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Long-Term Behaviour of Continuous Prestressed Composite Bridges

Comportement de longue durée des poutres mixtes continues précontraintes

Langfristiges Verhalten von vorgespannten kontinuierlichen Balken mit Verbundquerschnitt

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SUMMARY

The long-term analysis of continuous prestressed composite steel-concrete beams is presented. Referring to beams with thin slabs, the problem is solved in the viscoelastic domain by using a feasible algebraic formulation for the concrete creep constitutive law. In this way simple moment-rotation relationships, very useful for design, can be deduced. A case study considers the basic aspects of the problem, in particular the different structural responses to external loads and to prestressing.

RÉSUMÉ

L'analyse de longue durée des poutres continues précontraintes avec section mixte acier-béton est présentée. La résolution du problème est déterminée pour les poutres avec ta-bliers de faible épaisseur dans le domaine de la viscoélasticité linéaire en utilisant une formulation algébrique pour la loi de fluage du béton. L'application à un cas réel permet de discuter les aspects fondamentaux du problème, en particulier le comportement de la structure lors de l'application des charges ou de la précontrainte.

ZUSAMMENFASSUNG

Gegenstand der Arbeit ist die Analyse von vorgespannten kontinuierlichen Balken mit Verbundquerschnitt. Mit Bezug auf Balkenplatten mit geringer Dicke wird das Problem im Rahmen der linearen Viskoelastizität gelöst durch Anwendung von algebraischen Formulierungen des Zustandsgesetzes des Betons, die ein schnelles Abfassen der Verträglichkeitsgleichungen erlauben. Das Beispiel einer Anwendung gestattet es, die wichtigsten Aspekte des Problems zu erörtern, insbesondere die unterschiedliche Reaktion des Bauwerks auf äussere Kräfte und auf Vorspannung.

1. INTRODUCTION

Steel-concrete composite beams represent an efficient and economical choice for the construction of short or middle-span bridges. In design practice the simply supported scheme is commonly adopted in order to assure compressive stresses in the concrete slab which considerably increases the strength and the rigidity of the transverse sections. The simply supported scheme does not represent the most feasible choice for multispan bridges as it introduces some disadvantages exerting a marked negative influence on structural durability. The unreliability of the simply supported scheme is mainly connected to the presence of a large number of support devices and of elastic joints representing one of the most important factors of structural degrading. On the contrary, the scheme of continuous beam allows to considerably reduce the number of supports and to eliminate the elastic joints, assuring in the same time favourable moment redistributions at ultimate significantly increasing the structural bearing capacity. The main drawback of continuous beams is the presence of tensile stresses in the concrete slab in the zones near to the interior supports which can produce diffuse cracking and generate structural damaging to which a lifespan reduction is generally connected. In order to maintain the statical and economical advantages of continuous beams, contemporaneously eliminating cracking phenomena in the support zones, a feasible technique consists in prestressing the concrete slab. In this way we can guarantee the presence of only compressive stresses in the slab during the service stage, significantly improving the structural lifespan. On the other hand the evaluation of the prestressing effects has to be performed by means of a refined structural analysis in order to correctly compute the stresses connected to the presence of parasitic moments which markedly reduce the efficiency of prestressing. Moreover, steel-concrete composite beams exhibit a marked rheological inhomogeneity so that the state of stress produced by permanent loads varies in time as a consequence of the creep and shrinkage deformations arising in the concrete slab. In particular the mutual interaction existing between the stress distribution in the transverse sections and the moment distribution along the beams, has to be accurately investigated. A general and reliable procedure of structural analysis for continuous steel-concrete composite beams has not yet been satisfactorily developed so that, even if the Codes [1], [2], [3], give exhaustive informations about the creep constitutive laws for concrete, a reliable design procedure for the analysis of continuous composite steel-concrete beams is still lacking. This item is the main goal of the present paper, where the general formulation for the analysis of the state of stress in continuous prestressed steel-concrete beams is derived. The problem is approached in the linear viscoelastic domain, assuming for concrete the algebraic creep law proposed by Trost, [4] which drives, as widely proven in other works [5], [6], to very reliable results, allowing in the same time, to state simple design formulas which can be easily used in design practice. In order to avoid useless complications in developing the analytical process we shall assume that the slab can be regarded as perfectly flexible. This hypothesis, which is more realistic when the depth of the slab is small with respect to the height of the beam, does not represent a limitation for the solution, as its generalization to the case of deep slabs is a trivial task. On the contrary it allows to deal with simpler formulas which can be discussed in a direct and immediately understandable way.

