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Mechanical Models for Behaviour of Block Structures

Modèles mécaniques du comportement de structures rigides assemblées

Mechanische Modelle des Tragverhaltens von Blocktragwerken

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SUMMARY

The aim of this paper is to review and discuss recent studies on the mechanical behaviour of unconnected large block structures, like those of ancient temples. The main aspects which determine the structural behaviour will be discussed, namely, the identification of the load-bearing structures and the stability of the most probable mechanism. Recent results obtained with regard to free and forced motions will be summarized and their relevance with regard to the understanding of the actual structural behaviour under seismic actions, discussed.

RÉSUMÉ

L'article passe en revue les résultats de récentes études sur le comportement mécanique de structures composées de grands blocs rigides assemblés, comme ceux des temples antiques. On analyse les aspects qui déterminent le comportement dynamique; c'est-à-dire le problème de l'identification des éléments porteurs et la stabilité de l'ensemble. Une synthèse présente des résultats obtenus en étudiant la dynamique libre et forcée et leur implications afin de comprendre le comportement réel en présence d'une action sismique.

ZUSAMMENFASSUNG

Der Bericht schildert und diskutiert kürzlich gemachte Studien über das mechanische Verhalten von Tragwerken aus grossen Einzelblöcken, wie jene antiker Tempel. Die wichtigsten das Verhalten der Strukturen bestimmenden Faktoren werden besprochen, besonders die Identifikation der tragenden Teile und die Stabilität der wahrscheinlichsten Mechanismen. Eine Synthese präsentiert die betreffend freier und forcierter Dynamik gewonnenen Resultate und erklärt ihre Bedeutung für das Verständnis des Tragverhaltens unter seismischen Einwirkungen.

1. INTRODUCTION

The evaluation of vulnerability and risk with respect to environmental influences and accidental loads should be a prerequisite of any preservation policy for architectural heritage, in order to allocate rationally the available resources. Notwithstanding the active interest aroused in the accademic and professional communities on these themes, and the many studies developed in the last couple of decades, much still remains to be done.

In a previous paper [7] an extensive review has been presented on the mechanical models relevant in the study of the free and forced dynamics of structures made by large blocks without mortar, such as the structures of Grecian and Roman temples. The present paper will underline some aspects particularly important in view of applications aimed at conservation and restoration.

2. STRUCTURAL MODELLING

The main purpose of a consistent mechanical model is to identify the most relevant aspects of the response, with the minimum of a-priori restrictions, and to recognize for each given structure a "safe domain", i. e. a domain in the load space (space of the parameters of the external actions: tipically in case of seismic excitations, an "intensity" and a significative "frequency") within which the examined structure survives. This must be done taking into account all possible dynamics modes.

The structures dealt with in this paper are made of blocks, rigid by assumption, in contact with each other and with the support planes. Relevant aspects of the motion are rocking and relative rotations with consequent impacts, slidings with friction of blocks on one another, loss of equilibrium due to excessive rotations and/or relative displacements; these last ones determine permanent changes of the geometry.

The absence of connections between the elements and the consequent unilateral constraints give rise to several possible mechanisms with different centers of relative rotations and different planes of relative slidings; consequently, in each dynamic mode different values of mechanical and geometrical features appear.

Then, in the load space, a large number of evolutive regions with time dependent boundaries can be identified corresponding to the different dynamic modes; it is also possible that these regions overlap each others.

Which mechanisms are important depends on the present features of the structure; that can be a whole temple with a well preserved entablature, but also a surviving colonnade portion or even an isolated column.

In the following, it will be shown that all these structures can be assimilated to two mechanical models: an isolated column, possibly with an added mass at the top, or a combination of two columns supporting a lintel (trilith). This result gives a-posteriori significance to the great number of papers on the dynamics of slender rigid bodies, which have been spurred by the well known work by Housner[1] and include most of the papers purported to tackle the dynamics of columns and temples.

But, during the dynamics evolution, it can happen that, depending on the values of geometrical and mechanical parameters, one mode is dominant with respect to the other ones, which can be considered like some perturbations induced on the masses, on the geometry and on the restoring forces of the system. The first step is to analyze the most probable mechanism, at the instant in which the motion starts.

