Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte

Band: 71 (1994)

Artikel: Design and erection of long-span hypar-networks

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DOI: https://doi.org/10.5169/seals-54116

Design and Erection of Long-Span Hypar-Networks

Conception et montage de structures réticulées "Hypar" de grande portée

Entwurf und Montage weitgespannter Hypar-Netzwerke

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SUMMARY

Two types of prestressed networks will be examined: network, encircled by two inclined plane arches and hypar-network within an elliptical contour. Determination of initial form and stress-strain state of the network will be discussed. As examples of cable-networks the acoustic screens for song festival tribunes will be described.

RÉSUMÉ

La communication traite de deux types de surfaces réticulées par câbles de précontrainte: un réseau tendu entre deux surfaces courbes inclinées et un réseau de forme paraboloïde hyperbolique à bordure elliptique. L'auteur discute de la détermination de la forme initiale exempte de contrainte et de l'état contrainte-déformation dans le réseau. Il fournit comme exemple les structures réticulées par câbles prévues pour les écrans acoustiques de tribunes lors de concerts.

ZUSAMMENFASSUNG

Zwei Arten vorgespannter Seilnetze werden betrachtet: Ein Netzwerktyp, der zwischen geneigten Bogenebenen aufgespannt wird, und ein Hypar-Netzwerk mit elliptischer Berandung. Der Beitrag diskutiert die Bestimmung der spannungslosen Grundform und des Spannungs-Dehnungszustand im Netzwerk. Als Beispiele werden die Seilnetzwerke der akustischen Schirme für Tribünen eines Schlagerfestivals beschrieben.

1. PRELIMINARY REMARKS

For many types of long-span structures the round or oval layout may be statically most suitable. On the other side, for many assembly and sports buildings that form of building may be also more functional as common rectangular one. For instance, when we have in the center of building a great sports field with a running track, the round oval plan enables to locate the spectators' seats according to conditions of the best visibility. The same is valid for a number of other assembly buildings. Especially suitable for that kind of conditions may be buildings with the roof, formed as a surface with negative Gaussian curvature or so-called saddle-formed roofs. In the number of saddle-formed roof structures very important position have the cable-networks. In the following we shall investigate the most suitable types of suspended roofs, formed as prestressed cable-networks inside a closed contour beam.

2. INITIAL FORM OF NETWORKS

2.1 General remarks

The form of a prestressed network will be determined by the conditions of equilibrium of its nodes. In most cases the network will be formed by two families of crossing cables. Theoretically we have a plurality of possibilities for choise of configuration of network surface. In every case the Gaussian curvature of the network surface has to have a negative value in all its nodes.The most suitable roof structures for building practice to our mind are orthogonal or approximately orthogonal networks. In the following we shall investigate mainly the orthogonal networks. Approximately orthogonal may be considered the networks, formed by free mutually sliding cables at the time of prestressing the network. In the first case the horizontal component of inner force is constant on its length. In the second case invariable on the length of the cable is its inner force itself. In the first case the contact forces between carrying and stretching cables will be vertical, in the second case they will be applied in the direction of the bisector of the angle between the neighbouring sections of cables.

For simplification of our problem we may image the network as a system of two continous families of virtual cables, uniformly distributed on the length of spacing between the nodes. In this case the cross section areas of cables will be characterizied by the effective thicknesses of the families of carrying and stretching cables. In this case the condition of equilibrium will be presented as differential equations. The other possibility is to analyse the network as a discrete system. For networks with a great number of cables the real network may be replaced by a virtual one, which consists of a reduced number of cables; the cross section areas of substituting cables must be correspondingly increased. The conditions of equilibrium of nodes in that case will be algeabric equations.

2.2 Hypar-networks

The continous surface of a prestressed orthogonal network may be described by the elliptical equation

$$G_0 \frac{\partial^2 z}{\partial x^2} + H_0 \frac{\partial^2 z}{\partial y^2} - 0 \tag{2.1}$$

where G_0 and H_0 - the prestressing forces of the carrying and stretching cables correspondingly, suited to the width unit of the network.

