The Valeria of Roman Amphitheatres: The Colosseum

Les vélariums des amphithéâtres romains: le Colisée

Die Sonnensegel römischer Amphitheater

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SUMMARY

The paper reviews the current archaeological and structural assessments on the layout of the velarium. The aim is to suggest a methodology to understand the architectonic and structural choices, validating the various hypotheses through the structural analysis. The results obtained in this first step of the research will be used to narrow the field of the possibilities and concentrate further studies of better accuracy, on the models that look more reliable.

RÉSUMÉ

Les auteurs donnent une vue d'ensemble des évaluations archéologiques et structurales sur les tracés des vélaires de l'antiquité. Il s'agit de proposer une méthode pour comprendre les choix architecturaux et structuraux, en vérifiant les diverses hypothèses par des calculs statiques. La première phase de recherche fournit des résultats qui servent d'une part à réduire les variantes possibles et d'autre part à préciser les études supplémentaires, pour que les modèles présentent une meilleure fiabilité.

ZUSAMMENFASSUNG

Der Beitrag gibt einen Ueberblick über die gegenwärtige archäologische und bauliche Rekonstruktion von Sonnensegeln in der Antike. Ziel ist die Erarbeitung einer Methodik zum Verständnis der Entwurfskonzepte, für die unterschiedliche Hypothesen durch statische Berechnungen überprüft werden. Die im ersten Schritt erhaltenen Forschungsergebnisse dienen der Eingrenzung der Möglichkeiten und der Konzentration vertiefender Studien auf vielversprechendere Modelle.

1. The shape of the velarium: topographical and archaeological hypotheses

The architecture of the *Coliseum in specie ovi*, as it is called by Cassiodoro, was highly influenced by the existence on its building site of the older *stagna Neronis*, an artificial lake, in the *Domus Aurea*, placed at the centre of the city in the valley among the Celium, Palatinum, Velia and Oppium Hills.
The geometry of the plan is an oval with 4 centres of curvatures organised in 80 arch-bays repeated three times in the vertical and horizontal planes and connected with radial walls which bear the *cavea's* seats.
The two principal axes, of which the major one is aligned along the *Via Sacra,* measure 188 m and 156 m. Only the outer 50m where actually built over the ground with a degrading section 50 m tall at the top of the actic level of the outer wall. So that the area to be covered by the *velarium colore coeli* (Pliny, 9, 1, 24) measures approximately 23000 m^2.
The centre of the *Coliseum* is based on a Pythagorean triangle superimposed to the two axes and the centre of the valley whose measure is recognised by a specific topographical relation which organise the over all shape of the monument by an urban axis (fig. 1). This axis links the *ara maxima Herculis,* to the *Curiae Veteres,* two important knots of the *Roma quadrata* grid narrated by Tacitus, and it connects a number of significant centres of monuments, from different roman periods, as the *Apollum templum* by Augustus, the Flavi's labyrinth, the Adrian's *Adonea,* Constantinus's Arch, the Nero's octagonal room. This axis, called as the *urbis axis*, measures 19° N-NE (corresponding astronomically to the course of the sun of the XXI April, day assumed to be the Rome's birthday) and it is orthogonal with the ancient *via Lata.*
Such important considerations support the in situ exam over the moulding and the exam of seven architectonic pieces erratically dislocated on the archaeological site. They allow to formulate the hypothesis that these blocks of the same shape and two different sets of dimensions (average measures being 950x600x500 mm and 720x440x500 mm) were utilised as a further prop of the poles which hold the cables. The two different dimensions can suggest the idea that the poles had different functions. In particular it seems acceptable that the bays of the velarium (slack as they look in the Pompeii's fresco), were of the same extent as the lower structure, so to have a tent, in the shape of a Latin sail, covering each arch bay. The brackets on the outer side of the acticum level being three for each bay (fig. 2) there were three poles for each sail. We therefore think that the central pole was the one to which the principal cables, called *rudentes,* 80, one for each bay, were connected, and the only one working during the phase of mounting. Once the velarium had taken shape and place the forces were redistributed on the other two poles by means of secondary ropes.
The length of the poles is estimated to be about 12 m of which 7 to 7.5m above the moulding of travertine showing for each bay three rectangular holes of 400x500 mm in which the poles were slipped through and, resting on travertine brackets spanned 2.25 m., were fixed to the wall by a planking.
The geometry and structure of the velarium therefore seem to follow the same logic as the geometry and structure of the travertine construction below, conveying, through the repartition of the forces among the three poles, the flux of stresses toward the arch-pillars system.
To lay-out the cables net, mounting it and lifting it in the final position took approximately four days and the work of 300 men of the imperial fleet.