3. STARTING MECHANISMS

3.1. The column

Let us examine now the starting motion of a rigid column subjected to a given horizontal ground excitation; the governing forces are gravity and Coulomb dry friction.

Consider, first, a monolithic column of height h, base b, and mass m simply supported on a rigid ground (Fig. 1), in presence of a ground acceleration $a_o = k_o g$.

The relative values of the acceleration coefficient k_o, the static dry friction coefficient μ_s and the size ratio b/h decide whether the column remains at rest, or starts to rock, to slide or to slide-rock.

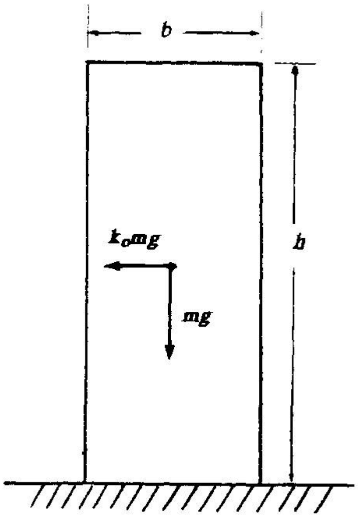


FIG. 1. *Monolithic column.*

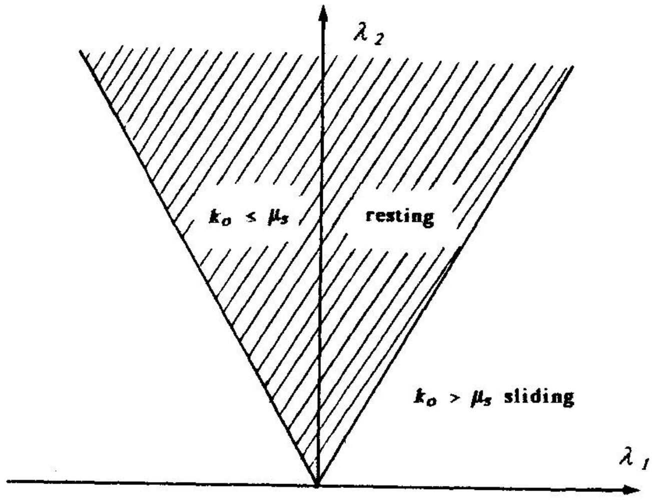


FIG.2. *Sliding regions in the plane (λ_1, λ_2) of horizontal and vertical reactions.*

Let us analyze separately the possible mechanisms.

First, assume that only sliding can occur. The column slides if k_o is larger than μ_s ; the region of "no motion" is the so-called Coulomb cone (Fig.2).

Instead, whether the rocks starts depends on the relative values of k_o and b/h. In fact, with respect to the possibility of rotation, the system is at rest in a potential well (Fig.3) similar to the situation of two inverted pendulums leaning on an oblique plane of slope equal to *arctg b/h*. (Fig.4).

