Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte

Band: 70 (1993)

Artikel: Strength assessment of ancient masonry vaults

Autor: Cauvin, Aldo / Stagnitto, Giuseppe

DOI: https://doi.org/10.5169/seals-53320

Strength Assessment of Ancient Masonry Vaults

Evaluation de la résistance d'anciennes voûtes en maçonnerie

Tragwiderstansbestimmung alter Mauerwerksgewölbe

Aldo CAUVIN
Prof. of Struct. Eng.
Univ. of Pavia
Pavia, Italy

Giuseppe STAGNITTO
Research Engineer

Pavia, Italy

SUMMARY

An overview of available methods for the safety assessment of ancient vaulted structures is given, specifying the limitations and fields of application of each approach. Suggestions are given for an appropriate use of the Finite Element Method. The validity of classical equilibrum methods based on the pressure line concept is emphasized and a simplified incremental method of non-linear analysis is proposed to evaluate the ultimate load and the influence of cracking in masonry arches and vaults.

RÉSUMÉ

L'article passe en revue les méthodes existantes pour l'évaluation de la sécurité d'anciennes structures voûtées. Il précise les limites et les domaines d'application de chaque méthode. Des propositions sont faites pour l'emploi de la méthode aux éléments finis. La validité des méthodes classiques de l'équilibre, basées sur le concept de lignes de pression, est soulignée. Une méthode simplifiée d'analyse non-linéaire est proposée pour évaluer la charge ultime et l'influence des fissures dans les arches et voûtes en maçonnerie.

ZUSAMMENFASSUNG

Es wird eine kritische Gesamtübersicht über die verfügbaren Methoden zur Bestimmung der Tragsicherheit alter Gewölbekonstruktionen und deren Anwendungsgebiet und -grenzen gegeben. Geeignete Einsatzmöglichkeiten der Finiten-Elemente-Methode werden vorgeschlagen Die Gültigkeit der klassischen, auf dem Drucklinienkonzept basierenden Gleichgewichtsmethode wird unterstrichen und eine vereinfachte inkrementelle Methode vorgeschlagen, um in nichtlinearer Berechnung die Traglast und den Einfluss der Rissbildung in gemauerten Bögen und Gewölben zu ermitteln.

-Introduction

Vaulted structures have been built for a long period of time using,until the advent of modern methods,materials unable to resist tension;in fact,apart from wood,which is a perishable and not always easily available material,the only available building materials were bricks and stone masonry;on the other hand,the only way to cover reasonably large spans using "no tension" materials was by adopting the "arch" scheme.
The dimensioning of arches and their derivatives,barrel and cross-barrel vaults has been performed in the past using purely empirical criteria,translated into geometrical rules.Only in more recent times rational verification methods were developed ,that is the well known equilibrium methods of graphical statics whose origins date back to the sixteenth century,but were formalized in a definitive way by Culmann [4]in the nineteenth century.
On the other hand,the modern evolution of elastic structural analysis and,most recently,of the finite element method,has provided a new tool of investigation also for this kind of structures.
However the finite element method,expecially if applied in the linear elastic field,could lead in these applications to unreliable results,as it ignores the influence of cracking,and of the highly anisotropic characteristics of the materials involved.
At last the development of nonlinear analysis methods permits to overcome ,at least partially,these limitations.
As will be later explained,the uncertainties which arise from these problems can be reduced by evaluating the results deriving both from "traditional" and "modern" methods of analysis:
in fact it is certain that traditional methods,being based on the "pressure line" concept retain a considerable validity as far as "no tension" materials such as masonry are concerned.
In this paper,using the experience accumulated by the authors in the field of ancient vaults restoration, the following topics are discussed:
-Validity of results given by the finite element method in the elastic field.
-Differences of behaviour between short barrel vaults on rigid supports and arches
-Possibility of adopting one dimensional schemes for vaults
-Importance of the "pressure line" concept
-Simplified methods of nonlinear analysis of masonry vaults and arches
-Procedures to be followed in structural analysis both of non restored damaged and restored vaults.
-Criteria to be followed for a rational restoration of vaults.
Examples of calculations performed on eighteenth century vaults are also,as available space permits, given.