2. PROBLEM FORMULATION

2.1 Sectional Analysis

Let us consider the section of Fig. 1.a, subjected to a bending moment M varying in time, and to an initial prestressing force N_{op}, applied by means of a steel cable having area A_{sp}. Assuming for concrete the subsequent algebraic creep law

$$\epsilon_c=\lambda\sigma_c/E_c+\mu\sigma_{co}/E_c+\epsilon_{sh} \qquad (1)$$

with

$$\lambda=-\phi/\rho, \qquad \mu=[\rho(1+\phi)+\phi]/\rho \qquad (2)$$

where $\phi>0$, $\rho<0$, ϵ_{sh}, are respectively the creep coefficient, the relaxation coefficient, and the shrinkage deformation, the compatibility equations between the slab and the cable and between the slab and the beam according to Fig 1.b assume the form

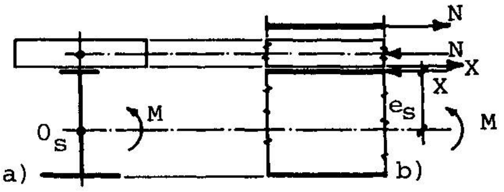

Fig.1 Composite section

$$(N-X)\omega\lambda+(N_o-X_o)\omega\mu+N\alpha(1-\omega)=\epsilon_1E_cA_c\omega \qquad (3)$$
$$(N-X)\omega\lambda+(N_o-X_o)\omega\mu-X(1-\omega)=\epsilon_2E_cA_c\omega+M\beta(1-\omega)/e_s$$

In eqs. (3) the subscript o is applied to the initial quantities (time t_o) and the subsequent parameters have been defined

$$\omega=[1+(1/np_s)(1+e_s^2/r_s^2)]^{-1}; \quad n=E_s/E_c; \quad p_s=A_s/A_c;$$
$$\alpha=(p_s/p_{sp})(1+e_s^2/r_s^2)^{-1}; \quad p_{sp}=A_{sp}/A_c; \quad \beta=(1+r_s^2/e_s^2)^{-1} \qquad (4)$$

Finally for ϵ_1, ϵ_2 representing the relative deformations imposed between the cable and the slab and between the slab and the beam the subsequent expressions hold

$$\epsilon_1=(N_{op}/E_cA_c)[(\alpha/\omega)(1-\omega)+(1-X_o/N_{op})]-\epsilon_{sh}; \quad \epsilon_2=-\epsilon_{sh} \qquad (5)$$

where the first term at second member of the first of eqs. (5) is related to the relative deformation between cable and slab imposed by prestressing. At initial time t_o, we have $\epsilon_{sh}=\phi=\rho=0$, and from eqs. (2) $\lambda=1$, $\mu=0$, so that eqs. (3) become

$$(N_o-X_o)\omega+N_o\alpha(1-\omega)=\epsilon_{1o}E_cA_c\omega; \quad (N_o-X_o)\omega-X_o(1-\omega)=\epsilon_{2o}E_cA_c\omega+M_o\beta(1-\omega)/e_s \qquad (6)$$

The solutions of eqs. (6), (3) drive to the subsequent expressions

$$N_o=-\beta\omega M_o/[e_s(\omega+\alpha)]+N_{op}; \quad X_o=-(\beta M_o/e_s)[1-\alpha\omega/(\omega+\alpha)]+N_{op} \qquad (7)$$

$$N=-X/\alpha-\beta M/\alpha e_s; \quad X=F_MM/e_s+F_M'M_o/e_s+F_PN_{op}+F_SN_s \qquad (8)$$

where $N_s=\epsilon_{sh}E_cA_c$ and functions F are given by the following relationships

$$F_M=-\beta[1-\alpha\omega\lambda/G]; \quad F_M'=\beta\alpha^2\omega\mu(1-\omega)/[(\alpha+\omega)G]$$
$$F_P=\omega[1+\alpha(1-\omega)\phi/G]; \quad F_S=\alpha\omega/G; \quad G=(1+\alpha)\omega\lambda+\alpha(1-\omega) \qquad (9)$$