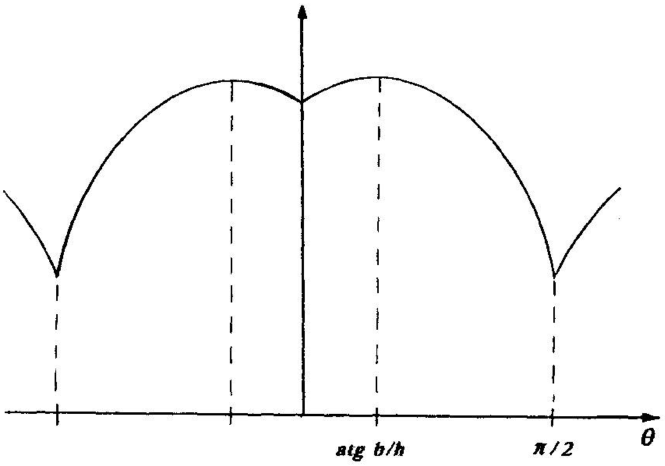


FIG. 3. *Gravitational potential energy.*

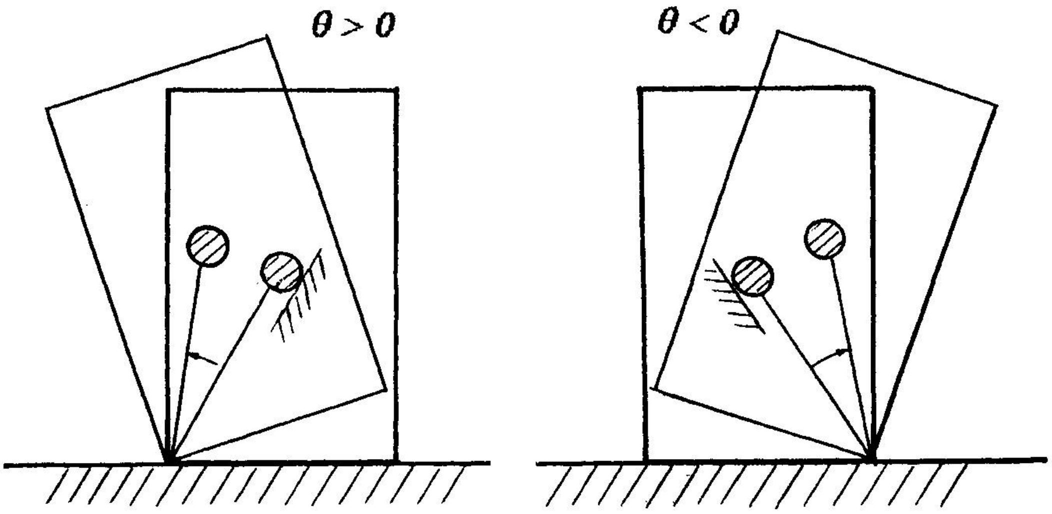


FIG. 4. *Equivalent systems for positive and negative angle.*

The regions of possible or impossible rocking are shown in Fig. 5, in the plane of the restoring gravitational moment M_r and unstabilizing moment M_u.

