Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte
Band: 70 (1993)

Artikel: Structural assessment of Leaning Tower of Pisa
Autor: Macchi, Giorgio / Ruggeri, Giovanni / Eusebio, Marina
DOI: https://doi.org/10.5169/seals-53323

Structural Assessment of the Leaning Tower of Pisa

Evaluation structurale de la Tour penchée de Pise

Einschätzung des Tragvermögens des Schiefen Turms von Pisa

Giorgio MACCHI
Professor
Univ. of Pavia
Pavia, Italy

G. Macchi has been Dir. of the Inst. of Construction, Fac. of Archit., Venice, from 1970 - 73, and became then Dean of the Fac. of Eng., Univ. of Pavia, where he is now Professor.

Giovanni RUGGERI
Civil Engineer
ISMES
Bergamo, Italy

G. Ruggeri, born 1955, received his degree at the Polytechnic of Milano and is now Head of the Mathematical Modelling Dep. of ISMES.

Marina EUSEBIO
Civil Engineer
ISMES
Bergamo, Italy

M. Eusebio, born 1961, received her degree at the Polytechnic of Milano. Her main interests are related to numerical analysis of civil structures.

Marco MONCECCHI
Civil Engineer
ISMES
Bergamo, Italy

M. Moncecchi, born 1965, received his degree at the Univ. of Pavia. His main interests are related to numerical analysis of civil structures.

SUMMARY

The safety of the Leaning Tower of Pisa requires preventive structural measures and their assessment on adequate models of the structure. Several models of analysis are compared and the inadequacy of the most simplified methods is proved, particularly in the interpretation of the role of the colonnade. The temporary prestressing at the first loggia is described and justified.

RÉSUMÉ

Le maintien de la sécurité de la Tour penchée de Pise nécessite des interventions au niveau structural par l'évaluation de modèles adéquats de la structure. Plusieurs modèles d'analyse sont comparés et l'inadéquation de méthodes les plus simples est démontrée, particulièrement dans l'interprétation du rôle de la colonnade. La précontrainte temporaire au niveau de la première loggia est décrite et justifiée.

ZUSAMMENFASSUNG

Die Sicherheit des Schiefen Turms in Pisa verlangt Eingriffe in die Gebäudestruktur und ihre Bewertung mittels geeigneter struktureller Modelle. Verschiedene Analysemodelle werden verglichen, und die Untauglichkeit der einfachsten Methoden wird dargestellt, insbesondere was die Funktionsbestimmung der Säulen betrifft. Die provisorische Vorspannung in der Ebene der ersten Loggia wird beschrieben und begründet.

1. INTRODUCTION

The increasing tilt of the Tower of Pisa* (the inclination is more than 5 degrees Southward) is causing a dangerous increase of the state of stress of the masonry in elevation.

The inclination, due to differential settlement of the subsoil, was already outstanding at the end of the construction, which took place from 1173 to 1370; the inclination was continuously growing in the following 600 years, and its rate, which was 3 seconds per annum in 1945, was estimated 6 seconds in 1990, and shows evidently a dangerous acceleration towards an overturning which apparently is not far. Moreover, as a consequence of the inclination, at the Northern edge of the cross section the vertical stress is near to zero, and the compression on the masonry is extremely high at the Southern edge.

The risk of a sudden compression failure of masonry (similar to the case of the Civic Tower in Pavia, 1989) is at least as high as the risk of overturning for a failure in the subsoil; and the state of stress is slightly increasing every day.

The cylindrical wall of the Tower is not homogeneous; between two marble linings of not constant thickness (between 250 and 400 mm) the core of the wall is made of rubble stones bonded with lime mortar (Fig.1).