The equation (2.1) will be satisfied for hypar

$$z = f_x \frac{x^2}{a^2} - f_y \frac{y^2}{b^2} \tag{2.2}$$

Corresponding prestressing forces will be determined by uniformly distributed contact loads p_0

$$G_0 = \frac{p_0 a^2}{2 f_x} \tag{2.3}$$

$$H_0 = \frac{p_0 b^2}{2 f_y} \tag{2.4}$$

The most suitable layout for the hypar will be the ellipse (Fig.1)

$$\frac{x^2}{a^2} + \frac{y^2}{b^2} - 1 = 0 \tag{2.5}$$

where a, b - semi-axes of the ellipse,

f_x, f_y - the rises of curvature of carrying and stretching cables correspondingly.

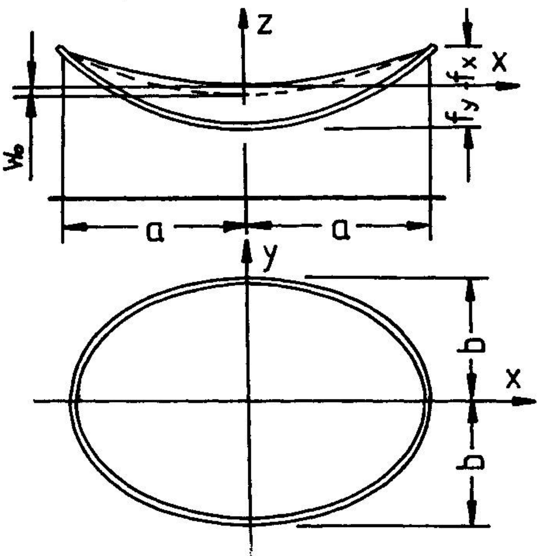

Fig.1 Hypar-network

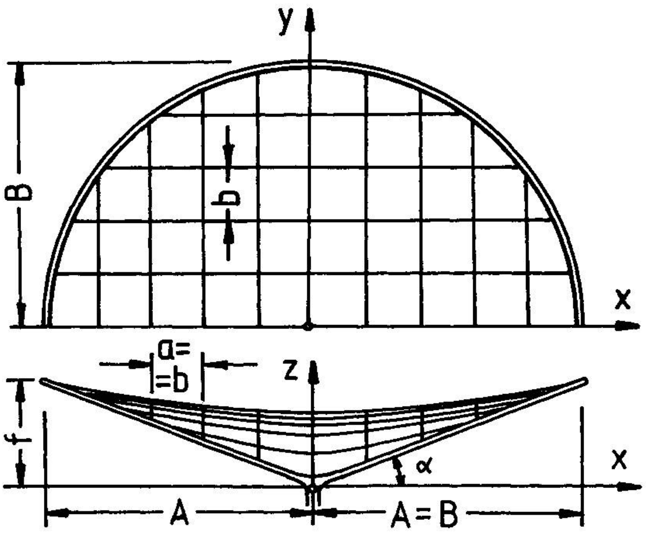

Fig.2 Orthogonal network with round contour

2.3 Orthogonale network with given form of contour

For the node i,k of an orthogonal weightless network the condition of equilibrium may be written in the form

$$G_{0i}\left(\frac{z_{i,k+1} - z_{i,k}}{a_{i,k}} + \frac{z_{i,k-1} - z_{i,k}}{a_{i,k-1}}\right) + H_{0k}\left(\frac{z_{i+1,k} - z_{i,k}}{b_{i,k}} + \frac{z_{i-1,k} - z_{i,k}}{b_{i,k-1}}\right) = 0 \tag{2.6}$$

where G_{0i} and H_{0k} - the prestressing forces for the i-th carrying and the k-th stretching cable correspondingly;

$a_{i,k}$; $b_{i,k}$ - the length projection of the k-th section of the i-th carrying cable and the i-th section of the k-th stretching cable correspondingly.

For the network with constant mesh dimensions the linear equation (2.6) may be written in the form

$$z_{i,k} = \frac{(z_{i,k-1} + z_{i,k+1}) + \lambda\,(z_{i-1,k} + z_{i+1,k})}{2\,(1 + \lambda)} \tag{2.7}$$

where $\lambda = \frac{H_{0k}a}{G_{0i}b}$.

The system (2.6) consists of equations, set up for all internal nodes of the network. The ordinates of contour nodes have to be given as the initial data of the problem. An orthogonal network, prestressed inside the contour, which consists of two inclined semicircle plane arches may be examined as an example (Fig.2).

