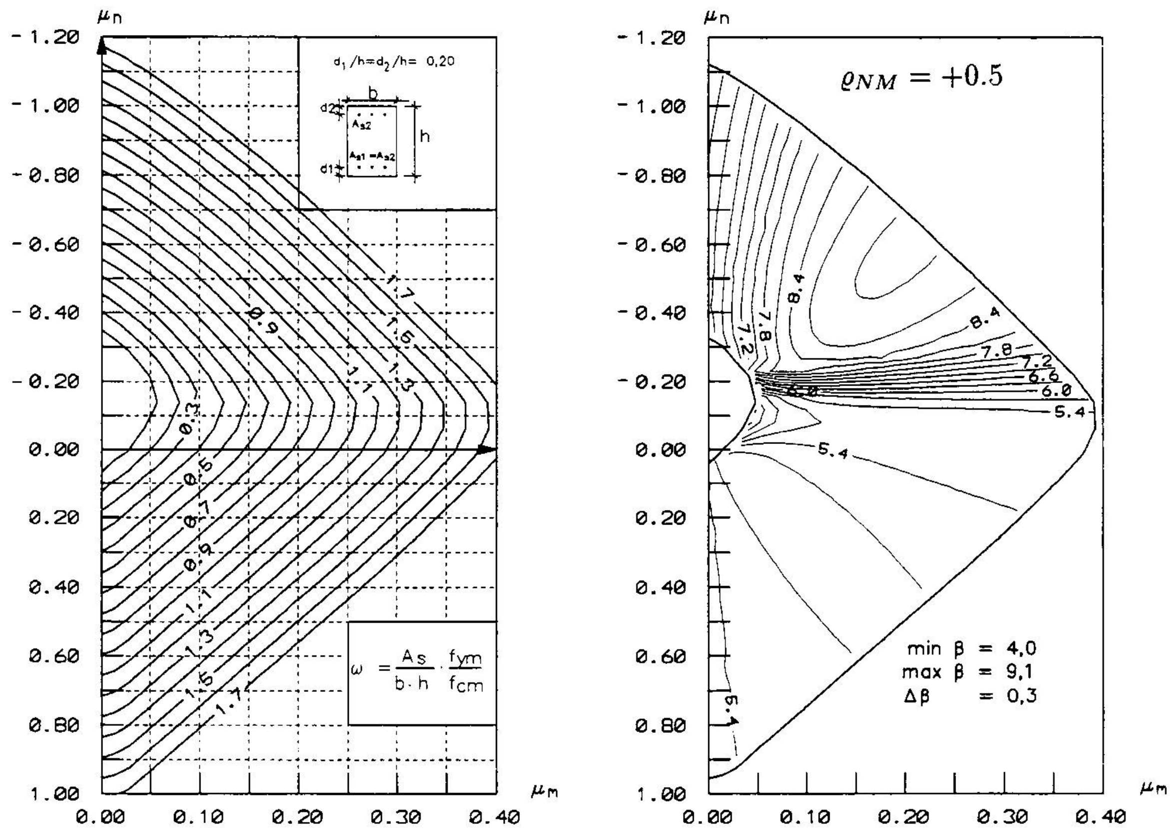

Figure 6: Interaction diagram and contour lines of β [T=1 year] for a symmetrically reinforced concrete cross-section

In Fig. 8 the safety index β is drawn for the limit state of cracking as a function of the reinforcement ratio $\rho = A_s/bh$. Here, the section is loaded only with a bending moment, considering different ratios of permanent (G) and variable (Q) loads. The deterministic design was carried out for SLS and ULS conditions.

Summarizing the results of the cross-section reliability analysis, it is evident that the reliability requirements of the cross-section design for reinforced concrete structures are fulfilled. It should be pointed out that a linear relation between the applied acting loads and the internal forces has been assumed. The extent to which these results are true for statically indeterminate structures with non-linear behavior has still to be investigated.

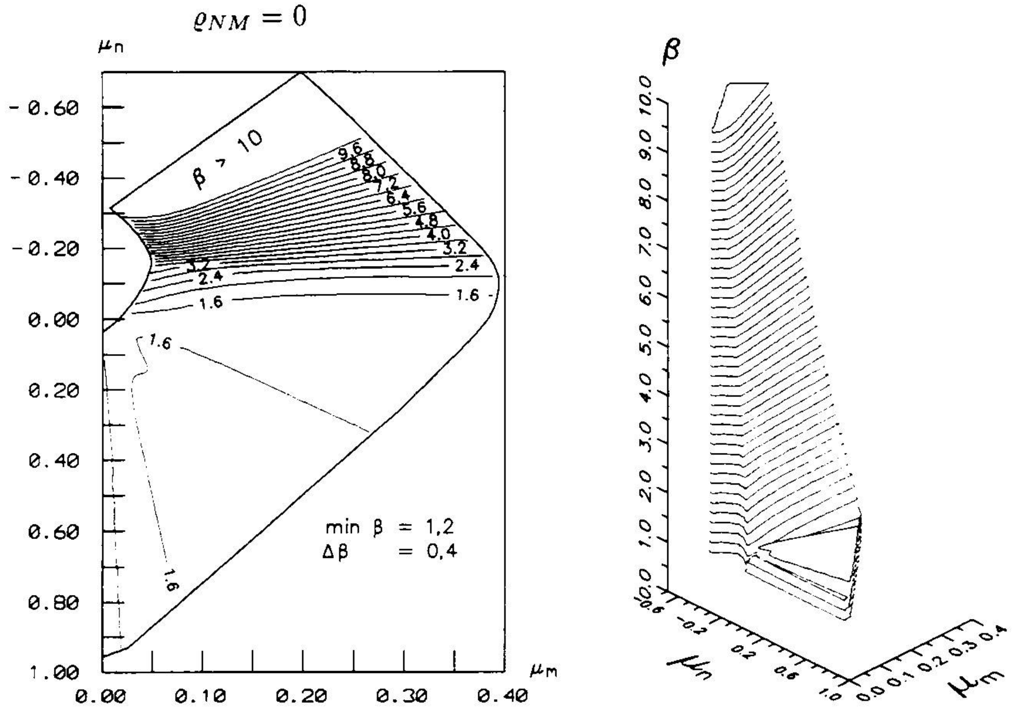

Figure 7: Contour lines and isoparametric plot of the reliability index β [T=1 year] for the limit state of steel stress limitation ($\sigma_s = 0.8 f_{yk}$)

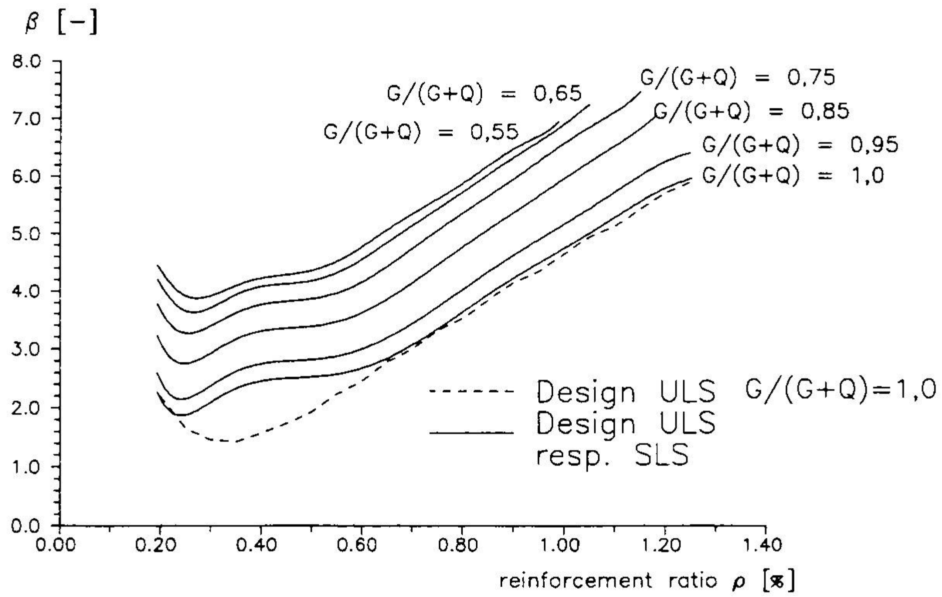

Figure 8: Reliability index β [T=1 year] for different ratios $G_k/(G_k + Q_k)$ for the limit state of cracking ($w_{lim} = 0.3$ mm)

4 Safety Analysis of an Indeterminate Reinforced Concrete Beam

Referring to EC 2 [3], system analysis may be performed by using non-linear methods. Useful safety concepts for nonlinear analysis are still discussed [1].The following example will investigate different proposals in order to compare the reliability in critical regions by using the Finite Element Reliability Method. Fig. 9 shows a statical system which may be taken as a symmetrical half of a two span reinforced concrete beam. The beam

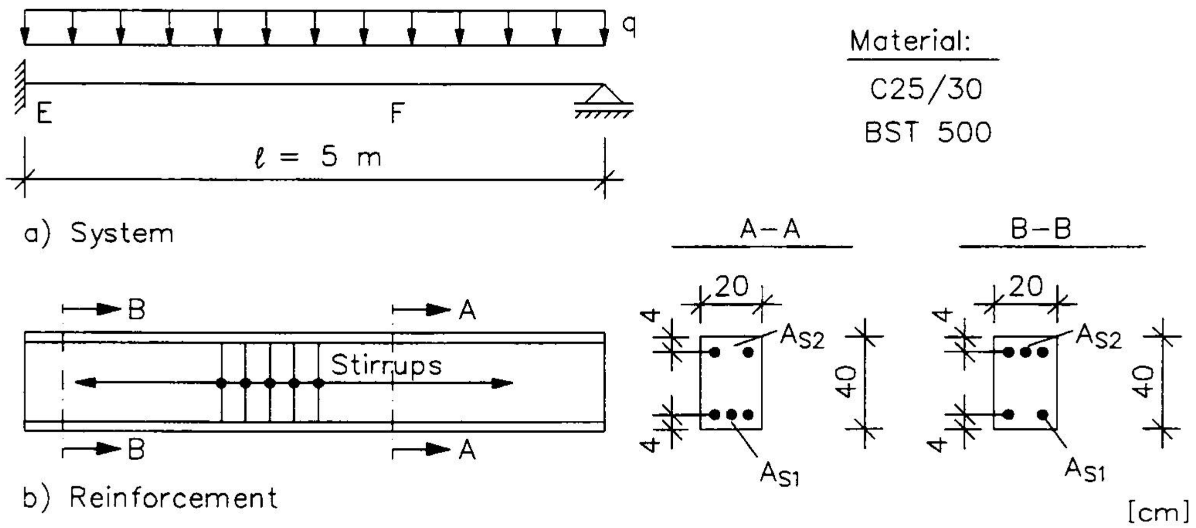

Figure 9: Reinforced concrete beam

is equally loaded with a permanent load q to cover the most unfavorable case. Using the linear elastic model for system analysis, design is carried out deterministically by SLS-criteria only (crack width = 0.3 mm, steel stress = $0.8 f_{yk}$). The design bearing capacity $q_{T,d}$ for the ULS is determinated by non-linear analysis, using different safety concepts for the material properties. In the following, to determine the bearing capacity, the investigated models with different safety concepts are discussed:

PLA plastic analysis with design values of material properties within critical zones. The assumption of design properties in the critical zones (plastic hinges) which are determinated by using the partial safety factors for steel ($\gamma_s = 1.15$) and concrete ($\gamma_c = 1.5$) covers the material uncertainty.

NLD non-linear analysis with design values of material properties for the whole structure. To determine the bearing capacity, the applied load q will be increased until the rotational capabilities[3] or instability of the system is reached, respectively. The assumption of design values for material covers the uncertainty.

NLMG non-linear analysis with mean values of material properties and gamma. The bearing capacity q_T is defined by reaching the rotational capabilities or instability of the system, respectively. The material uncertainty will be considered by

[3]The rotational capabilities are here defined by $\varepsilon_{cu} = -3.5$ ‰ and $\varepsilon_u = 20$ ‰ ($f_t/f_y = 1.0$).

applying a partial safety factor $\gamma_R = 1.25$ to q_T. That means, it has to be shown that

$$\gamma_F \cdot q \leq \frac{q_T}{\gamma_R}. \tag{15}$$

where γ_F summarizes the load safety factor. Eq. 15 may lead to a global safety factor concept because γ_R can also be used on the left side. However, it is much more reasonable and in the consequence of the well established partial safety concept to apply γ_R to q_T.

NLM non-linear analysis with mean values of material properties. To determine the bearing capacity, the applied load q will be increased until the rotational capabilities or instability of the system is reached, respectively. This is an analysis without any safety elements for the material. The results should point to the influence on the material uncertainty in general.

The probabilistic model used here was discussed in chapter 2.4. The random variables are taken as perfectly correlated (random field) over the system, between single variables (e.g. f_{y1} and f_{y2}) no correlation is assumed. The vector $\mathbf{x}$ of basic variables is given by

$$\mathbf{x} = (f_c, h, b, f_{yi}, A_{si}, d_i, q), \qquad i = 2E, 1F$$

The marked points F and E in Fig. 9 refer to the critical regions which will be observed. The point F is not fixed but depends on the actual stiffness of the system. First order reliability analyses are carried out for each design situation. The results have in accordance to the random variable description a reference period of one year. Fig. 10

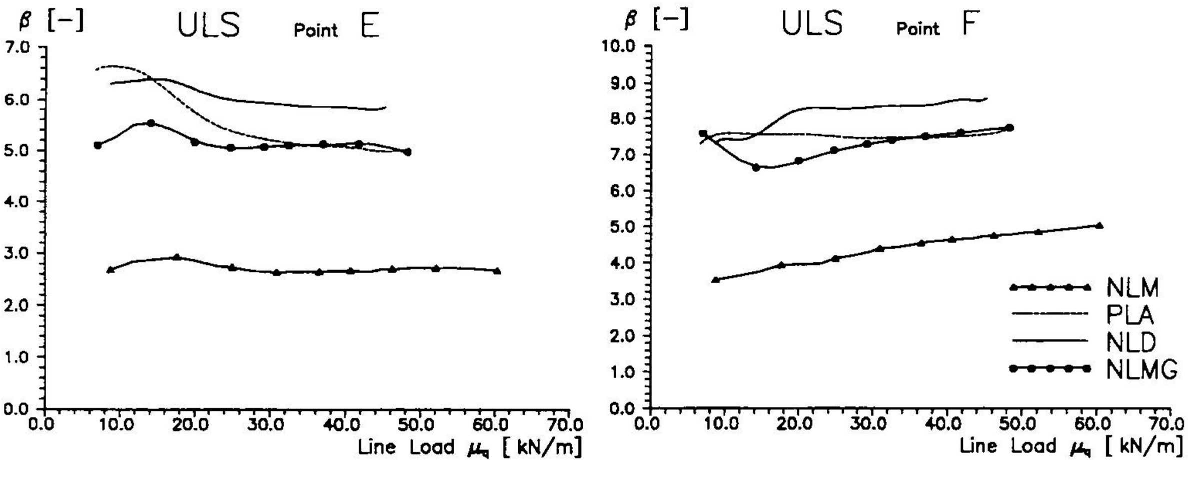

Figure 10: Reliability index β [T=1 year] for the ULS at points E and F

and 11 show the reliability index β, independently calculated for the critical system points E and F. Obviously, the assumption of NLM leads to a very low reliability for ULS which finally points to the requirement of material safety elements. The results for PLA, NLD and NLMG show that mainly independent of the applied safety model, a reasonable safety margin for the ULS is reachable while the values of γ may still be justified. However, the deterministic design using SLS requirements in conjunction with the applied material model for the ULS shows a sufficient reliability for the SLS.

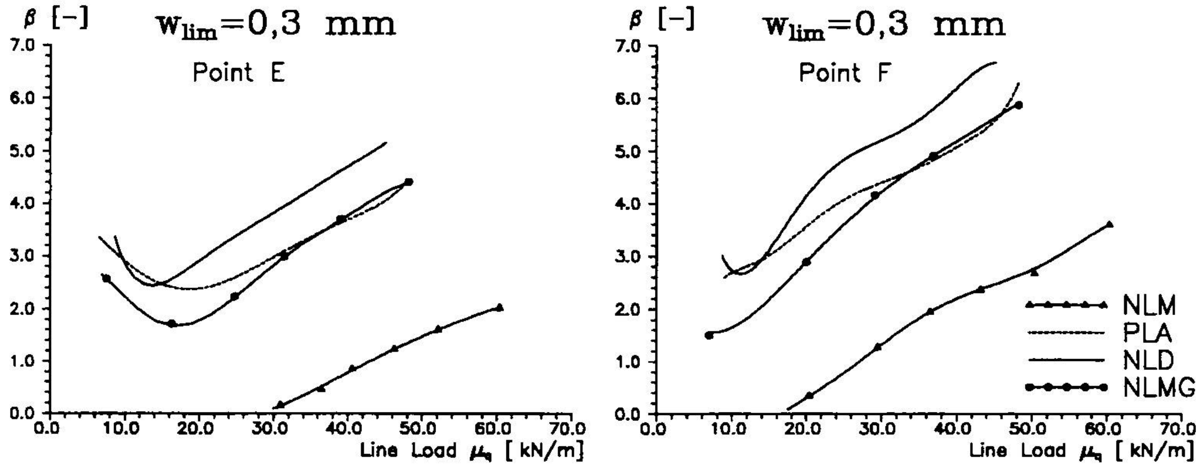

Figure 11: Reliability index β [T=1 year] for the SLS (cracking) at points E and F

5 Conclusions

The first order reliability method has been proven as a general tool to determine structural safety. An application of such safety assessment to reinforced concrete structures allows a comparative point of view to different deterministic design rules. As far as some investigated examples for nonlinear analysis of reinforced concrete structures shows, it should be noticed, that a homogeneous level of safety for the ULS may be reached with different (partial) safety concepts for design. Further investigations on this topic are still in progress. Namely the influence of M-N interaction and random field effects will be observed. In the future topics such as shear and prestressing should be made assessable to safety analysis of reinforced concrete, especially in conjunction with non-linear analysis for beams and plates.

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Upper bound for combination of action effects

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Janusz Murzewski, born 1928, got his civil engineering degree in 1951 and doctor degree in 1956 at the Cracow Technical University. 1963-70 chairman of Mathematics. since 1970 - chairman of Metal Structures & Structural Reliability Department of this University. Author of 10 books, monographs and more than 300 papers. Member of AMS, IABSE, GAMM and Polish Scientific Societies

Summary

The square-wave model of random actions with the Ferry-Borges & Castanheta combination rule is sufficiently exact but too difficult for practical design. The Turkstra rule is simpler but it gives lower bound estimates of action effects A new combination rule is also simple and it gives safe estimates. Combination values of nondominant actions depend on their repetition numbers relative to a specified reference period. The characteristic value of dominant action will be changed if a design working life of the structure is different from the reference period

1. Introduction

1.1 Random variations and time variations

Both permanent loads G and variable actions Q are random. It means that they are variable in population of construction works:

- of similar destination if occupancy loads are concerned,
- in the same climatic zones for wind action, air temperature and insolation, snow or icing.

Characteristic values G_k ,Q_k are enhanced by means of load factors γ_G and γ_Q for applications in partial factor design. The load factors cover uncertainties due to random variations of the permanent and variable actions.

Moreover the variable actions Q are variant in time. Combination factors ψ_o reduce characteristic values of simultaneous actions, except the dominant one, because their maxima will not probably occur in the same while. The characteristic values may be also reduced or enhanced if a design period is different from the reference period of the maximal variable actions .

