Modelling of the Oedometer Test by Neural Networks

Modélisation d'un essai oedométrique avec réseaux de neurones

Modellierung von Oedometerversuchen mit neuronalen Netzen

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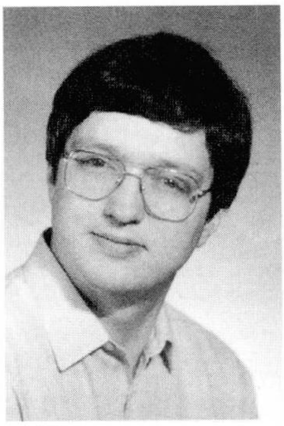

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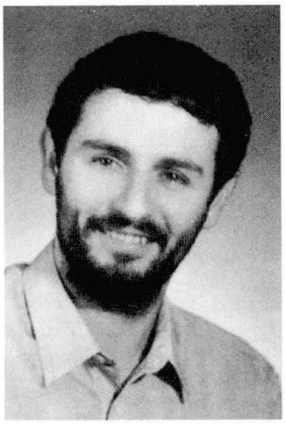

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SUMMARY
Constitutive modelling of non-linear material such as soil is a very difficult task. The development of artificial intelligence offers a new possibility in this field. A feed-forward neural network used in predicting the oedometer loading curve was trained by 40 oedometer curves obtained from tests made on marshland soil. The neural network was tested on 6 oedometer curves which were not included in the learning process. The oedometer curves and the clay parameter were evaluated. Good agreement between measured and predicted values was obtained, which implies that the neural networks can serve as an effective constitutive model as well as predicting sample behaviour.

RÉSUMÉ
Il est très difficile d'établir la relation entre contraintes et déformations pour des matériaux non linéaires, tels que les sols. Le développement de l'intelligence artificielle offre, avec les réseaux neuronaux, une nouvelle possibilité dans le domaine des modèles constitutifs. Les prévisions de la courbe oedométrique ont été établies avec un chargement, basé sur apprentissage, de 40 essais oedométriques du sol du Marais de Ljubljana. Les résultats ont été vérifiés avec 6 essais, non incorporés dans le processus de l'apprentissage. La bonne concordance entre les résultats expérimentaux et calculés permet d'utiliser le réseau neuronal comme modèle constitutif pour prévoir le comportement de l'échantillon.

ZUSAMMENFASSUNG
Das Aufstellen konstitutiver Beziehungen ist für nichtlineare Materialien, z.B. Erdstoffe, keine einfache Aufgabe. Die Entwicklung der künstlichen Intelligenz bietet als eine neue Möglichkeit den Gebrauch neuronaler Netze. Das Lernen von 40 Oedometerversuchen mit Erdstoffen aus dem Moor von Ljubljana ergab die Resultate für die Verdichtungskurve. Das neuronale Netz wurde mit 6 Oedometerversuchen, die nicht in den Lernprozess eingeschlossen waren, geprüft. Die Verdichtungskurven und die Parameter des Cam-Clay-Models wurden verglichen. Die Übereinstimmung der Messungen mit den Voraussagen des neuronalen Netzes zeigt, dass es als ein konstitutives Modell dienen kann, und die Voraussage des Erdprobenverhaltens ermöglicht.

1. INTRODUCTION

Oedometer tests are widely used for the determination of the compressibility characteristics of soils. For this reason a large amount of test data is available. We believe that the development of artificial intelligence enables us to convert these old files to active knowledge by using neural networks. Some results of the first part of our study on the use of artificial neural networks as a knowledge-based constitutive model for uniaxial soil behaviour are presented in the paper.

2. DATA BASE

During the last thirty years extensive in-situ and laboratory testing of Ljubljana marshland soils has been performed, mainly for the purpose of road and highway construction in very difficult soil conditions. Ljubljana marshland subsoil typically consists of two very soft silty and clayey layers underlain by stiffer silts or clays including sand and/or gravel. From the constructional point of view, the upper, up to 9 m thick, normally consolidated silty soil layer of very high compressibility (MH) is the most critical. The results of 46 oedometer tests made on samples of this silty soil were available. In addition to the oedometer curve $\sigma'-e$ (σ' is the effective vertical stress, e is the void ratio), for each sample the following characteristics are known: initial void ratio, e_0, natural water content, w_0, liquid limit, w_L, plastic limit, w_P, plasticity index, I_P, consistency index, I_C, specific weight, γ, and depth z from which the sample was taken. The extreme and the average values of these characteristics are shown in Tab. 1. Consistency characteristics of soil samples are presented in the plasticity chart (Fig. 1).

