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Generalised Collapse Analysis Method for Concrete Bridge Assessment

Mécanismes d'écroulement pour l'évaluation des ponts en béton

Kollapsanalyse zur Beurteilung von Betonbrücken

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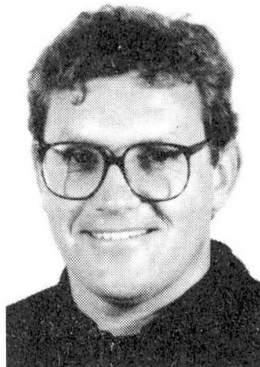
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SUMMARY

A generalised collapse analysis method for evaluating the ultimate load capacity of concrete bridge decks in flexure has been developed. By using computer graphics and solid-modelling concepts, many of the existing difficulties with plastic collapse and yield-line methods, which previously limited their application to simple slab and road configurations, have been overcome. In addition, work has progressed on incorporating a plasticity-based shear failure mechanism into the analysis. This additional module will generalise this concrete bridge assessment package still further.

RÉSUMÉ

Une méthode généralisée d'analyse des mécanismes de ruine a été développée dans le but d'évaluer la capacité de charge maximale des ponts en béton. Grâce à l'infographie, beaucoup de problèmes ayant trait à l'effondrement plastique et aux méthodes d'écroulement, problèmes qui jusqu'à alors avaient une application limitée à des simples configurations, ont été résolus. Par ailleurs, un mécanisme de rupture plastique par cisaillement a été introduit. Ce module supplémentaire permettra de généraliser l'application de ces programmes d'évaluation des ponts en béton.

ZUSAMMENFASSUNG

Eine verallgemeinerte Methode der Kollapsanalyse zur Abschätzung der maximalen Lastkapazität von Betonbrücken unter Biegung wurde entwickelt. Durch Anwendung von Computergrafik konnten viele der bestehenden Schwierigkeiten im Zusammenhang mit plastischem Versagen und Grenztragfähigkeits-Theorien der Platten ausgeräumt werden. Darüber hinaus wurde die Analyse durch einen auf der Plastizitätstheorie basierenden Versagensmechanismus für Schubbelastung erweitert. Durch dieses zusätzliche Modul wird das Programmpaket zur Abschätzung der Lastkapazität von Betonbrücken weiter verallgemeinert.



1. BACKGROUND TO THE PROBLEM AND SOLUTION

1.1 The U.K. Bridge Assessment Programme

In the U.K., assessment of all bridges is being undertaken to check their strengths. With over 50,000 bridges, of which more than 9000 are concrete, this is indeed a major task. Preliminary estimates[1] suggest that around 35% of the concrete bridges will need strengthening or replacement, costing of the order of \$1.2 billion.

1.2 The yield-line method as assessment tool

In order to assess this vast number of bridges, an automated, quick and accurate method of analysis is required. Unfortunately, up until recently, this goal has been elusive. The reason is that there are three main analysis techniques available to the engineer for assessing the strength of existing concrete bridges, all of which have limitations. The three methods are elastic, non-linear finite element and yield-line techniques. Elastic methods, although quick and in widespread use, are inherently conservative. This could have severe cost implications in such a large assessment programme. Non-linear finite element analysis is complicated and expensive. Further, it is excessively time-consuming. Yield-line analysis is tedious by hand and relies on the optimum failure geometry being known for the analysis to provide a safe result.

The work which has been conducted at Cambridge University over the past 5 years has been to develop a generalised flexural yield-line analysis package, called **COBRAS**, which overcomes the above disadvantages associated with yield-line analysis[2]. A concrete bridge may now be analysed by **COBRAS** and assessed for flexural load-carrying capacity within a couple of minutes.

2. THE GENERALISED COLLAPSE ANALYSIS METHOD

2.1 The basis of the method

Traditionally, automated yield-line analysis by computer has been carried out by programming the mathematical relationships amongst several variables and allowing the computer to optimise the solution in some way for one or more failure patterns. Complicated, but practical, failure patterns (such as fanning mechanisms) have been avoided due to the excessively complicated mathematical relationships which have had to be developed for each individual case.

The approach used here, however, has been to seek a general solution. It was recognised that any collapse mechanism can be reduced to what is fundamentally a problem of geometry. Using relatively recent developments in *computer graphics* and *solid modelling* theory, an analysis technique has been developed which can derive all the required geometrical relationships for the mechanisms, whilst incorporating features describing the component material properties and the applied loads. This is achieved as follows.

