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Autor: Kirkegaard, Poul Henning / Rytter, Anders
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Comparison of Vibration-Based Damage Assessment Techniques

Comparaison de techniques dynamiques de détection de dommages

Vergleich dynamischer Verfahren zur Schadenidentifikation

Poul Henning KIRKEGAARD
Assist. Prof.
Aalborg University
Aalborg, Denmark

Poul H. Kirkegaard, born 1962, got his M.Sc degree in Civil Engineering in 1988 at Aalborg University. After his Ph.D. degree in 1991 also from Aalborg University, his has been assistant professor in experimental dynamics there.

Anders RYTTER
Dr Eng.
Joint Research Centre
Ispra, Italy

Anders Rytter, born in 1959, received his M.Sc. degree in 1984 and his Ph.D. degree in 1993, both from Aalborg University, Denmark. From 1984 to 1994 he was employed as project engineer. Since April 1994 he has been working at Joint Research Centre, Ispra, Italy.

SUMMARY
Three different vibration-based damage assessment techniques have been compared. One of the techniques uses the ratios between changes in experimentally and theoretically estimated natural frequencies, respectively, to locate a damage. The second technique relies on updating of a finite element method based on experimentally estimated natural frequencies where the stiffness matrix is given as a function of damage size and location. The last technique is based on neural networks trained with the relative changes in natural frequencies. It has been found that all techniques seem to be useful. The neural networks based technique seems to be very promising.

RÉSUMÉ
Trois techniques différentes de détection de dommage basées sur les vibrations sont comparées. Une des techniques utilise le rapport entre les fréquences propres mesurées expérimentalement et déterminées théoriquement. La seconde technique repose sur une mise à jour d'une méthode par éléments finis, basée sur des fréquences propres déterminées expérimentalement, et où la matrice de rigidité est donnée comme une fonction de la taille du dommage et de sa location. La dernière technique est basée sur des réseaux neuronaux entraînés par les changements relatifs dans les fréquences propres. L'article montre que les trois techniques sont opérationnelles. La technique basée sur les réseaux neuronaux semble très prometteuse.

ZUSAMMENFASSUNG
Drei verschiedene Techniken zur Schadenidentifikation werden verglichen, die alle auf Schwingungsmessungen beruhen. Ein Verfahren verwendet das Verhältnis der Aenderungen von theoretisch und experimentell bestimmten Eigenfrequenzen, um einen Schaden zu lokalisieren. Ein anderes beruht auf der Nachführung eines FE-Modells gemäss experimentell bestimmten Eigenfrequenzen, wobei die Steifigkeitsmatrix von Ort und Ausmass der Schädigung abhängt. Das dritte Verfahren benützt neuronale Netzwerke, die mit den relativen Aenderungen der Eigenfrequenzen trainiert werden. Insbesondere letztere Technik erscheint sehr vielversprechend.

1. INTRODUCTION

Determination of location and size of damages in civil engineering structures, damage assessment, using vibration measurements is a problem which has received much attention, recently, see e.g. Rytter [1]. Such vibration based inspection techniques are particular needed for dealing with large structures, such as civil engineering structures and large space structures, since they do not need assess to the structures for the investigator, such as those techniques based on e.g. visual inspection, ultrasonic testing, eddy currents and acoustic emission. Further, research in vibration based damage assessment techniques has been initiated by a considerable demand for a more accurate non-destructive damage assessment technique.

One of the consequences of a structural damage, such as a crack, is a change in local stiffness which in turn results in a decrease in some or in all the modal quantities, e.g. natural frequencies. The most commonly applied vibration based damage assessment techniques are based on changes of natural frequencies only. By comparing changes of experimentally measured modal quantities in structures with patterns of changes predicted theoretical implies that the location and/or size of the damage can be obtained. However, this requires knowledge of the theoretical changes of the modal quantities in the structure due to different locations and sizes of the damage. Therefore, the damage assessment results are depending on how well the mathematical model describes the dynamic behaviour of the damaged as well as the undamaged structure. The problem of establishing such models has been considered in e.g. Gudmundson [2], Ostachowicz et al. [3] and Okamura[4]. Based on such models one can estimated the changes in the dynamic behaviour due to a crack. However, damage assessment from measured changes in dynamic behaviour is the inverse problem, i.e. how can information be obtained about location and size of a damage given some experimentally measured modal quantities. Many techniques have been proposed to solve this inverse problem. A review of such techniques can be found in e.g. Rytter [1].