Parameter	Minimum	Maximum	Average
e_0	1.191	4.989	2.66
w_0 [%]	68.2	206.3	100.8
w_L [%]	55.0	155.9	92.1
w_P [%]	37.0	82.6	48.9
I_P [%]	11.4	81.0	43.2
I_C	-2.97	0.47	-0.40
γ [kN/m^3]	12.8	15.9	14.5
z [m]	1.25	8.15	3.9

Table 1 The extreme and the average values of soil characteristics.

Oedometer curves, characterized by z, w_0, w_L and w_P, are given by 5 to 7 pairs of test stress σ' and corresponding void ratio e (Fig. 2). Most samples were tested in the stress range from 0 to 160 kPa. The maximum stress applied to any of the 46 samples was 300 kPa. Based on the collected data the table of data was formed as an input to the neural network training algorithm. Each row of the table corresponds to one point of one particular oedometer curve and represents an input-output pair. The following data were provided for each input-output pair:

z depth from which the sample was taken,
w_0 natural water content,

w_0 natural water content,
w_L liquid limit,
w_P plastic limit and
σ' testing effective vertical stress

as input parameters and

e corresponding void ratio as the output parameter.

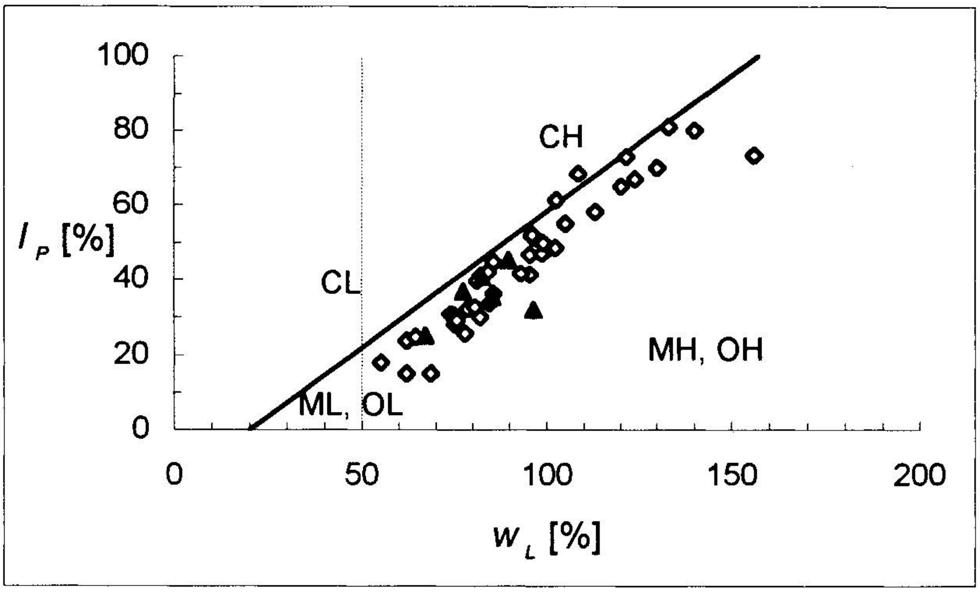

Fig. 1 Consistency characteristics of samples in plasticity chart.

Parameters I_P, I_C and γ were not included in the data base since they may be evaluated from the natural water content and consistency limits by the following equations:

$$I_P = w_L - w_P$$

$$I_C = \frac{w_L - w_0}{I_P}$$

$$\gamma = \frac{(\gamma_s + \gamma_w e_0 S_r)}{1+e_0} = \frac{(1+w_0)\gamma_w S_r}{(S_r \gamma_w/\gamma_s + w_0)} \approx \frac{(1+w_0)\gamma_w}{(\gamma_w/\gamma_s + w_0)}$$

where γ_s, γ_w, S_r are the specific weight of solid particles, the specific weight of water, and the degree of saturation, respectively. The degree of saturation S_r for the studied material is approximately equal to one.

The geological stress $\sigma_p' = \int_0^z \gamma(\zeta)d\zeta$ was substituted by the depth of sample z, since the estimation of the depth is considerably more accurate than the estimation of σ_p' due to the unknown exact distribution of γ. This substitution was feasible because the samples of the treated soil were taken from the upper layer and because the water table is practically at the surface, which makes the distribution of γ similar for all boreholes.