The combination of design action effects S_d is always more than the design value of action effect $\gamma_S S_k$ thanks to geometric summation of the standard deviations according to rules of the first-order second-moment probabilistic theory:

$$S_d = \sum_{j=1}^{m} c_j \gamma_G G_{j.k} + \sum_{i=1}^{n} c_i \gamma_Q \psi_{oi} Q_{i.k} = \sum_{j=1}^{m} c_j \overline{G}_j + \sum_{i=1}^{n} c_i \overline{Q}_i^* + \beta_S \left(\sum_{j=1}^{m} c_j \sigma_j + \sum_{i=1}^{n} c_i \sigma_i \right) \;; \tag{1}$$

$$\gamma_S S_k = \sum_{j=1}^{m} c_j \overline{G}_j + \sum_{i=1}^{n} c_i \overline{Q}_i^* + \beta_S \sqrt{\sum_{j=1}^{m} c_j^2 \sigma_j^2 + \sum_{i=1}^{n} c_j^2 \sigma_i^2} \;; \tag{2}$$

where $\overline{G}_j = G_{j.k}$ - mean and characteristic values of permanent loads,
$\overline{Q}_i^*$, $\psi_{oi} Q_{i.k}$ - combination values of variable actions,
σ_j σ_i - standard deviations for j=1, 2, ... m and i = 1, 2,... n,
β_S - a specified load index.

Some authors and codemakers mistake a reduction of S_d to the level $\gamma_S S_k$ with application of combination factors ψ_o. Perhaps additional reduction factors could be introduced to the linear combination of design values (1) in order to make the result S_d of partial factor design closer to the result of probabilistic design $\gamma_S S_k$ (2). Such a reduction factor ξ_j is foreseen for permanent actions only by the draft international standard of ISO: (*DIS2394,* 7.5.1). In addition another ξ factor could be defined for combination values of variable actions or a global ξ for both kinds of actions.The combination factors ψ for variable actions are better not to be amalgamated with ξ factors. The actual value of the global ξ would depend on the number $m+n$ of actions G_j and Q_i as well as proportions among them. The maximum value of the ξ factor occurs when only one action (either permanent or variable) is applied and $\xi = 1$. The minimum will occur when the moments of all $m+n$ particular action effects are equal

$$\xi = \frac{1+\beta_S v_S}{1+\beta_S v_S \sqrt{m+n}} \tag{3}$$

where $v_S = \sigma_j/\overline{G}_j = \sigma_i/\overline{Q}_i =$ const - coefficients of variation for $j = 1, 2 \ldots m$, $i = 1, 2 \ldots n$.

Further considerations will be limited to combination factors ψ_o applied to ultimate limite states of structures in persistent and transient situations.The subscript o will be omitted.

1.2 Pre-standardization of combination factors

International committee about bases for design of structures ISO/TC98 created in 1989 a working group on combination of actions SC2/WG5. This was preceded by a state-of-art report about load combination rules in codified design in ISO member countries (*Mathieu & Murzewski,* 1988). The report has shown that the rules are so different and heterogeneous that their harmonization is not possible.The load combination model of *Ferry-Borges & Castanheta* (1971) was recommended by the Committee as the basis for new unified rules. A special issue of International Journal *"Structural Safety"* devoted to load combinations was edited and combination models and applications have been developed by *Kanda, Murzewski, Nowak, Östlund, Shiraki, Wen* etc.(1993). During years 1989-94 seven drafts of new combination rules were discussed and the last one was submitted as Annex F to the final draft of revised international standard DIS2394: *"General principles on reliability for structures"* (1995). The Annex F after four modifications is a compilation of texts of drafts elaborated by the Working Group, the former edition of the IS2394 and informative documents to Eurocode 1: *"Basis of design and actions on structures"* (1993). The ISO draft standard will be refered further on as *DIS2394* with numbers of paragraphs of the main text or annexes. Similarly the the Eurocode 1. Part 1 will be refered as *EC1-1.*

Both Ferry-Borges & Castanheta model and the Turkstra rule are based on consideration of variations of actions in time. The Ferry-Borges & Castanheta model requires to calculate 2^{n-i} combination cases for each structural element. The *Turkstra* model takes only n cases into account. Combination factors ψ of the Eurocode 1 are associated rather with the *Turkstra* rule. The combination factors ψ of the Eurocode are specific for each variable action and they do not depend on other actions of the combination. It is not so supposed by the draft international standard (*DIS 2394*, F-3.1) .The ISO principles are as follows:

- "One action is chosen as the dominating action and is introduced by means of its characteristic value Q_{1k} .
- A second action is introduced with a reduced combination value $\psi_2 Q_2$, $\psi_2 \leq 1$, The combination factor ψ_2 depends on the characteristics of both the dominating action Q_1 and the nondominating action.
- A third action is introduced with a further reduced combination value $\psi_3 Q_3$, $\psi_3 < \psi_2$. The value of ψ_3 depends of all three actions. This process is repeated if necessary."

Involving 3 or more actions in one combination factor ψ seems to be too sophisticated. Perhaps 2 actions are sufficient as Ferry-Borges and Castanheta have assumed in their considerations but a practical combination rule should be still simpler as the Turkstra rule is.The problem will be discussed here for linear combinations of action effects. Reduction factors ψ_o for simultaneous actions will be analyzed for persistent and transient loading situations at the ultimate limit states of construction works. The subscript "o" will be omitted.

2. Characteristics of variable actions

2.1 Stochastic process of actions

Two moments $\underline{Q}$, σ_Q^2 of probability distribution should not be identified with "mean" $Q(t)$ and "variance" $\sigma_Q^2(t)$ determined during an observation time t for one selected construction work. The two moments will be equal one to another if the stochastic process of action is stationary and ergodic. An action process will be stationary if anticipated usage and environmental conditions do not change during the working life period (*Fig.1*). Much more difficult is to prove that the action process is ergodic. If it is even so, the random action $Q(t)$ has to be defined more precisely:

- If maximal values max $Q(t)$ are measured during a total observation period t_0, the mean max $\bar{Q}(t)$ always decreases with increasing t_0 and the variance $\sigma^2_{\max Q}(t)$ can be constant only for "stable" (in reference to maxima) short-term probability distributions of actions Q^*
- If original short-term values $Q^*=Q(t^*)$ are averaged in unit observation periods t^* (e.g. 10 minutes for wind velocities) its variance $\sigma_Q{}^*$ decreases with t^* according to an asymptotic formula $\sim\theta/t^*$ for $t^*\rightarrow\infty$ where θ is specific scale of fluctuation.
- If a random action is intermittent, the moments of its probability distribution are different for two cases: when only positive values are measured and when all values are measured. But if two exclusive actions occur periodically one after another, they may be characterized together as a continuous action.

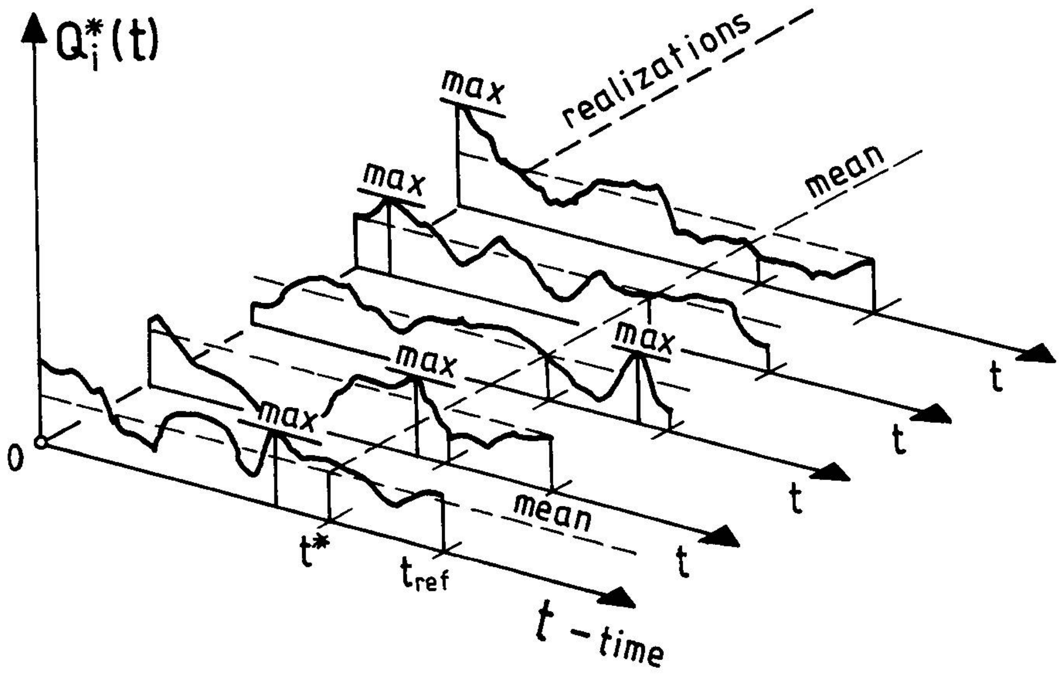

Fig. 1 Realizations of a continuous stationary and ergodic stochastic process

Characteristic values of maximal actions Q_k will be comparable if a constant reference period t_{ref} is selected for any kind of variable action and any country. A design period t_d is not necessarily equal to the reference period t_{ref}. The design period is identified with intended working life specified for construction works (*EC1-1*, Table 2.1; *DIS2394*, Table 2.1) which are classified as:

- temporary for 1-5 years,
- short life for 25 years,
- ordinary for 50 years,
- long life for 100 years.

Now the reference period t_{ref} is determined by codemakers of particular load standards. It is 50 years for wind action (*EC2-4*), the same for snow (*EC2-3*) although 1 year only is recommended by the Eurocode (*EC1-1*, 4.2.8). The reference period t_{ref}=50 years is better because:

- it is equal to the design period t_d for ordinary buildings and it is equal or close to conventional characteristic values of national standard specifications,
- asymptotic distribution functions of extreme values can be taken for 50 or more years with a much better accuracy than it would follow from the relation

$$F(Q \mid t_d) = [F^*(Q \mid t_o)]^r \tag{4}$$

where $F^*(Q \mid t_o)$ - the CDF of short term (e.g. one-year or "point-in-time") random variables
$r = t_d/t_o$ - repetition number of the short-term values during the design period t_d.

There are objections relative to equation (4). It requires that the extreme values Q^* be independent in not always well defined unit observation intervals t_0 and it happens that:

- the occupancy loads and other actions are autocorrelated for time intervals which may be longer than the short term periods t_0,
- There are many distribution functions F^* proposed for particular short-term actions and statistical tests do not give precise solutions (*Sedlacek*, 1992).

The situation is different in the case of extreme values which happen in a longer time period e.g. t_{ref}= 50 years. There are 3 types and only 3 asymptotic distributions of extreme values: the Gumbel (I), the Fréchet (II) and the Weibull (III). No empirical tests are necessary to verify this theorem of *R.A.Fisher and L.H.Tippett* (from *Gumbel*, 1954). The central parameter $\hat{Q}$ of any extreme value distribution has been called characteristic value in mathematical statistics. The characteristic maximum $\hat{Q}$ will be equal to the codified characteristic value Q_k (*EC1-1*, 1.5.3.14) if the prescribed probability of not been exceeded is exactly e^{-1}=0,368...
The probability that it will be exceeded once and only once during t_{ref} is the same . The upcrossing events are rare and the *Poisson* law may be applied. So the characteristic value $\hat{Q}$ will be exceeded on average once during the reference period of the Poisson sequence of events.

2.2 The Gumbel probability distribution of extreme actions

Preference should be given to the type I distribution for maximal actions during the reference period

$$F(Q) = \exp(-\exp\frac{\hat{Q}-Q}{u}) \tag{5}$$

where Q - characteristic maximum in the sense of mathematical statistics,
u - the Gumbel deviation - a parameter characterizing dispersion..

- The characteristic maximum $\hat{Q}$ will be equal to the mode $\tilde{Q}$, i.e. the most probable value during the reference period, for the Gumbel probability distribution,

$$f(Q) = dF(Q)/dQ = \max \rightarrow df(Q)/dQ=0 \rightarrow Q=\tilde{Q}=\hat{Q} \rightarrow F(\tilde{Q}) = e^{-1}. \tag{6}$$

- The characteristic maximum Q_t of the Gumbel distribution may be predicted for a period t longer than 50 years so that only the model maximum increases (*Fig*.2)

$$\tilde{Q}_t = \tilde{Q} + u \ln(t/50), \qquad u_t = u = \text{const.} \tag{7}$$

- The first and second moments of the Gumbel probability distribution are related to its parameters in a simple way:

$$\bar{Q} = \tilde{Q} + u\,C, \qquad \sigma^2 = u^2 \pi^2/6 \qquad \text{with C=0,5772... the Euler number.} \tag{8}$$

The normal coefficient of variation v and the Gumbel one υ are related as follows

$$v = (\upsilon\,\pi/\sqrt{6})/(1 + C\,\upsilon) = \upsilon/(0,780 + 0,450\,\upsilon) \;. \tag{9}$$

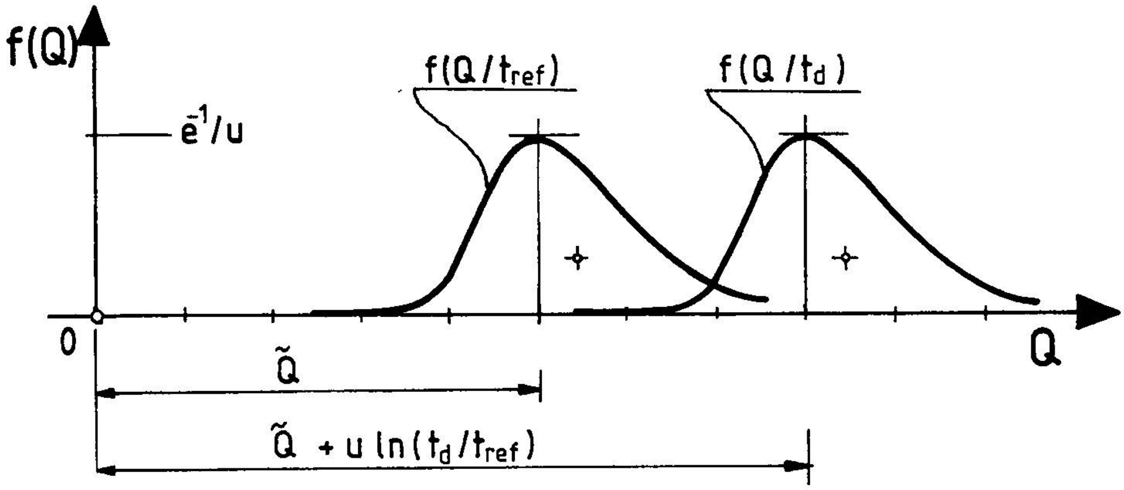

Fig.2. Modal values Q of extreme actions for the reference and design periods

If t<50 years, equation (8) is not necessarily exact. Short-term probability functions $F^*(Q)$can be quite different than their asymptotic distribution. A concept which enables to simplify the load model is to define a basic time interval θ and relative repetition number $r = t_{ref}/\theta$ so that the characteristic values be equal when estimated in two ways

$$\tilde{Q} - u \ln r = Q^* \rightarrow r = \exp\frac{\tilde{Q} - \hat{Q}^*}{u} \tag{10}$$

where $\hat{Q}^* = F^{*-1}(e^{-1})$ - inverse function to the CDF of short-term action from equation (4).

Thanks to the concept of basic time interval θ no extensive statistical investigations are necessary for probability functions of actions during 5-years, 1-year etc. Only the characteristic value $\hat{Q}^*$ is needed.

3. Combination rules for variable actions

3.1 Square-wave model of actions

It is assumed that random values of the same variable action Q_i are independent in any two basic time intervals θ_i, θ_j. That is the essential feature of the square-wave model of random action process. The equations (4), (5), (6), (7), (8), (9) will be actual if the Gumbel probability distribution is accepted for the variable actions and their combinations. Explanations and applications will be easier with this assumption however Ferry-Borges & Castanheta and Turkstra have considered their combination rules in more general formats.

Special numbering order of variable actions is important. Actions Q_1, Q_2, Q_3... Q_n are ordered in sequence of their repetition numbers $r_1 < r_2 < r_3 < \dots r_n$. according to the Ferry-Borges & Castanheta rule. There are other numbering rules, e.g. an action which gives the highest effect has number 1 and so on according to permutation rule recommended by some national standards, e.g. the Polish standard *PN-82/B-02000*. The numbering order is not important for applications of the Turkstra rule.

One variable action Q_c, c=1, 2, 3,... , is taken as dominant for each combination case. Its characteristic value will not be reduced (i.e. ψ_c=1) unless the design period t_d is different from the reference period t_{ref}. But nondominant actions Q_i are reduced with combination factors ψ_i<1, $i \neq c$, and they do not depend on the design period t_d. They depend on either the reference period t_{ref} or a basic interval θ_j of another variable action Q_j not necessarily the preceding one. The international draft standard does not give exact advice for this point.

There is no difference between the Ferry-Borges & Castanheta, the Turkstra and the new combination rule in the case of two variable actions only. The differences can be shown when at least three simultaneous variable actions Q_1, Q_2, Q_3 occur.

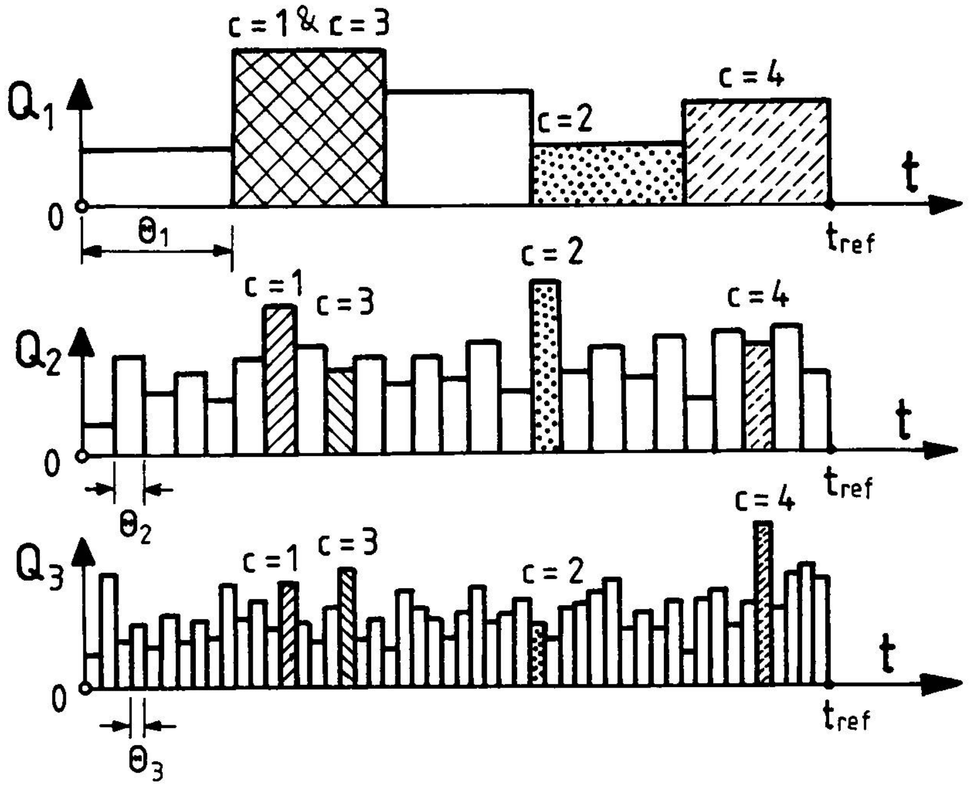

Fig.3. Three variable actions with different basic time intervals

3.2 The Ferry-Borges & Castanheta combination rule

The combnation rule is such that after the dominant action has been chosen, another variable action is selected, not necessarily the next as a sub-dominant one. It is selected from actions with shorter basic intervals. Then again a sub-sub-dominant action may be selected etc. if there are more variable actions in the combination. An extension (*Murzewski*, 1983) of the original Ferry-Borges & Castanheta combination model consists in numbering not only actions: i=1, 2, 3,... n but also their combinations: c = 1, 2, 3,... 2^{n-1} in such a way that periodic order of the combinations is revealed. A current number m=1, 2, 3, ... helps to indicate the column where dominant action can be found from the matrix of combination factors $[\psi_{ic}]$

$$
\begin{array}{lll}
\psi_{ic} = 1 + u_i \ln(t_d/t_{ref}) & \text{for } c = 2^i\,(m\text{-}1/2) & \\
\psi_{ic} = 1 - u_i \ln(t_{ref}/\theta_i) & \text{for } c < 2^i\,(m\text{-}1/2) & \qquad (11) \\
\psi_{ic} = 1 - u_i \ln(\theta_j/\theta_i) & \text{for } c > 2^i\,(m\text{-}1/2) \text{ and } j>i & \\
\psi_{ic} = 1 - u_i \ln(t_{ref}/\theta_i) & \text{for } c < 2^i\,(m\text{-}1/2) \text{ and } j<i &
\end{array}
$$

where $\upsilon_i = u_i/\tilde{Q}_i$ - the Gumbel coefficient of variation.