The aim of this paper is to investigate three different damage assessment techniques proposed to solve the inverse problem. These three different techniques are chosen since they represents three different damage assessment principles. One of the techniques uses the ratio of changes in experimentally and theoretically estimated natural frequencies, respectively, to locate a damage, see Cawley et al. [6]. The second technique relies on updating of an FEM where the stiffness matrix is given as a function of damage size and location, i.e. both location and size of the damage are estimated, see Shen et al. [7] and Rytter et al. [11]. The last technique is based on neural networks trained with the theoretically relative changes in natural frequencies as input and size and location of a crack as output, see Kirkegaard et al. [8].

Section 2 gives a description of the three different vibration based damage assessment techniques is given. In section 3 the described techniques are compared in an example with a straight hollow section steel cantilever beam. At last in section 4 conclusions are given.

2. DAMAGE ASSESSMENT TECHNIQUES

The following chapter presents the three different damage assessment techniques compared in this paper.

2.1 Cawley & Adams' Damage Detection Technique

In this section a damage assessment technique relying on the measurement of small changes in natural frequencies and upon adequate theoretical predictions of these frequency changes is presented. This technique can be used to give an estimate of the damage location. However, the technique does not give any indication of the quantity of the damage. The technique (briefly the CA-technique) is based on that fact that for small frequency changes the ratio of frequency changes in two modes is a function of the location of the damage only, see Cawley et al. [6]. Using an FEM to model a damage in a structure, theoretical values of the ratio of frequency changes in two modes can be obtained for various damage scenarios and modes. Observing the structure, both at virgin state and after a damage has been estasblihed the actual ratio of the frequency changes. Although some allowance must be made for inadequacy of the damage model used to estimate the theoretical frequency changes, the ratio of experimentally obtained frequency changes in two modes and the ratio of analytically obtained frequency changes in two modes, respectively, should have a close relation for the same damage scenarios and modes. In Cawley et al. [6] it is proposed to handle this matching problem by using an error e_{rij} quantity given by

$$e_{rij} = \frac{\frac{\Delta\omega^a_{ri}}{\Delta\omega^a_{rj}}}{\frac{\Delta\omega^m_i}{\Delta\omega^m_j}} - 1 \qquad \frac{\Delta\omega^a_{ri}}{\Delta{\omega^a}_{rj}} \geq \frac{\Delta\omega^m_i}{\Delta\omega^m_j} \tag{1}$$

$$e_{rij} = \frac{\frac{\Delta\omega^m_i}{\Delta{\omega^m}_j}}{\frac{\Delta\omega^a_{ri}}{\Delta\omega^a_{rj}}} - 1 \qquad \frac{\Delta\omega^a_{ri}}{\Delta\omega^a_{rj}} < \frac{\Delta\omega^m_i}{\Delta\omega^m_j} \tag{2}$$

where $\Delta\omega^a_{ri}$ and $\Delta\omega^m_i$ are analytically and experimentally obtained changes of the ith natural frequencies at location r, respectively.

Thus the total error e_r assuming the damage located at position r may be calculated as

$$e_r = \sum_{i=1}^{N_f}\sum_{j=1}^{N_f} e_{rij} \tag{3}$$

N_f is the number of measured modes. Normalising the total error e_r with respect to the minimum total error e_{min} implies a normalized error E_r given by

$$E_r = \frac{100 e_{min}}{e_r} \tag{4}$$

By this definition, the normalized error E_r will always be 100 for the position r which gives the smallest error e_{rij}.

One major disadvantage of the CA-technique is that the technique will locate a damage even if the natural frequencies have changed slightly, due to e.g. the temperature effects or the measurement noise.

2.2 Damage Assessment by Calibrating a FEM

In recent studies the damage assessment problem has been solved by a model updating procedure (briefly the UP-technique). A very used approach is to estimate the elements in the stiffness matrix for all the potential damage locations, see e.g. Smith et al. [9] and Richardson et al. [10]. The largest reduction in e.g. stiffness, compared with the stiffness of the undamaged structure, is giving the most likely damage location. These techniques belong to a general category of system identification techniques where parameters of an apriori analytical structural model such as its mass, stiffness, and damping are adjusted to minimize the difference between the analytically predicted and experimentally measured dynamic characteristics. If the stiffness matrix is given as a function of damage size and location it is also possible to estimate the magnitude of the damage, see e.g. Shen et al. [7] and Rytter et al. [11].