-Problems arising from the use of the FE method in analysis of masonry vaults

The use of the FE method for the analysis of vaulted structures permits to model geometrically the structure in a very precise way and thus to define boundary conditions quite accurately(fig.1).
However this kind of analysis is not in itself enough to evaluate the structural safety for the following reasons:
A-The material does not resist tension and therefore cracks,altering the distribution of stresses,are present.Besides the material behaviour is highly nonlinear ,nonelastic and anisotropic.
Large cracks due to past settlements can be accounted for by introducing discrete cracks in the mesh according to the survey of visible cracks[2].However non visible tension cracks can only be accounted for by performing a nonlinear analysis.Nonlinear FE methods using "layered" elements have been prepared for reinforced concrete shells[9].
However nonlinear analyses are reliable provided that constitutive laws for the materials can be accurately defined.The problem is already difficult to solve for concrete which is a much more homogenuous material than masonry;the problem of definition of constitutive laws for masonry in two dimensional state of stress as is found in shells seems very difficult to solve.The problem can be simplified if it is possible to consider an equivalent one dimensional state of stress as will be explained later.
B-Commercial FE graphic postprocessors usually represent results in the form of "isostress"lines.
This is all right if ,as normally happens,the program is used to analyze materials with symmetrical behaviour such as steel.In the case however of material subject to cracking the given results both are inaccurate and do not permit to give an interpretation of the structural behaviour.