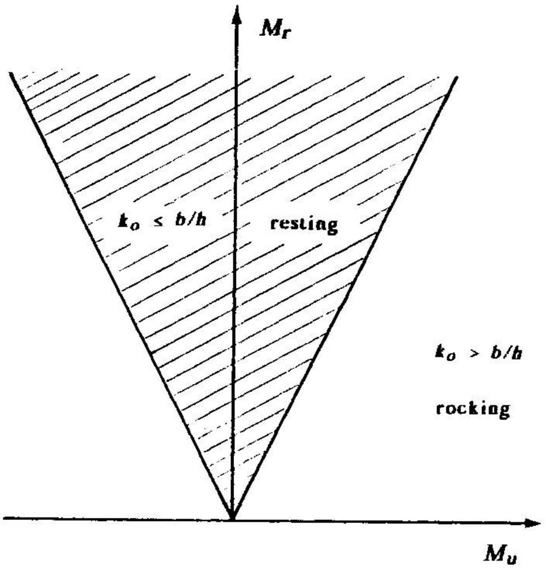


FIG. 5. *Rocking regions.*

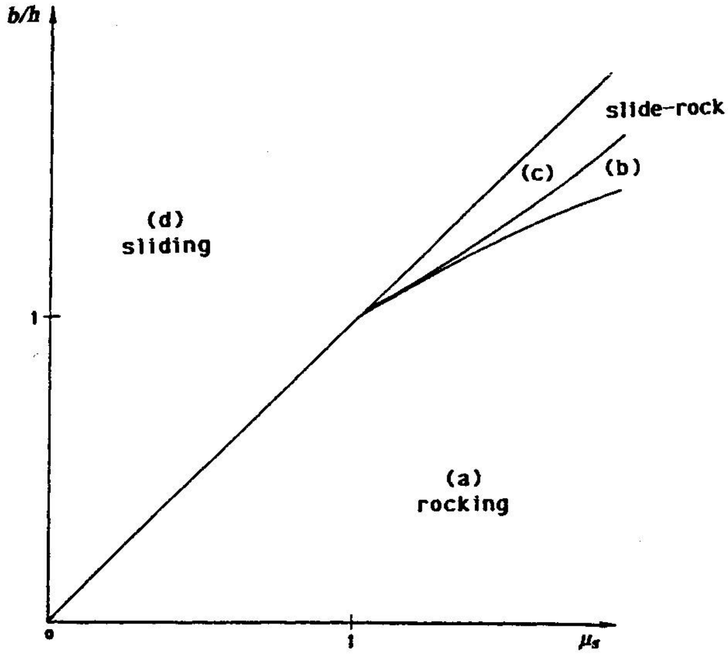


FIG. 6. *Regions of rocking, sliding or sliding-rocking.*

Assume now that rocking and sliding are both allowed. If k_o is smaller than μ_s and b/h, obviously the column does not move. Otherwise, different situations arise depending on whether b/h is smaller, equal or larger than μ_s, as illustrated in Fig. 6. In fact, it has been demonstrated [3] [10] that, if b/h is larger than μ_s, the motion always starts as a sliding; on the contrary, if $b/h \leq \mu_s$, regions of sliding, coupled sliding-rocking and rocking appear when $\mu_s > 1$. Discarding this latter case, which is unrealistic in the contact between stone blocks, it can be said that the starting motion of a monolithic column is either sliding or rocking.

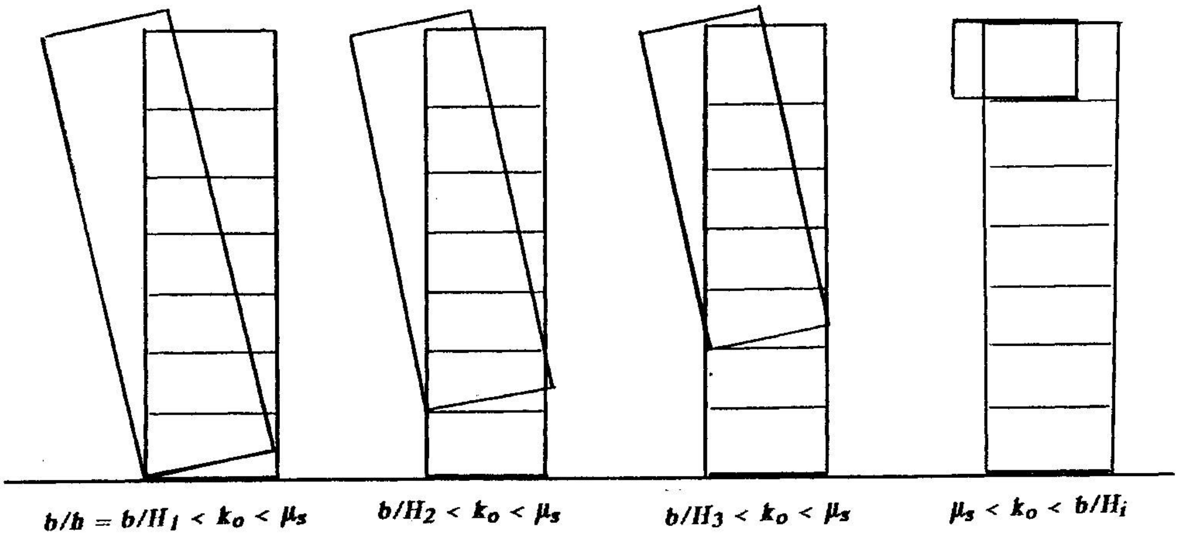


FIG. 7. *Multiblock column.*

3.2. The multiblock column

In case of a multiblock column, either relative slidings or rotations between adjacent blocks can be activated. But relative movements in different contact joints imply different size ratios b/H_j: in fact, H_j must the height above the involved joint (Fig. 7).

The number of activated d.o.f. then depends on the value of k_o (Fig. 8): assuming that $b/H_j < \mu_s$ when $j < i$ and $b/H_j \geq \mu_s$ when $j \geq i$, only rocking can start in the joints below the *i-th*, sliding in the joints above.