The solution of eqs. (6), (3) for the unprestressed sections ($p_{sp}=0$) can be obtained from eqs. (7), (8), (9) by assuming, according to eqs. (4), $\alpha=\infty$. In this way we obtain

$$N_o=N_{op}=0; \quad X_o=-\beta(1-\omega)M_o/e_s; \quad N=0; \qquad (10)$$

$$X=\bar{F}_MM/e_s+\bar{F}_M'M_o/e_s+\bar{F}_SN_s \qquad (11)$$

$$\bar{F}_M=-\beta[1-\omega\lambda/\bar{G}]; \quad \bar{F}_M'=\beta\omega\mu(1-\omega)/\bar{G}; \quad \bar{F}_S=\omega/\bar{G}; \quad \bar{G}=1+\omega(\lambda-1) \qquad (12)$$

The absolute values of functions F versus ω are reported in Fig. 2 for $t=\infty$, $\beta=0.55$ assuming α as parameter. The function $|F_M|$ is decreasing as it points out the effect of the viscoelastic restraint represented by the slab in reducing the bending stresses in the steel beam. High α values are related to high deformabilities of the slab and consequently to small values of F_M. In particular for $\alpha=\infty$, $\omega=1$ the slab is perfectly deformable so that no restraints are applied to the beam and consequently $F_M=0$. Functions F_P and F_S are increasing as they are connected to the effect of the elastic restraint represented by the beam in preventing the slab deformations due to prestressing and shrinkage. For $\omega=0$ the beam is perfectly deformable making the slab unrestrained

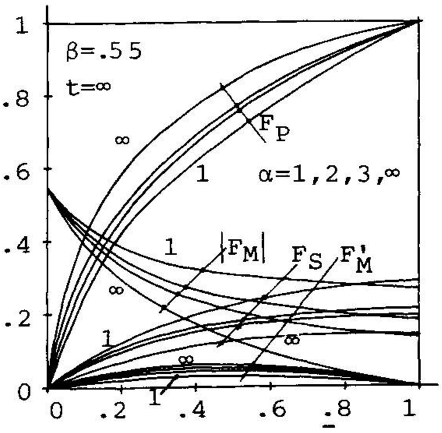

Fig.2 Functions $F, \bar{F}$

so that $F_P=F_S=0$, while for $\omega=1$ the beam is rigid and the functions F_P, F_S take their maximum values. It is interesting to observe that for $\omega=1$ we have $F_P=1$ whatever is α, while F_S takes values which are increasing with α. This can be explained taking into account that the imposed prestressing force N_{op} does not depend from the cable deformability while the shrinkage effects in the slab are increased by increasing the cable deformability. Finally function F_M' which is related to the algebraic form assumed for the creep law takes zero values for $\omega=0, \omega=1$ independently on α as these two cases, related to a perfectly deformable or to a perfectly rigid slab do not require the definition of creep deformations for evaluating the problem solution. The parameter α affects the functions F in a not negligeable way. We can see that the functions $\bar{F}$, represented in Fig. 2 by the diagrams with $\alpha=\infty$, cannot be assumed as representative of the totality of the obtained results so that in performing the structural analysis we have to take into account that the transverse sections of the beams have to be considered variable, by using for the calculation of the F functions $\alpha=\infty$ in the unprestressed zones and the proper value of α, depending on p_{sp} in the prestressed zones.

2.2 Structural Analysis

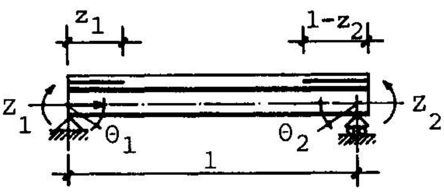

Fig.3 Rotations at beam edges

The evaluation of the moment distribution in continuous beams can be conveniently performed by means of the moment-rotation relationships. Referring to Fig. 3, indicating by g the uniformly distributed load and by N_{op} the common prestressing force acting in the two zones $0\le z\le z_1$, $z_2\le z\le l$, for the moment M, the curvature $1/r$ and the end rotations Θ_1, Θ_2 of the steel beam we can write

$$M=Z_1(1-z/l)+Z_2 z/l+M_g;\quad 1/r=(M+Xe_s)/E_sJ_s;\quad \Theta_1=\int_l (1-z/l)\frac{1}{r}dz;\quad \Theta_2=\int_l (z/l)\frac{1}{r}dz \tag{13}$$