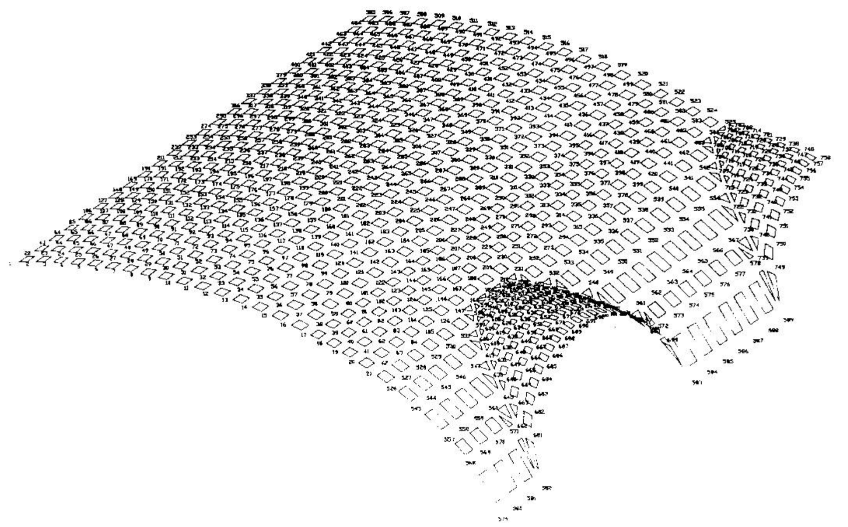

Fig.1-FE mesh of a barrel vault

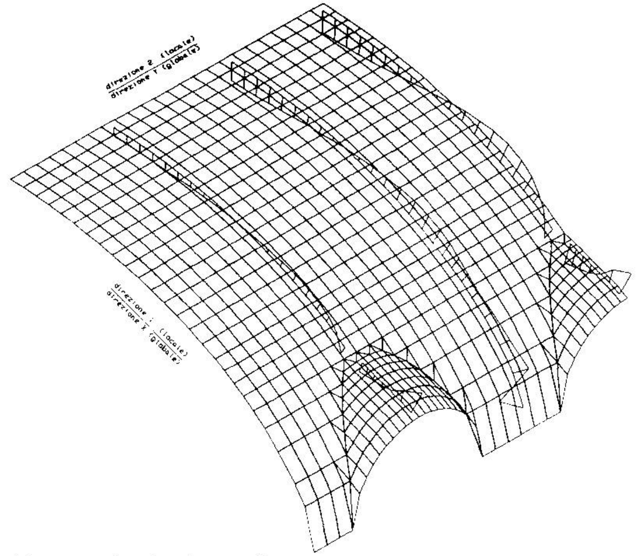

Fig.2-Moments in barrel vault

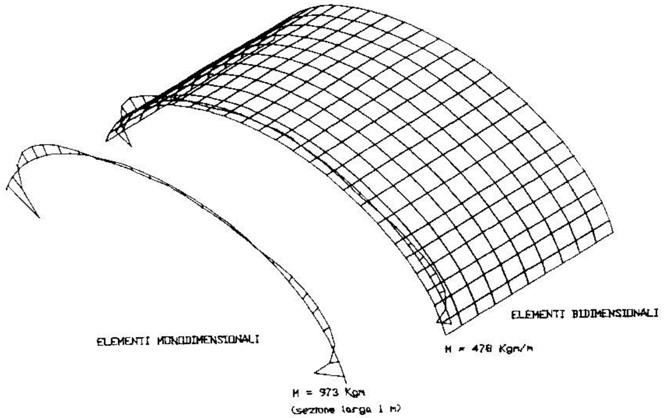

Fig.3-Torsional effect in barrel vault

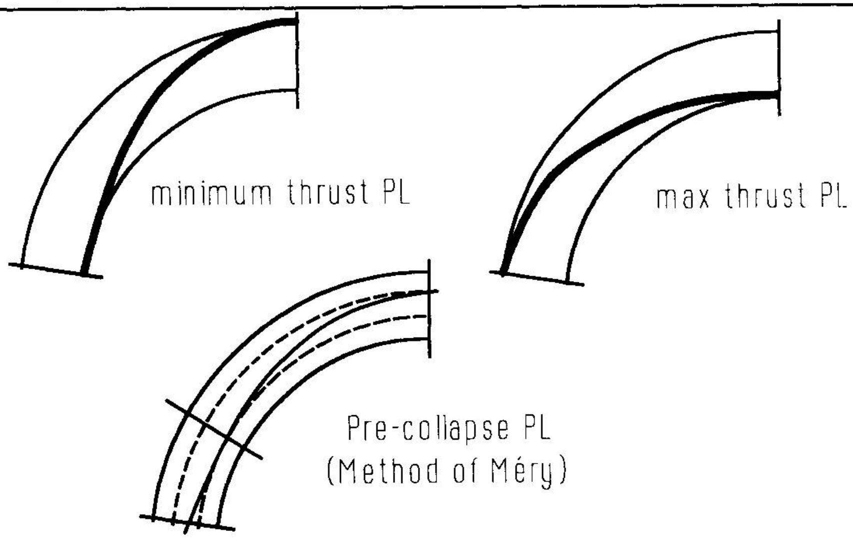

Fig.4-Limit pressure lines in arches

For this reason a specialized post-processor which permits to represent the diagrams of action effects along given alignments was prepared.The program was written in autolisp(the particular form of lisp language which can be used within the AUTOCAD program)and therefore can exploit the enormous grafic possibilities of the AUTOCAD system.Details concerning this subject can be found in [11].On fig.2 an example of this kind of representation is given.
This kind of representation permits to interpret in a very intuitive way the structural behaviour;in particular it permits to establish when a simplified structural scheme(such as a plane one-dimensional one)can be reasonably adopted to perform nonlinear analyses.
This postprocessor also permits to represent the state of stress in terms of isostatic lines.This reppresentation can also be useful for the same reason.
C-FE analyses permit to evaluate the state of stress and strain but not ,at least directly,the structural stability of a vaulted structure made of a "no tension" material.This subject will be considered later.Let us observe now the following facts concernig historical structures:
-Mean stresses are normally(but not always) very low and much below the material resistance.Besides when higly stressed zones are found,they have a very limited extension and are therefore unable to produce a danger of global collapse.
In fact the empirical rules used by ancient builders were normally very conservative;on the other hand the very fact that these structures have survived for so many years is a symptom of the presence of low stresses.
-Large deflections are normally due to foundation settlements which occurred in the past and which are usually stabilized;deflections due to applied loads are normally very limited and therefore not critical in the verification process.
It may be concluded that results from FE analyses,although necessary and important are not usually the crucial part of this process as far as structures composed of "no tension" materials are concerned.