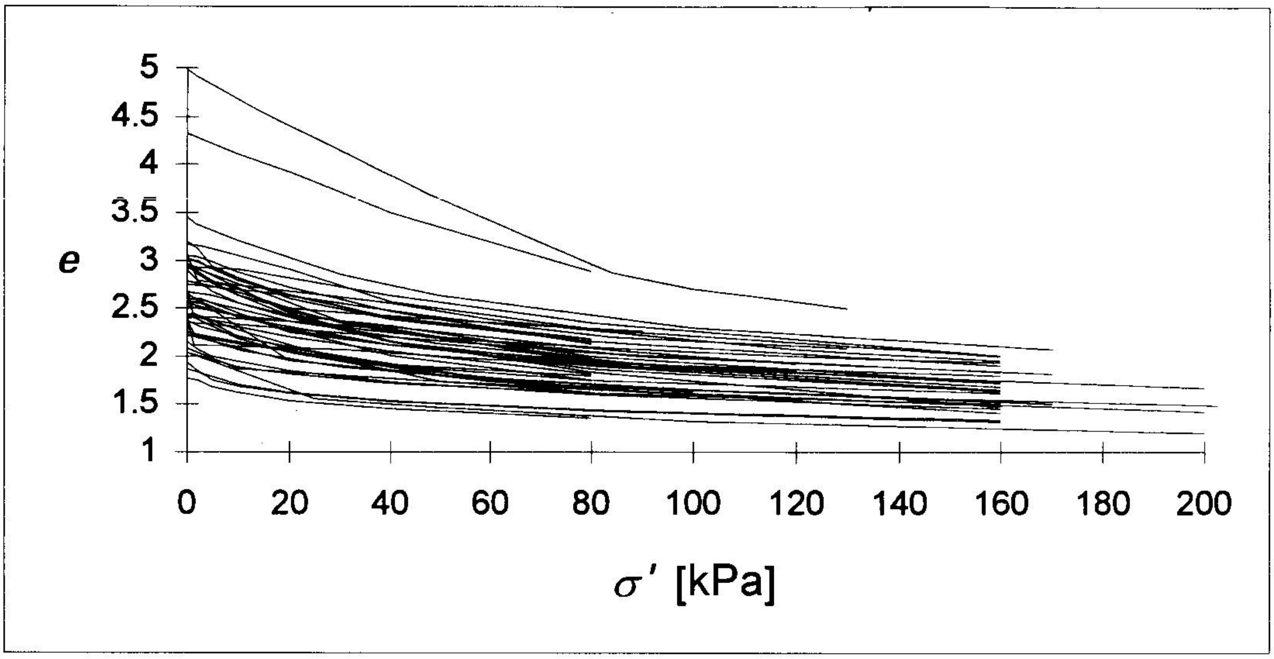

Fig. 2 Oedometer curves used in this study.

The available data were split into two parts. Forty oedometer tests, represented by 203 input-output pairs, were used as training data set. The remaining six oedometer tests were selected to test the performance of trained neural networks.

The preliminary calculations had shown that predicted oedometer compression lines did not follow the well known behaviour of soil sample in uniaxial compression conditions. For stresses greater than the pre-consolidation pressure the compression line is usually a straight line (if plotted in the logarithmic scale). The oedometer compression lines predicted by the neural network did not exhibit such behaviour. Their plots in the logarithmic scale were curved from zero to the final stresses (Fig. 3). To improve the solution, a number of additional data points were linearly interpolated between the measured values in the straight part of oedometer compression lines. This extended data base consisting of 408 input-output pairs was used for the training of neural networks.

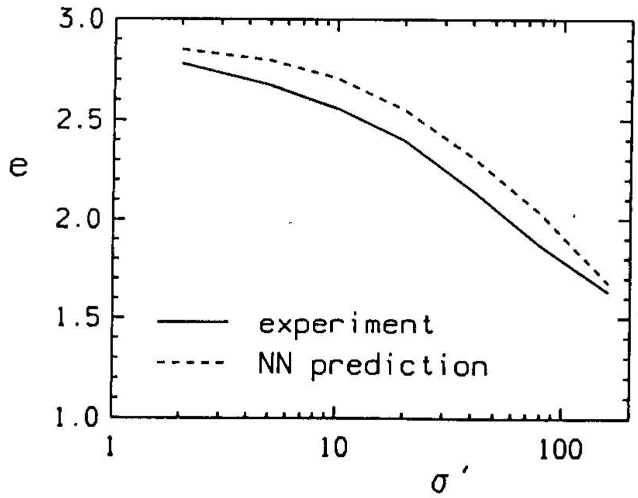

Fig. 3 Observed and preliminary determined oedometer compression lines.