The danger is particularly high at the level of the first *loggia*, where the section of the wall is substantially reduced by the helicoidal stairs and by local cavities. Moreover, the very stiff marble facing is pressing the underlying rubble masonry with a very high compressive stress (Fig.2). In the same zone, 2 long vertical cracks have been recently found on the external facing; they may be a sign of splitting under vertical compression. Finally, the presence of radial tensile stresses caused several circumferential cracks in the ceiling and on the steps of the stairs.

Such a complex state of stress had to be carefully analyzed in order to ascertain the present risk of structural collapse, and provide a sound basis for the temporary strengthening of the elevation (circular prestress, applied in 1992).

The analysis has been first performed on the critical zone, and then on the entire body of the Tower subject to self weight, wind, circular prestress, and on the structure affected by cracks and damages in the colonnade.

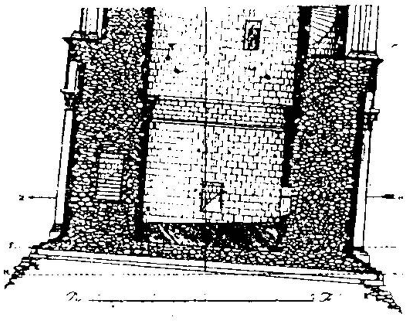


Fig.1 - Internal structure of the Tower

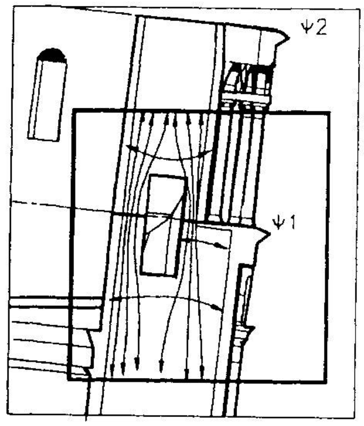


Fig.2 - Critical zone of the Tower at the first loggia

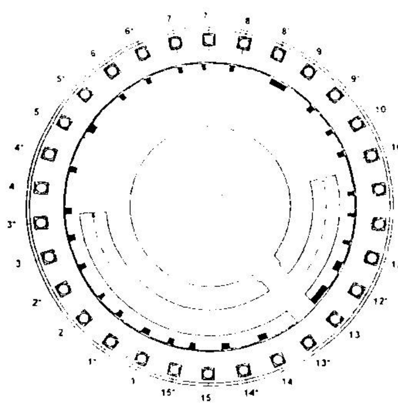


Fig.3 - Buche pontaie (recesses) at the first loggia

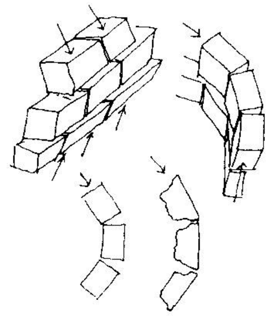


Fig.4 - Facing failure mechanisms

---

* The structural assessment of the Leaning Tower was carried out by the Consorzio Progetto Torre di Pisa (BONIFICA - ISMES - ITALSONDA - RODIO - TREVI).

The present report deals with the studies and interventions done by the International Committee installed in 1991 by the Italian Government for the safeguard of the Tower, after the closure which followed the collapse of the Civic Tower in Pavia. Frequent reference is made to investigations and computations published by previous Commissions, [1, 2, 3], since 1838. Very little consideration was previously given to the structural strength, even in the report 7 of 1988, where the previous Project Team described the stabilization project which was proposed at the time.

2. STATE OF STRESS AND STRENGTH AT THE FIRST LOGGIA

The recognition that a structural strengthening was urgently needed led the new Committee to conceive a reversible temporary prestress, dimensioned by preliminary analysis of state of stress; in the mean time, the refined 3D analysis was prepared.

The preliminary design was based on two simple approaches:

- a conventional linear elastic beam approach;
- a local bidimensional Finite Element analysis.

Both approaches were based on the existing knowledge 3 of the geometry and of the material densities of the Tower, as the refined survey could not by ready within the first year.