-Possible simplifications of structural schemes:reduction to arch structures

As known ,in the classical methods of analysis,the study of vaulted structures is reduced to one dimensional problems:in other words the vault is imagined as composed of arches and each type of arch is studied separately.
One might wonder whether this method can,at least to some extent and for the simplest vaults(such as barrel vaults)be accepted.
Let us consider the barrel vault of fig.3 which has been analyzed both using two-dimensional finite elements and considering it as a series of independent parallel arches.In the first case the maximum moments along the arches are much smaller (about 50% less for the central arch)due to the collaboration of adjacent arches which in the second case has been disregarded.
Apparently the Finite Element calculation is more "exact".It must be noticed however that,with reference to the central arches, this collaboration takes place mainly because of the torsional stiffness of "fibers" orthogonal to the "arches".In fact,if the torsional stiffness of finite elements is assumed as zero,the results are exactly the same as for the one dimensional analysis.
Assuming an homogeneous elastic material,the torsional stiffness of transverse fibers in a shell structure like this is considerable(as the ratio between the sides of the shell is about one).
It is not easy however to evaluate torsional stiffness of a brick panel and to which extent this stiffness can be relied upon,as,to our knowledge,experimental evidence is lacking.
Common sense suggests however that this stiffness be both low and unreliable.
We might conclude therefore that the one-dimensional approach,at least in these cases,is to be preferred ,not only because it is on the safe side,but also because it is after all the more likely to approach reality.

-The individuation of "pressure lines"

As known the individuation of the possible pressure lines is essential to assess the overall equilibrium of an existing arch structure made with a "no tension" material,as stated by the Hayman's principle[1].
If a linear elastic behaviour is assumed the pressure line is univocally individuated and can be derived from the elastic analysis.In the same way it is possible to derive the pressure line corresponding to a nonlinear analysis.However,given the uncertainties in the evalution of the "true" pressure line,it is also important in the diagnosis and restoration process to consider limit pressure line such as those represented in fig.4:
-The pressure line corresponding to maximum thrust