There are 2^{n-1} combinations to check for each structural element in the case of the Ferry-Borges & Castanheta rule. It is perhaps too many for practical design. However still more combinations (if n>2) are required to be checked for each structural element, namely $n!$, in the case of the permutation rule. But only n combinations are necessary with the Turkstra rule.

3.3 The Turkstra combination rule

The concept of Turkstra is that all nondominant actions are taken in their instantaneous values. If the square-wave model (*Fig.3*) and the Gumbel probability distribution are assumed, the values $\psi_{ic}Q_i$, $i \neq c$, are determined for their basic time intervals θ_i.
The combination factors ψ_{ic} are as follows for dominant and nondominant actions:

$$
\begin{array}{lll}
\psi_{ic} = 1 + \upsilon_i \ln(t_d/t_{ref}) & \text{for } c = i & \\
\psi_{ic} = 1 - \upsilon_i \ln(t_{ref}/\theta_i) & \text{for } c \neq i & \qquad (12)
\end{array}
$$

The Turkstra combination factors ψ_{ic} for some nondominant actions are lower than corresponding factors according to the Ferry-Borges & Castanheta rule. Thus the Turkstra rule will underestimate the action effects.

3.4 New combination rule

A new rule for combination of actions provides also only n different combinations of actions as the Turkstra rule does but it gives safe upper bound estimates of action effects. The concept of the new combination rule is such that maxima of nondominant actions, $\psi_{ic}\tilde{Q}_i$ for $i \neq c$ are determined during the basic interval θ_c of dominant action if this time is longer than the basic interval θ_i of the action Q_i,

$$
\begin{array}{lll}
\psi_{ic} = 1 + \upsilon_i \ln(t_d/t_{ref}) & \text{for } c=i\,, & \\
\psi_{ic} = 1 - \upsilon_i \ln(t_{ref}/\theta_i) & \text{for } c>i\,, & \qquad (13) \\
\psi_{ic} = 1 - \upsilon_i \ln(\theta_j/\theta_i) & \text{for } c<i\,. &
\end{array}
$$

The new combination factors for some nondominant actions are higher than corresponding factors according to the Ferry-Borges and Castanheta rule. That is why it gives always a safe upper bound of the load effect.

3.5 Numerical example

Combination factors ψ_{ic} are calculated and shown in Tables 1, 2, 3 for three variable actions: Q_1 - occupancy load, Q_2 - snow in winter or temperature increase in summer, Q_3 -wind. Snow and elevated temperature are exclusive events with durations of no more than half a year that is why they are taken as one variable action with two variants. It is a new concept how to treat intermittent actions with long periods of absence.

The Gumbel coefficients of variation of the actions are equal: $\upsilon_1=\upsilon_2=\upsilon_3=0{,}160$;
they correspond to the normal coefficients of variation (9): $v_1=v_2=v_3=0{,}160\ \pi/\sqrt{6}=0{,}188$;
The coefficients are equal because there are equal load factors: $\gamma_S=$ 1,50 (*EC1-1*, Table 9.2).
If also the load index is accepted (*EC1-1*, Table A.2 and A3.2) $\beta_S = 0{,}7 \cdot 3{,}8 = 2{,}66$,
the value $v = 0{,}188$ agrees with the Eurocode load factor: $\gamma_S = 1 + 2{,}66 \cdot 0{,}188 = 1{,}50$.

The design period is equal to the reference period: $t_d = t_{ref} =$ 50 years
:
and the basic intervals of the variable actions are : $\begin{cases} \theta_1 = \text{5 years for occupancy load,} \\ \theta_2 = \text{1 year for snow/temperature,} \\ \theta_3 = \text{1 week for wind.} \end{cases}$

The new ψ_i values are more likely than $\psi_1=0{,}7$ and $\psi_2=\psi_3=0{,}6$ which would follow from the Turkstra and the Eurocode combination factors (*EC1-1*, Table 9.3): $\theta_1=2{,}32$ and $\theta_2=\theta_3=0{,}83$.

i \ c	1	2	3	4
1	1	0,775	1	0,775
2	0,843	1	0,618	0,618
3	0,614	0,614	0,544	1

Table 1. Combination factor matrix according to Ferry- Borges & Castanheta

i \ c	1	2	3
1	1	0,775	0,775
2	0,618	1	0,618
3	0,235	0,235	1

Table 2. Combination factor matrix according to Turkstra

i \ c	1	2	3
1	1	0,775	0,775
2	0,843	1	0,618
3	0,544	0,614	1

Table 3. Combination factor matrix according to the new rule

4. Conclusions

4.1 One reference period t_{ref} for all variable actions and a well defined characteric value Q_k are necessary to make reasonable comparison, unification or differentiation of numerical values. The value t_{ref}= 50 years should be mentioned as a standard in Eurocode 1. It is better than t_{ref}= 1 year for reasons explained in sub-chapter 2.1.

4.2 The codified characteristic value Q_k should be equal to the characteristic extreme value Q in the reference period t_{ref} as it is defined in mathematical statistics: a fractile with intended probability of not been exceeded: e^{-1} = 0,3678... instead of the recommended value 0,98 (*EC1-1*, 4.2.8). So defined characteristic value $Q_k = Q$ may be easily changed if the design period t_d differs from the reference period t_{ref} .

$$\psi_d Q_k = [1 + \upsilon \ln(t_d / t_{ref})] \tilde{Q} \tag{14}$$

Equations (8) and (9) relate the modal value $Q_k = \tilde{Q}$ and the Gumbel coefficient of variation $\upsilon = u/\tilde{Q}$ with the normal parameters: $\bar{Q}$ and v

4.3 A value $\gamma_Q \psi_d Q_k$ may be introduced to ultimate limit states design with the load factor γ_Q.

$$\gamma_Q = 1 + (C + \beta_S \pi/\sqrt{6})\, \upsilon \qquad \text{with } C = 0{,}5772... \tag{15}$$

The product $\gamma_Q \psi_d$ gives a little different value than the exact design value Q_d according to probabilistic theory

$$Q_d = Q_k \{1 + [C + \beta_S \pi/\sqrt{6} + \ln(t_d / t_{ref})\ \upsilon]\} \tag{16}$$

4.4 The new combination rule (13) gives safe estimates for combination values of variable actions. They are upper bounds for the Ferry-Borges & Castanheta combination values. The new combination rule requires n trials to evaluate the maximum action effect for each structural element, so many as the Turkstra rule does but less than 2^{n-1} according to the Ferry-Borges & Castanheta. The exemplary combination factors ψ_{ic} (*Table* 3) have been determined for likely basic intervals θ.

4.5 A joint effect of independent permanent and variable actions is reduced thanks to geometrical summation of standard variations. No general rule can be found how to take advantage of that in partial factor design except perhaps a simple rule given for the case of a permanent load combined with one variable load (*Murzewski*, 1993). No reduction factor is used in the design (like ξ from *DIS2394*, 7.5.1) i.e. the upper bound value $\xi = 1$ is used.

4.6 Uncoupled reliability-based format may solve the above problem and simplify the design. Separate load and resistance indices β_S ,β_R can be calibrated in two ways:

- conventional way (*EC1-1*, A-3) such that constant split indices β_S β_R are specified for each safety class of construction works with the same propoportion β_S / β_R = const. The joint reliability index β may be variable for each design case,

$$\beta = a_S \beta_S + \alpha_R \beta_R \tag{17}$$

The sensitivity factors α_S, α_R depend on proportions of standard deviations σ_S/σ_R or coefficients of variation v_S/v_R ;

- optimal way such that the β_S and β_R values depend on the safety class and the coefficients of variation v_S and v_R. of the action effect or resistance, respectively. The separate indices β_S and β_R may be derived from minimum failure probability taken as the objective function of the optimization procedure (*Murzewski*, 1989, 1994,1995b).

The commonly known approach to probabilistic design (*Rshanitsin*, 1978; *Madsen, Krenk & Lind*, 1986; *Thoft-Christensen & Murotsu*, 1986) is based on maximum failure frequency as the objective function. The split indices β_S, β_R and design values S_d R_d are coupled in result of such calibration method, i.e. β_S depends on v_S and v_R and vice-versa - β_R depends on both v_S and v_R .

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Influence of skewness on reliability verification and safety factors

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Summary

The assumptions on the probability distribution of resistance, format of reliability verification and determination of numerical values of partial factors, adopted in Eurocodes, are discussed with regard to the influence of skewness. Error estimates of design reliability conditions and examples of determination of design resistance from tests in the cases of non-conforming skewness are shown. The results are compared with those obtained by a suggested design reliability condition involving an explicit occurrence of the coefficient of skewness of resistance.

1. Introduction

The present Eurocodes are developed as level 1 codes employing the limit state concept in conjunction with a partial factor method [1]. The not exceedance of all relevant limit states is verified comparing the design values of action effects and resistance. Adopting design models, the reliability condition is expressed in terms of the design values of actions, material properties and geometrical data given by their representative values and partial factors. The target level of reliability is achieved adjusting appropriate numerical values to partial factors. Calibration of partial factors is primarily based on comparison to historical and empirical design methods with amendments via a simplification of the first-order reliability method (FORM) [1]. Further development towards a probabilistic justification of numerical values of partial factors and more precise reliability verification format is envisaged.

The application of FORM, utilized in Eurocodes, is the common one - as a level 2 reliability method representing basic random variables and their functions by the first two moments. The representation sets a level of approximation allowing for further simplifications, among others (cf. [2]):

- Assumptions made on probability distributions lead to closed-form or simplified expressions for reliability verification.
- A convenient separation of action effects and resistance in the design reliability condition is adopted.

- Resistance is assumed in a product form with the log-normal distribution of basic random variables and thereby also of the resistance.
- A direct determination of the design resistance from the characteristic value of the product resistance, without explicit determination of design values for individual basic variables, is applied for steel structures (EC 1993) and is often used in connection with design by testing.

However, in application of FORM a more complete probability information can be used. There are good reasons for inclusion of the third moment, i.e. the coefficient of skewness, assuming three-parameter probability distributions of resistance and possibly of some basic random variables. Tichý [3] pointed out that neglect of the third moment may cause considerable errors in determination of the probability of failure. In many practical cases neither basic variables nor resistance itself possess values of the coefficient of skewness approximately equal to three times the coefficient of variation, which is characteristic for the log-normal distribution adopted for resistance in Eurocodes [1]. Long term investigations show that the statistical distributions of strength of higher strength steels and concretes tend to negative skewnesses [4]. This is important for checking the resistance of a compact cross-section which is dominated by the material property. Negative skewnesses were also found on studying strength functions modelling column buckling [5] and post-buckling of plates [6], mainly due to the type of probability distribution of initial deflection.

For utilization of the information on skewness in codification a simple separated form of reliability verification with an explicit occurence of the coefficient of skewness, at least in the fundamental case of reliability margin, is a necessary preliminary. From the by Tichý [3] suggested invariant first-order third-moment method there does not appear to issue a simple (formal) separation of parameters in reliability condition. Recently, for the fundamental case of safety margin the problem has been successfuly treated by Mrázik [7] or in [8] by a FORM-based asymptotic analysis. Let us note, that neither Tichý's method [3], nor Mrázik's approach are FORM oriented. Obviously, the resistance side of reliability condition, while implemented into the procedure for determination of design resistance from tests, directly influences numerical values of partial factors.

A question arises about the determination of the coefficient of skewness of resistance. Since large samples are needed to assess its value, prior knowledge from investigations of model strength functions have to be gained, if necessary. A suitable tool for identification of the model resistance by moments offer an application of the solution of inverse reliability problem [9], based on the first-order reliability index. The procedure was checked against the results obtained by the simple Monte Carlo simulation [9] and non-trivial cases were already treated, cf. [6].

In this contribution, the format of reliability verification is discussed. Especially, error estimates for a design reliability condition adopted in Eurocodes [1] and the one suggested in [8] are shown. In the cases of skewness non-conforming with the assumption of Eurocodes, examples of determination of design resistance from tests as well as the corresponding numerical values of partial factors are presented.

2. Reliability verification

2.1 Design reliability conditions

Consider a reliability problem given by the fundamental case of safety margin

$$Z = R - S \tag{1}$$

where R denotes resistance and S action effects. For normally distributed R and S, FORM procedure coincides with the well known closed-form solution yielding the design reliability condition

$$\mu_S - \alpha_S \beta_t \sigma_S \le \mu_R - \alpha_R \beta_t \sigma_R \tag{2}$$

where

$$\alpha_R = \frac{\sigma_R / \sigma_S}{\sqrt{1 + (\sigma_R / \sigma_S)^2}} , \qquad \alpha_S = -\frac{1}{\sqrt{1 + (\sigma_R / \sigma_S)^2}} \tag{3}$$

are called the FORM weight factors or sensitivity factors. The preset target value of the reliability index β_t is related to failure probability by

$$P_f = \Phi(-\beta_t) \tag{4}$$

where Φ is the standardized normal distribution function. μ, σ, v, a denote the mean value, standard deviation and coefficients of variation and skewness of a random variable or function indicated in subscript position. Assigning to the weight factors suitable constant values a convenient separation of action effects and resistance is achieved. The empirically-based values

$$\alpha_R = 0{,}8 \ , \qquad \alpha_S = -0{,}7 \tag{5}$$

recommended in [2] imply

$$\mu_S + 0{,}7\beta_t \sigma_S \le \mu_R - 0{,}8\beta_t \sigma_R \tag{6}$$

Under the assumption of the log-normal distributions of R and S, another closed-form solution to the reliability problem can be obtained ,cf. [2]. Assigning again to the weight factors the values (5) and assuming that the coefficients of variation of R and S are small a counterpart to the design reliability condition (6) can be found as

$$\mu_S \exp(0{,}7\beta_t v_S) \le \mu_R \exp(-0{,}8\beta_t v_R) \tag{7}$$

In Eurocodes a combination of design values of action effects and resistance, obtained for different assumptions on probability distributions, in reliability verification is admitted [1]. Thus, for self weight usually taken with normal distribution and log-normal resistance, the design reliability condition may read, cf. (6), (7)

$$\mu_S + 0{,}7\beta_t\sigma_S \leq \mu_R \exp(-0{,}8\beta_t v_R) \tag{8}$$

In order to gain an insight about the influence of the coefficient of skewness of resistance a_R upon the reliability verification, the case of S normal and R three-parameter log-normal was studied [8]. By an asymptotic FORM-based analysis a design reliability condition with an explicit occurrence of a_R was suggested [8]:

$$\mu_S + 0{,}7\beta_t\sigma_S \leq \mu_R - (0{,}8 - 0{,}3a_R)\beta_t\sigma_R \tag{9}$$

2.2 Error estimates

On designing a structural element, the actual reliability measure β_c may differ from the target one. Let us check the design reliability conditions (8) and (9) in an idealized situation. Following [8] we assume that the design is economical, i.e. the equality in the reliability condition is reached, and further, that the, say actual, probability distributions of actions effects and resistance are normal and three-parameter log-normal, respectively. The differences β_c-β_t then issue from:

- Non-conformity of the assumed probability distributions with those used in the derivation of design reliability condition.
- Adopted simplifications.

The value of $\beta_c = |\Phi^{-1}(P_f)|$ is obtained by the solution of the reliability problem

$$Z = R - S \geq 0 \tag{10}$$

with presumed actual distributions of R,S adjusted to the parameters issuing from the considered economical design. The probability of failure P_f is found by importance sampling technique with sample size n=50.000. For an illustrative presentation of the calculated β_c-β_t, a suitable parametrization of the reliability problem (10) and design reliability conditions under consideration are performed.

Following [8], R and S are standardized to $\tilde{R},\tilde{S}$ and (10) is rearranged to

$$Z^{st} = \beta + \frac{\tilde{R}}{[1+\frac{1}{(\sigma_R/\sigma_S)^2}]^{1/2}} - \frac{\tilde{S}}{[1+(\sigma_R/\sigma_S)^2]^{1/2}} \geq 0 \tag{11}$$

where

$$\beta = \frac{\mu_R - \mu_S}{\sqrt{\sigma_R^2 + \sigma_S^2}} \tag{12}$$

Then it can be shown that for sampling of $\tilde{R},\tilde{S}$ and evaluation of (11) altogether three parameters β, σ_R/σ_S and a_R are needed [8].

The assumed equality in the design reliability condition sets a relationship between the parameters. Thus, in the case (9) we readily find

$$\beta = \beta_t \frac{0{,}7 + (0{,}8 - 0{,}3a_R)\sigma_R / \sigma_S}{\sqrt{1 + (\sigma_R / \sigma_S)^2}} \tag{13}$$

Treatment of (8) is not so straightforward. Subsequently we divide (8) by σ_S, introduce the coefficients of variation v_R, v_S and eliminate v_S employing the equality sign. Then we express β by v_R, v_S, σ_R/σ_S and substitute for v_S the obtained expression, which finally yields

$$\beta = \frac{1}{\sqrt{1 + (\sigma_R / \sigma_S)^2}} \{0{,}7\beta_t + \frac{\sigma_R / \sigma_S}{v_R} [1 - \exp(-0{,}8\beta_t v_R)]\} \tag{14}$$

We see that in this case, besides of σ_R/σ_S, a_R, β_t, moreover the coefficient of variation v_R have to be considered as a parameter.

The error estimates β_c-β_t are calculated for σ_R/σ_S varying from 0,1 to 1,0 ; a_R= 0,5 , 0,25 , 0 , -0,25 , -0,5 ; β_t = 3,8 and v_R = 0,05 , 0,11 , 0,17 . The value of β_t is in Eurocode 1 [1] introduced as reliability level "appropriate for most cases". The choice of v_R is taken after Annex Z of ENV 1993-1-1, where the aforementioned values are attributed, according to test observations, to limit states of excessive yielding or gross deformations, local buckling and overall instability, respectively.

The results of checking the design reliability condition (8) for v_R=0,05 , 0,11 , 0,17 are shown in Figs.1,2,3. We see that with increasing v_R the curves fall deeper in the unsafe side, but the non-uniformity of approximation is smaller.