An evaluation of the ratio between the elastic moduli E of the marble facing and the rubble masonry was needed. The modulus of the facing was derived by previous tests done with double flat-jacks, and assumed to be E=50000 MPa. The modulus of the inner masonry was taken from dilatometer tests previously performed in some cores (direct tests on the cored material were not reliable) and was evaluated E=7000 MPa. Nevertheless the consideration of extreme values of the experimental results led to modular ratios ranging from 4.5 to 16.7, so that a sensitivity analysis was done; the possible error around the mean values of the stresses was estimated 25% in the facing and 35% in the inner masonry.

Corings operated by previous Commissions 3 proved that the internal masonry is interrupted by several cavities; nevertheless, coring in the critical zone was considered dangerous, so that a local evaluation of the quality of the masonry was only based on non-destructive tests (sonic tomography, radar measurement across the wall, thermography). The results were alarming, but could not provide any further precise information for a non-homogeneous modelling of the masonry.

However, a very important local deficiency as been well documented and was taken into account: a presence of 27 large recesses (the mean size was approximately 360x230x200 mm), the *buche pontaie*, left at the level of the first loggia during construction, for the accommodation of the scaffolding (Fig.3). Such recesses reduce by 18% the resisting horizontal section of the marble facing, and increase by 13% the edge stresses under permanent load.

The contribution of the colonnade has been neglected in this phase (if perfectly connected, with its high modulus of elasticity, it would considerably increase the moment of inertia of the critical section; its role will be discussed later on).

Taking into account the recesses (but not yet the internal cavities) the conventional linear elastic beam approach led to the following values of the vertical compression in the maximum lean plane:

- in the marble facing: σ_z max = 7.6 MPa
- in the rubble masonry: σ_z max = 1.1 MPa

Such values may not appear excessive for good marble stones and for good masonry. However, the situation is alarming. In fact:

- the marble stones of the facing have normally only a limited contact at the bedjoints; this may highly increase the local stresses or allow local buckling (Fig.4);
- the strength of the internal rubble masonry is highly dependent on the quality of the mortar, and very little on the strength of the undressed stones; its knowledge is insufficient at the time (it has been differently estimated by different Committees on the basis of few tests, but 4 MPa seems to be a reasonable conservative compressive strength; dilatometric tests showed cases of high permanent strain at 5 MPa);
- the uncertainty of the modular ratio may lead to values 35% higher than calculated; the cavities reduce the local strength;
- concentration of stresses near the openings may lead to premature failure (the collapse in Pavia occurred under a mean compression of 1.1 MPa, when the masonry mean strength was 2.8 [5]).

However, the main concern is due to the high pressure that the external marble facing applies on the underlying masonry, through a marble slab only 160 mm thick (Fig.2). Therefore, a more detailed study of the state of stress in that region has been carried out by Plane Strain Finite Element analysis (Fig.5).

Where the marble facing is pressing on the pavement of the loggia a vertical stress of 5.9 MPa was found, and under it, in the infil masonry, 3.0 MPa, value which is approaching the estimated strength. Radial tensions up to 0.3 MPa were found on the steps and roof of the helicoidal stairs, and may explain the existing circular cracks.

In order to provide an increased margin of safety of the elevation of the Tower during the envisaged intervention on the foundations, a temporary and reversible circumferential prestress of the critical zone has been studied in 1991 by Macchi and Leonhardt, and implemented by VSL/Preco in 1992 (Fig.6).

A circumferential prestress of 2100 KN is applied by 18 0.6" unbonded strands, specially anchored. The intervention creates a slight state of multiaxial compression, and so slightly increases the strength to vertical compression of the infil and of the facing; it counteracts and it provides also a passive strength preventing external local spalling or dilatation in critical zones.