This implies that the following optimization problem can be formulated, see Rytter et al. [19]

$$\min_{r,a} \quad log(\sum_{i=1}^{N_f}(\frac{\frac{\omega_i^{mp}}{\omega_i^{mv}}}{\frac{\omega_{ri}^{ap}}{\omega_{ri}^{av}}} - 1)^2 W_i) \tag{5}$$

$$s.t \quad |\frac{\omega_{ri}^{ap}}{\omega_{ri}^{av}} - \frac{\omega_i^{mp}}{\omega_i^{mv}}| \leq \beta_i \tag{6}$$

where ω_{ri}^{ap} and ω_i^{mp} are analytically and experimentally obtained values for the damaged structure (periodical measurement), of the ith natural frequency, respectively. ω_{ri}^{av} and ω_i^{mv} are analytically and experimentally obtained values, for the undamaged structure (virgin state measurement), of the ith natural frequency, respectively. W_i and β_i are weighting parameters established from the accuracy of the estimates of the natural frequencies. In Rytter [1] it is shown that the formulation of the optimization problem (6) is important in order to avoid or reduce the problem of local minima.

2.3 Damage Assessment by using Neural Networks

Above it is explained how the damage assessment problem can be solved using optimization. Such a procedure based on minimization of a measure of the difference between measured data and the corresponding predictions obtained from a mathematical model implies a comprehensive search which is computationally expensive and nearly impossible for complex structures. Therefore, a pattern recognition scheme could be needed to decipher the complex pattern of dynamic behaviour changes that occurs due to a damage. However, recently, artificial neural networks are proving to be an effective tool for pattern recognition in a variety of applications, see e.g. Hertz et al. [12] and among these also for damage assessment. In Kirkegaard et al. [8] a neural network (briefly the NN-technique) has been trained with the relative changes in natural frequencies obtained by an FEM. The network is then used to estimate location and size of a crack in a beam from measured natural frequencies.

2.3.1 Neural Networks

Many different types of neural networks have been proposed by changing the network topology. Examples of those are e.g. the Hopfield network, Hopfield [13], the Kohonen network, Kohonen [14] and the so-called multilayered perceptron (MLP) network. The MLP trained by the back-propagation algorithm is currently given the greatest attention by application developers, see e.g. Hertz [12]. The MLP network belongs to the class of layered feed-forward nets with supervised learning. A multilayered neural network is made up of one or more hidden layers placed between the input and output layers, see fig. 1.

Each layer consists of a number of nodes connected in the structure of a layered network. The typical architecture is fully interconnected, i.e. each node in a lower level is connected to every node in the higher level. Output units cannot receive signals directly from the input layer. During the training phase activation flows are only allowed in one direction, a feed-forward process, from the input layer to the output layer through the hidden layers. The input vector feeds each of the first layer nodes, the outputs of this layer feed into each of the second layer nodes and so on. Associated with each connection between node i and node j in the preceding layer $l-1$ and following layer l is a numerical value $w_{lj,i}$ which is the strength or the weight of the connection.

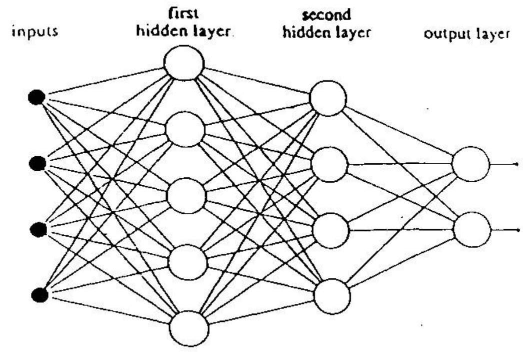

Fig. 1: Principle of a multilayer perceptron neural network.

At the start of the training process these weights are initialised by random values. Signal pass through the network and the jth node in layer l computes its output according to

$$x_{lj} = f(\sum_{i=1}^{N_{l-1}} w_{lj,i}x_{l-1,i} + \theta_{lj}) \tag{7}$$

for $j = 1,..,N_l$ and $l = 1,..,k$, where x_{lj} is the output of the jth node in the lth layer. θ_{lj} is a bias term or a threshold of the jth neuron in the lth layer. The kth layer is the output layer and the input layer must be labelled as layer zero. Thus N_0 and N_k refer to the numbers of network inputs and outputs, respectively. The function $f(\cdot)$ is called the node activation function and is assumed to be differentiable and to have a strictly positive first derivative. For the nodes in the hidden layers, the activation function is often chosen to be a so-called sigmoidal function

$$f(\beta) = \frac{1}{1+e^{-\beta}} \tag{8}$$

The activation function for the nodes in the input and output layers is often chosen as linear.