3. NEURAL NETWORKS

An approximation of an unknown mapping $f(\mathbf{X})$ of several variables is sought. There are two possible ways, how to solve the problem:

- Traditionally the problem is solved in two steps. Firstly, an approximation function $g(\mathbf{X},\mathbf{C})$ is chosen. Subsequently, unknown parameters $\mathbf{C}$ are evaluated by the least squares method.
- Alternatively, the neural network may be used to approximate the unknown mapping. For this purpose a feed-forward neural network is usually used.

The general characteristics of neural networks which naturally act as associative memory, are able to generalize, and are highly fault tolerant, imply that the alternative is often better than the traditional method. Moreover, if there are many independent variables, the choice of the approximation function is a very difficult task. As a result, several authors ([1], [3], [5], [8], [10], [12]) reported on their successful use of neural networks in the approximation of functions. It has been proven that any continuous mapping can be approximated by a network with at least one hidden layer [4], [6].

In our research the feed-forward neural network was used. One or several hidden layers contained different number of neurons. A typical neural network is shown in Fig. 4. The activation function was chosen to be a sigmoidal with no bias term

$$\mathrm{sig}(x) = \frac{1}{1+\mathrm{e}^{-x}}$$

In training procedure a classical back-propagation algorithm was used [11]. In order to improve the efficiency of learning, the adaptive step size algorithm [7] or simulated annealing may be used [9], [13]. To avoid a possible over-learning for some experimental points, the experiments for which the error was lower than a specified fraction of allowed error were excluded from the learning.

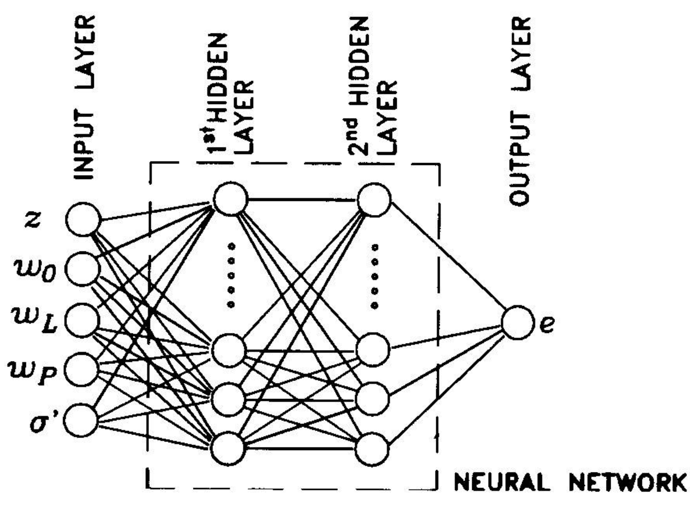

Fig. 4 Feed forward neural network

4. RESULTS

Neural networks of different size were tested. The number of hidden layers was taken to be 1, 2 and 3. Although the networks having two or three hidden layers learned the mapping rapidly, the preferred number of hidden layer is one. Networks with two and three hidden layers lacked the ability to generalize. Therefore, only the networks with one hidden layer and different number of neurons were used.

Firstly, the network was trained using 408 training input-output pairs. After the networks had been trained, the success of the training was tested using a set of testing data. One of the oedometer curves was eliminated from the testing set, since the error was notably larger than in the other five oedometer curves. It was later found that this had occurred because of an evident experimental error. Maximum and average normalized errors in predicting the void ratio are shown in Tab. 2. Considering the accuracy that is usually met in geotechnical engineering, the errors are relatively small. However, the results were considerably improved if the augmented learning was performed. When using the basic learning, the oedometer curve was predicted from z, w_0, w_L and w_P only. Additional data could be obtained from the classification tests, i.e. e_0, which represents the void ratio at the beginning of the test ($\sigma' = 0$). Therefore, the training of the network using this additional input-output pair (z, w_0, w_L, w_P, $\sigma' = 0$ and e_0) may proceed. The performance of the neural network trained by the augmented training was better. The maximum error decreased in most cases from about 30% to a little more than 10%. Similarly, the average error decreased from more than 10% to less than 5%.