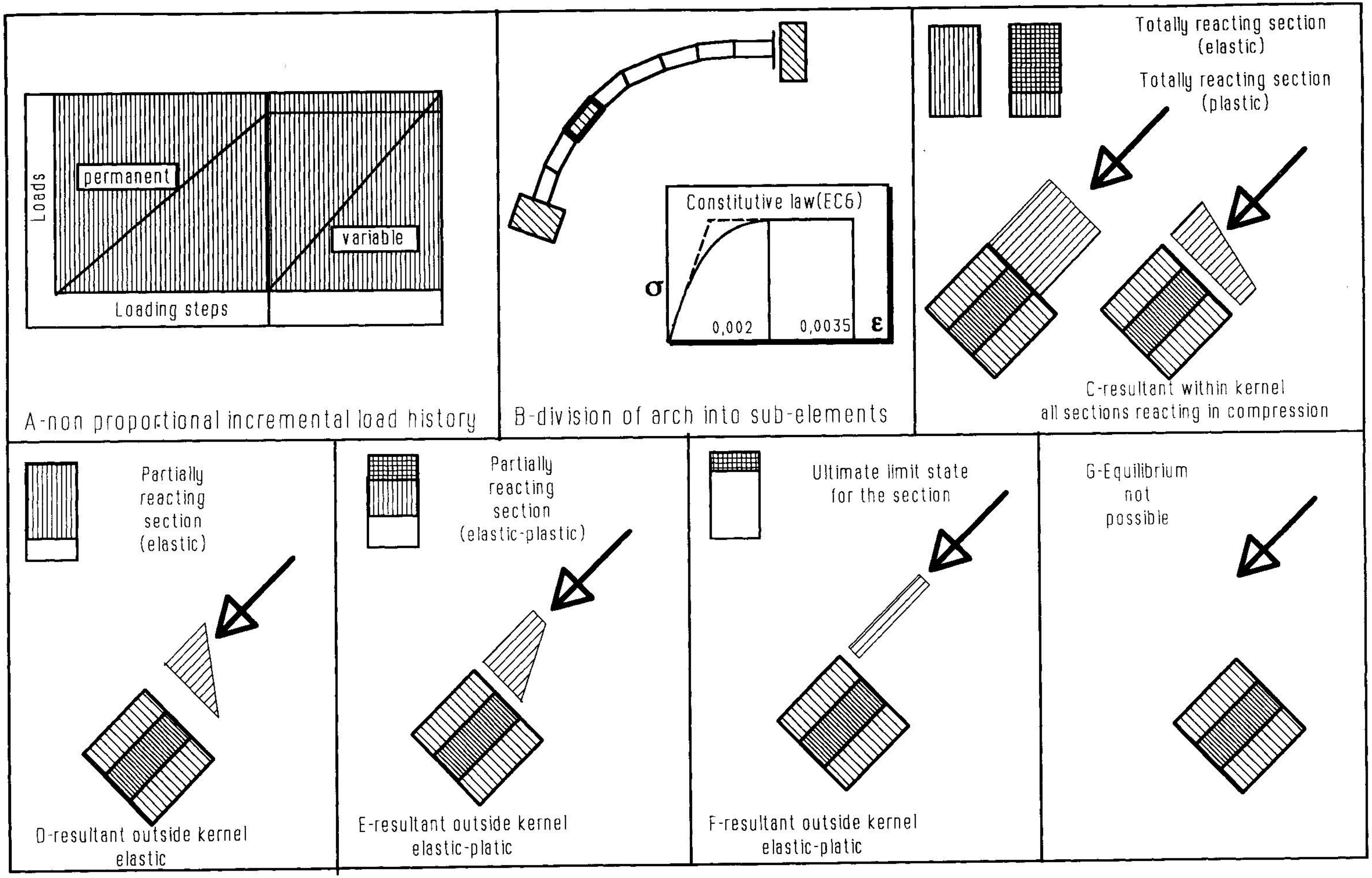

Fig.5-Simplified nonlinear analysis

-The pressure line corresponding to minimum thrust
-The pressure lines corresponding to the pre collapse phase in which critical sections begin to(or are already) cracked.The pressure line based on the classical method of Mery belong to this category.
In these cases elastic analysis cannot help and the equilibrium methods of graphical statics must be used[4].
In particular the method for the tracing of the only funicular polygon connecting the applied loads and passing through three determined points must be applied.
However,in the computer era,graphical constructions are considered too laborious , time consuming and not easily integrated in the design or analysis process.
To overcome this inconvenience an analytical procedure has been elaborated by the authors and a computer program prepared for the instant tracing of pressure lines passing through three predetermined points[12]
In this way the limit solutions which "brackets" the unknown physical situation can be rapidly determined.

-Simplified nonlinear analysis of masonry arches and vaults.

A simplified procedure for nonlinear analysis of masonry was prepared which permits to obtain,in the case of one dimensional plane structures the following results:
-Evaluation of the collapse load
-Evaluation of second order effects in slender arches
-Evaluation of the influence of cracking.
The program is based on the same phylosophy which was adopted by the first author in the preparation of programs for nonlinear analysis of Reinforced Concrete Structures[5],[7],[8],which can be summarized as follows(while details on the procedure are reported in [6]):
-The solution procedure is incremental,non proportional;permanent loads are applied first,then variable loads are applied step by step until collapse of the structure(fig.5/A)
-A "smeared" cracking "concentrated"plasticity approach is adopted,that is:
-Each element is divided into sub-elements(fig.5/B)
-The stiffness of each element is reduced,during the loading process,to take account of cracking,according to the criteria which are summarized in figg.5/D-E.
-The stiffness of each element is reduced to take into account second order effects
-Plastic hinges are introduced in critical sections when full plasticization is reached in a critical section(figg.5/C,F)
-The structural equilibrium is checked section by section and step by step(fig.5/G)
Although the accuracy of this approach should not be overemphasized,it can give useful informations in the verification process of a vault structure such as a reasonably accurate evaluation of collapse load(certainly more accurate than the one given by conventional "plastic"methods),of the "trend" in redistribution of action effects due to cracking,and of the importance of second order effects.
A constitutive law such as the one suggested by EC6(fig.5/B)[3]can be adopted for masonry,as it is appropriate for the degree of accuracy required by the method.

Use of the "pressure line" concept for rational restoration of barrel vaults. An example.