3. GLOBAL ANALYSIS

A Finite Element 3D global analysis has then provided, in 1992, a comprehensive knowledge of the effects of permanent loads, wind, and circumferential prestress, which were only imperfectly known through the preliminary studies. Under the guidance of the Committee, ISMES first built a numerical

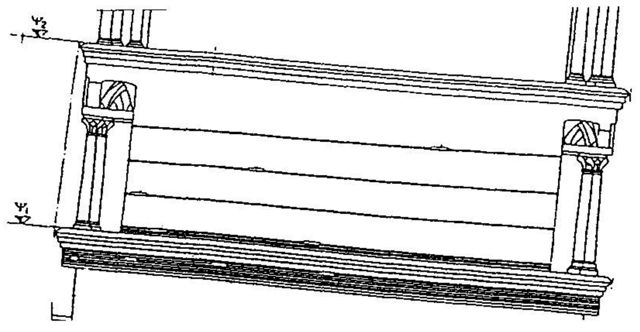


Fig.6 - Circumferential temporary prestress

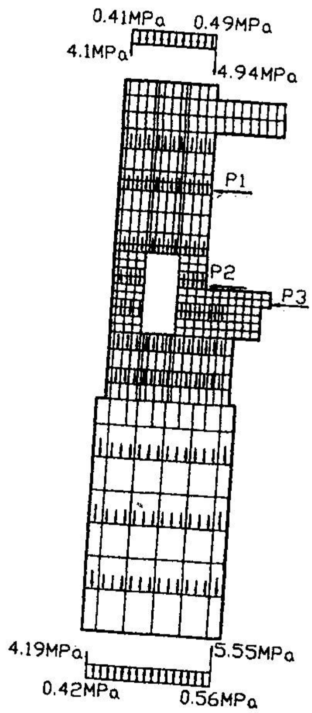


Fig.5 - Local bidimensional F.E. approach at the first loggia

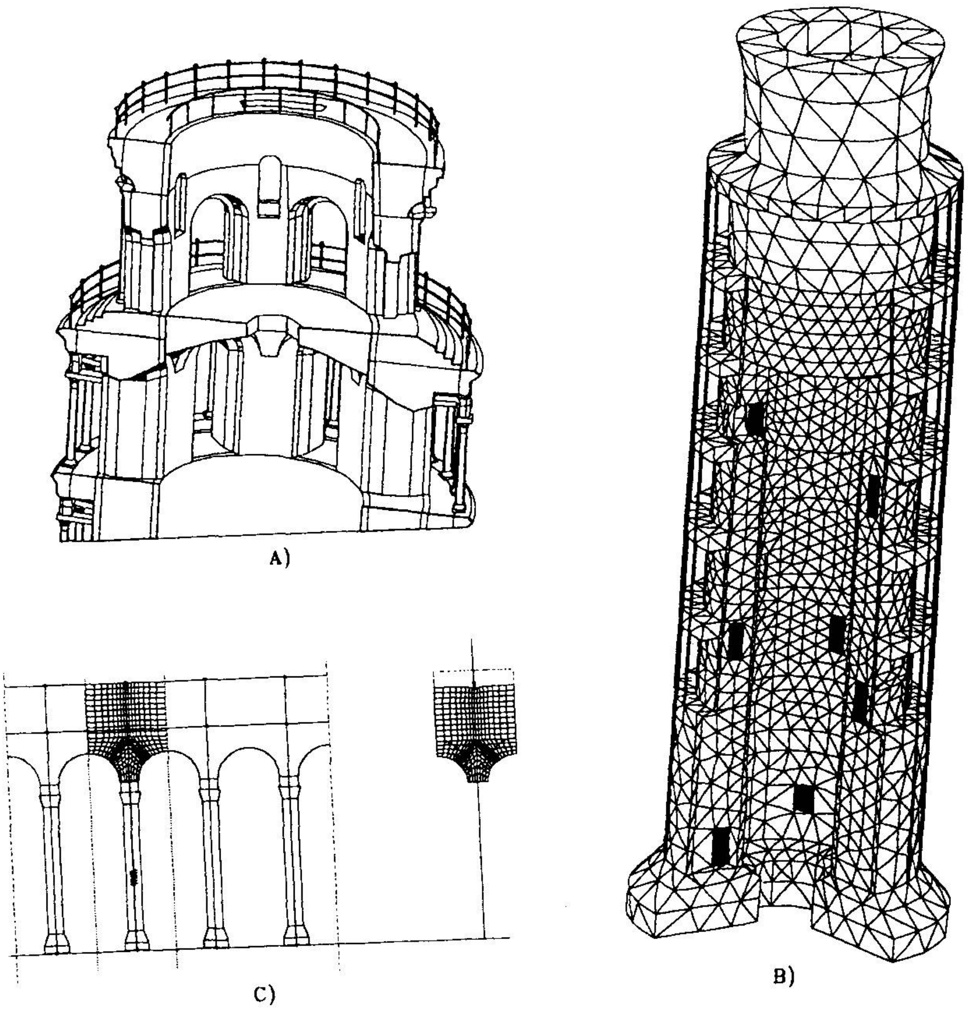


Fig.7 - Modelling of the Tower: numerical model (A), F.E.M. (B), substructuring of the colonnade system (C)

model of the Tower (Fig.7A), and then a Finite Element mesh (Fig.7B) of tetrahedral 10 nodes element (simulating the Tower and a block of linear elastic soil), for a total of 67000 degrees of freedom.

The model takes into account all the relevant openings and stairs, and 7 different densities for parts of the structure (from 18.2 to 2.7 KN/m^3). A reduced level of refinement of the mesh in the upper part allowed a considerable reduction of the complexity of the model; in the most critical zones (at South, for the lowest 4 sections) the refinement has been improved by increasing the order of polynomial shape function [6]. Nevertheless, the complexity would be excessive if the colonnade would be modelled in detail; therefore (Fig.7C) the elements of the external colonnade (columns and arches) are simulated by trusses of equivalent axial stiffness.

The analysis has been repeated with a second mesh taking into account the main cracks observed in the structure, and neglecting all columns ("3D global damaged" analysis).

The two 3D analyses were precious in providing information otherwise not available:

- the position of the zones of maximum stress (in Fig.8 the vertical stress contours show that the critical section at the first loggia is not in the plane of maximum lean, but at SW, at the edge of the door);