No. of neurones	Basic learning		Augmented learning	
	Δe_{max} [%]	$\overline{\Delta e}$ [%]	Δe_{max} [%]	$\overline{\Delta e}$ [%]
30	31.1	13.9	11.0	3.6
35	20.9	8.9	12.5	4.3
40	36.3	15.8	15.8	4.8
45	31.2	14.4	12.6	4.2
50	38.1	16.9	10.5	3.9
60	39.4	15.7	13.0	5.7
75	33.3	13.2	22.0	3.8

Table 2 The maximum and the average error in void ratio prediction on a testing set.

The analysis of the soil behaviour including in-elasticity and consolidation is often performed by the finite element method. One of the most successful material models is the Cam-Clay model. One of the parameters (λ) of the model is the slope of the normal compression line in the ($\ln \sigma' : e$) plane.

Parameter λ was evaluated from the actual testing curves and the curves obtained by neural networks. The normalized errors are larger than in the case of void ratio. However, it appears that the largest errors correspond to extremely small values of λ. Thus, the absolute error of λ is relatively small. The normalized errors of λ for different number of neurons in the hidden layer are shown in Fig. 5. Bars represent the maximum and lines the average error values. In this case the error of λ does not exceed 12.0% and the average error is 6.5% for the network with 35 neurones in the hidden layer.

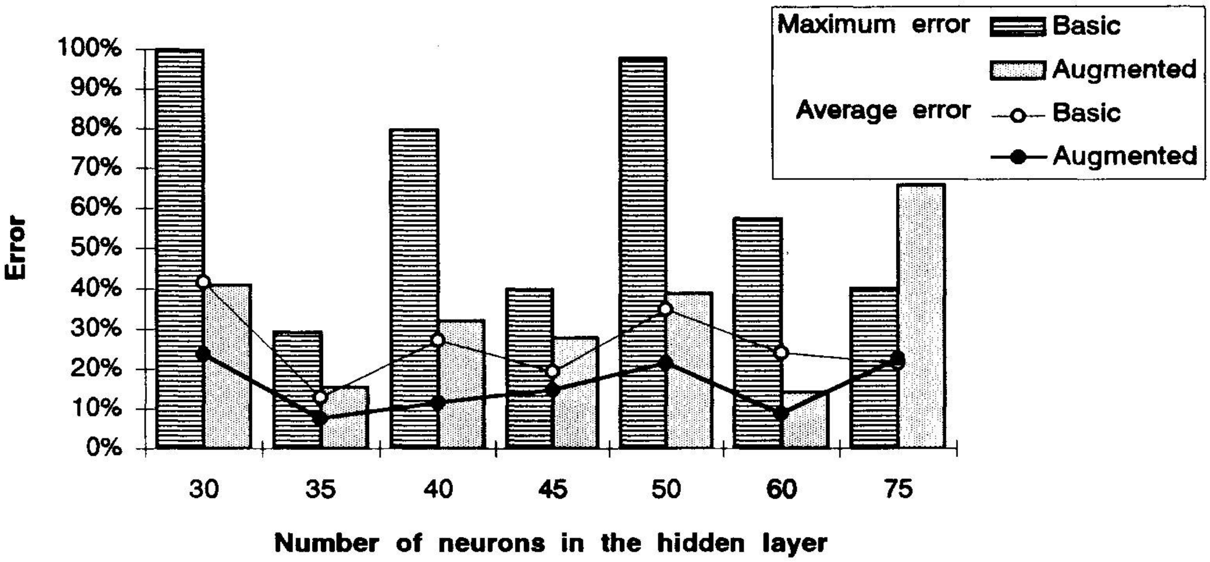

Fig. 5 Normalized errors of λ.

The overall performance of the neural network was optimal if the number of neurons in the hidden layer was 35. The actual oedometer curves and those obtained by the neural network with 35 neurons in the hidden layer are shown in Fig. 6. It is evident that in the case of the oedometer curve which was excluded from the testing set the discrepancy occurs because of an experimental error. Hence, the neural network may also be used as a warning unit embodied in the testing device. When the difference between the expected (neural network) and the measured values of void ratio is too large, the warning ought to be issued.

5. CONCLUSIONS

The neural networks were trained to approximate the oedometer curves obtained from the database of 46 laboratory experiments. Forty of them were employed as a training set, while the remaining six made up the testing set. The basic and the augmented training were performed.

The results have shown that the prediction of the void ratio is very reliable. The errors were generally lower than 15% for the basic training and lower than 5% for the augmented training. The error was larger in the case when the Cam-Clay parameter λ was predicted. However, if λ is to be used in a constitutive model, all available data (including testing data set) should be used in the training procedure.