The first stage of creating an artificial neural network to model an input-output system is to establish the appropriate values of the connection weights $w_{lj,i}$ and thresholds $\theta_{lj}$by using a learning algorithm. A learning algorithm is a systematic procedure for adjusting the weights in the network to achieve a desired input/output relationship, i.e. supervised learning. The most popular and successful learning algorithm used to train multilayer neural networks is currently the Back-propagation routine, see Rumelhart [15]. The so-called Back-propagation algorithm employs a gradient descent search technique for minimizing an error normally defined as the mean square difference between desired and actual outputs from the network.

During the training phase, representative examples of input-output patterns are presented to the network. Each presentation is followed by small adjustments of weights and thresholds if the computed output is not correct. If there is any systematical relationship between input and output and the training examples are representative of this, and if the network topology is properly chosen, then the trained network will often be able to generalise beyond learned examples. Generalisation is a measure of how well the network performs on the actual problem once training is complete. It is usually tested by evaluating the performance of the network on new data outside the training set. Generalisation is most heavily influenced by three parameters: the number of data samples, the complexity of the underlying problem and the network architecture. Currently, there are no reliable rules for determining the capacity of a feed-forward multilayer neural network. Generally, the capacity of a neural network is a function of the number of hidden layers, the number of processing units in each layer, and the pattern of connectivity between layers. However, it is shown in Cybenko [16] and Funahshi [17] that one hidden layer is sufficient to approximate all continuous functions. The process of computing the gradient and adjusting the weights and thresholds is repeated until a minimum of the error is found. However, it is generally true that the convergence of the Back-propagation algorithm is fairly slow. Attempts to speed learning include variations on simple gradient search, line search techniques and second order techniques, see e.g. Hertz [12] and Billings [18].

2.3.2 Use of Neural Networks for Damage Assessment

When an artificial neural network is used for damage assessment the basic idea is to train the neural network in order to recognise the behaviour of the damaged as well as the undamaged structure. Subjecting this trained neural network to information from vibration tests should imply information about damage state and location. The network is trained with patterns of the changes in quantities describing the dynamic behaviour that occur due to a damage. This implies that each pattern represents the computed changes of e.g. the response spectrum, natural frequencies, mode shapes etc. due to a damage of a particular size at a particular location. The patterns of the quantities describing the dynamic behaviour are used as inputs and the damage location and size as outputs to train the neural network. Then the trained network subjected to measured patterns of the quantities describing the dynamic behaviour can be used to determine the location, size and of a damage. The training of a neural network with appropriate data containing the information about the cause and effect is a key requirement of a neural based damage assessment technique. This means that the first step is to establish the training sets which can be used to train a network in a way that the network can recognise the behaviour of the damaged as well as the undamaged structure from measured quantities. Therefore, ideally, the training sets should contain data of the undamaged as well as the damaged structure in various damage states. These data can be obtained by measurements, model tests or through numerical simulation, or through a combination of all three types of data. This possibility of using all obtained information, or only a part, in a neural network based damage assessment technique is a capability which is not available in traditional damage assessment techniques.

3. EXAMPLE

In this example the three different damage assessment techniques are applied to a hollow section steel cantilever, see fig 2.

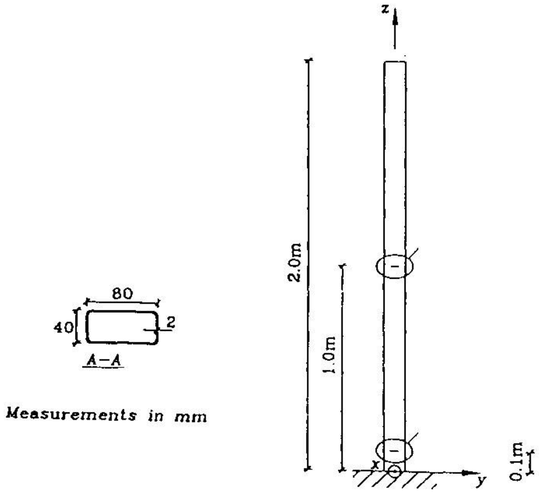

Fig. 2: Test structure.