The bridge is considered to be made up in plan of polygonal areas, each one of which has associated attributes. These attributes, or features, would include a concrete layer, a dimension layer, steel reinforcement layers, loading layers, and so on. See Figure 1.

Having modelled the bridge in this way, these polygons may be *intersected* to provide a distinct 'patchwork' on the bridge of areas where all attributes are identical. Then, a 3-dimensional failure mechanism is applied to the model and the deformed shape of the structure is found. See figure 2. Contained within this new solid bridge model are full details of all the information necessary to describe the structural parameters of the material components and dimensions of the bridge, the external loading acting on the bridge, and the required geometric information needed for the collapse mechanism analysis using the work method. This includes the location of all the yieldlines, the details of fixity at each of the boundaries and the relative rotations between adjacent rigid plate elements of the failure

surface. Further details of this approach are presented in Reference 2.

By changing the position of some of the solid-model vertices, a rapid “step-like” iteration of the solid failure mode shape can be performed in a search for the critical global collapse mechanism with the lowest factor of safety.

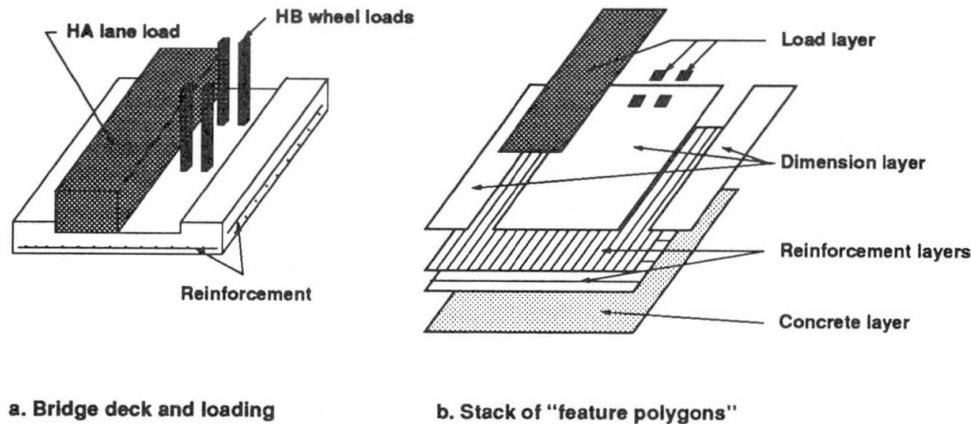


Figure 1: Bridge feature polygons

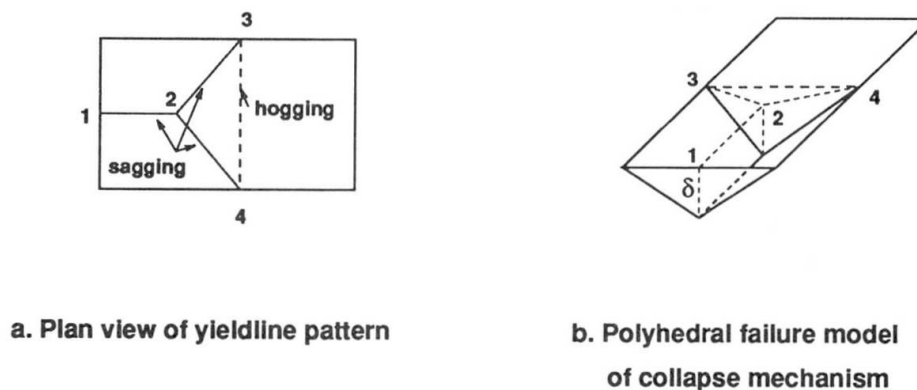


Figure 2: Modelling the failure mechanism

2.2 Other program features

The **COBRAS** program calculates the “theoretical” moment capacity of the actual section allowing for all the orientations, depths and types of reinforcement that cross the selected yieldline overcoming the need to adopt Johansen’s stepped yield criterion and also avoiding any necessity to use the affinity theorems to account for orthotropic reinforcement layouts.

Since most code measures of ductility are related to the geometry and material components along the yieldlines of the structure, each of which is fully defined in the solid bridge model in the **COBRAS** program, the rotation capacity at all yieldline sections can be checked directly.

2.3 Testing of the **COBRAS** program

COBRAS has been tested against a wide variety of theoretical, numerical and test data. Details of these favourable correlations are to be found in Reference 2. It is noteworthy that

in comparison with theoretical textbook solutions, where simplifications for ease of hand-calculation have been made, more critical failure mechanisms than the 'critical' one assumed in the textbook, have been found[2].