2. The shape of the velarium: structural hypotheses

The structural elements which can be still seen today on the monument and the historical and archaeological documents, presented in the previous paragraph, lead to convincing hypotheses about the construction of the velarium and its assemblage and rising in place; however they are not sufficient to define completely the structure and the building technology used. The structural analysis can be usefully applied to enlighten these aspects and therefore to reduce the uncertainty associated to different lay-outs , pointing out among these, only the ones which fulfil the stress requirements according to materials and technology available at the time. Confronting the different configurations proposed by various authors, it seems possible to define a number of points which form a common core for further discussion:
a) the main cables present a radial lay-out, one for each bay of the outer travertine arched structure, and they were probably made of hemp, this being the material commonly used by sailors. Most likely diameters were

in the range from 40 to 60 mm (the cross section being measured when in tension) and each cable was tensed by a capstan controlled by two to three men. This hypothesis, better than the one with 240 cables, allows to have homogeneous bays and even distribution of loads and stresses (if not for the slight variation connected with the change in curvature along the oval) therefore avoiding asymmetries not liked by Romans and making the structures of the velarium congruent with the lower structure.

b) 80 capstans were placed above the roof of the giant portico in front of the central pole of each bay, which held the pulley of the principal cable; during the mounting phase, the central pole bore all the load of one bay, and it was probably sustained by two other poles put in a shoring; once the cables were tensed and the velarium put in place, before removing the shoring, the load on the principal cable was shared onto the two lateral poles by means of secondary cables (see fig. 3) tensed by winches set into blocks of travertine at purpose shaped. The presence of the boundary-stones, with four holes, placed at the ground level outside the monument, has been interpreted by some authors as an anchorage for the cables. It seems unlikely for a number of reasons: a huge increase in the amount of materials, a difficulty in the coordination of the operations due to lack of visual connection, the hindrance that these outer cables could cause during the flux and defluxion of the audience.

c) the canvases were directly tied to the principal cables in roughly rectangular pieces of width slightly longer than the distance between the two adjacent cables and length of 3 to 4 m., so to have elements of workable dimensions (20 to 30 m^2 was the average dimension of Latin sails) and to allow independent movements of the different pieces under the action of the wind, so to reduce the global pressure;

d) the inner ring to which are connected the low points of the principal cables, repeats, with smaller dimensions, the shape of the oval plan of the structure, sharing the same centres of curvature. The four arcs in which the oval is divided by the change of curvature have the same length , but different angles, the one with smaller radius being 1.84 rad, the other 1.30 rad. Considering the thrust of the cables (20 on each arc equally spanned) like a pression uniformly distributed over these arcs, one obtains an hoop thrust which differs with respect to the mean value (calculated for even angles of $\pi/2$ rad) for a $\pm$ 17% . As it is no possible in such a structure to have variable thrust this means that the mean value is reached by means of a change in sag in the cables: for the cables in the area with smaller radius of curvature the sag will be reduced of 17% and on the other it will be increased of 17% so to have an opposite increase and reduction of the hoop thrust in each area. The inner ring will show a saddle shape similar to many roofs of modern stadiums.

Starting from these points of common agreement a number of questions remains open:

1- the actual configuration of the principal cables, the covered surface, the limitations imposed by the visibility line and the possibility of shadowing the podium related to the position of the sun;

2- the connection of the canvases to the cables: the possibility of moving them along the cable means a further load due to the secondary mesh of rope;

3- the configuration and realisation of the inner ring made of hemp or iron rods connected by small rings;

4- the actual dimension of the poles, the need of struts during the mounting phase and the compatibility of the stresses;

5- the action of wind and rain, at least in their mild summer manifestations, and their compatibility with the global lay-out of the velarium and the dimension of its structural element.