Then, for multiblock columns, the rocking of the whole column is in all cases activated whenever k_o is larger than b/h, while activation of the other d.o.f. requires larger and larger values of k_o, so that relative slidings are improbable in the starting motion for usual geometric ratios of ancient columns and values of k_o.

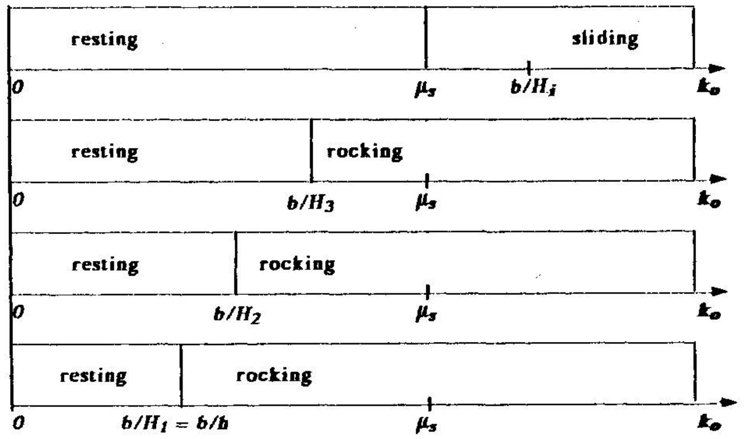
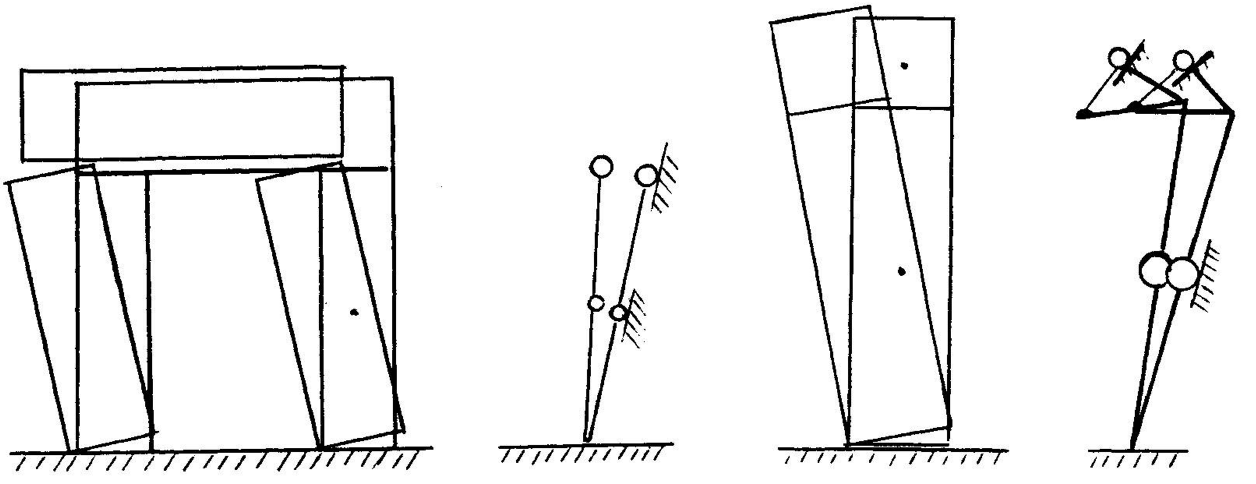


FIG. 8. *Starting modes for multiblock column.*

3.3. The trilith and the temple

The same situation above occurs for a trilith (Fig. 9 a); it starts with an one d.o.f. mechanism, the rocking of the two columns, whose behaviour is also governed by Fig. 6 [9]. It can be easily demonstrated that the motion of a colonnade in the plane defined by the columns, and the motion of a whole temple in the plane of the excitation (taking into account the axial simmetry of the columns and assuming efficient connection between the entablatures) can be modelled like that of a trilith.