Combining eqs. (8), (9), (12), (13), for the left edge rotation Θ_1 we derive

$$\Theta_1=\delta_{11}Z_1(c_{11}+\bar{c}_{11})+\delta_{12}Z_2(c_{12}+\bar{c}_{12})+\delta_{11}Z_{10}(d_{11}+\bar{d}_{11})+\delta_{12}Z_{20}(d_{12}+\bar{d}_{12})+$$
$$+3/2\delta_{11}N_{op}e_s p_{11}+3/2\delta_{11}N_s e_s s_{11}+gl^3/24E_sJ_s(c_{10}+\bar{c}_{10});\quad \delta_{11}=2\delta_{12}=l/3E_sJ_s \tag{14}$$

$$c_{11}=(1+F_M)[1+f_{11}(\zeta_1)-f_{11}(\zeta_2)];\quad \bar{c}_{11}=[1-c_{11}/(1+F_M)](1+\bar{F}_M);$$
$$c_{12}=(1+F_M)[1+f_{12}(\zeta_1)-f_{12}(\zeta_2)];\quad \bar{c}_{12}=[1-c_{12}/(1+F_M)](1+\bar{F}_M);$$
$$c_{10}=(1+F_M+F_M')[1+f_{10}(\zeta_1)-f_{10}(\zeta_2)];\quad \bar{c}_{10}=[1-c_{10}/(1+F_M+F_M')](1+\bar{F}_M+\bar{F}_M')$$
$$d_{11}=c_{11}F_M'/(1+F_M);\quad d_{12}=c_{12}F_M'/(1+F_M); \tag{15}$$
$$\bar{d}_{11}=\bar{c}_{11}\bar{F}_M'/(1+\bar{F}_M);\quad \bar{d}_{12}=\bar{c}_{12}\bar{F}_M'/(1+\bar{F}_M)$$
$$f_{11}=\zeta(\zeta^2-3\zeta+3);\quad f_{12}=\zeta^2(3-2\zeta);\quad f_{10}=\zeta^2(6-8\zeta+3\zeta^2)$$
$$p_{11}=F_P[\zeta_1(2-\zeta_1)+(1-\zeta_2)^2];\quad s_{11}=\bar{F}_S[\zeta_2-\zeta_1][2-(\zeta_1+\zeta_2)]+p_{11}F_S/F_P;\quad \zeta=z/l$$

The rotation Θ_2 can be obtained from eq. (14) by changing Z_1 with Z_2 and putting $\zeta_1=1-\zeta_2$ in eqs. (15). Applying eq. (14) at time $t=t_0$ ($\lambda=1$, $\mu=0$) we obtain the initial elastic moment rotations relationships which allow to write the compatibility equations of the continuous beam and to perform the structural analysis at initial time evaluating the moments Z_{10}, Z_{20}. As a second step we apply eq. (14) and calculate the moments Z_1, Z_2 at time t. The problem is so reduced to the execution at different stages of two elastic structural analyses, the first, at initial time t_0 connected to the actual values of the elastic parameters of the various parts, the second, at time t, connected to a varied value of the elastic modulus of the concrete part, taking also into account the effects produced by the time variability of the state of stress connected to the initial values Z_{10}, Z_{20} entering in eq. (14).

3. NUMERICAL EXAMPLE

The preceding relationships have been applied for the analysis of the continuous beam of Fig. 4, for which we have α=3.02; β=0.683; ω=0.2556 together with the subsequent values for the various parameters: ϕ=3; ρ =-0.88; μ=0.6; $\epsilon_{sh}=20 \cdot 10^{-5}$; N_{op}=8500 kN; g=53.3 kN/m. In Fig. 5 the relative moments $M/(gl^2/8)$ are represented for various loading conditions at time t_o and for t=∞. We observe that the moment M_g produced by external load is scarcely influenced by creep so that we can consider it constant in time without significant error. On the contrary the parasitic moment due to prestressing which is about $-0.52\ gl^2/8$ at initial time grows up to $-0.81\ gl^2/8$ with an increase of 55%. This fact, together with the developing of the moment produced by shrinkage, produces an increase of about 22% of the initial total moment on the central support. The time variability of the absolute values of the stresses acting in the continuity and in the span sections is reported in Fig. 6, where an increase of the beam stresses together with a marked decrease of the slab stress can be observed. In order to simplify the plotting of the diagrams, the stresses have been normalized to their maximum value 184.2 MPa, coinciding with the compressive stress σ_{s2} acting

