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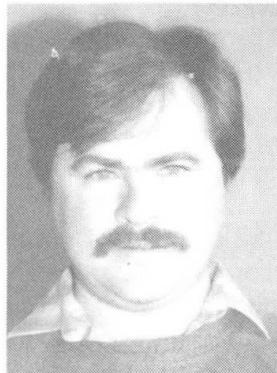
Stability of a Steel Bridge Girder Strengthened by Prestress

Stabilité d'une poutre de pont renforcée par précontrainte

Stabilität eines durch Vorspannung verstärkten Stahlbrückenträgers

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SUMMARY

Steel bridges with plate girder substructures may be strengthened and stiffened during their lifetimes by prestressing the bottom flange. In the absence of live loading, the pre-stressing operation may produce large compressive stresses in the bottom flange, so that instability of this flange is a necessary design consideration. The paper describes a method of buckling analysis of plate girders during prestressing, and compares the solutions with those of the U-frame approach deployed for assessing the stability of steel through girders.

RÉSUMÉ

Au cours de la vie utile des ponts à poutres métalliques, il est possible de renforcer et augmenter la rigidité de ces dernières par précontrainte de leurs membrures inférieure. La mise en précontrainte en l'absence de surcharge produit d'importantes contraintes de compression dans les membrures inférieures, qui doivent être prises en compte lors du dimensionnement. L'auteur décrit une méthode d'analyse du voilement des poutres métalliques soumises à la précontrainte et compare les solutions obtenues avec celles d'un modèle en cadre en U mis au point pour examiner la stabilité de poutres à âme pleine en acier.

ZUSAMMENFASSUNG

Stahlbrücken mit Blechträgern können während ihrer Nutzungsdauer verstärkt und versteift werden, indem ihr Untergurt vorgespannt wird. Beim Fehlen der Verkehrslast können beim Vorspannprozess grosse Druckspannungen im Untergurt auftreten, so dass die Instabilität des Gurtes ein notwendiges Bemessungskriterium darstellt. Der Beitrag beschreibt eine Methode zum Beulnachweis von Blechträgern unter Vorspannung und vergleicht die Lösungen mit denen eines U-Rahmen-Modells, das zur Stabilitätsuntersuchung von Massivstahlträgern hergeleitet wurde.



1. INTRODUCTION

In many steel bridges whose substructure consists of steel I-section plate girders, the bottom flange and lower portion of the web may yield if the bridge is required to support heavier loads than that for which it was initially designed. The onset of inelasticity may be prevented by welding a steel plate to the bottom flange, but prestressing the girder is a viable alternative since it both delays yielding of the bottom flange and produces a camber in the bridge which may control deflections.

Post-tensioning of steel beams and composite steel-concrete girders has been considered by the author [1,2,3], and it was concluded that instability of the flange which is compressed during prestressing may occur. Although a closed form solution was presented for a composite beam [3], recourse generally has to be made to a computer program for solving the problem. The restraint provided by the superstructure in a steel plate girder bridge means that the buckling mode must necessarily be distortional [4] as shown in Fig. 1. The computer program must therefore be required to account for the distortion or flexing of the web during buckling.

In this paper, an efficient method for the analysis of elastically restrained beam-columns which distort during buckling is modified to incorporate prestressing forces produced by tensioning high strength steel bars or prestressing strand near the bottom flange. The analysis is very efficient computationally, and allows the prestressing force to cause buckling to be calculated for a range of beam geometries, superstructure restraints and prestressing eccentricities. Because of the number of significant parameters involved, an exhaustive parameter study is prohibitive and only a specific girder is examined in detail. The results are compared with a simplified U-frame model used for analysing steel through girders, and the latter approach is shown to be a little conservative.

2. DISTORTIONAL BUCKLING MODEL

An efficient analysis for the distortional buckling of I-section beam-columns with elastic restraints was presented by Bradford [5]. The model assumed that the beam-column was simply supported and subjected to uniform bending and compression, so that the buckling deformations could be represented by a sine curve with a given number of half-wavelengths. In addition, it was assumed that only the web distorted during buckling, with the flanges buckling as rigid bodies. This is applicable to a prestressed plate girder bridge, since the webs are invariably slender and the flanges are stocky.