3. CALCULATION OF DISPLACEMENTS AND INNER FORCES OF HYPAR-NETWORK

The stress-strain state of the network is to be described by means of conditions of equilibrium and equations of deformations compaibility [1]. For the case of action of vertical loads p we may write (Fig.1)

$$G\frac{\partial^2(z+w)}{\partial x^2} + H\frac{\partial^2(z+w)}{\partial y^2} = p \tag{3.1}$$

$$\frac{G-G_0}{E\,t_x}\left[1+\left(\frac{\partial z}{\partial x}\right)^2\right]^{\frac{3}{2}} = \frac{\partial u}{\partial x} + \frac{\partial w}{\partial x}\left(\frac{\partial z}{\partial x} + \frac{1}{2}\frac{\partial w}{\partial x}\right) \tag{3.2}$$

$$\frac{H-H_0}{E\,t_y}\left[1+\left(\frac{\partial z}{\partial y}\right)^2\right]^{\frac{3}{2}} = \frac{\partial v}{\partial y} + \frac{\partial w}{\partial y}\left(\frac{\partial z}{\partial y} + \frac{1}{2}\frac{\partial w}{\partial y}\right) \tag{3.3}$$

where u, v - displacements in directions of axes x and y correspondingly,
G, H - the final horizontal forces of the network,
E - modulus of deformation of cables,
t_x; t_y - effective thicknesses of the carrying and the stretching cables correspondingly.

For elimination horizontal displacements we may equate the network edge displacements to displacements of the contour beam, caused by action of horizontal forces of the network.

After approximation of the deflection function for the network and using Bubnoff-Galjorkin procedures we get for the relative deflection $\zeta_0 = \frac{w_0}{f_x}$ a cubic equation

$$(1+\psi+4\xi)\zeta_0^3 + 3[(1-\alpha\psi)+2(1-\alpha)\xi]\zeta_0^2 + \{2[(1+\alpha^2\psi)+(1-\alpha)^2\xi] +$$
$$+ (1+\alpha^{-1})(1+\chi_x)(1+\vartheta\xi)p_0^*\}\zeta_0 = (1+\chi_x)(1+\vartheta\xi)p^* \tag{3.4}$$

where $\alpha = \frac{f_y}{f_x}$; $\chi_x = \frac{5f_x^2}{3a^2}$; $\chi_y = \frac{5f_y^2}{3b^2}$; $\psi = \frac{a^4 t_y(1+\chi_x)}{b^4 t_x(1+\chi_y)}$;

$\vartheta = 1 + \frac{1}{\psi}$ - geometrical factors;

$\xi = \frac{5E\,t_y a^3\sqrt{\frac{a}{b}}}{72E_c J_c(1+\chi_x)}$ - parameter of contour rigidity;

$E_c J_c$ - the bending rigidity of contour beam;

$p_0^* = \frac{9p_0 a^4}{10E\,t_x f_x^3}$ - parameter of the network pretension;

$p^* = \frac{9pa^4}{10E\,t_x f_x^3}$ - loading parameter

The horizontal components of the network inner forces may be presented as follows

$$G = G_0 + \frac{5E\,t_x f_x^2 \zeta_0[(2+\zeta_0) + 2(1-\alpha+\zeta_0)\xi]}{9a^2(1+\chi_x)(1+\vartheta\xi)} \tag{3.5}$$

$$H = H_0 - \frac{5E\,t_y f_x^2 \zeta_0[(2\alpha-\zeta_0) - \frac{2}{\psi}(1-\alpha+\zeta_0)\xi]}{9b^2(1+\chi_y)(1+\vartheta\xi)} \tag{3.6}$$

4. ESTIMATION OF BEHAVIOUR OF HYPAR-NETWORK

Hypar-network with elliptical contour, supported by the vertical columns, is characterizied by some specific qualities. The contour without any outer horizontal supports may freely widen or narrow under action of cable forces, caused by loading the network. Therefore the contour stiffness and other system parameters has decisive import for the stress-strain state of the network. In the following will be described some aspects of behaviour of the network and its collaboration with the contour beam:

1) Dependence of relative deflection of the network on load parameter has obviously nonlinear character (Fig.3), similar to analogous relationship for the plane suspended systems;