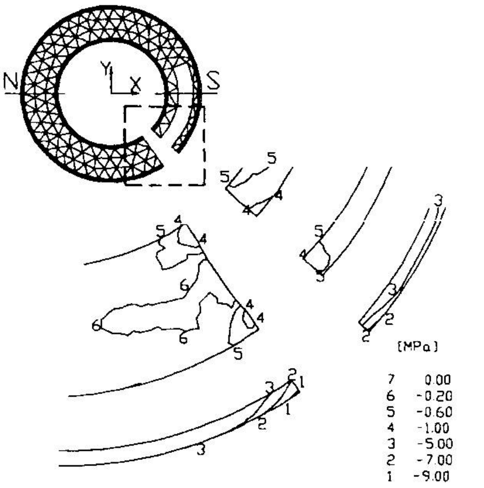


Fig.8 - Contours of the vertical stresses at the first loggia

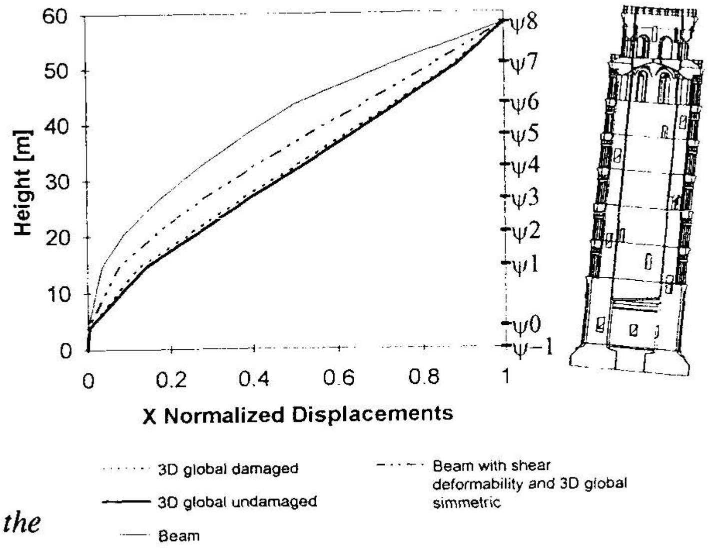
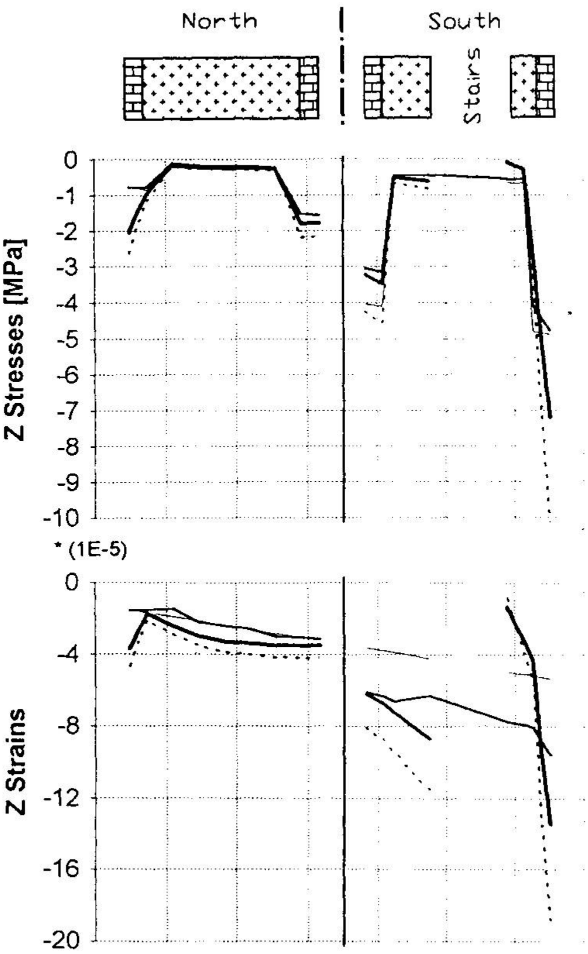
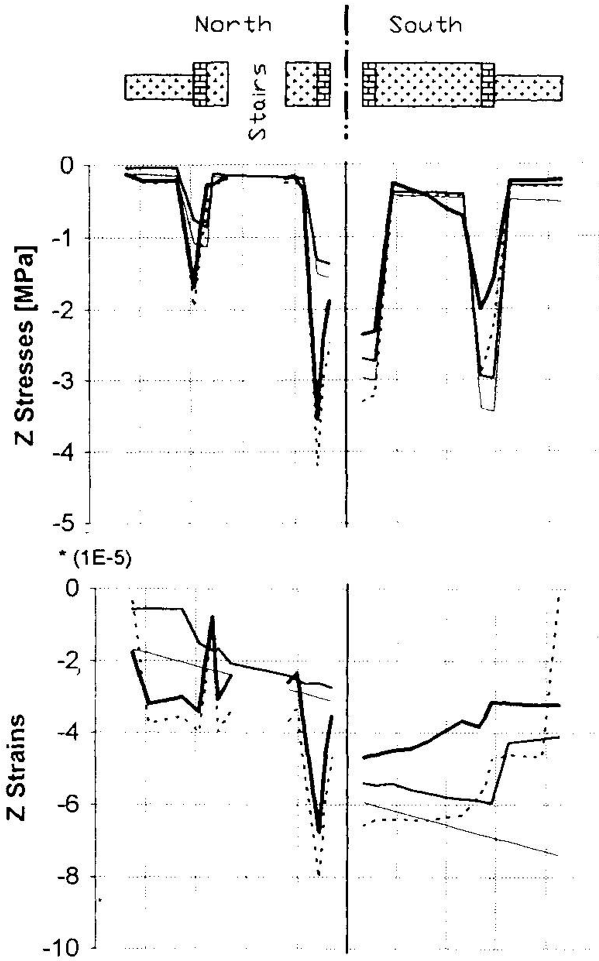