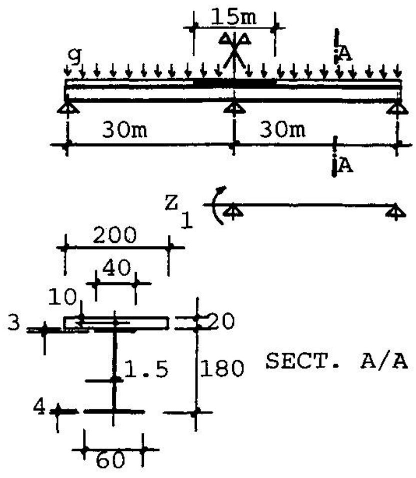

Fig.4 Case study

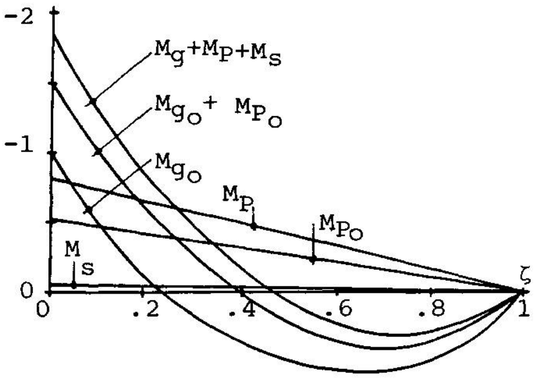

Fig.5 Bending moments at initial and final time

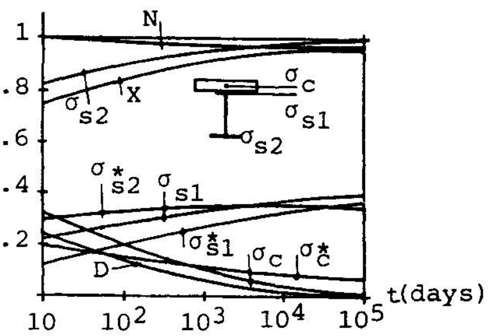

Fig.6 Stresses in transverse sections

at time t=∞ at the lower edge of the beam in the continuity section. The stresses in the concrete slab have been amplified by the factor 10 to obtain a clearer representation. In the same figure the values of the prestressing force N, of the shear force X and of their diference D=N-X are reported using N_{op} as normalization factor. The stresses in the beam are greater in the continuity section, in particular the highest value is reached by σ_{s2} as at the lower edge of the beam both the prestressing force and the bending moment generate compressive stresses. The reduction of the stress in the concrete slab is quite higher in the continuity section where the prestressing produces a marked increase of the shear X strongly reducing the slab compressive stress which is practically vanishing for t=∞. On the contrary in the span section the reduction of the concrete stress is connected to the increase of the slab deformation produced by creep and is governed by the functions $\bar{F}_M$, $\bar{F}_M'$ which have finite values for t=∞. Regarding the prestressing and the shear forces N, X we can observe that the first is practically constant in time (N/N_{op}=0.97) as the presence of the elastic restraint represented by the steel beam drastically reduces the creep deformations of the slab together with the loss of the prestressing force. For the same reason X markedly increases in time reaching the final ratio X/X_o=1.30=0.96 N_{op}, so that the difference D=N-X representing the total axial force acting in the slab is practically vanishing at final time together with the final concrete stress.

4. CONCLUDING REMARKS

The scheme of continuous beam, reducing the number of supports devices and the elastic joints between adjacent decks represents a favourable choice in preventing the risks of structural damaging. The prestressing of the slab in the continuity zones, avoiding the developing of cracking phenomena allows to significantly increase the structural durability and the related lifespan. The behaviour of continuous steel-concrete prestressed beams is markedly influenced by concrete creep producing notable variations in the state of stress and in the moment distribution acting along the beams. In order to correctly design the various structural elements a reliable procedure for the long term structural analysis is consequently needed. The procedure developed in the present work, based on an algebraic formulation of the creep law represents an useful tool for the analysis and the design of continuous prestressed beams as it allows to obtain good results by means of simple calculations very similar to the ones governing the elastic structural analysis. The application to an actual case study has shown that the parasitic moment produced by the prestressing force significantly increases in time, together with the shear force acting between the slab and the beam, so that the compressive stress in the concrete slab is highly reduced in time. This fact clearly states the need of proceeding by means of a refined long term structural analysis in order to evaluate in a reliable way the final stress in the concrete slab and to check the structural safety regarding the efficiency of prestressing in preventing cracking effects and in improving structural lifespan.

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