The motion of a colonnade out of its plane can be assimilated to that of a higher column. In conclusion, the most probable mechanism for any kind of monumental structure, either monolithic or multiblock, can be modelled at the instant of starting motion like a single column, i.e. an inverted pendulum with appropriate values of masses and possibly added masses on the top.



(a) in-plane motion

(b) out of plane motion

FIG. 9. *One d.o.f. trilith mechanism.*

4. DYNAMIC EVOLUTION AND FAILURE MODES

In time, the dynamical behaviour of a multiblock structure can become more and more different from that of a monolithic column. In fact, the variations of the inertia forces and of the restoring gravitational moments, with respect to the instantaneous value of the excitation, change the "threshold" conditions at different heights of the column and can allow the starting of new mechanisms. But, if rocking has already started, the above variations must be evaluated in such a mechanism for each element of the structure.

The dynamic analysis of the rocking response is then mandatory for any multiblock structure; the actual effects of the impact depend on the number of the blocks and on their geometry [4][5].

Let us refer to a column under a harmonic excitation. The dynamics of the rocking mode for small angles and any kind of periodic response (symmetric or not, and with any number of impacts per period) is governed by a damped forced Hill equation [5][7], in both cases of monolithic and multiblock column:

$$u'' + p(\tau)\,u' + q(\tau)\,u = f(\tau) \qquad (1)$$

Equation (1) is typical of a system with parametric resonance. It is impossible to obtain closed form solutions for $f(\tau) = 0$; the identification of stable oscillations for the column would require that all kinds of periodic response are analyzed, with given initial conditions.

This result seems discouraging: fortunately, conditions of existence and of stability coincide; therefore, it is sufficient to perform numerical investigations to obtain the regions of either stable (periodic) or unstable (exponentially increasing amplitude) motions as functions of the parameters (ω, K_S) of the excitation.

This is shown in Fig.10 for $b/h = 0.2$ and $h = 10m$, geometrical features of the multiblock columns of the E-3 Temple in Selinus (Sicily).

Fig. 10 is limited to values of ω in the range *0-6 rad/sec*: in fact, the response amplitude increases systematically with K_S and decreases quickly with ω increasing, tending asynptotically to zero; the amplitude becomes neglegible for ω larger than *6 rad/sec*.

If the same column is subjected to a generic stationary excitation, the linear character of the system between two impacts and the small values of the amplitude of the stable oscillations allow to obtain the response by summation of the responses to each harmonic component defined by the Fourier spectrum of the excitation.

For ω *>6 rad/sec*, the global amplitude of the oscillation is negligible; the angular velocities and the relative slidings consequent to the impacts are of the same order of magnitude.