3. SHEAR FAILURE MODEL

3.1 Justification for the model

The above yield-line analysis package is aimed at determination of the flexural failure capacity of existing concrete bridges. However, there exists the need to carry out checks on the shear capacity of concrete bridges for a full assessment. If the flexural check has been carried out by realistic yield-line analysis, then it is reasonable to use a similar approach for the shear analyses, lest conservative shear checks should condemn the bridge.

For this reason, it was decided to incorporate a shear checking module into COBRAS. This shear model is based on a plasticity approach, so that the actual collapse mechanism is modelled in a compatible manner with the flexural analysis.

3.2 Shear model details

Figure 3 shows a possible shear failure mechanism for a concrete beam, loaded by any $P(x)$. A bi-linear yield-line is permitted to occur, with relative rotation between the two rigid blocks causing compressive and tensile yield-lines. Relative vertical and horizontal translation between the two rigid blocks also occurs. This model can therefore cover the range from pure flexure (relative translations are zero) to pure shear (relative rotation is zero) in the beam.

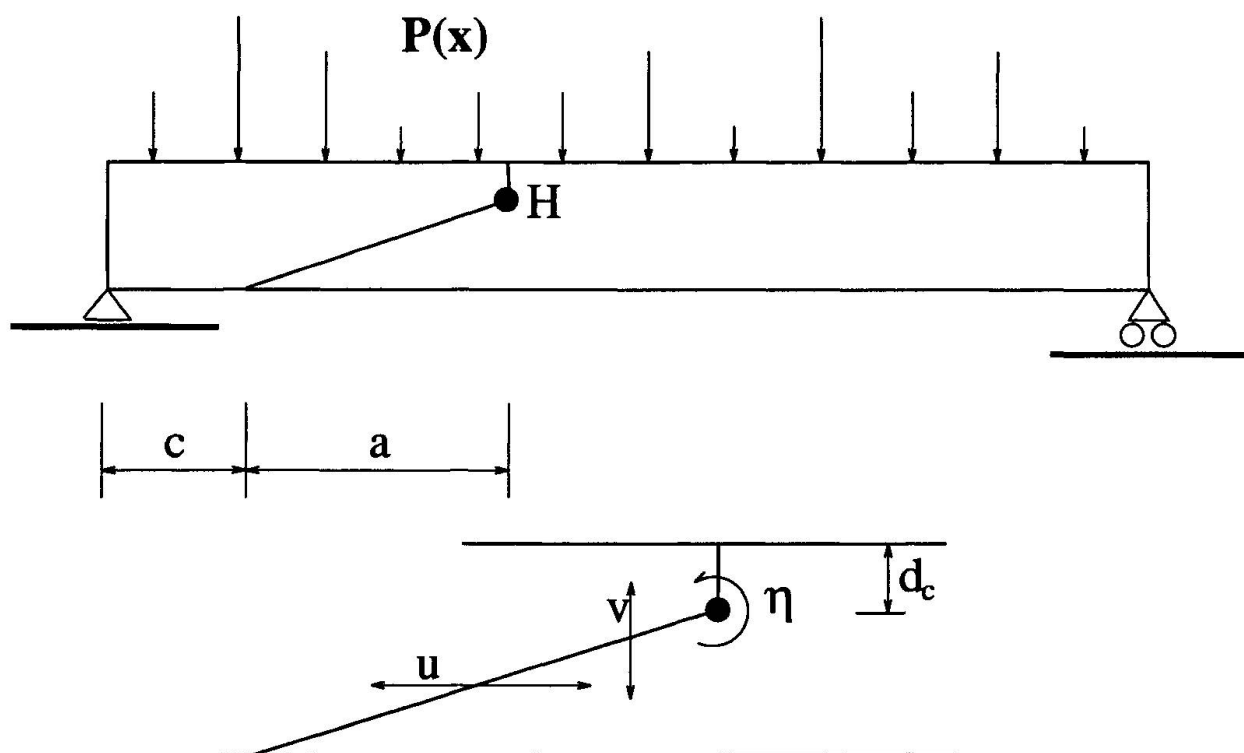


Figure 3: Possible shear failure mechanisms for a beam under general loading.

The variables which are to be optimised in each case are

1. The horizontal distance to the base of the inclined shear crack, c ,
2. The horizontal length of the inclined crack, a ,

3. The depth into the beam of the hinge, d_c ,
4. The rotation applied to the hinge, η ,
5. The relative vertical translation, v , and
6. The relative horizontal translation, u .