3.1 Analytical and Experimental Results

The experimental data used in this example are estimates of the lowest three bending natural frequencies. Real line cracks were obtained in the test beams by attaching a sinus-varying load to the beams by means of a shaker. The frequency of the sinus was either the actual first or second bending natural frequency. The cracks lengths were measured by means of a microscope mounted on an electrical measurement rail. Two different crack locations were considered. In one beam a crack was initiated at z=1.0 m and in an other beam at z=0.1 m. The cracks were initiated as small narrow laser cuts (width $\approx$ 0.15 mm). The experimental determination of the bending natural frequencies was obtained from free decays. The free decays were introduced by removing a well-defined static load from the beam momentary. The accelerations were measured at z=0.5 m, z = 1.4 and z=2.0 m. The natural frequencies were estimated by a minimization of the least square error between the response obtained by an analytical expression of a linear and viscously damped system and the measured response. A more throughout description of the experimental procedure and the experimental results can be found in Rytter et al. [1].

The analytical estimated changes of the natural frequencies are estimated by an FEM of the beam. The development of a crack at a certain location of a beam corresponds to a sudden reduction of its bending stiffness at the same location. The crack divides the original non-cracked beam into two shorter beams, connected, at the crack location, by a very infinitesimal portion of beam with different characteristics. The characteristics in bending modes can be modelled by a torsion spring. The spring stiffness is estimated by using a fracture mechanical model, see e.g. Okamura[4] and Gomes et al. [5]. The FEM was calibrated by using experimental data from the non-cracked beam. This calibration was performed to secure that the FEM describes the beam in the best possible way. The quality of the predictions from any technique of damage assessment is critically dependent on the accuracy of the damage model, see e.g. Rytter [1].

3.2 Results

In the following it is explained how the three different damage assessment techniques were implemented in order to detect and locate the cracks in the beams.

3.2.1 The CA-Technique

In order to use the CA-technique the errors given by (4) were calculated using analytical and experimental values of the three lowest natural frequencies. The analytical values were estimated due to a crack of 0.06m placed in intervals of 0.025 m between $z = 0.0m$ and $z = 2.0m$. The crack locations estimated using the CA-technique are shown in fig. 3 with a dotted line.

3.2.2 The UP-Technique

The applicability of the UP-technique was tested by solving the optimization problem formulated in (5)-(6). Again the three lowest natural frequencies were used. This means that the optimization variables were size a and location z of the crack. The initial value of a and z were taken as 0m and 1.5 m, respectively. The crack locations and sizes estimated using the UP-technique are shown in figure 3 and 4, respectively with a dashed line.

3.3.3 The NN-Technique

The neural network results were obtained using a hierarchical, two step approach. This implies that the relative changes of the bending natural frequencies of the 3 lowest modes and the location of the crack are used as input and output, respectively, in one network. In an other network the crack location and the relative change of the bending natural frequencies of the 3 lowest modes are used as inputs and the size of the crack as output. A hierarchical approach was used since it was found in Kirkegaard et al. [8] that better results could be obtained instead of using only one big network. The training sets were generated for cracks located in intervals of 0.025 m between z = 0 m and z = 2.0 m, respectively. The cracks depths were in intervals of 0.004 m between 0.02 m and 0.140 m. By a trial-and-error approach it was found for the first network that a 4 layers neural network with 3 input nodes, 8 nodes in each of the two hidden layers and 1 output node gave the network with the smallest output error. For the second network it was found that a 4 layers neural network with 4 input nodes, 8 nodes in each of the two hidden layers and 1 output node gave the smallest output error. Results from the networks trained with analytical data and subjected to experimental data are shown with a dashed-dotted line in figure 3 and 4, respectively.

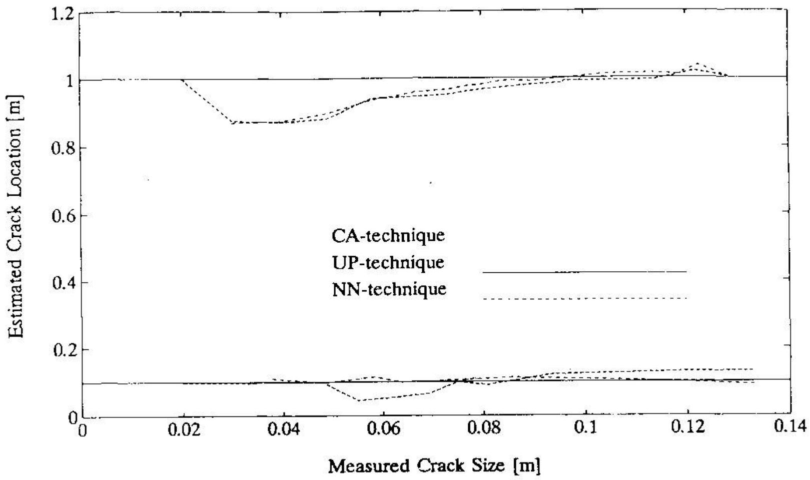

Fig. 3: Estimated crack location.