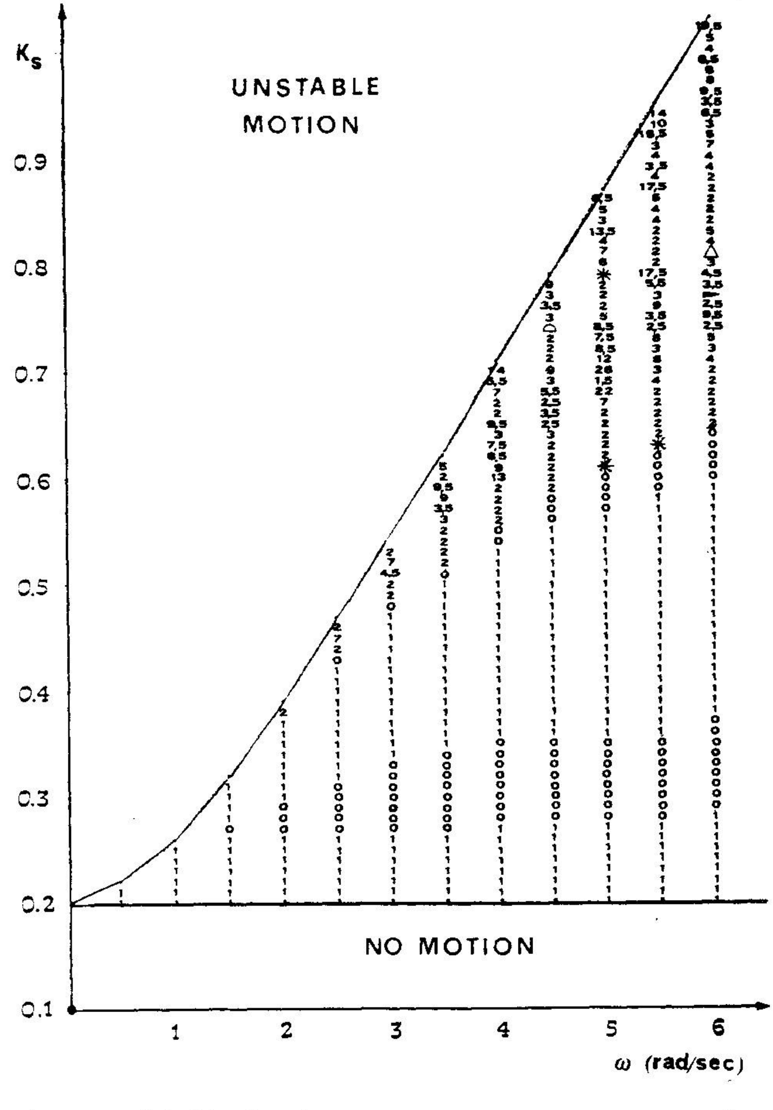


FIG.10. *Multiblock column: stable or unstable motions [5,7].*

The amplitude of the column response becomes significant for values of angular frequency below ω = *6 rad/sec* and increases with ω tending to zero. In this range new mechanisms can start.

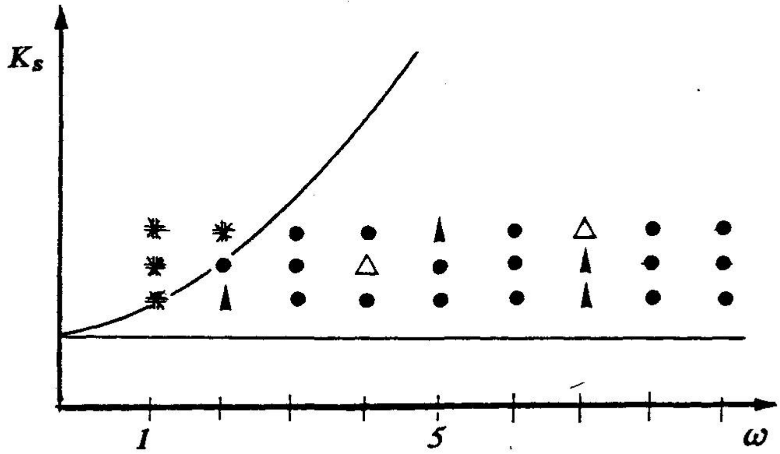


FIG. 11. *Trilith: stable or unstable motions*
* *Immediate overturning,* ▲ *overturning after one or more impacts,* △ *collapse due to excessive slidings,* ● *bounded motions.*

An analysis of the relative importance of slidings displacements on the upper elements of the structure has been performed studying the behaviour of a trilith under harmonic excitation [9]. It has been assumed that relative slidings are possible only between the lintel and the two columns. The geometric features are derived from a colonnade of the E-3 Temple in Selinus. The numerical investigation performed confirms that before the first impact only the rocking mechanism is activated; after this instant, relative slidings appeared for all values of K_s and ω.

Examples of the distributions of the maximum angular responses of the column in simple rocking and of the maximum sliding displacements in the trilith motion, in a given time interval, are shown in Fig. 12 and Fig. 13 respectively.
In the latter, the average absolute relative sliding S_m is reported, defined as

$$S_m = 1/t \; \left[\int_0^t (|S_l| + |S_r|) \right] dt \qquad (2)$$

where $S_l(t)$ and $S_r(t)$ are the instantaneous relative slidings respectively on the left and the rigth support of the lintel. The average displacement S_m increases monotonically with time.