3. Structural analysis

To work out the structural analysis we need to define the mechanical features of the material and the distribution and entity of actions. The nominal values mentioned below have been assumed from old codes or from materials which are today similar to the old ones:

	cross sect.	weight	failure stress	elastic modulus
hemp cables	2000 mm^2	30 N/m	30 - 40 N/mm^2	300 - 400 N/mm^2
oak wood poles	0.12-0.20 m^2	1000 -2400N/m	40 - 60 N/mm^2	12000 N/mm^2
iron bars	900-2500mm^2	70-200N/m	200 N/mm^2	210000 N/mm^2
linen clothes	1. m^2	10 N.		

To verify the first point mentioned above, the lower bound for the sag, not to interfere with the visibility line of the upper seats, is 25 m. (fig. 4). Although some of the archaeological reconstructions show the velarium as practically flat, the analysis shows that the upper bound can be not less than 7 - 11 m for cables spanning from 40 to 60 m. The minimum of the last two measures, 40 m., is conditioned by the extension and duration of the shadowing over the lower seats, while the maximum span is bounded by the increase in load with relation to the strength of the cable. The analysis proves that for the longer span tested (61 m) is not possible to have the overload of two additional cables to movement the canvases and therefore in this way is implicitly solved the second point of the previous paragraph (fig. 5).
The geometry and cross section of the inner ring is related to the length of the cable on one side and to the horizontal component of the trust (T_h) on the other, assuming that the sag in the cables along the ring will be variable as already discussed at the point d) of the previous paragraph. The hoop stress in the ring will be:

$N = \frac{40 T_h}{\pi} = 12{,}7 T_h$, and varies , for a sag of 9 m, from 171.5 KN to 381 KN for a span ranging from 40 to 60 m, while for a sag of 15 m the corresponding values are 127 KN to 222.5 KN. For the span of 61 m the case of reduced load due to unmovable canvases has also been considered, producing a hoop force of 222.5 KN and 159 KN, for sags of 9m and 15m respectively. Taking into account the variability of the distance between cables at the lower points as a function of their span, an average weight of the inner ring on each cable of 500 N can be assumed.
According to the span and sag ranges considered, the cable reaches the pole forming an angle with the horizontal variable between 18° and 29°, while dimensional analysis implies that the position of the capstan, placed as close as possible to the pole, will give to the cable coming from the pulley an angle with the vertical line of approximately 20°. This means that the resultant of the thrust will form with the axis of the pole an angle of 40° to 46°. Therefore the horizontal and vertical components will be respectively in the range of 1.18 T to 1.30T and 1.41T to 1.30 T, giving place to the following generalised stresses in the poles:

$M = O \cdot 6.6 + V \cdot 0.3, \quad S = O, \quad N = V$ .The stresses for the most severe case vary from 11.3 to 14.23 N/mm^2 for cross section of 500x400 mm, and from 24. to 31.6 N/mm^2 for cross section of 400x300 mm. Confronting this values with the ultimate strength value it can be seen that the pole is able to resist those loads with a safety factor variable between 3.6 and 1.3 depending on the cross section.

4. Conclusions

The results obtained represent the first iteration of a process of trial and error that should be replicated introducing the variation suggested by new information and their critical analysis. However they are useful to bound the field of the possible solutions. In particular:
- taking into account that the decrease in load (from 25 to 15 m sag) is of about 18 %, while the increase in thrust is far from the limit value, it can be thought that the more likely shapes were the ones with sag variable from 7 to 15 meters, depending on the span. It is worth to mention that, when observed from below at a distance of approximately 50 m from the lowest point, over an overall length of 180 m. entities of the sag from 10 to 15 m could have given the impression of a flat velarium.
- while for the smaller spans and bigger sag is still possible to think that the ring would be made of hemp ropes (requiring a section of approximately 15000 mm^{2}with a weight 265 N/m) , when the longer span are considered, and always for sags close to the upper limit value, the iron rods appear convenient, involving much smaller section and therefore a weight approximately estimated in 180 N/m.
-at least during the mounting phase, for the longer spans of the cables and smallest cross section of the poles a strut was required to lift up the cables net.