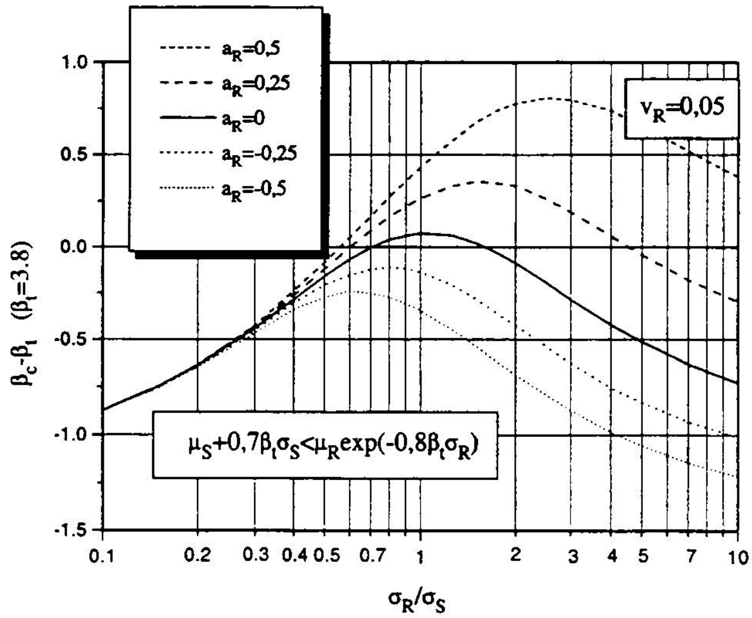

Fig. 1. Error estimates of the reliability verification according to Eurocode 1 - condition (8), β_t =3,8 , v_R=0,05.

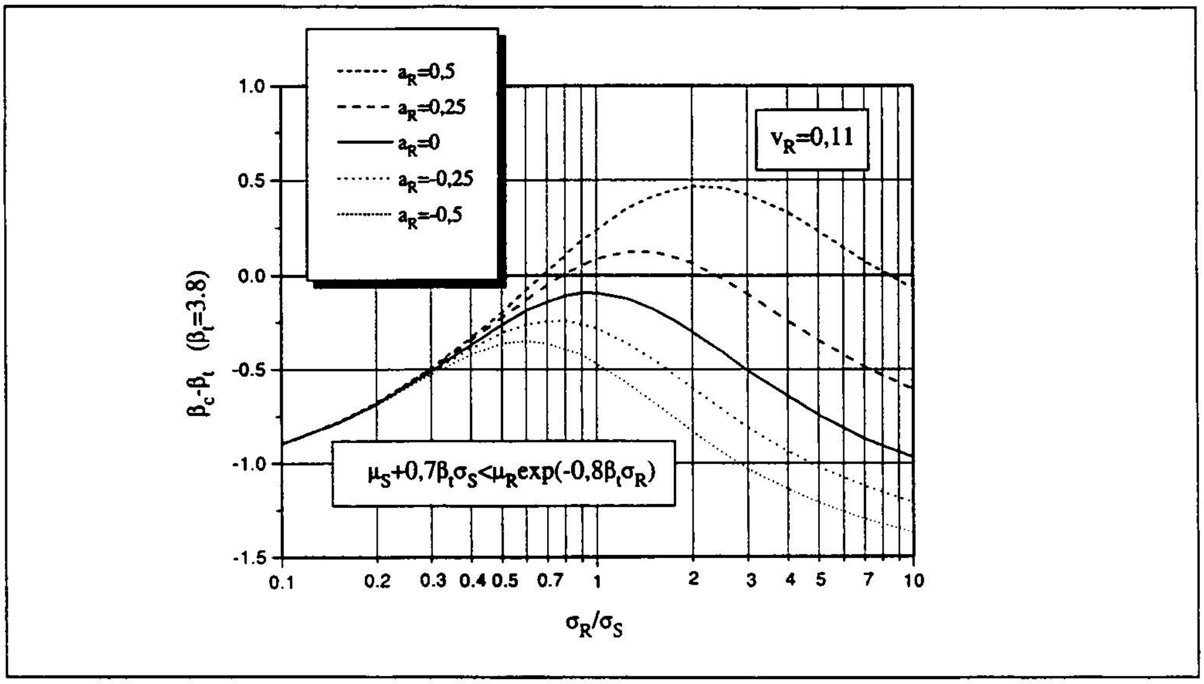

Fig. 2. Error estimates of the reliability verification according to Eurocode 1 - condition (8), β_t =3,8 , v_R=0,11.

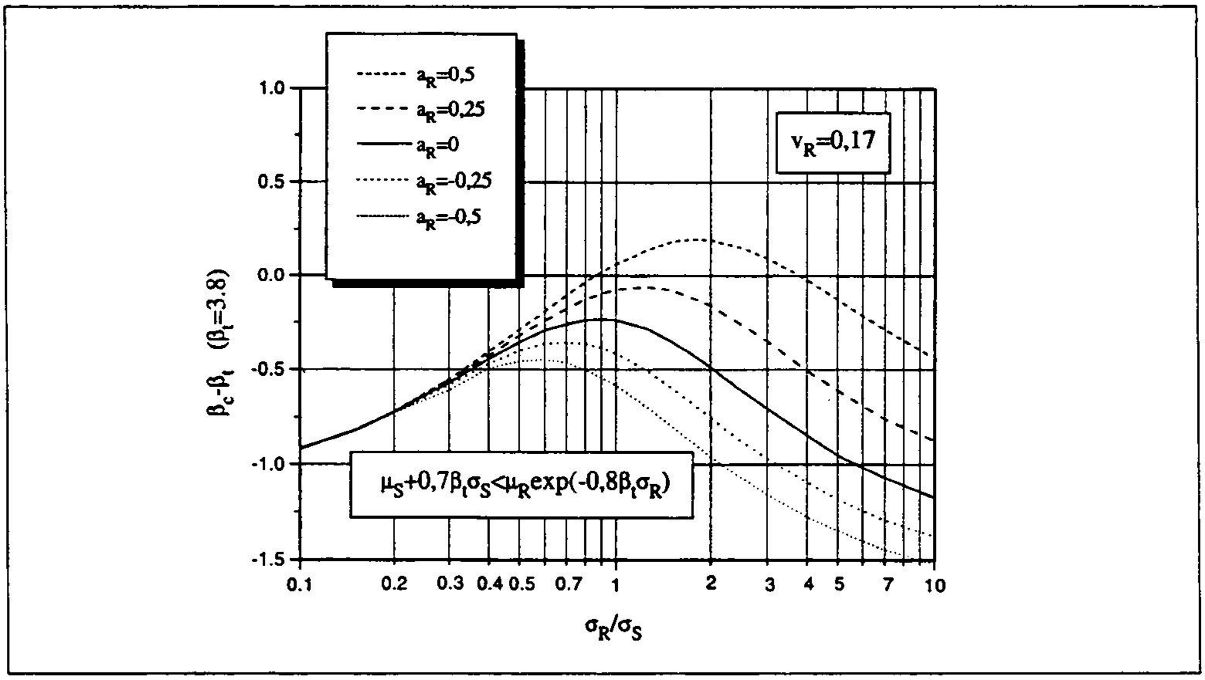

Fig. 3. Error estimates of the reliability verification according to Eurocode 1 - condition (8), β_t =3,8 , v_R=0,17.

Fig. 4 shows the corresponding results for the in [8] suggested design reliability condition (9). The error estimates of (9) were in [8] calculated by an explicit formula issuing from an asymptotic approximation of β_c.

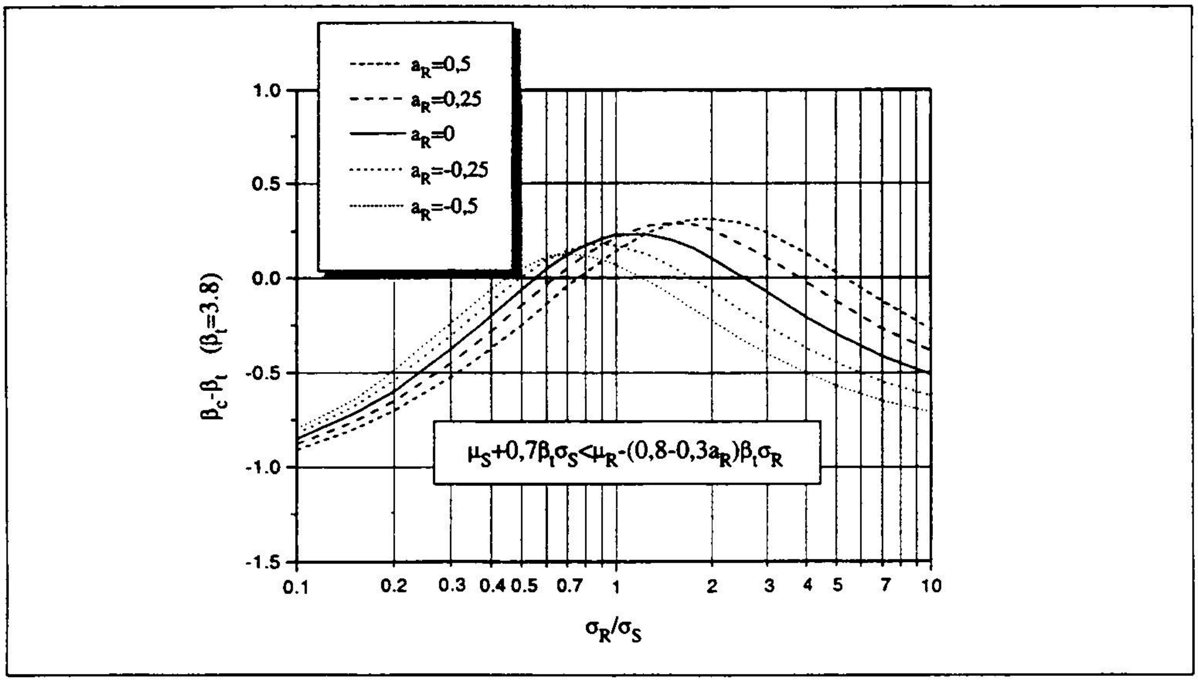

Fig. 4. Error estimates of the suggested reliability verification - condition (9), β_t =3,8.

3. Partial factors

In Eurocodes basic variables are introduced by their representative values usually defined as:

- characteristic values with a prescribed or intended probability of beeing exceeded
- nominal values

The design values are introduced indirectly, by the representative values and a set of partial factors and load combination factors.

One of the aforementioned simplifications admits a direct determination of the design resistance by testing expressing it by the characteristic value r_k - the 5% fractile of a product resistance and partial factor γ_R as

$$r_d = r_k / \gamma_R \tag{15}$$

From the viewpoint of practical utilization, it is preferable to relate the design value of resistance to the value r_n of strength function obtained for nominal values of parameters. Then the partial factor γ_R^* is defined

$$\gamma_R^* = \frac{r_n}{r_d} \tag{16}$$

On studying the numerical values of partial factors, we resort again to idealized situation. The procedure for determination of design resistance from tests is applied to a strength function identical to a basic random variable considering a perfect correlation between the design model and experiments. In this connection we may immagine about the study of the yield strength of steel specimen. The statistical characteristics are supposed to be evaluated by an almost infinite number of tests and thereby the statistical uncertainty can be neglected. S is assumed with normal probability distribution and R implied by tests as three-parameter log-normal. Then the right-hand sides of the considered design reliability conditions (8) or (9) represent the design resistance and the assumed probability distributions imply the characteristic values of resistance, thus yielding γ_R by (15).

For the reliability verification (8) with the log-normal distribution of resistance according to Eurocode 1 the corresponding partial factor denoted γ_R^{EC} is

$$\gamma_R^{EC} = \frac{\mu_R \exp(-1{,}645 v_R)}{\mu_R \exp(-0{,}8\beta_t v_R)} = \exp((0{,}8\beta_t - 1{,}645)v_R) \qquad (17)$$

Thus, for given β_t, it depends only on v_R. Some numerical values of γ_R^{EC} are for β_t =3,8 shown in Table 1.

γ_R^{EC} - (17)			
v_R =	0,05	0,11	0,17
γ_R^{EC} =	1,072	1,150	1,268

Table 1. Partial factor γ_R^{EC} according to Eurocode 1 (8).

Considering the suggested condition (9), the normal distribution $N(\mu_R,\sigma_R)$ can be attributed to the resistance. The related partial factor denoted γ_R^{aR} is

$$\gamma_R^{aR} = \frac{\mu_R - 1{,}645\sigma_R}{\mu_R - (0{,}8 - 0{,}3a_R)\beta_t\sigma_R} = \frac{1 - 1{,}645 v_R}{1 - (0{,}8 - 0{,}3a_R)\beta_t v_R} \qquad (18)$$

Naturally, in addition the coefficient of skewness a_R has appeared. Examples of evaluations of γ_R^{aR} are for β_t =3,8 shown in Table 2. We see that unusually high values of partial factors were obtained for v_R=0,17 and small and negative skewnesses. Due to different r_k values, we do not intend to compare the partial factors γ_R^{EC} and γ_R^{aR}.

γ_R^{aR} - (18)					
a_R =	0,5	0,25	0	-0,25	-0,5
v_R = 0,05	1,047	1,064	1,082	1,101	1,120
= 0,11	1,100	1,153	1,200	1,252	1,308
= 0,17	1,242	1,355	1,491	1,657	1,865

Table 2. Partial factor γ_R^{aR} (18) *corresponding to the suggested reliability verification* (9).

A meaningful comparison offer the values of partial factor γ_R^* (16) determined in correspondance with (8) or (9) (distinguished by superscript EC and aR, respectively). Obviously, the ratio

$$\frac{\gamma_R^{*EC}}{\gamma_R^{*aR}} = \frac{1-(0{,}8-0{,}3a_R)\beta_t v_R}{\exp(-0{,}8\beta_t v_R)} \tag{19}$$

equates to the reversed ratio of design resistances, thus, estimating a relative exploitation of a structural element when designed according to (8) or (9). Numerical results are shown in Table 3.

$\gamma_R^{*EC}/\gamma_R^{*aR}$ - (19)					
a_R =	0,5	0,25	0	-0,25	-0,5
v_R = 0,05	1,020	1,004	0,987	0,971	0,954
= 0,11	1,018	0,974	0,930	0,886	0,842
= 0,17	0,973	0,891	0,810	0,726	0,648

Table 3. Ratio (19) *of* γ_R^* *values calculated according to Eurocode 1* (8), *and the suggested reliability verification* (9).

4. Conclusions

The influence of skewness of resistance upon reliability verification and partial factors has been studied.

- The error estimates of reliability verification (8) according to Eurocode 1, expressed in terms of the difference between the actual and target reliability indices, show a high non-uniformity of approximation with respect to the skewness, Figs. 1,2,3.
- Checking of the suggested design reliability condition (9), with an explicit occurrence of the coefficient of skewness, shows that the scatter can be diminished to the level obtained for the case of normally distributed action effects and resistance, Fig.4, cf.[8].

The procedure for the determination of design resistance from tests has been applied in an idealized situation, employing the original assumption of the log-normal distribution of resistance adopted in Eurocode 1 [1] and the suggested normal distribution expressing the influence of skewness.

- An assumption on probability distribution to some extent predetermines the partial factors, Tables 1,2.
- The conjunction of extremely high coefficient of variation with small and negative skewnesses leads to high - unrealistically appearing partial factors, Table 2.
- The results presented in Table 3 show that generally the approach of Eurocode 1 may lead to optimistic assessments - smaller values of partial factors of resistance.

As stated in Eurocode 1, p.65 [1] "the same level of formal reliability can be obtained in many different ways". Thereby any improvement should be considered within an overall safety format of a code. Let us mention some points of possible further development in the spirit of this contribution:

- combinations of various types of probability distribution in reliability verification involving the influence of skewness
- prior knowledge of the coefficient of skewness for classes of structural elements obtained from realistic models by e.g. the approach of [9]
- implementation of the prior knowledge of statistical characteristics and assumed probability distributions into the procedure for the determination of design resistance from tests.

To cope succesfully with the outlined problems a broader cooperation on the topic is necessary.

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Safety, Economy and Legal Aspects of Limit State Design - Experience in Eastern Europe

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Summary

The transition from the Allowable Stress (A.S.) design standards to the Limit States (L.S.) design method in Eastern Europe was accomplished a quarter of century ago. At that time it raised problems and disturbances, that are actual nowadays, too. This paper is a review of their results and proposals, which could be perceived now as a "background" for the present application of Structural Eurocodes in Western Europe and in the world over.

1. Introduction

The qualitative methodological differences between the deterministic A.S. and the probability based L.S. design methods cause considerable quantitative differences to the safety and economy of the bearing structures. Here we have in mind especially the "Partial Factors" of the "Actions on Structures". The safety of the structures depends not only on scientific/ technical "Design Rules" of the Structural Codes, but also (in many cases - first of all) on organisation/ procedure "Legal Rules". These two aspects - on one hand "Safety and Economy" and on the other hand "Legal" aspects - have a fundamental meaning to the application of the Structural Codes in every country.

2. The safety and Economy Aspects

2.1 The Point of the Matter

The safety and economy of the bearing structures are insured at two levels: **(1)** By structural modelling of the structural systems/ forms. In these cases it is possible that the more economical (involving less material) structures might be also more safe; **(2)** By structural dimensioning/ calculating of the separate cross sections/ elements. In theses cases the safety and economy are always in inverse interrelation - bigger cross sections are less economical (with more material) and more safe. Of course and vice versa.
The sparing of structural materials by dimensioning of the cross sections: **(1)** is independent of

the design and execution expenditures, insurance and interest of bank credits, etc, but it affects positively all of them, and **(2)** it is an important ecological problem - less steel plants, cement mills, energy, etc.

The advantages and disadvantages of the L.S. design - in comparison with the A.S. design - are manifested mainly by dimensioning of the cross sections. The advantages are mainly theoretical: **(1)** many individual Partial Factors (instead of one), revealing in this way economic reserves, and **(2)** methodological improvement of the structural codes by probabilistic approach. The disadvantages are mainly practical: **(1)** less economical (bigger cross sections, involving more structural material) in many dimensioning cases, and **(2)** the application of the theoretical probabilistic approach is used together with unsystematic deterministic interventions.

These disadvantages are treated in detail analytically and illustrated graphically in [6 - Part One], [1], [2], [3], [4], [5]. Here they are shown pointedly only on fig. 1, 2 and 3.

2.2 Safety - Economy Comparisons

The parametric studies reveal the following characteristics, shown on fig. 1 and fig. 2:

- The L.S. method is insignificantly more economical (respectively "less safe") than the A.S method only when the temporary design forces have the same sign (direction of action) with the permanent forces and exceed them a little. I.e. heavyweight structures - concrete and usually building structures.
- The L.S. method is less economical ("more safe") than the A.S. method in the cases when temporary design forces have the same sign as the permanent forces and are respectively bigger in absolute value. I.e. for lightweight structures - steel, timber and usually bridges, towers, masts etc.
- The L.S. method is less economical ("more safe") in all cases when the temporary forces have the reverse sign of permanent forces and have a bigger absolute value than them. I.e. for all structures - more in lightweight, less in heavyweight.

2.3. On the Initiations and Improvements of L.S. Design

The problem here is to eliminate the less economical cases of the L.S. method. It is possible by the parametric approach [6 - Part Two]:

- First way - in individual cases - by an algorithm for eliminating these individual cases which are less economical.
- Second way - for all cases - by increasing adequately the design resistance, so that to avoid less economical cases when the design forces have the sign of the permanent forces.
- Third way - especially for the cases with reverse sign ($\alpha < 0$) - by individual structural modelling in such a way that the design forces have the sign of the permanent forces.