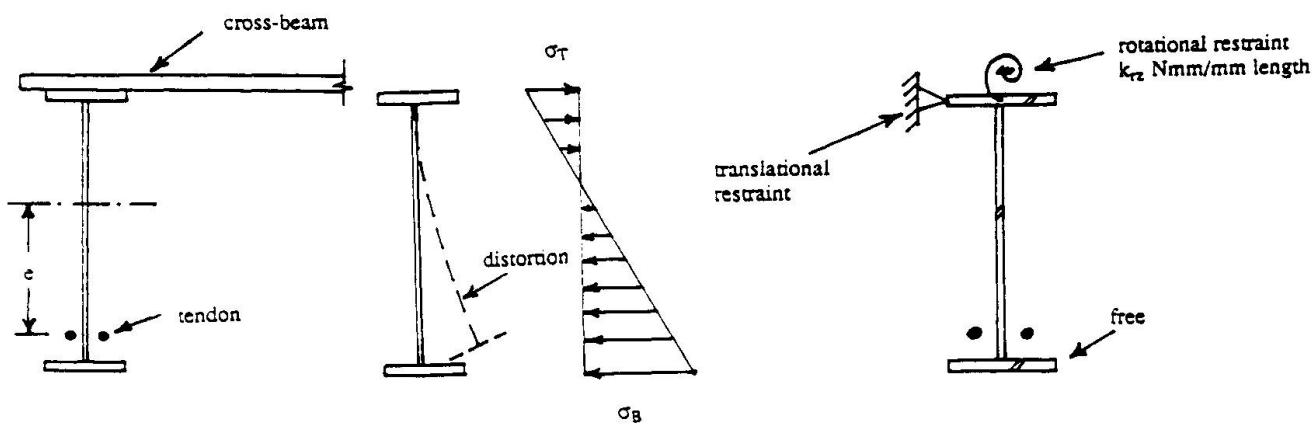


Fig. 1 Prestressed girder and buckling

Fig. 2 Top flange restraint model

The distortional buckling analysis was also able to account for elastic restraints at the level of the top or bottom flange. For the problem under consideration, the bridge superstructure effectively prevents lateral buckling displacement and in-plane rotation of the top flange of the plate girder, but provides elastic restraint against twist of the top flange of the girder. This continuous elastic elastic restraint (see Fig. 2) has a stiffness of $k_{rz} = 4EI_s/L_s$ [6], where E is Young's modulus and I_s is the second moment of area of the superstructure per unit length of bridge girder taken about a longitudinal axis parallel to the girder, and L_s is the lateral spacing of the girders.

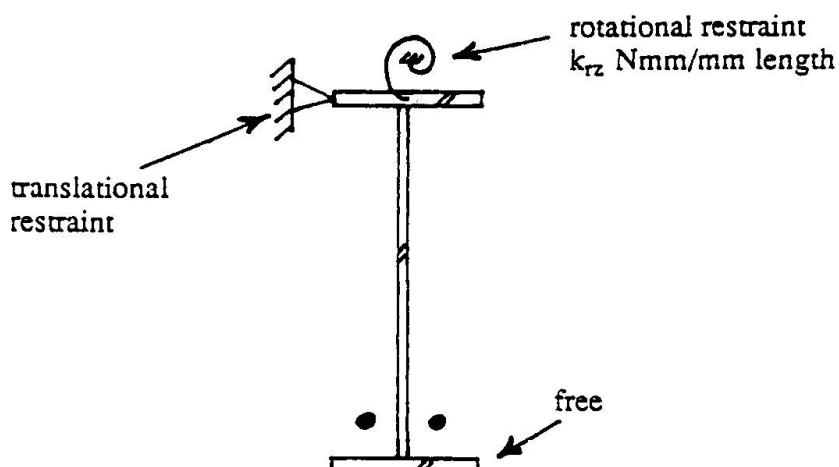


Fig. 2 Top flange restraint model

The vector of the four buckling degrees of freedom $\{q\}$ consists of the lateral displacements and twists of the top and bottom flanges. The strain energy stored during buckling may then be written as

$$U = \frac{1}{2} \{q\}^T ([k] + [k_r]) \{q\} \quad (1)$$

where $[k]$ is the elastic stiffness matrix and $[k_r]$ is the restraint stiffness matrix. These matrices have been derived explicitly by Bradford [5].

The beam-column is assumed to be subjected to a moment M and an axial force N , and factored by the load factor λ until buckling occurs. If the girder is subjected to a prestressing force P , then these actions are clearly

$$\lambda M = \lambda Pe \quad (2)$$

and

$$\lambda N = \lambda P \quad (3)$$

where e is the eccentricity of the prestressing force below the centroid of the plate girder. The actions λM and λN do work V during buckling, which can be written as

$$V = \frac{1}{2} \{q\}^T \lambda [g] \{q\} \quad (4)$$

where $[g]$ is the stability matrix derived explicitly by Bradford [5].