Fig.9 - Normalized horizontal displacements of the center line under permanent load





3D global undamaged ······ 3D global damaged —— 3D global symmetric —— Beam

Fig.10 - Stresses and strains at the first loggia: comparison among different models

Fig.11 - Stresses and strains at the second loggia: comparison among different models

- the maximum of the stresses (10.0 MPa vertical compression under permanent load on the undamaged model, value which shall be increased to take account of the *buche pontaie*);
- the state of stress due to a 50 years wind (0.2 MPa only);
- the circumferential and radial stresses, and the extension of the beneficial effects of prestress;
- the effect of the existing damages, mainly of a possible lack of continuity of columns (this effect will be studied in the following Chapter 4).

4. DAMAGED MODEL AND SIMPLIFIED APPROACHES: COMPARISONS

The comparison of the "3D global damaged" analysis and the "3D undamaged" one had the aim of defining two boundaries between which the real state of stress should be, as a consequence of the existing cracks and of the fact that many columns have been substituted in recent times (and probably most of them in the past) so that their contribution to bearing the loads is doubtful. This is proved by the fact that the "3D global undamaged" analysis underestimates the compression at the South edge (meanly 1.8 MPa less) and overestimates the compression at the North edge (2.1 MPa more) in comparison of the values experimentally measured by means of flat-jacks.

The main results of the "damaged" analysis are the following:

- the existing cracks do not influence the global behaviour;
- a total absence of columns would lead to a considerable increase of the maximum vertical stress (the value of 10.0 MPa above mentioned at the edge of the door at the first loggia under permanent load is increased up to 13.5 MPa).

Therefore, it would be very important to state the (intermediate) real situation.

In general, the 3D model showed behaviours and states of stress which would be hardly found by simpler means which could not take into account the effects of the openings, of the non symmetric loading, and of the warping of the horizontal sections.

For this reason, in order to better understand a so complex structural behaviour, the Tower has also been analysed at lower levels of sophistication, so that a comparison has been made between the following analyses:

- 3D global undamaged;
- 3D global damaged;
- 3D global symmetric (without stairs and openings);
- Beam with shear deformability;
- Beam.

Figures 9, 10, 11, 12 show some of the most interesting results.

The inadequacy of the simple beam model is shown in Fig.9, where the deflection shapes are compared (under permanent load only): it gives flexural deflections only, when the "3D globals" act at the top as shear cantilevers, and show a peculiar contraflexure; intermediate is the behaviour of the simplified "3D symmetric", whose solid walls are less sensitive to shear; the latter may be well simulated by the beam with correction for shear deformability. The deflections are normalized to 1; in fact, "3D damaged" gives higher deflections than "3D undamaged".

Figures 10 and 11 compare the strain and stress distribution across the Tower, in the plane of maximum lean, at the first loggia $\psi 1$ and a second loggia $\psi 2$; the former has the stair at South, the latter at North. In both cases the stress distribution is far from the linear distribution of the plane section hypothesis ("beam" approach), and high vertical stress concentrations appear at the edges of the helicoidal stair opening. The "3D damaged" shows the higher stresses due to the absence of the columns.

Finally, Fig.12 compares the load in each column, at the different levels, according to the different assumptions; column 3* is at the Northern edge, column 11 at the Southern one.