The reason of these ways is based on the normative and objective position, that the safety of the previous A.S. design method is sufficient, based on practical experience.

2.4. To Conclude the Safety - Economy Aspects

The above mentioned comparisons really disturb the initiation of the L.S. design because it is less economical than the previous A.S. method - on account of needles higher safety than A.S.

method which is proved in practice to be sufficiently safe.

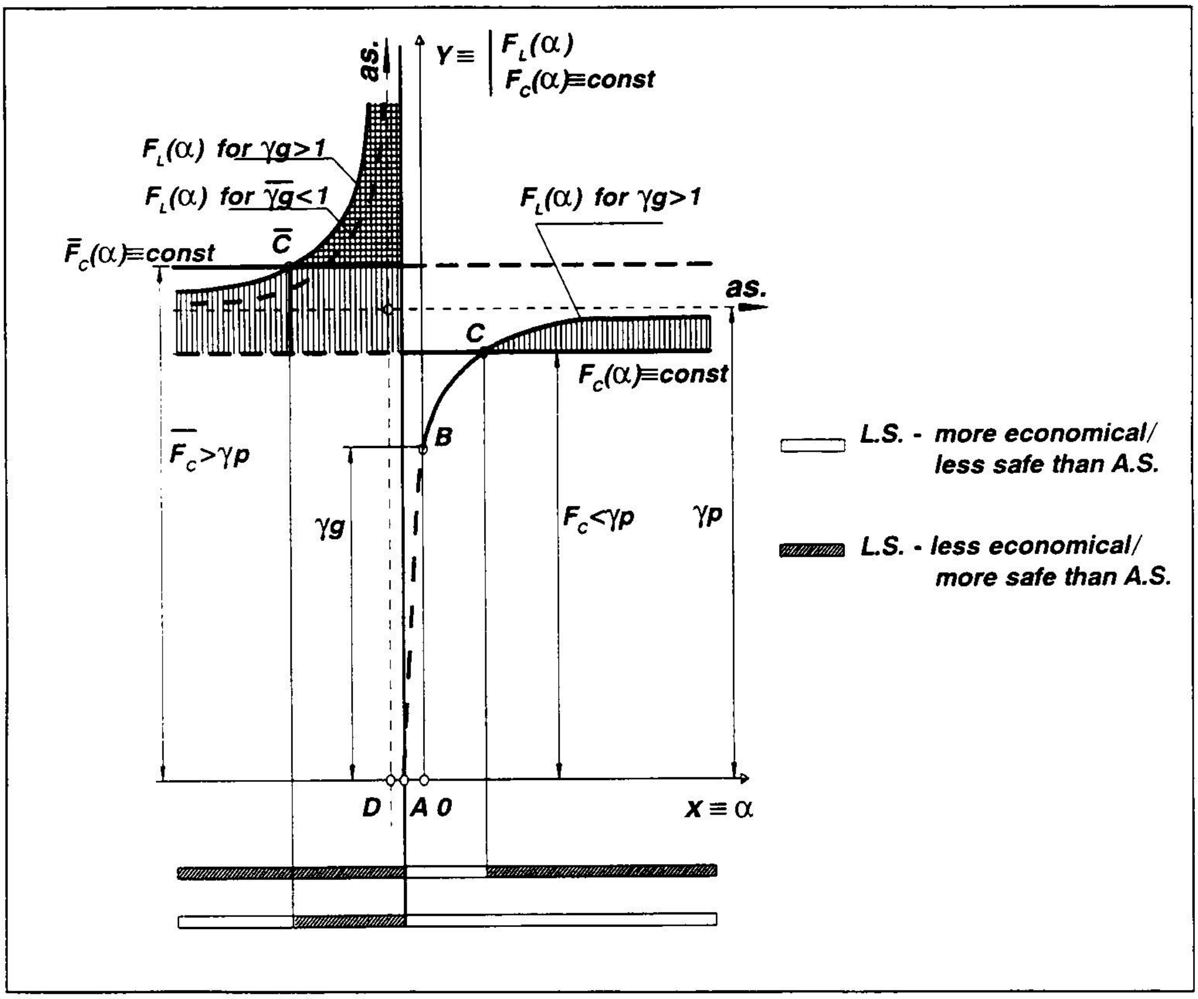

Fig. 1 Characteristic feature of the "Safety and Economy Reciprocity" between L.S. and A.S. design by dimensioning of particular cross sections/ elements in parametric study: α -Ratio of normative (without partial factors) dimensioning the temporary to the permanent forces; F_L - ratio of the acting forces (with partial factors) after L.S. to the forces after A.S.; F_C - ratio of the bearing capacities after L.S. to the capacities after A.S.; γ -partial factors.

The comparisons made for different dimensioning cases - actions, structural materials, types of buildings and structures - give the following practical possibilities: **(1)** for safety/ economy evaluations of existing structures (cross-sections/ elements designed in the past by A.S. method) - to reveal the cases where they are not safe according to the new legitimate L.S. method, and **(2)** for improvements of the L.S. design - by elimination/ reduction of the zones/ cases where it is less economical than the past A.S. method. This comparison (in text, graphics and formulas) would be helpful by the application of the Structural Eurocodes in the individual countries as a "Structural Codes Background". They could be placed in Eurocode 1 - as an **addendum "Recommendations for Evaluation of the L.S. Design Regarding the Compatibility of the Previous A.S. Design Practice".**

3. The Legal Aspects

3.1 The Point of the Matter

For one and the same earthquake impact the differences in different countries are very indicative: in one country were killed 3 persons, in another - 3 000, and in a third - 30 000 (well known cases). The reason for these extremely great differences is not in the lack of "Design Rules" in Structural Eurocodes, but the lack of "Legal Rules"(lows in force).
For example: **(1)** first of all - the role of the central, regional and local state administrations for safety and reliability of the bearing structures of all buildings, etc, **(2)** no construction - without special structural design/ project, **(3)** types and degrees of structural engineers - consultants, designers, controllers, experts, builders, operators, **(4)** certification of the qualification of structural engineers, **(5)** regulation of structural engineering design activities,

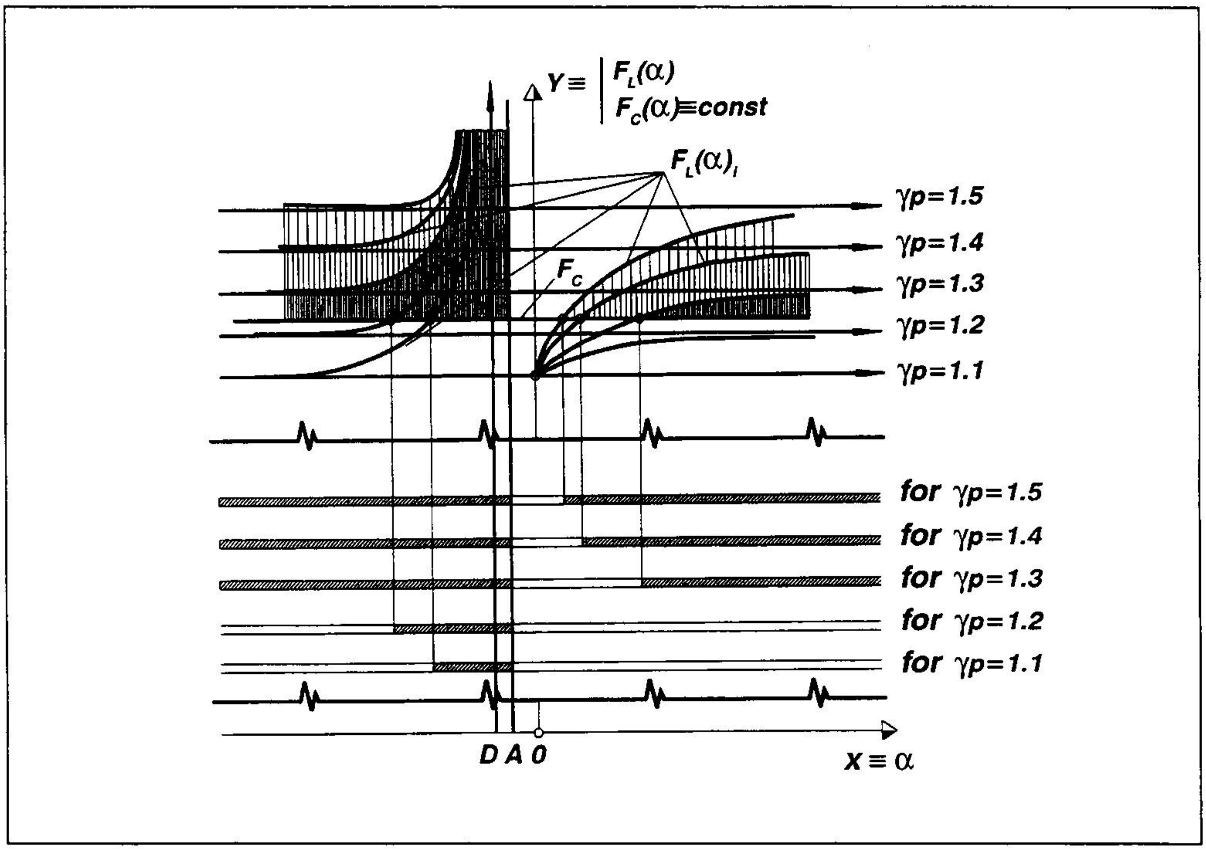

Fig. 2 Safety - Economic Relations between L.S. and A.S. design by cases with different F_L*, but one* F_C *(see fig. 1)*

(6) qualitative and quantitative criteria - system and priorities, **(7)** conjuncture factors - specificity and significance, **(8)** design parts of the structural projects - kinds and contents, **(9)** design process - functions and responsibilities of the designers, controllers and experts, **(10)** design teams - interaction and responsibilities, **(11)** role and responsibility of CAD and licenses, **(12)** control by the state - central, regional and local - structural engineering administrations, **(13)** collaboration with structural engineering societies, associations, unions, chambers, **(14)** structural project as an intellectual product - authorship, rights, **(15)** expert appraisal of the structures of buildings, etc. for the purpose of insurance, leasing, purchase, etc. We have prepared Legal Rules, available at any interest.

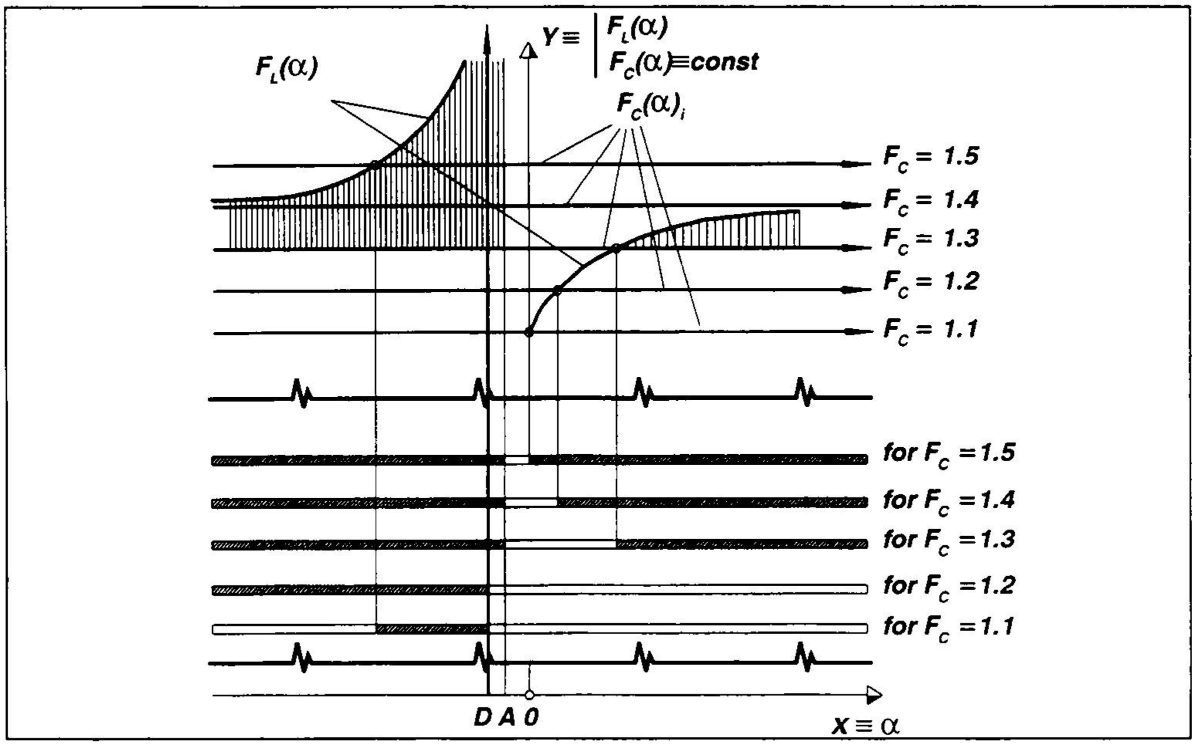

Fig. 3 Safety - Economic Relations between L.S. and A.S. design by cases with different F_C, *but one* F_L *(see fig. 1)*

3.2 To Conclude the Legal Aspects

The Legal Aspects concern "Procedure Rules": **(1)** for structural design processes and activities, and **(2)** for structural engineering administration and control. Through them only the Structural Eurocodes could run throughout all the investment activities (planning, design, construction, etc) to achieve their **final economical social goals - Safety and Economy, etc of buildings and all civil engineering works as basic conditions for a qualitative life and work of the people, for a sustainable function and development of the society**. This will mainly manifests the necessary common interest of the structural/ civil engineers and the society/ government administration. These "Legal Rules" could be placed also in Eurocode 1 as another **addendum "General Recommendation for the National Legal Acts to Compulsory Complete Application of the Structural Eurocodes"**.

After the legal acceptation of L.S. Design, respectively of the Structural Eurocodes, almost all of the existing structures (buildings, bridges, etc.), designed according to the A.S. method will turn out to be insufficiently safe from legal point of view. In this case it will be necessary: **in the above mentioned addendum to include a closing mark to legalise the existing bearing structures**. The legal alternative - analyse all of them, to close them for operation or their strengthening - is not acceptable.

4. The Structural Eurocodes and Eastern Europe

The technical standardisation system of the former socialist block, including the structural standards (codes) does not exist any more. For association and integration of these countries into the European Community the basic prerequisite is the harmonisation of the standards, and particularly of the Structural Eurocodes.

The only organised system in the world now for the development and harmonisation of the Structural Eurocodes is TC 250 at CEN. The Countries in Eastern Europe have to join this system as a matter of necessity. But on the other hand their knowledge and experience in this field could be of great use for the western countries as well.

The safety, economy and legal aspects of the bearing structures are of great importance for all countries in the world over. The protection of the society against the subjective errors and administrative negligence, concerning the safety and reliability, serviceability and durability, effectiveness and economy, natural disasters, technological accidents, etc. of the buildings, bridges and all other civil engineering works (by means of Structural Codes - for Structural Design Rules, and Legal Norms - for Structural Engineering Guidance and Control) is professional mission of the structural engineers and social duty of the state administration.

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Seismic soil structure interaction analysis and the codes of practice

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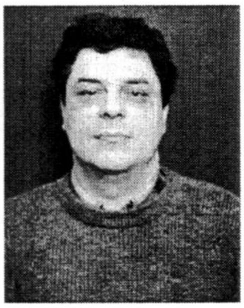

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ABSTRACT: This paper presents the main aspects that must be addressed in a SSI analysis and some of the advanced analysis techniques. Recommendations for considering the SSI effect into the seismic codes, based on simplified SSI methods are discussed. It is recommended that the already existing experience in SSI analysis, developed by the nuclear industry to be reflected into general seismic building codes.

1. Introduction

Due to the nuclear industry, the Soil Structure Interaction (SSI) phenomenon was beginning to be understood around 1970 and was considered to have significant effects on the dynamic response of the structure. Today it is known that SSI effects may govern the seismic structure response in case of relatively rigid buildings and soft soil conditions.

An important amount of research effort have been spent in this field during the 1975 - 1982 period. The result of this effort was the development of various analysis techniques and tools so called "state of the art of the industry". For the nuclear industry, these techniques became standard procedures and they were included into codes and regulations, like ASCE 4-86, US Standard Review Plan, etc. so there is a lot of experience concerning the SSI analysis techniques.

In Chapter 2 are briefly presented aspects related to the hazard level of the seismic design force, as they are reflected into building codes. Some of the basic features of the SSI problems using a very simple model, are presented in Chapter 3. Chapter 4 presents an example analyzed as follows :

- ignoring SSI effects,
- using advanced SSI methods (3D complex frequency response),
- using simplified SSI methods.

2. Hazard levels and soil structure interaction provisions in building codes

The item focuses on probabilistic definition of the key factors involved in the assessment of seismic design force according to Eurocode 8, ASCE 7 and ASCE 4 codes. The difficulty of establishing the overall reliability level of seismic design force is due to the imperfect probabilistic definition of the partial factors involved, Table 1:

$$F_b = a_g \, S \, \beta(T) \, \eta \, \frac{1}{q} W = S_e(T) \frac{1}{q} W = S_d(T) \, W$$

where:

F_b is the seismic base shear
a_g - (effective) peak ground acceleration at a site
S - soil factor
$\beta(T)$ - normalized acceleration response spectrum for 5% damping
η - damping correction factor for elastic response
q - behavior factor (response modification factor) to reduce the base shear from elastic level to the first yielding (ultimate strength level, not allowable stress level)
$S_e(T)$ - elastic response spectrum
$S_d(T)$ - design response spectrum
W - gravity load.

Peak (or effective peak) ground acceleration hazard induced by:		Soil factor[4)]	Probability of non-exceedance of response spectra	
Source magnitude	Attenuation law[5)]		Soil-dependent normalized elastic response spectra	Response modification factor [6)]
(T = 50 yr.) 0.5 prob. of exceedance in 50 yr.	Mean	Mean	0.5[2)]	0.5
(T=475 yr.) 0.1 prob. of exceedance in 50 yr. [1)]	Mean plus one standard deviation	Mean plus one standard deviation	0.9[3)]	0.9

Table 1. Hazard levels of the factors involved in the assessment of seismic design force

Note. Mean and mean plus one standard deviation values may be roughly considered respectively equal to 0.5 (median) and to 0.85 fractile of the distribution.

1) ASCE 7-93 and Eurocode 8
2) ASCE 4-95 draft and Eurocode 8
3) ASCE 4-86
4) ASCE 7-95 draft
5) 6) Probability-based definition is missing in building codes

The peak acceleration value at a site corresponding to a specified return period is generally defined in codes by a single value, even any recorded earthquake and corresponding attenuation analysis prove that a site must be characterized at least by two values: (I) the mean and (ii) mean plus one standard deviation value. The soil factors (recently introduced by the ASCE 7-95) have different hazard levels : (i) mean value for the constant spectral acceleration branch of the response spectrum and (ii) mean plus one standard deviation value for the constant velocity range of the response spectrum.

The normalized elastic response spectrum is defined as : (i) a median spectrum in Eurocode 8 and in the draft of ASCE 4-95 code, but as (ii) a mean plus one standard deviation spectrum in ASCE 4-86 code.

The calibration of the safety level of seismic design force explicitly requires a clear probabilistic definition of the all partial factors involved in the assessment of the force. Even the hazard level induced by the source magnitude to the peak (or effective peak) ground acceleration and the hazard level of the normalized acceleration elastic response spectra are usually indicated, however, the probabilistic background of the response modification factor (due to the inelastic behavior) is always missing. Generally this factor is the product of two factors:

$$q = q_{\mu}\, q_{ov}$$

where:

q_{ov} is the over strength factor

q_{μ} is factor to reduce the base shear from elastic level to the collapse level.