Invoking the principle of stationary potential energy [7] produces

$$\frac{\partial(U-V)}{\partial \{q\}} = \{0\} \quad (5)$$

so that, for nontrivial buckling displacements $\{q\}$,

$$|[k] + [k_r] - \lambda [g]| = 0 \quad (6)$$



Equation 6 may be solved by standard eigenvalue routines for the buckling load factor λ and the mode shape $\{q\}$. Because the stiffness and stability matrices are only 4 by 4, solution of the eigenproblem in Eq. 6 is instantaneous on a personal computer.

3. THROUGH GIRDER BUCKLING MODEL

Through girders are inverted bridges of the type considered herein, so that the deck is connected to the bottom flange and the top flange is subjected to compression under gravity loading. These through girders may be analysed for instability of their compression flanges by a U-frame approach [8].

The main assumption in the U-frame approach, which is an inverted U-frame for this study, is that the bottom flange is restrained elastically only by the web with a continuous translational restraint stiffness α_t , and furthermore that the bottom flange is a strut subjected to a constant compressive force. The value of α_t may be calculated by applying a pair of equal and opposite unit forces to the girders at the bottom flange level, and calculating the deflection of the bottom flange Δ . The flexibility of the web and the rotation at the attachment to the superstructure contribute to Δ , so that

$$\Delta = \frac{h^3}{3EI_w} + \frac{h^2}{k_{rz}} \quad (7)$$

where I_w is the second moment of area of the web per unit length of girder ($t_w^3/12$) and h is the distance between the flange centroids. The translational stiffness α_t can then be calculated from the flexibility Δ by

$$\alpha_t = \frac{1}{\Delta} \quad (8)$$

A strut with a continuous elastic translational restraint is analysed in Bleich [9]. Based on this analysis, it is shown in Oehlers and Bradford [8] that the critical force to cause buckling of the bottom flange is

$$N_{cr} = 2\sqrt{EI_F\alpha_t} \quad (9)$$

where I_F is the second moment of area of the bottom flange ($b^3t_f/12$). Equating the critical load in the bottom flange N_{cr} to the force in the bottom flange produced by the prestressing force P at eccentricity e yields

$$P_{cr} = \frac{2\sqrt{EI_F\alpha_t}}{1 + \frac{eh}{2r^2}} \quad (10)$$

where r is the major axis radius of gyration of the plate girder ($\sqrt{I/A}$).

4. BUCKLING STUDY

A steel plate girder bridge with a web depth $h = 1700\text{mm}$ and thickness $t_w = 15\text{mm}$ and flange widths $b = 300\text{mm}$ and thicknesses $t_f = 25\text{mm}$ has been studied. The eccentricity of prestress e is at 800mm . Under a prestressing force P (in MN), the stress in the bottom flange will be $64.7P \text{ MPa}$ and the camber is $ML^2/8EI = 0.029PL^2$ where L is in metres.

Figure 3 shows the prestressing force to cause buckling versus the buckling half-wavelength as a function of the torsional restraint at the top of the flange for the first harmonic. For all torsional restraint values except zero, the curves have a minimum and are garland shaped. The curves may be plotted for a given length of attachment of tendons to the plate girder L_t by translating the garlands $l, 2l, \dots$ to the right to create a series of cusps as are familiar in local buckling studies and presented in Bulson [10].

Consider the plate girders spaced 10m apart with transverse cross beams in the deck, typically each of second moment of area $I_s = 2.5 \times 10^7 \text{ mm}^4$ spaced at 4m intervals. The torsional restraint is $4EI_s/L_s = 2.0 \times 10^9 \text{ Nmm}$ or $k_{rz} = 5.0 \times 10^5 \text{ Nmm/mm}$ length of girder. A prestressing force P of 2.46 MN in a girder of length $L = 35\text{m}$ will produce a precamber of $L/400$ and a bottom flange compressive stress of 166 MPa. Such a prestressing force is obtainable from tendons of total effective area 2730 mm^2 stressed to 900 MPa. Clearly from Fig. 3, buckling will occur for attachment lengths L_t greater than 7.5m. For a strength analysis using limit states or LRFD principles, this length would have to be reduced even further.