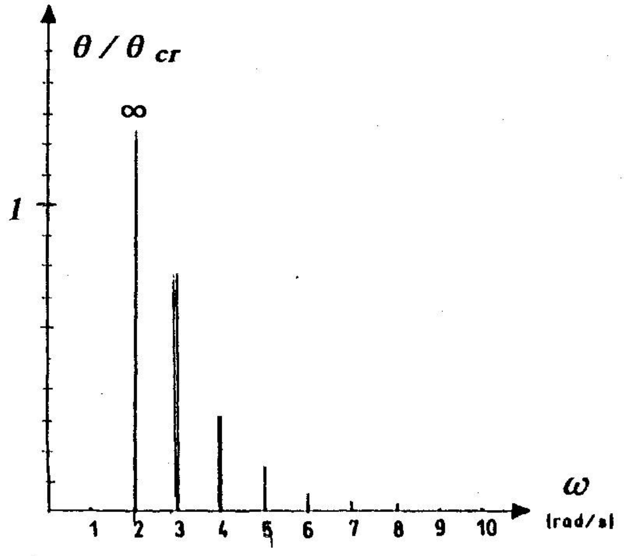


FIG. 12. *Typical distribution of max. rotation for the column of Fig. 10.*

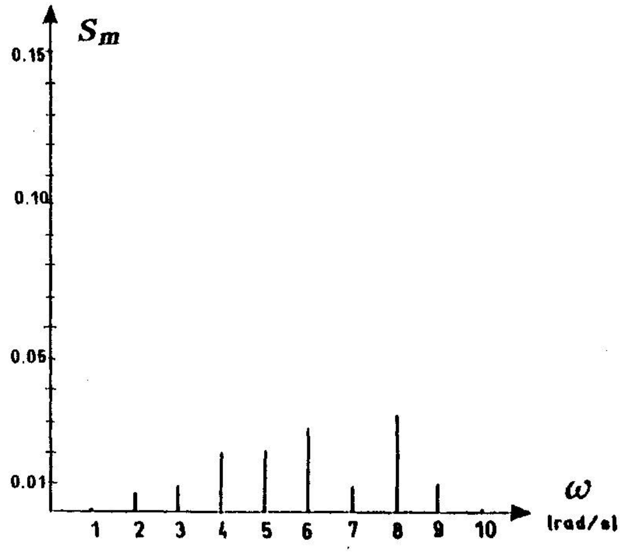


FIG. 13. *Typical distribution of max. relative displacement for the trilith of Fig. 11.*

Comparison between Fig. 12 and Fig. 13 shows that the responses in the uncoupled rocking and coupled sliding-rocking vary with ω in a very different way.

5. CONCLUDING REMARKS

In conclusion, the dynamical behaviour of block structures, in particular under earthquake-type loads, is a well developed field of research, which has already yielded very significative results (cf. the numerous references listed in [7]). Much less developed appear the applications of these studies to actual problems: even in the few cases in which the importance of the problem almost forced a systematic program of studies parallel to the actual works, like the restoration of the Parthenon, the dynamic aspects have been somewhat undervalued, and the structural analysis followed the quasi-static approach [2].

The present paper was conceived as a modest attempt towards bridging this gap.

The results synthetically illustrated in the preceding sections show that overturning by overall eccessive rotation is a rather improbable mode of collapse under seismic loads, both for isolated columns made of stocky blocks and well preserved temples. On the other hand, rocking does not lead to any apparent damage of the structure, unless the repeated impacts weaken the material and cause fractures and "spalling", a process that may be enhanced by previous wheatering. This phenomenon has not been studied yet in sufficient depth, but some preliminary results [8] seem to indicate that it becomes relevant under extreme conditions only.

On the contrary, sliding causes permanent displacements [6][9] that alter the geometry. However, impacts and sliding displacements dissipate energy and therefore tend to stabilize the structure.

For all these reasons, it appears that blocks must not be tied or rigidly connected to each other: the only sensible "maintenance" policy is the elimination of permanent displacements and, possibly, the restoration of the damaged basis of column so to preserve the original size ratio.

Further studies are still very much needed with an eye not only at academic results but also at real possibilities of applications.

ACKNOWLEDGEMENTS

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