The $1/q_\mu$ factor can be defined either as (I) the median factor or as (ii) a factor having a specified probability of exceedance. Moreover, the values of q_μ are clearly dependent on the spectral content of the seismic input. For wide frequency band motions it is generally independent on the structure period but for narrow frequency band motions having a clear predominant period it is a function of the ratio of the structure to the soil predominant periods.

The two-earthquake methodology used in the aseismic design of the nuclear power plants (NPP), buildings and other structures designated as essential facilities claims to assess the two-hazard levels of the seismic design force from various combinations of individual hazard levels of the factors it depends. The hazard level of each of these partial factors involved in the assessment of seismic design force must be compatible to the hazard level of the remaining factors in the product.
Last but not least, the partial safety factors used by Eurocode 1 and ASCE 7 within the ultimate state design are as follows:

$$G_k + \gamma_I A_{ed} + (0.3 \div 0.8)\, Q_k \qquad \text{(EC 1)}$$

$$1.2D + E + (0.5 \div 1.0)\, L + 0.2\, S \qquad \text{(ASCE 7)}$$

where G or D indicates the dead load, A_{ed} or E - the earthquake load, Q or L - the live load and S - snow load. The subscript k denotes the characteristic values. The importance factor γ_I in EC1 depends on the building category: from 0.8 - minor importance up to 1.4 - vital importance for civil protection.

Eurocode 1, Part 5, Chapter 6 specifies that soil-structure interaction should be considered in the case of: structures with massive or deep seated foundation, slender tall structures and structures supported on very soft soil. For these cases natural periods, damping, mode shapes, etc. will differ from those of the fixed base structures.

To account for interaction effects (when the effects are on the safe side) for regular buildings, the draft ASCE 7-95 code reduce the seismic base shear V as follows:

$$V^* = V - \Delta V$$

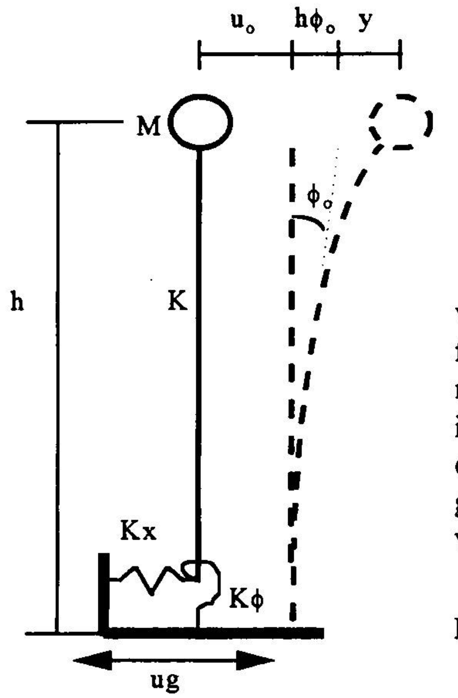
$$\Delta V = [\, C_s - C_s^*(\frac{0.05}{\beta})^{0.4}]W \; < 0.3V$$

$$\beta^* = \beta_0 + 0.05(\, T^*/T)^3)$$

where : C_s and C_s are the overall seismic coefficients determined without and with SSI effect,
T^*, $T^*>T$, - the natural periods of flexible supported building and rigid supported building,
β^*, β - the damping coefficient with and without SSI effect,
W - the effective gravity load.

3. Soil Structure Interaction

To illustrate the SSI effect a simple model consisting of a single mass M, lumped at a height h above the base and structure stiffness K, will be used, Fig.1. For the case of a horizontal excitation the equation of motion for the mass point is:



$$Mu + Ky = 0 \tag{1}$$

$$u = u_G + u_0 + y + h\Phi_0$$

$$Ky = K_x u_0$$

$$Khy = K_\Phi \Phi_0$$

where K_x is the horizontal spring representing the foundation translation stiffness, K_Φ is the corresponding rocking spring, u is the absolute displacement of mass, y is the structural deformation and u_0 and Φ_0 are the deformation of the foundation springs, and u_G is the ground displacement in the free field. Equation (1) can be written as:

$$M\,(1 + K / K_x + Kh^2 / K_\Phi)\,y + Ky = -M\,u_G \tag{2}$$

Figure 1. Simple Model

The natural frequency of the structure on a rigid base (without SSI) is :

$$\omega_o = (K / M)^{1/2} \tag{3}$$

Taking into account the flexibility of the foundation, the frequency becomes:

$$\omega = \frac{\omega_o}{(1 + K / K_x + Kh^2 / K_\Phi)^{1/2}} \tag{4}$$

Assuming the structure internal damping D_{st} of hysteretic type which is frequency independent and the soil internal material damping D_s, also hysteretic and dashpots C_x, C_Φ associates with the foundation sprongs K_x and K_Φ (to reproduce the loss of energy by radiation), then the effective damping D of the system at its natural frequency ω is given approximately by [8]:

$$D = D_{st}\left(\frac{\omega}{\omega_0}\right)^2 + D_s\left[1 - \left(\frac{\omega}{\omega_0}\right)^2\right] + D_s\left(\frac{\omega}{\omega_0}\right)^2\left[\frac{K}{K_x}\,\frac{\omega C_x}{2K_x} + \frac{Kh}{K_\Phi}\,\frac{\omega C_\Phi}{2K_\Phi}\right] \tag{5}$$

As it could be expected, the flexibility of the soil results in a decrease of the natural frequency, indicating that the system is more flexible.

The magnitude of this change is a function of relative stiffness of the structure with respect to soil, as indicated by terms K/K_x and Kh^2/K_Φ. Equation (5) shows the soil contribution to the effective damping of the soil-structure system. The amount of increase depends mainly on the magnitude of the last term, representing the radiation damping. From the analysis of this simple dynamic system, it can be seen that the main effects of soil structure interaction are:

- a decrease of the natural frequency of the system, depending on the relative stiffness of structure with respect to the soil;
- a change in the effective damping of the system; the main factor contributing to the increase in damping is the lose of energy by radiation of waves from the foundation;
- the appearance of the rotational component of motion at the base.

In order to estimate the magnitude of interaction effects it is necessary to know the values of terms K_x, C_x, K_Φ, C_Φ, K_z and C_z, which represent the dynamic stiffness of the foundation. These values are function of soil material, foundation shape, embedment depth and also are frequency dependent. A comprehensive review of the SSI methods was done by Roesset [8].

4. Example

The following example illustrates the principal SSI problems that should be addressed. The dynamic structure model is presented in Figure 2. In Tables 2a and 2b are presented the structure inertial and stiffness characteristics.

The SSI analysis has been performed using two parallel methods:

a) advanced method - using complex frequency domain analysis
b) simplified method - using modal analysis with a spring base model.

The seismic excitation was defined at free field level base from seismic hazard analysis. The maximum peak ground acceleration is 0.195g.

Elevation		Shear center		A	S_{shx}	A_{shy}	I_x	I_y	I_t
from	to	X(m)	Y(m)	(m^2)	(m^2)	(m^2)	(m^4)	(m^4)	(m^4)
10.0	13.2	14.07	12.93	232.3	158.3	169.9	9580	18410	28020
13.2	22.2	13.53	9.40	86.6	53.6	37.45	4911	10033	13323
22.2	28.2	15.73	5.46	121.9	67.1	66.4	4773	10231	12582
28.2	31.2	16.31	3.55	111.3	54.0	51.3	5808	6545	9754
31.2	36.0	15.90	8.38	137.2	86.7	92.7	5463	6851	9800
36.0	43.7	14.25	10.65	0.41	0.0	0.0	1.1	1.8	2.9
43.7	46.0	14.25	10.65	0.41	0.0	0.0	1.1	1.8	2.9

Table 2.a Stiffness properties

Elev.	Mass center		e_x	e_y	M_{vert}	M_x	M_y
(m)	X(m)	Y(m)	(m)	(m)	tones	tones	tones
10.0	14.51	11.04	0.04	1.89	3536	3311	3311
22.2	12.90	9.96	2.83	4.50	2833	2833	2833
31.2	15.06	10.30	1.23	1.51	2173	2398	2398
36.0	14.26	10.36	0.01	0.29	1981	1885	1912
48.0	14.25	10.65	0.00	0.00	138	138	138

Table 2.b Inertial properties

The seismic waves produce shear and volume strain deformation in soil material. The non-linear effect produced by the seismic waves in the soil material is called the primary nonlinearity. The dynamic foundation stiffness taking into account the soil profile layout, soil dynamic properties, primary non-linearity, foundation characteristics (shape, embedment, etc.) was computed using SUPELM computer code [7]. The dynamic foundation stiffness includes also the damping: material damping and radiation damping.

The soil profile is presented in Table 3. The dynamic soil properties are based on site measurements of shear wave velocity and lab tests. The Seed & Idriss curves G-γ and D-γ, representing the variation of the dynamic shear modulus G versus shear strain deformation γ and material damping D_s versus shear strain γ respectively corresponding to send material were used in analysis .

Layer	Height [m]	Unit weight. [t/m^3]	V_s [m/s]	G [t/m^2]	Damping %	Poisson
1 Sand+ Gravel	1.5	1.8	196.4	6943.1	2.7	0.40
2 Sand+ Clay	4.0	1.75	156.2	4269.7	11.0	0.43
3 Sand+ Cl+Grav	7.5	1.80	203.0	7417.6	12.5	0.42
4 Sand+Gravel	6.0	1.85	287.0	15238.3	10.0	0.38
5 Sand	5.0	1.90	338.8	21235.3	9.0	0.38
6 Sand	10.0	1.95	478.5	44647.6	9.0	0.36
7 Sand	100.0	2.0	565.0	63845.0	7.0	0.35

Table 3 Iterated soil properties profile

The next important problem is to determine the seismic motion corresponding to the foundation level. This step is called kinematic interaction. The result of the kinematic interaction is the modified free field motion corresponding to the foundation level. This step was performed using KININT program [7].

The last problem was to determine the soil-structure dynamic response. The structure response has been solved using advanced complex frequency analysis model EKSSI [7], simplified spring base model and without SSI effect - i.e. fixed base structure. Based on complex frequency dependent foundation stiffness matrix, equivalent soil springs constants have been calculated to be used in simplified method.

Comparison between the floor response spectra computed at elevation 36.0 Figure 3, shows a good agreement between advanced and simplified method. Comparison between fixed base structure and spring base structure are presented in terms of maximum displacements, accelerations and base shear forces in Tables 5 and Table 6

Elevation	Maximum Displacements			Maximum Acceleration		
(m)	X(cm)	Y(cm)	Z(cm)	X(g)	Y(g)	Z(g)
10.0	1.14	1.01	0.16	0.132	0.143	0.08
17.2	1.52	1.20	0.25	0.166	0.172	0.09
22.2	1.89	1.33	0.28	0.212	0.198	0.10
31.2	2.10	1.57	0.35	0.268	0.251	0.12
36.0	2.00	1.70	0.30	0.273	0.277	0.11
48.0	2.20	2.20	0.30	0.802	0.963	0.11

Table 5a. Seismic response (with SSI effect)

Shear (X)	Shear (Y)	Vertical (Z)	Overturning Moment		
			M_x	M_y	M_t
kN	kN	kN	kNm	kNm	kNm
39110.0	39540.0	23800.0	692100.0	473100.0	475200.0

Table 5b. Global force at foundation level (with SSI effect)

Elevation	Maximum Displacements			Maximum Acceleration		
(m)	X(cm)	Y(cm)	Z(cm)	X(g)	Y(g)	Z(g)
10.0	0.0	0.0	0.0	0.0	0.0	0.0
17.2	0.033	0.030	0.002	0.190	0.211	0.03
22.2	0.058	0.048	0.005	0.230	0.270	0.07
31.2	0.110	0.090	0.007	0.380	0.450	0.11
36.0	0.130	0.100	0.010	0.450	0.530	0.12
48.0	0.760	0.550	0.018	2.500	1.270	0.41

Table 6a. Seismic response (without SSI effect)

Shear (X)	Shear (Y)	Vertical (Z)	Overturning Moment		
			M_x	M_y	M_t
kN	kN	kN	kNm	kNm	kNm
40680.0	45840.0	11560.0	852000.0	757000.0	370000.0

Table 6b. Global force at foundation level (without SSI effect)

The analysis of these results shows:

- the soil-structure system frequencies are 2.14 Hz and 2.45 Hz for horizontal translation and 4.52 and 5.48 for rocking;
- the soil-structure system mode shapes correspond to rigid body translation and rocking;
- the fix base structure first modes are 7.10 Hz. and 8.35 Hz;
- the SSI effect increases the damping of the soil-structure system and decreases the seismic force and structure elastic deformation;
- simplified SSI method using spring base model can produce good results if the spring constants are properly calibrated [3], [5];

- the SSI effect consists in the reduction of natural frequencies, rigid body displacement response, and in the increase of system damping, reduction of global seismic base force and changes in the distribution of seismic forces (see accelerations)
- for higher frequency (over 3.0 Hz) the simplified method produces conservative results due to the fact that the soil stiffness and damping characteristics were considered frequency independent.

5. Conclusions

In the calculation of seismic design force using building codes, the hazard level of each partial factors involved must be consistent.
Design requirements concerning SSI effect, developed by nuclear industry, started to penetrate in a simplified form the general seismic building codes - ASCE 7-95 and EC1.
Without proper analysis, SSI is hardly predictable; the effects could be on both sides: favorable and adverse to the structure.
The SSI experience accumulated in the nuclear industry design should be used in establishing simplified design requirements applicable for regular buildings.
Further studies and numerical test are beneficial for comparison between the simplified and advanced SSI methods.

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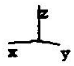
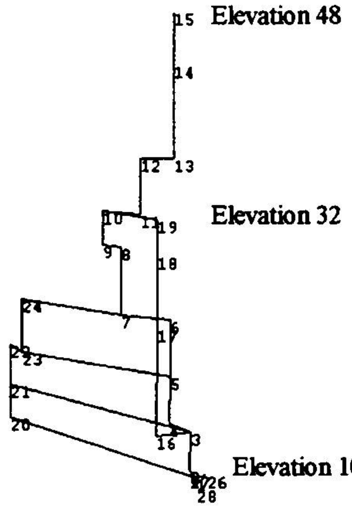
FIGURE 2 Dynamic model



Mode	Fix base Model Freq, Hz	Spring base model Freq, Hz
1	7.10	2.14
2	8.35	2.45
3	10.89	4.52
4	11.48	5.48
5	24.46	7.16
6	25.04	8.24
7	29.60	9.51
8	32.05	10.70
9	-	27.55
10	-	30.10

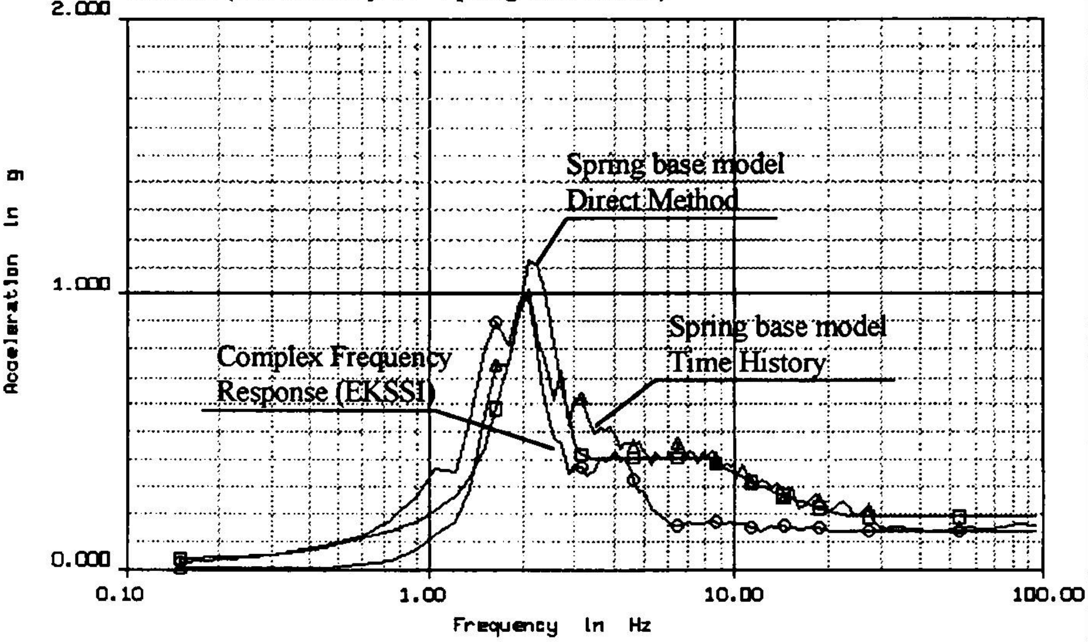


Translation Spring Damping = 0.30
Rocking spring Damping = 0.15

Global Foundation Stiffness

DOF	Value t/m
Horizontal Translation	6.85E+05
Vertical Translation	3.72E+06
Rocking	4.48E+08
Torsion	1.62E+08

FIGURE 3. Floor Response Spectrum, Elevation 36, Damp.= 0.02
Comparison between advanced method EKSSI and simplified method (modal analysis - spring base model)



Statistical procedures for design assisted by testing

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Summary

Three parameter asymmetric distributions, characterised by the mean μ, standard deviation σ and independent coefficient of skewness α, are considered to present necessary statistical techniques for estimating characteristic and design values of basic variables from test data of limited size. It is shown that the resulting estimates for characteristic strength may considerably depend on the applied method and on available prior knowledge; possible asymmetry of the distribution should be considered whenever the coefficient of skewness exceeds $\pm$ 0,5.

1. Introduction

The Eurocode 1 [1] provides in Section 8 "Design assisted by testing" application rules for design procedures performed on the basis of tests. Design values for a material property, a model parameter or a resistance value may be determined from tests in either of the following two ways:

a) by assessing a characteristic value, which is divided by a partial factor and possibly by an explicit conversion factor,
b) by direct determination of the design value, implicitly or explicitly accounting for the conversion aspects and the total reliability required.

A simple statistical technique for assessment of material quality from tests is described in the informative annexes A and D of the Eurocode 1 [1], further information is available in ISO/CD 12491 [2] and revised ISO 2394 [3]. The methods included in [1], [2] and [3] are based on Bayesian approach assuming symmetrical normal distribution and vague prior information. It is, however, noted in the above mentioned Annex D that in practice there may be prior knowledge available indicating that the distribution type is of more favourable nature (for instance lognormal distribution with zero origin). There may be also partial prior knowledge about the mean and standard deviation based on previous experience which may lead to more accurate design values.

The aim of this contribution is to suggest possible extension of basic statistical methods recommended in [1], [2] and [3], particularly to show effect of population asymmetry and to propose operational procedures and appropriate provisions which could be included in an expected revision of the Eurocode 1 [1]. Presented procedures follow from previous studies concerning estimation of fractiles assuming general lognormal distribution [4], [5] and effects of distribution asymmetry in structural reliability and statistical quality control [6], [7], [8] and [9].

2. Statistical techniques

2.1 Basic probabilistic concepts

From the probabilistic point of view the characteristic or the design value of a resistance variable like the strength of concrete can be defined as a specified fractile of appropriate probability distribution. Fractile x_p is generally defined as a value of a random variable X satisfying the following relation

$$\mathrm{P}\{X < x_p\} = p \tag{1}$$

where capital X denotes a random variable and small x its particular realisation, p denotes specified probability. For the characteristic strength often the probability $p = 0{,}05$ is assumed. However, for the design strength lower probabilities, say $p \cong 0{,}001$, are to be considered. On the other hand the design value of non-dominating variables may correspond to greater probabilities, say $p \cong 0{,}10$.