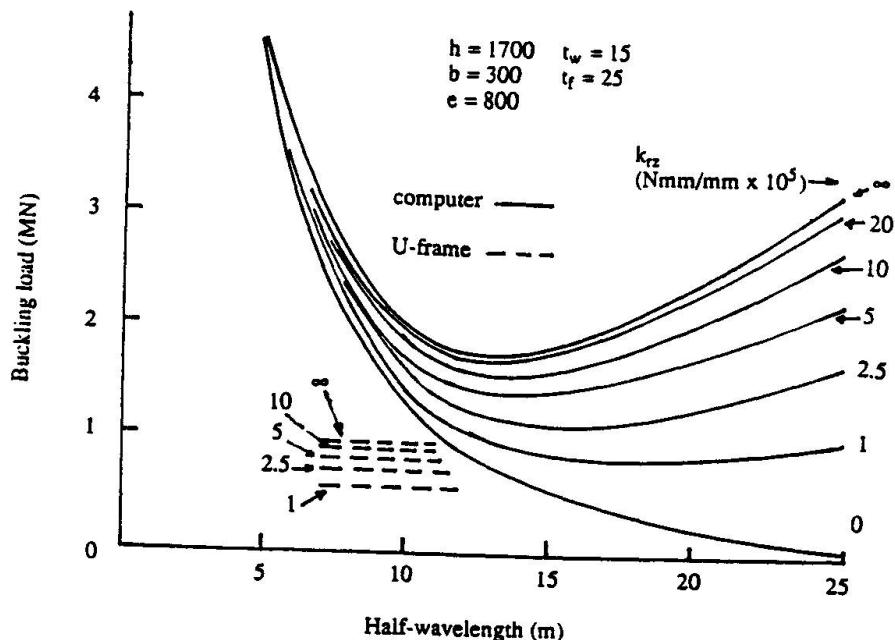
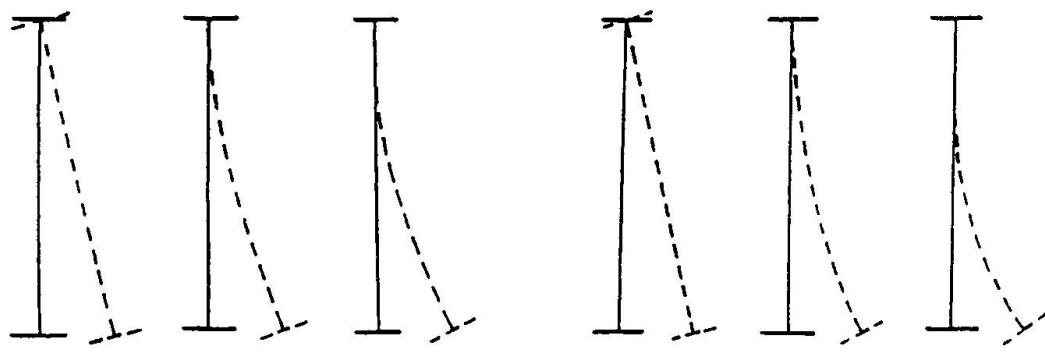


Fig. 3 Buckling loads



$k_{rz} \times 10^5 = 0 \quad 5 \quad \infty$

Fig. 4 Buckling modes for $l = 12.5 \text{ m}$

Fig. 5 Buckling modes for $l = 7 \text{ m}$

Figure 3 also demonstrates the substantial effect that the restraint of the deck has on the minimum buckling load. The effect of this torsional restraint can also be incorporated into the inverted U-frame approach, as noted in the previous section. The buckling loads calculated from Eq. 10 are also plotted in Fig. 3. While being a reasonable and conservative prediction of the nadir of the computer-generated curves, they are quite conservative for the shorter tendon attachment lengths L_t which would be met in practice. The buckling modes are plotted in Fig. 4 at $l = 12.5\text{m}$ for $k_{rz} = \infty$, 5×10^5 and 0 Nmm/mm , and in Fig. 5 for $l = 7\text{m}$. The increased cross-sectional distortion as k_{rz} increases is obvious.



5. CONCLUDING REMARKS

A computer-based method for analysing the elastic lateral-distortional buckling of prestressed plate girders in steel bridges has been described. Prestressing is an efficient means of strengthening and stiffening existing steel bridges, and instability of the plate girder during the stressing operation is a necessary consideration in the absence of substantial gravity loads. The effects of the torsional restraint provided to the top flange by the stiffness of the cross beams in the deck was shown to be considerable, but quite less than a rigid torsional restraint.

Because of the large number of significant parameters that affect the buckling solution, only one bridge girder was considered for illustrative purposes. This girder was liable to instability under the quite large prestressing forces required to camber the bridge and to impart practical compressive stresses in the bottom flange. Under this combination of geometry and loading, it was shown that limitations had to be placed on the distance between the attachment of the tendon to the plate girder. The U-frame model affords a convenient, albeit conservative, model of the elastic buckling which can be used in lieu of the computer solution.

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