Besides the clear proof of the large errors to which the plane section approach may lead, some interesting information arises:

- the loads on the columns are much more uniform than expected by the simplified methods of analysis;
- the columns at North are more efficient than foreseen by "beam";
- the helicoidal stairs have influence on the column load.

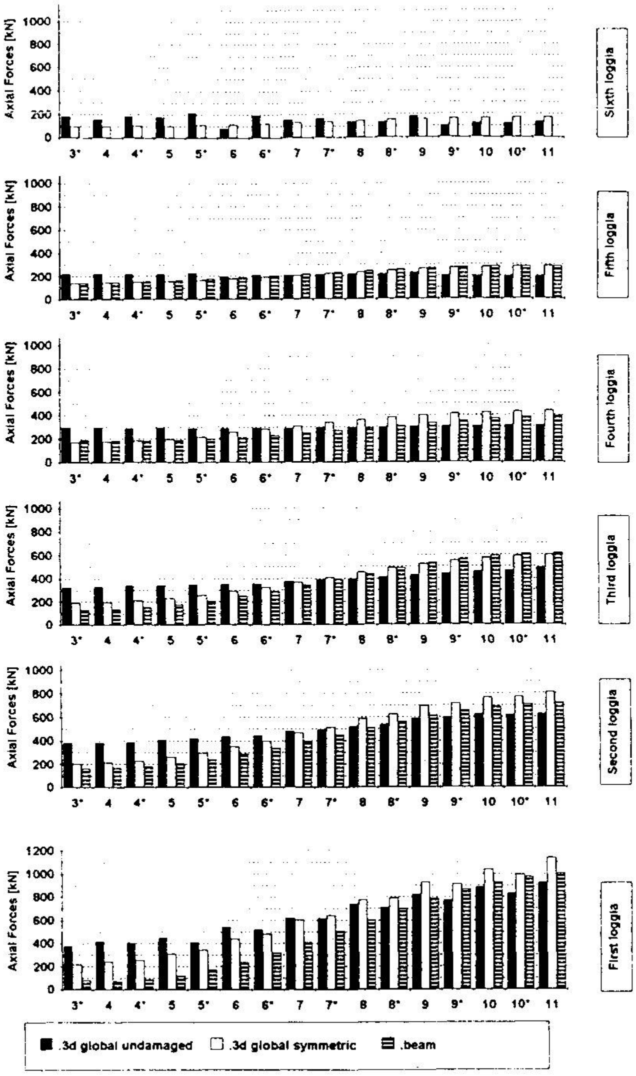


Fig.12 - Axial forces in the colonnade: comparison among different models

5. CONCLUSIONS

Refined Finite Element simulation, even if limited to linear elastic constitutive laws, may provide precious information on the behaviour of complex masonry structures and on their safety against brittle compression failures. The results may be a valid help in the study oh strengthening measures. Simplified approaches can only be useful for preliminary studies, and may lead to neglect important factors of the structural behaviour.

6. REFERENCES

1. GHERARDESCA, A., *Appendice alle considerazioni sulla pendenza del Campanile della Primaziale Pisana.* Miscellanee Artistiche, Pisa, Nistri, 1838.
2. *Relazioni compilate dalla Commissione Tecnica per lo studio delle condizioni presenti del Campanile di Pisa*, Firenze, Tip. Galileiana, 1913.
3. POLVANI, G. et al., *Ricerche e studi su la Torre Pendente di Pisa e i fenomeni connessi alle condizioni d'ambiente*, Firenze, Istituto Geografico Militare, 1971.
4. BARTELLETTI, R. et al., Stabilization of the Leaning Tower of Pisa, *Proceedings of the 13th IABSE Congress*, Helsinki, 1988.
5. MACCHI, G., Monitoring medieval structures in Pavia, *Structural Engineering International*, Vol. 3, n. 1, 1993.
6. ISMES, *FIESTA: software system for static analysis of solid structures based on the p-version of the finite element method*, Bergamo, 1992.