When assessing strength of building materials, usually a limited number of observations is available only. Moreover, relatively high variability (coefficient of variation up to 0,25) and mostly a positive distribution asymmetry should be expected. That is why applied statistical techniques should be chosen cautiously, particularly when design strength corresponding to small probability is investigated.

In the following a lower fractile x_p ($p < 0{,}5$) of a random variable X is considered only. It is assumed that the population mean μ is unknown and sample mean m is available. The standard deviation σ is assumed to be either known or unknown. In the later case the sample standard deviation s is used. The coefficient of skewness α is always assumed to be known from previous experience. Two basic statistical methods to estimate fractiles are used most frequently: the coverage method and prediction method. When previous observations of a continuous production is available Bayesian approach can be used.

2.2 Coverage method

The classical coverage method is based on the key notion of the confidence level γ (often assumed 0,75, 0,90 or 0,95) for which the one-sided estimate $x_{p,\mathrm{cover}}$ of a lower p-fractile is determined in such a way that

$$\mathrm{P}\{x_{p,\,\mathrm{cover}} < x_p\} = \gamma \tag{2}$$

If the population standard deviation σ is known, the lower p-fractile estimate $x_{p,\mathrm{cover}}$ is given as

$$x_{p,\mathrm{cover}} = m - \kappa_p\ \sigma \tag{3}$$

if the population standard deviation σ is unknown and the sample standard deviation s is used then

$$x_{p,\mathrm{cover}} = m - k_p\ s \tag{4}$$

The estimation coefficients $\kappa_p = \kappa(\alpha, p, \gamma, n)$ and $k_p = k(\alpha, p, \gamma, n)$ depend on the coefficient of skewness α, on the probability p corresponding to the desired fractile x_p, on the confidence level γ and on sample size n. Explicit knowledge of the probability γ, that the estimate $x_{p,cover}$ shall lay on the safe side from the actual value x_p, is the most important advantage of the method. To take account statistical uncertainty the value γ = 0,75 is recommended in [3]. However, when unusual reliability consideration is required, higher confidence level 0,95 seems to be appropriate [5], [6]. In the documents [1] and [3] only the normal distribution is considered without taking into account possible asymmetry of the population distribution.

It may be shown [4] that if the population standard deviation σ is known, then the estimation coefficient $\kappa(\alpha, p, \gamma, n)$ may be well approximated using formula:

$$\kappa(\alpha, p, \gamma, n) = - u_p + u_\gamma / \sqrt{n} \tag{5}$$

where u_p is p-fractile of standardised lognormal distribution having the coefficient of skewness α, and u_γ is γ-fractile of standardised lognormal distribution having the coefficient of skewness $\alpha / \sqrt{n}$. If the population standard deviation σ is unknown, then the coefficient $k(\alpha, p, \gamma, n)$ may be expressed as

$$k(\alpha, p, \gamma, n) = - t(\alpha, p, \gamma, \nu) / \sqrt{n} \tag{6}$$

where $t(\alpha, p, \gamma, \nu)$ is γ-fractile of the generalised noncentral t-distribution having the coefficient of skewness α, corresponding to the probability p and with $\nu = n$-1 degree of freedom. The noncentral t-distribution, describing distribution of the p-fractile of lognormal distribution with the coefficient of skewness α, is a modification [4] of well known noncentral t-distribution derived from normal distribution. Extensive numerical tables for both estimation coefficients (σ is either known or unknown) are available in the Klokner Institute of CTU Prague.

2.3 Prediction method

According to the prediction method [10] the lower p-fractile x_p is assessed by the prediction limit $x_{p,pred}$, determined in such a way that a new value x_{n+1} randomly taken from the population would be expected to occur below $x_{p,pred}$ with the probability p, thus

$$\mathrm{P}\{x_{n+1} < x_{p,\,pred}\} = p \tag{7}$$

The prediction estimate $x_{p,pred}$, defined by equation (7), asymptotically approaches the unknown fractile x_p with increasing n, and from this point of view $x_{p,pred}$ can be considered as an assessment of x_p. It can be also shown that the prediction estimate $x_{p,pred}$ correspond approximately to the coverage method assuming the confidence level γ = 0,75 [8].

If the population standard deviation σ is known, the lower p-fractile estimate $x_{p,cover}$ is given in terms of the sample mean m as

$$x_{p,pred} = m + u_p\,(1/n + 1)^{1/2}\,\sigma \tag{8}$$

where $u_p = u\,(\alpha, p, \nu)$ is p-fractile of standardised lognormal distribution having the coefficient of skewness α. If the population standard deviation σ is unknown and the sample standard deviation s is used then

$$x_{p,pred} = m + t_p\,(1/n + 1)^{1/2}\,s \tag{9}$$

where $t_p = t(\alpha, p, \nu)$ is p-fractile of a generalised Student t-distribution having the coefficient of skewness α for $\nu = n - 1$ degrees of freedom.

2.4 Bayesian approach

When previous observations of a continuous production is available an alternative technique is provided by Bayesian approach [1], [2] and [3]. Let m is the sample mean, s the sample standard deviation determined from a sample of the size n. Besides from previous observations the sample mean m' and sample standard deviation s' determined from a sample, which values and the size n' are unknown, are available. Both samples are assumed to be taken from the same population having theoretical mean μ and standard deviation σ. Hence both samples can be considered jointly. Parameters of the combination of both samples are [2], [3]

$$n'' = n + n'$$

$$\nu'' = \nu + \nu' - 1, \text{ when } n' \geq 1, \quad \nu'' = \nu + \nu' \text{ when } n' = 0$$

$$m'' = (mn + m'n') / n''$$

$$s''^2 = (\nu s^2 + \nu' s'^2 + n m^2 + n' m''^2 - n'' m''^2) / \nu'' \tag{10}$$

Unknown values n' and ν' may be estimated using formulae for the coefficients of variation $V(m')$ and $V(s')$, which may be written as

$$n' = [\sigma / (\mu \; V(m'))]^2, \quad \nu' = 1 / (2 \; V(s')^2) \tag{11}$$

Obviously, both values n' and ν' may be chosen individually (generally $\nu' \neq n'$-1) depending on previous experiences concerning degree of uncertainty in estimating the mean μ and standard deviation σ.

In accordance with [2] and [3] the Bayesian estimate $x_{p,\text{Bayes}}$ is given by a formula similar to equation (9) used by prediction method assuming that σ is unknown

$$x_{p,\text{Bayes}} = m'' + t_p \, (1/n'' + 1)^{1/2} \, s'' \tag{12}$$

where $t_p = t(\alpha, p, \nu'')$ is again p-fractile of the generalised Student t-distribution having the coefficient of skewness α for ν'' (generally different from n'' - 1) degrees of freedom.

When applying the Bayesian technique for determining strength of building materials, an advantage may be taken of the fact, that long term variability of the strength is usually stable. Thus, uncertainty in determining σ is relatively small, the value $V(s')$ is also small and ν' given by (11) and ν'' given by (10) is high. This may lead to a favourable decrease of the resulting value t''_p and to an favourable increase of the estimate for the lower fractile x_p (see equation (12)). On the other hand uncertainty in determining μ and $V(m)$ is usually high and previous information may not significantly affect the resulting n'' and m''.

If no prior information is available, then $n' = \nu' = 0$ and the characteristics m'', n'', s'', ν'' equal the sample characteristics m, n, s, ν. Equation (12) reduces to the previous expression (9) . In this special case the Bayesian approach leads to the same procedure as prediction method and equation (9), in the case of known σ equation (8), are to be used. It should be noted that this special case of Bayesian technique with no prior information is considered in the informative annex D of the Eurocode 1 [1] and in ISO documents [2] and [3].

3. Comparison of coverage and prediction method

To estimate the characteristic and design strength the coverage and prediction method are applied most frequently. These methods are compared here (see also [8]) assuming normal distribution (lognormal distribution with $\alpha = 0$) of the population. Table 1 shows the coefficients κ_p and $u_p(1/n+1)^{1/2}$ used in equations (3) and (8) for selected values of n and γ. It follows from Table 1, that differences between both coefficients are dependent on number of observations n as well as on confidence level γ. For $\gamma = 0{,}95$ and small n the coefficient κ_p of the coverage method is by almost 40% higher than the corresponding coefficient $u_p(1/n+1)^{1/2}$ used in the prediction method. If $\gamma = 0{,}75$ is accepted (as recommended in [2] and [3]) than the differences are less than 10%. Generally, however, the prediction method would obviously lead to higher (less safe) characteristic values than the classical coverage method for the confidence level $\gamma \geq 0{,}75$ (see also [8]).

Coefficients		Number of observations n								
		3	4	5	6	8	10	20	30	∞
κ_p	$\gamma = 0{,}75$	2,03	1,98	1,95	1,92	1,88	1,86	1,79	1,77	1,64
	$\gamma = 0{,}90$	2,39	2,29	2,22	2,17	2,10	2,05	1,93	1,88	1,64
	$\gamma = 0{,}95$	2,60	2,47	2,38	2,32	2,23	2,17	2,01	1,95	1,64
$-u_p(1/n+1)^{1/2}$		1,89	1,83	1,80	1,77	1,74	1,72	1,68	1,67	1,64

Table 1. Coefficients κ_p and $u_p(1/n+1)^{1/2}$ for $p = 0{,}05$ and known σ.

If the standard deviation σ is unknown, equations (4) and (9) are to be compared. Table 2 shows the appropriate coefficients k_p and $t_p(1/n+1)^{1/2}$ for the same number of observations n and confidence levels γ as in table 1. Obviously, differences between the coefficients corresponding to different confidence levels γ are much more significant than in previous case of known σ. For $\gamma = 0{,}95$ and small n the coefficient k_p used by the coverage method is by almost 100% greater than the coefficient $t_p(1+1/n)^{1/2}$ used by the prediction method. For $\gamma = 0{,}75$ both coefficients are nearly the same. The coefficient k_p is, however always slightly greater than $t_p(1/n+1)^{1/2}$ except for $n = 3$ (see also [8]). Like in the previous case of known σ, the prediction method would generally lead to greater (less safe) characteristic strengths than the classical coverage method. The difference increases with increasing confidence level.

Coefficients		Number of observations n								
		3	4	5	6	8	10	20	30	∞
k_p	$\gamma = 0{,}75$	3,15	2,68	2,46	2,34	2,19	2,10	1,93	1,87	1,64
	$\gamma = 0{,}90$	5,31	3,96	3,40	3,09	2,75	2,57	2,21	2,08	1,64
	$\gamma = 0{,}95$	7,66	5,14	4,20	3,71	3,19	2,91	2,40	2,22	1,64
$-t_p(1/n+1)^{1/2}$		3,37	2,63	2,33	2,18	2,00	1,92	1,76	1,73	1,64

Table 2. Coefficients k_p and $t_p(1/n+1)^{1/2}$ for $p = 0{,}05$ and unknown σ.

4. Effect of asymmetry

Actual asymmetry of population distribution may have significant effect on results of fractile estimation, particularly when small samples are taken from a population with high variability [6]. Assuming general three parameter lognormal distribution with independent coefficient of skewness α effect of population asymmetry on 0,05-fractile estimate is shown below for two confidence levels considering three coefficients of skewness α = -1,00, 0,00 and +1,00. Table 3 shows the coefficient k_p for selected numbers of observations n and confidence γ = 0,75. Table 4 shows the coefficient k_p for the same numbers of observations n as in table 3, but for the confidence level γ = 0,95.

Coefficients of skewness	Number of observations n								
	3	4	5	6	8	10	20	30	∞
α = -1,00	4,31	3,58	3,22	3,00	2,76	2,63	2,33	2,23	1,85
α = 0,00	3,15	2,68	2,46	2,34	2,19	2,10	1,93	1,87	1,64
α = 1,00	2,46	2,12	1,95	1,86	1,75	1,68	1,56	1,51	1,34

Table 3. Coefficients k_p for p = 0,05, γ = 0,75 and unknown σ.

Coefficients of skewness	Number of observations n								
	3	4	5	6	8	10	20	30	∞
α = -1,00	10,9	7,00	5,83	5,03	4,32	3,73	3,05	2,79	1,85
α = 0,00	7,66	5,14	4,20	3,71	3,19	2,91	2,40	2,22	1,64
α = 1,00	5,88	3,91	3,18	2,82	2,44	2,25	1,88	1,77	1,34

Table 4. Coefficients k_p for p = 0,05, γ = 0,95 and unknown σ.

Comparing data given in both tables 3 and 4 it follows that the effect of distribution asymmetry on the estimate $x_{p,cover}$ considerably increases with increasing confidence level γ. Generally the effect decreases with increasing n, nevertheless, it never vanishes even for $n \to \infty$. Detailed analysis [8] shows that when assessing characteristic strength of concrete corresponding to the 0,05-fractile, actual asymmetry of probability distribution should be considered whenever the coefficient of skewness is greater (in absolute value) than 0,5.

Differences between estimates obtained assuming general lognormal distribution with a given coefficient of skewness $\alpha \neq 0$ and corresponding estimates assuming normal distribution with α = 0, increases also with decreasing probability p associated with the estimated fractile x_p (see also [8]). This is one of the reasons why design value of strength, corresponding to a very small probability p (say 0,001), should not be generally determined directly from test data. Direct assessment could be applied only in those cases when sufficient number of observations and a convincing evidence on appropriate probabilistic model (including information on asymmetry) are available. When such an evidence is not accessible, the design value should be preferably determined by assessing a characteristic value, which is divided by a partial factor and possibly by an explicit conversion factor, as recommended in Eurocode 1 [1].

Effect of asymmetry on the coefficient t_p used in the prediction method is shown in table 5 for the same coefficients of skewness α = -1,00, 0,00 and +1,00 as before. However, in Table 5 values of the coefficient t_p are given for various degrees of freedom ν and not for the sample size n. The reason for this arrangement is possible use of indicated values in the method based on the Bayesian approach.

Coefficients of skewness	Coefficients - t_p for degrees of freedom ν								
	3	4	5	6	8	10	20	30	∞
α = -1,00	2,65	2,40	2,27	2,19	2,19	2,04	1,94	1,91	1,85
α = 0,00	2,35	2,13	2,02	1,94	1,86	1,81	1,72	1,70	1,64
α = 1,00	1,92	1,74	1,64	1,59	1,52	1,48	1,41	1,38	1,34

Table 5. Coefficients - t_p for p = 0,05 and unknown σ.

Similarly as in the case of classical coverage method the effect distribution asymmetry decreases with increasing n, here with increasing value of the degrees of freedom ν, nevertheless, it never vanishes even for $\nu \rightarrow \infty$ (see Table 5).

5. Example

A sample of n = 5 concrete strength measurements having the mean m = 29,2 MPa and standard deviation s = 4,6 MPa is to be used to assess the characteristic value of the concrete strength f_{ck} = x_p, where p = 0,05. Using coverage method it follows from equation (4) and table 2 that for the confidence level γ = 0,75

$$x_{p,\,cover} = 29{,}2 - 2{,}46 \times 4{,}6 = 17{,}9 \text{ Mpa} \tag{13}$$

and for the confidence level γ = 0,95 it holds

$$x_{p,cover} = 29{,}2 - 4{,}20 \times 4{,}6 = 9{,}9 \text{ Mpa} \tag{14}$$

If the prediction method is used, it follows from equation (9) and table 2

$$x_{p,pred} = 29{,}2 - 2{,}33 \times 4{,}6 = 18{,}5 \text{ Mpa} \tag{15}$$

Thus, using the prediction method (which is recommended in [1], [2] and [3]), the estimate for the characteristic strength is only slightly greater than the value obtained by the classical method assuming the confidence level γ = 0,75 given by equation (13). However, when the confidence level γ = 0,95 is required, then the prediction method lead to the estimate which is greater by almost 90% than the value given by equation (14).

When information from previous production is available Bayesian approach can be used. Assume the following prior information

$$m' = 30{,}1 \text{ MPa},\ V(m') = 0{,}50,\ s' = 4{,}4 \text{ MPa},\ V(s') = 0{,}28 \tag{16}$$

It follows from equations (11)

$$n' = \left(\frac{4{,}6}{30{,}1}\ \frac{1}{0{,}50}\right)^2 < 1\ ,\ \nu' = \frac{1}{2}\ \frac{1}{0{,}28^2} \approx 6 \tag{17}$$

The following characteristics are therefore considered : $n' = 0$ and $\nu' = 6$. Taking into account that $\nu = n - 1 = 4$, equations (10) yield

$$n'' = 5, \quad \nu'' = 10, \quad m'' = 29{,}2 \text{ MPa}, \quad s'' = 4{,}5 \text{ Mpa} \tag{18}$$

and finally it follows from equation (12)

$$x_{p,\,\mathrm{Bayes}} = 29{,}2 - 1{,}81 \times \sqrt{\frac{1}{5}+1} \times 4{,}5 = 20{,}3 \text{ MPa} \tag{19}$$

where the value $t_p = 1{,}81$ is taken from Table 5 for $\alpha = 0$ and $\nu = 10$. The resulting characteristic strength is therefore greater (by 10 %) than the value obtained by prediction method. Also other available information (see annex D in [3]) on application of Bayesian approach clearly indicates, that when previous experiences are available this technique can be effectively used. Particularly in the case of a high variability of strength or in the case of assessment of existing structures Bayesian approach may be valuable.

For commonly used (low strength) concrete a positive asymmetry of probability distribution (with the coefficient of skewness up to 1) is often observed. It is assumed that the sample of $n = 5$ concrete strength measurements, analysed above, is taken from a population with lognormal distribution having the coefficient of skewness $\alpha = 1$. Using the classical coverage method for the confidence level $\gamma = 0{,}75$, equation (4) and coefficients given in Table 3 yield

$$x_{p,\,\mathrm{cover}} = 29{,}2 - 1{,}95 \times 4{,}6 = 20{,}2 \text{ Mpa} \tag{20}$$

For the confidence limit $\gamma = 0{,}95$ it holds

$$x_{p,\mathrm{cover}} = 29{,}2 - 3{,}18 \times 4{,}6 = 14{,}6 \text{ Mpa} \tag{21}$$

These values are greater by 13% and 47% respectively, compared to the previous case (equations (13) and (14)) when asymmetry was disregarded; thus, due to positive asymmetry more favourable estimates are obtained. Similarly using equation (9) the prediction method would yield the estimate for the characteristic strength as

$$x_{p,\,\mathrm{pred}} = 29{,}2 - 1{,}74 \times \sqrt{\frac{1}{5}+1} \times 4{,}6 = 20{,}4 \text{ MPa} \tag{22}$$

where the value $t_p = 1{,}74$ is taken from Table 5 for $\alpha = 1{,}0$ and $\nu = 5 - 1 = 4$. The resulting strength is by 10% greater than the previous value obtained for the normal distribution ($\alpha = 0$) given by equation (15) and again approximately equal to the value obtained by the classical coverage method assuming the confidence level $\gamma = 0{,}75$ given by equation (20). However, when the confidence level $\gamma = 0{,}95$ is required, then the prediction method lead to the estimate which is greater by almost 40% than the value given by equation (21).

When Bayesian approach is used, then it follows from equations (12), (17), (18) and Table 5

$$x_{p,\,\mathrm{Bayes}} = 29{,}2 - 1{,}48 \times \sqrt{\frac{1}{5}+1} \times 4{,}5 = 21{,}9 \text{ MPa} \tag{23}$$

which is the value by 8% greater than the corresponding estimate obtained in equation (19) for the coefficient of skewness $\alpha = 0$.