One possible source of the error stems from the fact that the experiments had been performed by three different laboratories. Every laboratory has its own characteristics, and due to those differences the errors, which have to be eliminated, occur. Some of the gross errors of the measurement have already been found and eliminated by the neural networks.

The data concerning the unloading part of the oedometer tests were not as complete as those for loading. After the data are obtained, the network will be trained for that part, too. It is our goal to use a neural network as a constitutive model, replacing the explicit models, such as Cam-Clay, used in FEM codes today.

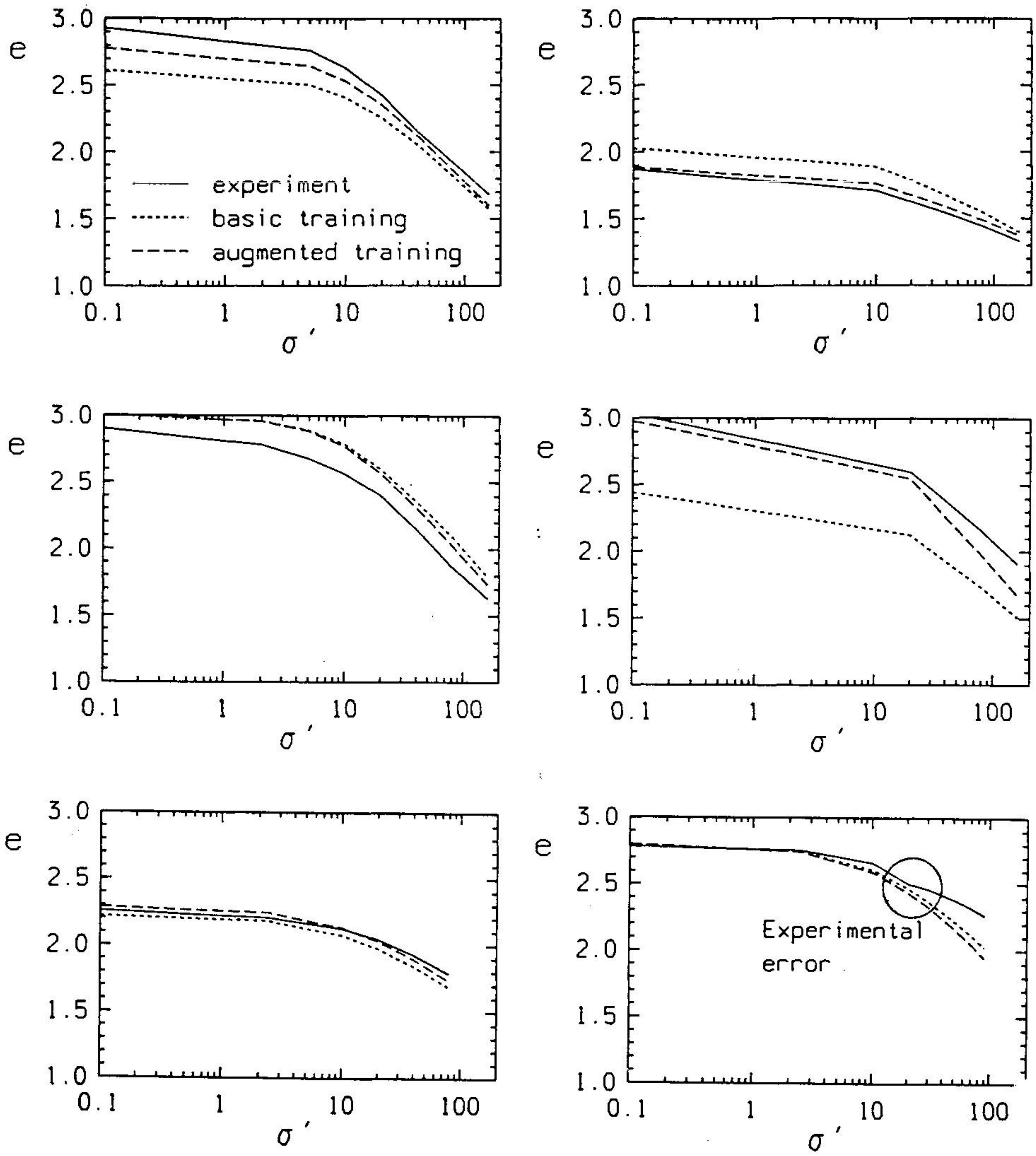

Fig. 6 Oedometer curves of the testing set.

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