It is not possible here to discuss in due depth the reduction of the global pressure of the wind obtained by the cuts in the canvases, further studies and possibly tests on a model in the wind tunnel being necessary to achieve reliable knowledge of the phenomena involved. The effects, however, could have been partially absorbed by relatively small sub vertical stays tied at the lower levels of the seats. The stays would have

been slack o slightly tensed, so not to affect the configuration under dead loads only. The increment of vertical action due to the rain water or the wind flowing among the poles, these do not seem to invalidate the above results. For the rain, the load previously taken into account for the canvases, already included a waterproof treatment by linen oil and a water film 0.5 mm thick. On the other end the cuts in the canvases and their slope allow to assume that, even during a fairly intense rain, the increase on the effects will not be, on average, greater than 35% or 70% according to the different configurations which allow or not to roll up and down the canvases. This simple analysis, therefore, with such an increase significantly reducing the security factors mentioned above, suggests that very likely the span of the principal cables would not have exceeded the 50 to 55 m, and that for such configurations the option of fix canvases was the most likely.
As for the wind, with a simplified equivalent static analysis can be determined that un upper bound for the speed of the wind will have been 14-15 m/s. Such a speed would have caused an increase to the vertical force that in the depression area would equal the dead load effects, while in the pression area would reduced the safety factor to 1.5, which seems not reliable. For the same speed the horizontal component seems to be not of much influence the increase or decrease in the horizontal thrust being of a 10% of the values discussed above.