It should be, however, noted that possible negative asymmetry, which may occur in the case of some high strength materials, would cause an unfavourable effect on resulting fractile estimates, particularly when design value corresponding to small probabilities ($p < 0{,}001$) are considered.

Thus, using different statistical techniques and the same sample data the resulting estimate for the 5% characteristic strength is within a broad range from 9,9 MPa up to 20,3 for the coefficient of skewness $\alpha = 0$ (normal symmetrical distribution) and, within a range from 14,6 up to 21,9 Mpa for the coefficient of skewness $\alpha = 1$. Generally, it follows from the above numerical example and from numerical values given for various coefficients of estimation that resulting estimates for both the characteristic and design strength considerably depend on the applied method and on available prior knowledge.

6. Conclusions

(a) Design values of strength should be preferably determined by assessing a characteristic value, which is divided by a partial factor and possibly by an explicit conversion factor; direct assessment from test results could be used only in those cases when convincing evidence on appropriate probabilistic model is available.

(b) Considerably different estimates for characteristic and design strength may be obtained depending on applied statistical technique, specified probability, population asymmetry, sample size and in the case of coverage method also on accepted confidence level.

(c) Classical coverage method of fractile estimation with a given confidence level is recommended; in common cases the confidence level 0,75 may be accepted (which yields almost the same results as the methods recommended in the latest version of Eurocode 1), in special cases when increased reliability is required, higher confidence level (0,95) should be considered.

(d) When previous observations of a continuous production are available an alternative technique provided by Bayesian approach can be effectively used.

(e) Possible asymmetry of the population distribution should be considered by any estimation method whenever the coefficient of skewness exceeds $\pm 0{,}5$.

7. References

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Comparison of statistical evaluation models

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Summary

This paper deals with statistical evaluation models for resistance and material testing. It is shown that for a limited number of tests, say 1 to 4, as normal in daily practice, the model presented in Annex D 'Design Assisted by Testing' of Eurocode 1 [1] can result into unrealistic low design values. As an alternative, a sophisticated model which makes use of prior knowledge is presented. Also attention is payed to the evaluation on basis of a design model. The presented models are illustrated by examples.

1. Introduction

In most cases a structural engineer uses design formulae or data available in codes to establish design values of resistance properties of structural elements or materials. But in the following cases the engineer has to chose for a design based on experimental models:

- When no theoretical models or data are available, or the actual circumstances are not covered by existing models.
- When design formulae might give very conservative results and tests might lead to a more economic solution.
- To develop new design formulae.

When the choice is made for design by testing, the structural engineer is confronted with a lot of problems which has to be covered. In Annex D 'Design Assisted by Testing' of Eurocode 1 [1], the engineer can find guidelines which may be valuable for the planning and evaluation of tests. The evaluation model described in that document is based on a statistical analysis of test results and the partial safety factor design. One major issue the engineer has to deal with, is the fact that the number of tests should be sufficient for a valuable statistical interpretation. This implicates that the design by testing might be a very expensive and time consuming method. To study the possibility of using a smaller number of tests, TNO Building and Construction Research has carried out a review of a sophisticated statistical model. This so called Bayesian approach makes use of prior knowledge about the distribution of the test results. In this study also attention has been given to the evaluation on basis of a design model. The following chapters will give an overview of the available statistical evaluation models and will illustrate the possibilities with an example of a beam-column connection of a storage racking structure.

2. General considerations

2.1 Planning of test series

The planning of a test series is an important part of the design by testing, because correct choices have to be made to get valid results. To start with, the objective of the test series has to be formulated. Than a qualitative analysis has to be carried out in which e.g. the expected behaviour (parameters of influence, fail mechanism), boundary conditions, loading conditions, environmental conditions, time effects and differences between testing and reality are investigated. On basis of these results a relevant test arrangement has to be defined. This includes the specification of the type of specimen, the definition of the execution of the tests, the choice of environmental conditions, the method of observation and recording, the method of evaluation, the number of tests, the selection procedure of specimens and the design of the test rig. The development of the planning of a test series is not an easy task and requires appropriate theoretical knowledge, experience in testing and engineering judgement.

2.2 Execution of tests

After the planning of the test series has been worked out, the specimens have to be produced and selected, the test rig has to be build and the test programme has to be carried out. To ensure that the results are valid, the chosen measurement techniques should be in accordance with the required tolerances. One should be aware that the execution of tests is in accordance with the planning. If there is a discrepancy between the testing and the original planning, e.g. the occurrence of an unexpected failure mechanism, the whole planning of the test series has to be reconsidered. One should also be aware of uncontrolled reinforcements of e.g. the supports and unexpected environmental effects.

2.3 Evaluation of test results

After the tests are finished, the results have to be evaluated. The behaviour during loading and the failure mechanism of the tests have to be analyzed in general and the design values have to be determined. In the past several models to determine those design values were proposed, which are in many cases rules of thumb. E.g. according to the Dutch design recommendations of storage racking structures published in the seventies, the design strength of a beam-column connection as discussed in chapter 4, is equal to the factored value (0.67) of the lowest result of three tests. Nowadays it is generally excepted that a model based on the statistical theory is more in accordance with the partial safety factor design. A model based on the classical statistical theory is available, but also models based on the Bayesian theory which makes use of prior knowledge, are worked out for a single test series or a family of tests. In the following chapter a description of those models is given.

3. Description statistical evaluation models

3.1 Classical approach

According to the classical approach [2], [3] and [4], the design value of the resistance R is in case a normal distribution of the test results might be assumed, equal to:

$$R_d = \eta \frac{R_k}{\gamma_M} \tag{1}$$

where:
η is the conversion factor;
γ_M is the partial factor for the design;
R_k is the characteristic value based upon n results.

The conversion factor η takes into account the differences between testing conditions and actual ones. This factor is strongly dependent on the type of test and type of material. The value is mostly determined on basis of engineering judgement. The partial factor for the design γ_M is dependent on the field of application. The value should be taken from codes. The characteristic value R_k includes the statistical uncertainty. The value is determined by:

$$R_k = m_R - k_n s_R \tag{2}$$

where:
m_R is the mean value of the results;
k_n is the coefficient depending on the number of results n;
s_R is the standard deviation of the results.

For the classical approach the characteristic value is normally based on the 5 % fractile. If there is a complete lack of knowledge about the standard deviation, the value of k_n has to be taken from table 1 for the case that the standard deviation is unknown. If on the other hand, the standard deviation is fully known from prior knowledge, the value of k_n has to be taken from table 1 for the case that the standard deviation is known.

Table 1 - Values of k_n based on a 5 % fractile

standard deviation	n							
	3	4	6	8	10	20	30	∞
unknown	3.15	2.68	2.34	2.19	2.10	1.93	1.87	1.64
known	2.03	1.98	1.92	1.88	1.86	1.79	1.77	1.64

In the procedure given above a normal distribution of the test results is assumed. But in several applications other distributions are found, which leads to more economic design values. In case of a lognormal distribution the same procedure as given above can be followed if log values of the test results are used.

3.2 Bayesian approach

According to the Bayesian approach [2], [3] and [4], the design value of the resistance R in the case that a normal distribution of the test results might be assumed, equal to:

$$R_d = \eta \{m_R - t_v s_R \sqrt{1 + \frac{1}{n}}\} \tag{3}$$

where:
t_ν is the coefficient of the Student distribution.

The value of t_ν follows from table 2, where $\nu = n - 1$. The product $\alpha\beta$ corresponds to a fractile $P(\Phi)$ as indicated in table 2. The reliability index β is related to the failure probability for which a target is given by the code. The FORM weight factor α follows from a first order reliability method. In a design where the uncertainty of R is dominating, a value of $\alpha = 0.8$ should be used. Also other distributions of the test results than a normal distribution can be used.
Instead of using the direct method to determine the design value by equation (3), it is also possible to use the partial safety factor design as formulated with equation (1). The characteristic value R_k is then defined by equation (3) with $\alpha\beta = 1.64$. It is also possible to calculate the partial factor for design from $\gamma_M = R_k / R_d$.
It is known that the Bayesian approach is sensitive for the value of the standard deviation, specially if only a small number of test results is available. Too small or too large standard deviations might result into unsafe or uneconomic design values. An advantage of the Bayesian theory is that the prior knowledge can avoid unrealistic design values.

Table 2 - Values of t_ν

$\alpha\beta$	$P(\Phi)$	ν							
		2	3	5	7	9	19	29	∞
1.64	0.05	2.92	2.35	2.02	1.89	1.83	1.73	1.70	1.64
2.33	0.01	6.97	4.54	3.37	3.00	2.82	2.54	2.46	2.33
2.58	0.005	9.93	5.84	4.03	3.50	3.25	2.86	2.76	2.58
3.08	0.001	22.3	10.2	5.89	4.79	4.30	3.58	3.40	3.08

3.3 Prior knowledge

In literature [2], [3] and [4], the Bayesian approach which takes prior knowledge into account, is discussed. This approach establishes a prior distribution function for the unknown distribution parameters of the resistance R. On basis of this prior distribution in combination with the test results, a posterior distribution of the resistance R is derived. The prior (normal) distribution function can be represented by the following parameters:
$m(m_R)$ which is the mean value of the mean of the resistance R;
$s(m_R)$ which is the standard deviation of the mean of the resistance R;
$m(s_R)$ which is the mean value of the standard deviation of the resistance R;
$s(s_R)$ which is the standard deviation of the standard deviation of the resistance R.
It is noted here that if a lognormal distribution is chosen, the coefficient of variation V_R has to be used instead of the standard deviation s_R.

For practical applications it is important to know how the above given parameters of the prior distribution have to be determined. For many applications no prior knowledge about the mean of the resistance R is available. This implicates that $s(m_R)$ will have a large value and that the choice of the value for $m(m_R)$ is not relevant. On the other hand it is mostly possible to formulate prior knowledge about the standard deviation. This can be done by engineering judgement, but it is advised to determine the values for $m(s_R)$ and $s(s_R)$ of a group of comparable series of tests already available. The procedure which combines the prior distribution and the results of the considered tests, to determine the posterior distribution represented by the parameters m'', s'', ν'' and n'', is described in [4]. With these parameters the design value of the resistance R can be calculated with equation (3).

3.4 Evaluation on basis of a design model

It is also possible to evaluate tests on basis of a design model. More types of specimens with known varying parameters, e.g. plate thickness, beam height and yield strength, are included in the evaluation. These parameters might be deterministic or random. A mathematical relation (the 'design model') between those parameters has to be formulated. It must be kept in mind that the design model represents one failure mode that occurs in the tests. The result of the analysis is a design function for a given reliability level.
A description of the procedure that has to be followed is out of the scope of this paper. An overview is given in [4] and a detailed step by step procedure is described in [5]. The authors have added to this procedure the using of prior knowledge, as is reported in [6].

4. Example of a connection of a racking structure

4.1 Tests

To demonstrate the statistical evaluation models, an example is worked out. To optimize the economical solution of the design of a storage racking structure, design by testing of the components is preferred. The cantilever bending test on the beam-column connection is a standard test for which the planning and execution of the test is fully described in [7].

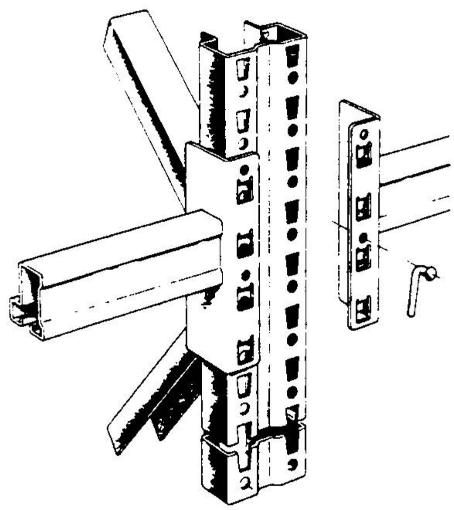

Figure 1 - Beam-column connection

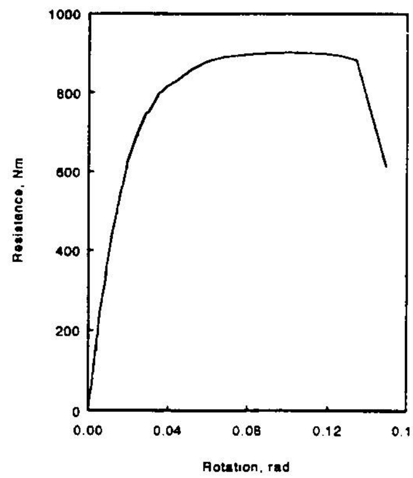

Figure 2 - Typical moment-rotation diagram

In figure 1 a beam-column connection is shown. The column is a cold-formed C-section with a continuing perforation pattern. The beam is also a cold-formed section. At the end of the beam a connector is welded, which has hooks or other devices which engage in the perforation. A typical moment-rotation diagram as a test result, is shown in figure 2.
For 6 types of specimen, A to F, with two plate thicknesses of the column and three beam heights, test series were carried out. The results are presented in table 3. It is assumed that the physical behaviour of the connections can be described by two parameters. One is the steel thickness t. The other one is the distance h, which is defined as the distance between the upper hook and the location of the connector where the beam rotates during loading (near bottom side of connected beam). Here it is assumed that the resistance moment of the connection is the maximum force in the hook times the distance h.

Table 3 - Overview measured resistances beam-column connections in Nm

Type of specimen	A	B	C	D	E	F
t, mm	2.0	2.0	2.0	2.5	2.5	2.5
h, mm	77	125	165	77	125	165
R_{test}, Nm	311	740	890	314	787	927
	353	740	820	323	837	950
	328	723	930	298	693	953
	337	693	773	310	870	1000
m_R, Nm	332	724	853	311	797	958
s_R, Nm	17.5	22.2	70.2	10.4	77.1	30.6
V_R	0.0528	0.0306	0.0823	0.0333	0.0968	0.0320

4.2 Results of interpretations

The test results given in table 3 are interpreted according to the statistical evaluation models. It is decided to assume a lognormal distribution, because the evaluation on basis of a design model is completely based on this type of distribution. For the interpretations according to each model, the following considerations have been made:

I Classical approach. It is assumed that there is a complete lack of prior knowledge. The characteristic value R_k is based on the 5 % fractile and the partial factor for design is taken equal to $\gamma_M = 1.25$.

II Bayesian approach without prior knowledge. In case of the determination of the design value R_d a reliability index of $\beta = 3.6$ is chosen and a FORM weight factor of $\alpha = 0.8$ is used. In case of the determination of the characteristic value R_k the product $\alpha\beta$ is chosen equal to 1.64, which corresponds to a fractile of 5 %.

III Bayesian approach with prior knowledge. The same considerations as mentioned for model II are used here. No prior knowledge for the mean value should be formulated, because significant differences between the resistances of the types of specimen (A to F, see table 3) might be expected. For the prior distribution function only the parameters of the coefficients of variation are determined on basis of the six series given in table 3: $m(V_R) = 0.0546$ and $V(V_R) = 0.524$.

IV Evaluation on basis of a design model without prior knowledge. The same considerations as mentioned for model II are used here. On basis of an engineering judgement it can be stated that the parameter h has a linear influence and the parameter t has a squire root influence (both parameters are assumed deterministic):

$$R = C \cdot h^{1.0} t^{0.5} \tag{4}$$

The analysis determines the design value of the factor C.

V Evaluation on basis of a design model with prior knowledge. The same considerations and design model as mentioned for model IV are used here. For the prior distribution function the same parameters as given for model III are taken.

The calculations have been carried out with the program SCEPTRE developed by TNO [6]. The design values for the six considered series are graphically presented in figure 3.

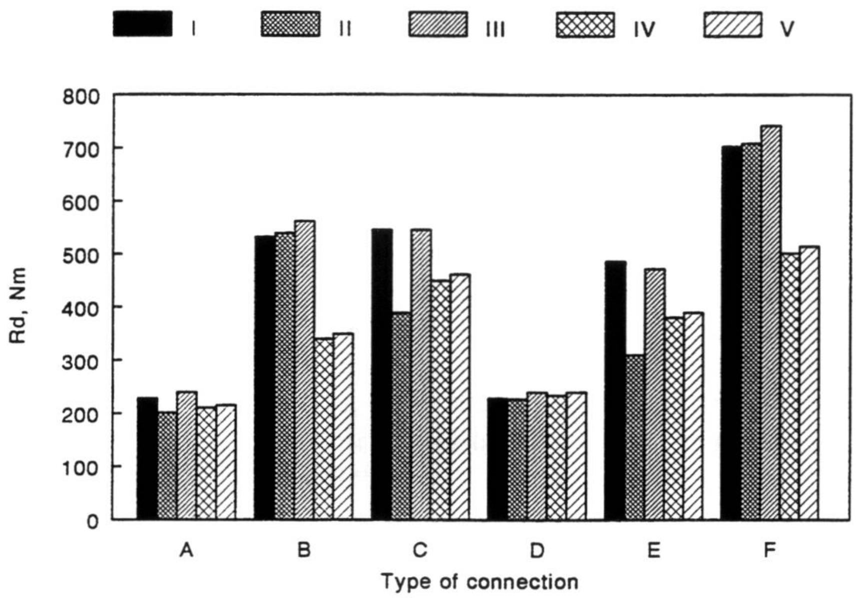

Figure 3 - Design values of types of specimen A to F according to models I to V

4.3 Remarks

The design values according to model II in a few cases significant lower (for type C and E) than those calculated by model I and III. The calculations indicate that the partial factor for design is for those two types is also very high. It can be stated that for this kind of test a total number of 4 results might give very conservative design values in case of model III. Model I is not effected by this lack of prior knowledge.

In case of the evaluation on basis of a design model it can be seen that the design values are significantly lower than those determined according to the other approaches. This indicates that the assumed design model according to equation (4) does not fully describe the physical behaviour. This discrepancy can be caused by several facts. E.g. the fact that several hooks are loaded is not taken into account. If more is known about the physical behaviour, the proposed design formulae can be reformulated. But it is noticed here that the prescribed formulae that can be used according to the theory of [5], are limited and that it might be impossible to give a correct description of the physical behaviour.

It is also marked that in case of the evaluation on basis of a design model no influence of prior knowledge can be seen. This is caused by the fact that the prior knowledge is based on exactly the same test results as those used in the analysis. This means that the prior knowledge is not independently from the evaluated test results.
Another item not discussed in this example is the choice of the lognormal distribution of the test results. From calculations not presented here, it is observed that in case of a normal distribution the design values are mostly lower and the differences between the approaches are more pronounced. So the chosen lognormal distribution leads to more economic design values.

5. Conclusions and recommendations

The following conclusions and recommendations are drawn:

- In Annex D of Eurocode 1 [1], the structural engineer can find valuable guidelines for the planning and evaluation of tests. The planning, the execution and the evaluation of tests require appropriate knowledge and experience.
- The well-known classical approach, the Bayesian approach and the evaluation on basis of a design model are discussed in this paper. For the last two prior knowledge can be incorporated. Also the possibility of using different distributions (normal and lognormal) of the test results is pointed out.
- In case of a small number of test results the Bayesian approach without using prior knowledge can give unrealistic design values. A more sophisticated model using prior knowledge might be a useful alternative. In case of the classical approach the use of a fixed partial factor of design is also a kind of prior knowledge.
- Prior knowledge should be formulated independently from the considered results.
- It is shown that the evaluation on basis of a design model is rather complicated. This is mainly due to the fact that a valid physical model have to formulated. It is noted that the possibilities of the prescribed formulae given in [5], are limited.

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