An example of the use of pressure lines of barrel vaults is given in fig.6.
Eighteenth century masonry vaults,having a span of 10 meters,had to be repaired and strengthened. The rubble infill originally used to level the exterior part of the vault(fig.6/A) had to be removed,a r.c. cap was added and a light overstructure was built in place of the rubble infill(fig.6/B).
In addition to FE analyses,the pressure lines for the two loading cases were traced and it was noticed that,despite the great reduction in permanent load,and the increase in arch cross section,the state of stress in critical sections had really worsened because of the increased excentricity of the pressure line with reference to the arch axis(figg.6/A1,A2,6/B1,B2).In fact the ancient builders were well aware of the concept of "funicular arch" and designed the vault accordingly.
An edge r.c. beam was therefore added to increase the section height of the arch near the abutment.
An alternate solution would have been to evaluate the distribution of overloads in the new situation,corresponding to an assumed pressure line,approaching as much as possible the arch axis.
This can be done using well known equilibrium criteria.

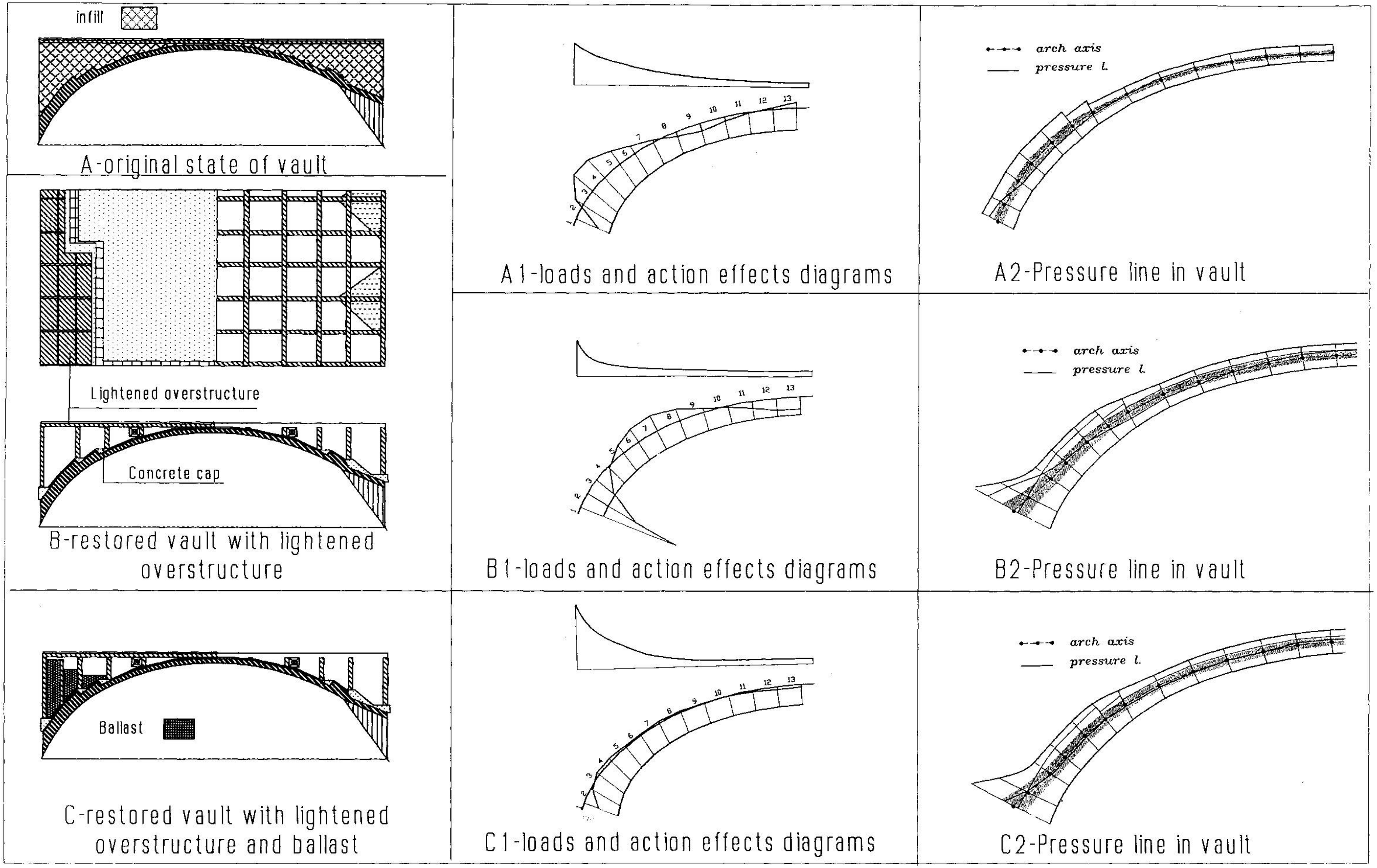

Fig.6-Example of restoration design of a masonry eighteenth century barrel vault

As a result,it was verified that by filling some of the cells of the overstructure with sand ballast a much more favourable pressure line could be obtained(fig.6/C,6/C1,6/C2)

Conclusions

The assessment of the degree of safety of masonry vaults is a complex procedure which requires the use of different approaches:
-The FE method in the linear elastic field permits to evaluate the state of stress and displacement of the structure assuming realistic structural schemes and boundary conditions.However cracking and nonlinear behaviour cannot be accounted for and the overall equilibrium of a structure made of "no tension"material cannot be directly established.
Suitable graphic representation of results permit in many cases to adopt realistically one-dimensional simplified structural schemes,which improve structural understanding,and simplify analysis.
-Whenever this is possible,the tracing of pressure lines using graphical or,better,analytical procedures permit to assess the structural stability and,in case,to alter permanent loads,during design of restoration, to increase this stability.
-A simplified nonlinear approach can also be useful to evaluate the ultimate load and the influence of cracking and second order effects.