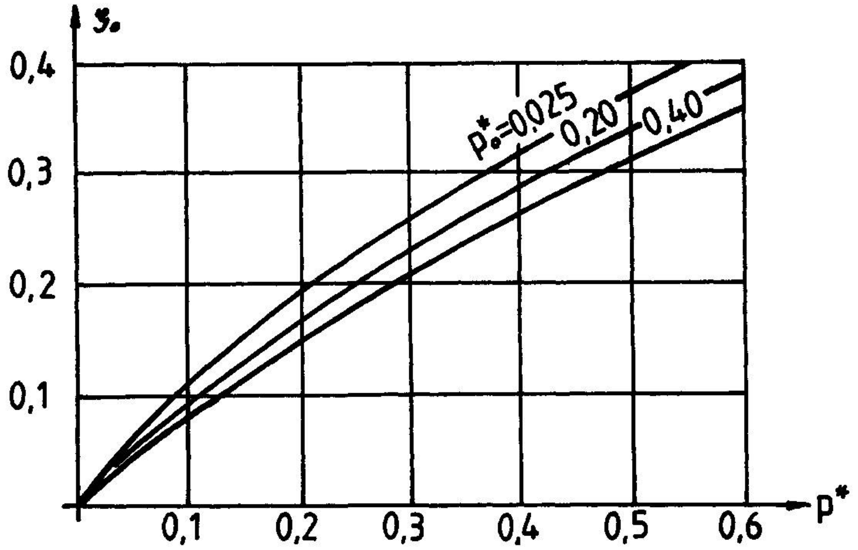

Fig.3 Dependence of displacements on the load parameter

2) Inner forces of carrying cables under action of network loads will increase by all values of bending rigidity of the contour. Inner forces of stretching cables will decrease in the case of great and increase by small bending rigidity of the contour beam. Therefore the bending moments of flexible contour beam are comparatively small.

3) Dependence of network deflection on the parameter of contour rigidity is shown on the Fig.4. It is remarkable, that by very great and very small rigidity of the contour beam, deflection of the network does not change remarkably.

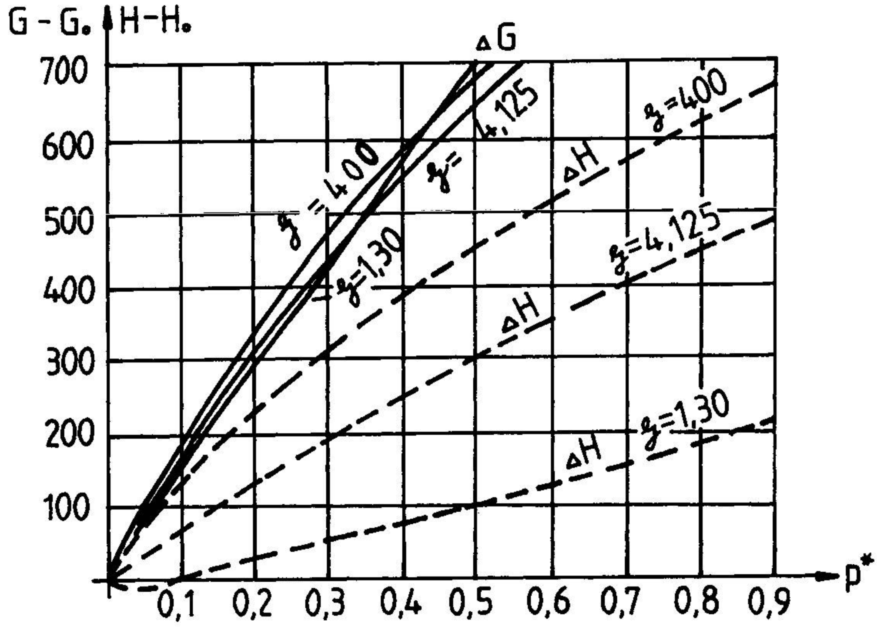

Fig.4 Dependence of cable forces on the load parameter

5. ERECTION EXPERIENCE OF SADDLE-FORMED SUSPENDED ROOFS IN ESTONIA

Two saddle-formed network structures have been erected in Estonia as acoustic screens for song festival tribunes. The first of them was erected in Tallinn in 1960. It is inclined network, prestressed inside the contour, formed by two plane arches and supported by massive counterforts. The bearing structure of the acoustic screen of the tribune in Tartu erected in 1993 is an hypar-formed network within a contour, constructed as a spatial tubular rod with axis, having elliptic and parabolic projections. The contour is supported by three plane supports, connected with the contour and foundation by linear hinges. The supports do not resist symmetrical horizontal displacements of the contour. Therefore the interaction between the network and the contour is of particular importance for balancing outer loads. Because of acoustic requirements the cladding of both screens in Tallinn and in Tartu was made from timber panels. For tribune in Tartu the collaboration between the network and timber shell is taken into account.

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