It is seen from figure 3 an 4 that the estimated location and size of the two cracks obtained by the three techniques are reasonably correct for all crack sizes. Especially, the estimates obtained by the NN-technique are interesting. By using the NN-technique the location and size of the crack can be estimated on-line. Figure 3 also shows that the influence from errors on the measured natural frequencies becomes smaller for increasing crack length.

4. CONCLUSIONS

Results from an example with a hollow section steel cantilever demonstrate the capability of three different vibration based damage assessment techniques. The techniques rely on the measurements of small changes in natural frequencies and upon adequate theoretical prediction of these frequency changes. It is explained how the damage assessment problem can be solved using optimization. It is found that the size and location of cracks can be predicted using the three techniques. Especially, the estimates obtained by a neural network based technique seems to be encouraging. By using the neural network based damage assessment technique an on-line technique is established. In the following work the neural based technique for detecting and locating damages in civil engineering structures based on vibration measurements has to be investigated.

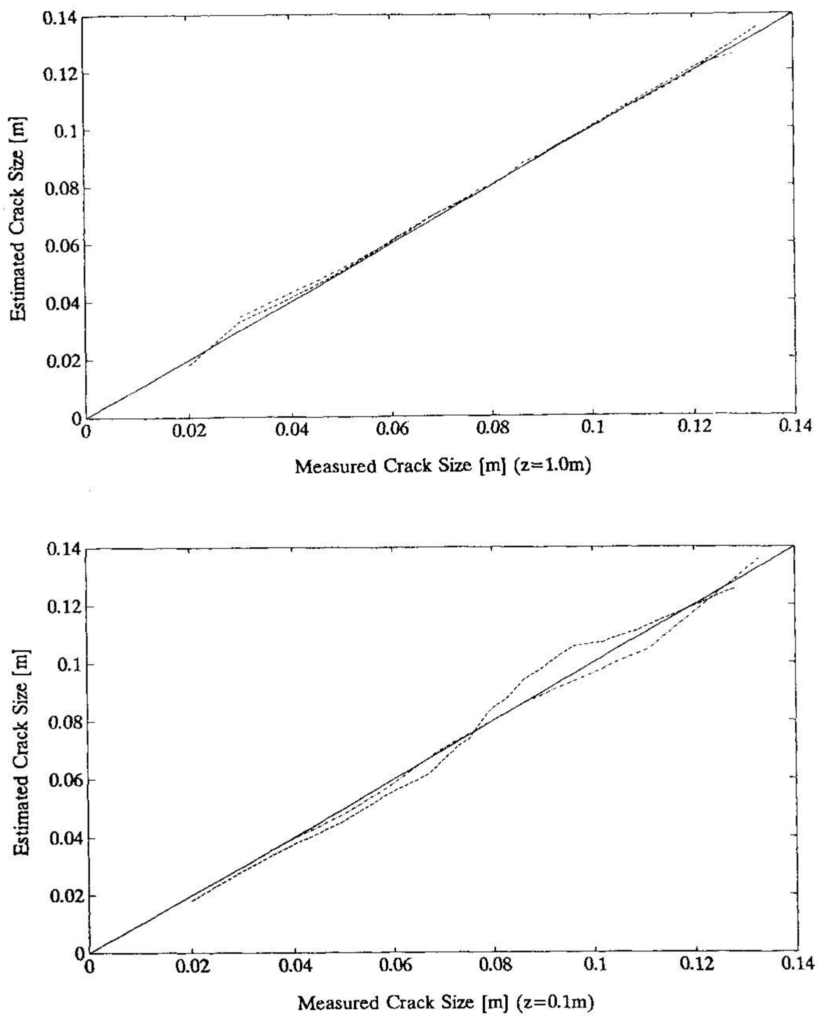

Fig. 4: Estimated crack size.

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