-The use of more sophisticated FE nonlinear approaches is hampered by the difficulty to establish realistic two-dimensional constitutive laws and,in particular,to assess torsional behaviour.

References

1-Heyman J.,The care of masonry buildings.The engineer's contribution.,from the volume:structural repair and mantainance of historical buildings,edited by C.A.Brebbia,Computational Mechanics Publications Birkhauser,Basel 1989
2-Ignatakis C,Stylianidis K.,Stavrakakis E.,Design of intervention in domes.The importance of consideration of cracking,from the volume:structural repair and mantainance of historical buildings,edited by C.A.Brebbia,Computational Mechanics Publications Birkhauser,Basel 1989
3-Eurocode n.6,Unified Common Rules for Masonry Structures,Commission of the European Communities,Industrial Processes,Building and Civil Engineering,Predraft document,1991
4-Culmann Karl,Traité de Statique Graphique,Dunod,Paris,1880
5.-Cauvin A,Analisi non lineare di telai piani in CA.,Giornale del Genio Civile;fascicolo 1,2,3-1978
6-Cauvin A, Stagnitto G.,"MAS-NL" a computer program for the nonlinear analysis of one dimensional plane masonry structures,Research report,Department of Structural Mechanics,University of Pavia,Pavia,Italy,1993
7.-Cauvin A,Incremental Nonlinear Analysis of r.c. frames using microcomputers,from"Microcomputers in Engineering",edited by Schrefler B.A. and Lewis R.W.,Pineridge Press,Swansea,1986
8.-Cauvin A,Nonlinear analysis of Coupled Shear Walls in Tall Buildings,from"Numerical methods for nonlinear problems",edited by Taylor,Owen,Hinton,Damjanic,Pineridge Press,Swansea,1986
9.- Scordelis A.C.,Finite Element Analysis of reinforced concrete structures,Proceedings of the Conference on Finite Element Methods in Civil Engineering,Montreal 1972.
10-Cauvin A,Stagnitto G.,Problems concerning strength assessment and repair of Historical Vaulted Structures,to be published on the proceedings of the IASS Symposium"Public Assembly Structures from Antiquity to the Present",Istambul,1993
11-Cauvin a.,Stagnitto D.,Stagnitto G.,Postprocessing of Finite Element Analyses in "AUTOCAD"environment.Research report,Department of Structural Mechanics,University of Pavia,Pavia,Italy,1993
12-Cauvin A.,Stagnitto G.,Tracciamento mediante computer delle linee delle pressioni negli archi(Computer tracing of pressure lines in arches)Research report,Department of Structural Mechanics,University of Pavia,Pavia,Italy,1993

Aknowledgements

This research was prepared with the partial financial support of MURST(Italian Ministry for University and Scientific Research)