3.3 Internal energy dissipation

3.3.1 General dissipation formulation

It may easily be shown[3] that the energy dissipation rate per unit length in the concrete along a yield-line under generalised (compatible) motion between two rigid blocks is given by

$$\dot{D} = \frac{1}{2}f_c\delta(1 - \sin \alpha) \quad (1)$$

in the range $0 \leq \alpha \leq 2\pi$, where f_c is the *effective* compressive strength of concrete, δ is the instantaneous relative displacement between the two rigid blocks and α is the angle that this displacement vector makes with the yield-line. Note that the tensile strength of concrete is considered negligible in this particular case.

Where steel bars cross a line of discontinuity, the internal energy dissipated in the steel (which is assumed to be yielding), W_s , is given by

$$W_s = A_s f_y \delta |\cos(\alpha - \gamma)| \quad (2)$$

where A_s is the area of steel crossing the yield-line, f_y is the yield strength of the steel and γ is the angle the steel makes with the yield-line.

3.3.2 Energy dissipation in the concrete

Let us consider the yield-line portion above the hinge point, H, in Figure 3. The combined relative displacement vector, δ , will vary along the length of this yield-line, as will the angle it makes with the yield-line, α . As an illustration of how the calculations would be carried out, consider a point half-way along this yield-line (that is, a point $d_c/2$ in depth into the beam). At this point, the horizontal displacement vector has magnitude $(\frac{1}{2}d_c\eta + u)$ and the vertical displacement vector is v . The relative displacement vector, δ , is

$$\delta = \sqrt{(\frac{1}{2}d_c\eta + u)^2 + v^2} \quad (3)$$

at an angle to the yield-line of

$$\alpha = \arctan\left(\frac{\frac{1}{2}d_c\eta + u}{v}\right). \quad (4)$$

The instantaneous energy dissipation, \dot{E}_c , at this point is then

$$\dot{E}_c = \frac{1}{2}b\delta(1 - \sin \alpha). \quad (5)$$

The total energy dissipation in the concrete along this yield-line will then be given by

$$E_c = \int_0^{d_c} \dot{E}_c dx. \quad (6)$$

In order to carry out this integration, Simpson's Rule with four intervals is used for each line.



In order to calculate the energy dissipation in the concrete along the tensile (sloping) yield-line, a similar approach is used.

3.3.3 Energy dissipation in the steel

All steel will undergo stretching due to the three relative displacements occurring. Energy in the longitudinal (flexural) steel will be dissipated as follows.

$$E_s = \sum_{i=1}^{ns} A_s f_y |d_i - d_c| \eta + \sum_{i=1}^{ns} A_s f_y u \quad (7)$$

where ns is the number of flexural steel bars crossing the yield-line and d_i is the depth from the top surface to the steel bar.

Where steel shear stirrups are present, additional energy E_{ss} will be dissipated, calculated in a similar way.

The final energy dissipation in the system is then found to be

$$ED = E_c + E_s + E_{ss}. \quad (8)$$

3.3.3 External work done by the loads

The external work done by the loads is calculated based upon the vertical distances through which the various loads on the bridge move. This is easily calculated as

$$WD = \int_0^l P(x)w(x)dx \quad (9)$$

where $w(x)$ is the variation along the beam in notional displacement, as defined by the particular combination of variables specified above.

3.4 Comments on the shear analysis

This shear analysis method is presently being implemented into the **COBRAS** package. Included in the features are provisions for the curtailment of reinforcing bars and multiple beam-and-slab portions failing in a single mechanism.

Comparisons with existing test data for simple beams have been promising. Generally, the correct failure type (pure or flexural shear) is predicted. However, the quantitative predictions for shear capacity are (as usual in plasticity theory) heavily dependent on the 'effectiveness factor' chosen for the strength of the concrete. This calibration work continues at present.

4. CONCLUSIONS

A new generalized collapse mechanism analysis method for evaluating the *ultimate strength* of concrete bridges has been developed. Computer graphics and solid modelling techniques have been employed to overcome many of the existing difficulties with traditional yieldline theory.

This program provides a means by which researchers, designers and highway authorities can more effectively assess the strength of concrete bridges under the influence of overload and/or deterioration of the material components.

More recently, the problem of shear failure in beam-and-slab bridges has been addressed by considering a plasticity-based shear failure mechanism for a concrete bridge. The model has been preliminarily shown to be suitable for such analysis, although work on its implementation continues.

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