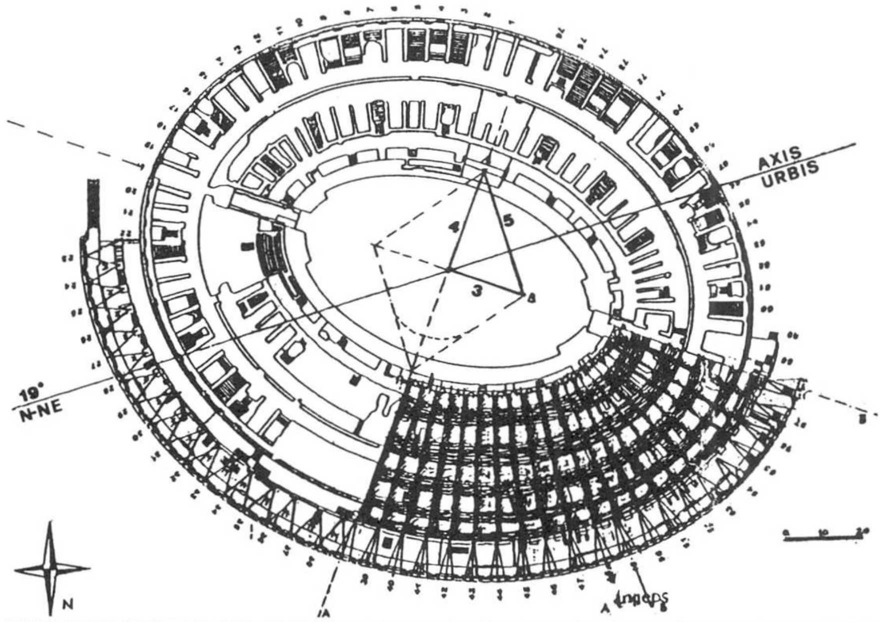

Fig. 1 : Plan of the Coliseum with the lay-out of the velarium

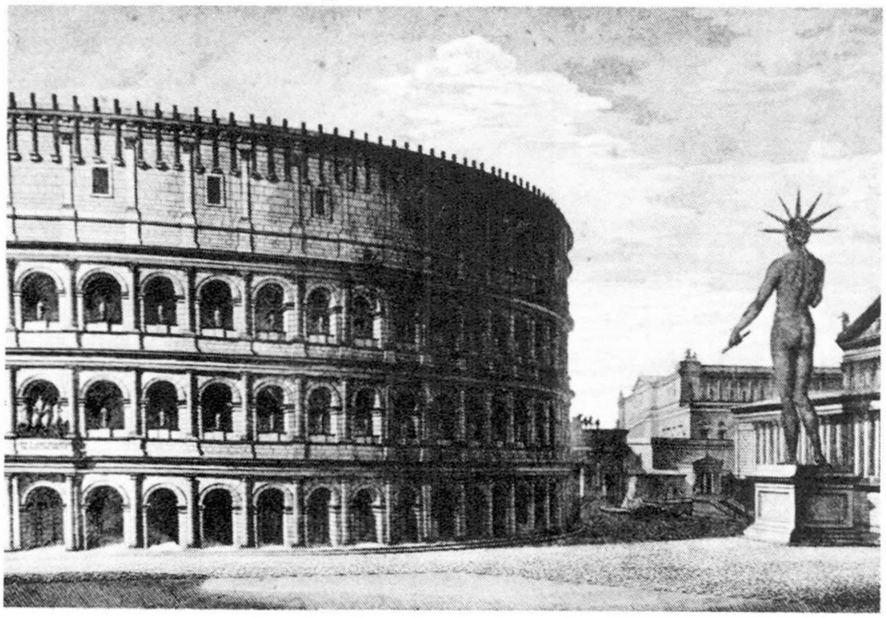

Fig. 2: The historical reconstruction of Canina (1860).

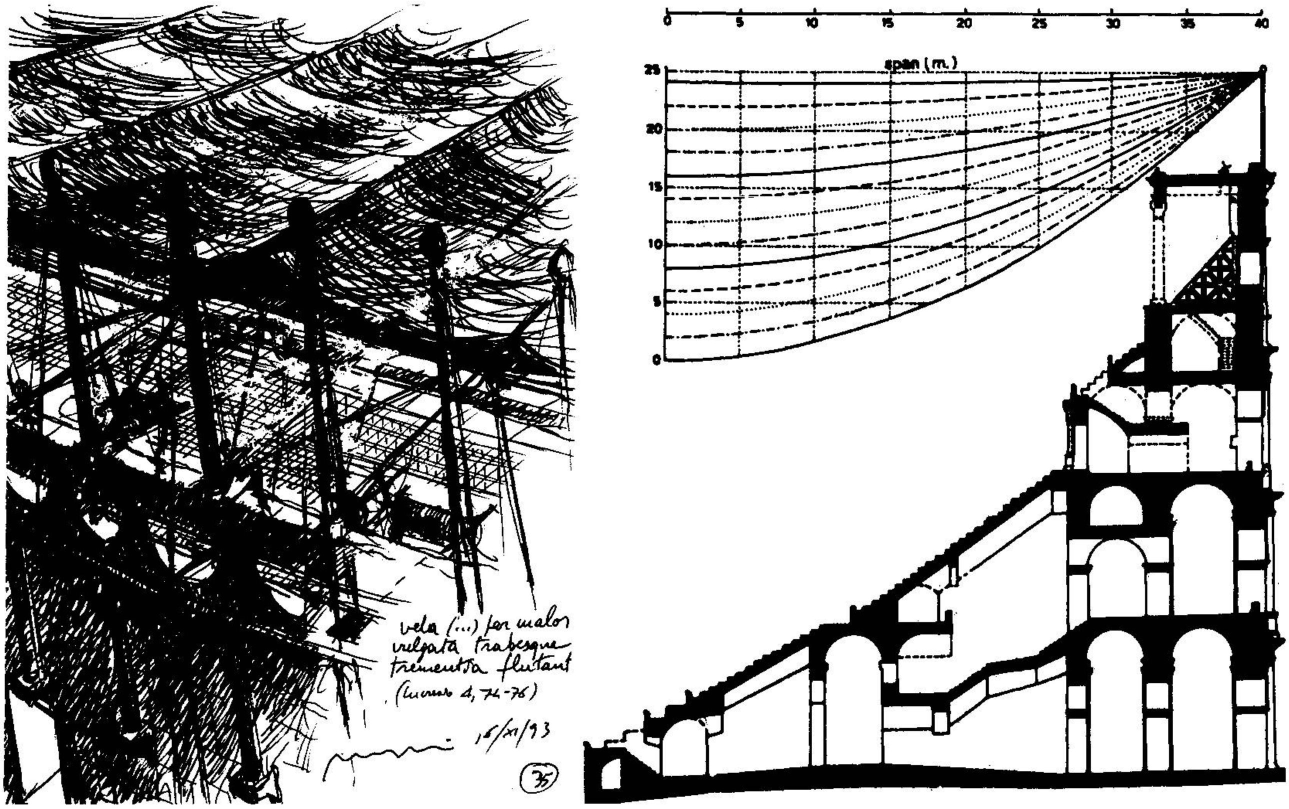

Fig. 3 and 4 : Possible shapes of the velarium