Modeling Masonry Behaviour using Distinct Elements

Modélisation de la maçonnerie par la méthode des éléments distincts

Modellierung von Mauerwerk mittels distinkten Elementen

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SUMMARY

Masonry made out of natural stones and bricks is among the most common building materials which at present need to be preserved. However, there is still a big gap between actual material behaviour and a sound understanding of how to model or analyze its behaviour. One way to describe discontinuous material behaviour is by using Distinct Elements; a 'continuum' is subdivided into distinct blocks which are able to undergo large motions and rotations. This numerical method is introduced to masonry and critically reviewed.

RÉSUMÉ

La maçonnerie de pierres naturelles et de briques représente un type de construction commun qui mérite d'être conservé de nos jours. Pourtant, nous manquons encore de modèles analytiques qui décrivent véritablement le comportement de la maçonnerie. L'utilisation des élément distincs offre un tel modèle d'analyse. Dans la méthode des éléments distincts, un 'continuum' est divisé en différents blocs qui peuvent soutenir de grands déplacements et rotations. Cettte méthode numérique est appliquée à la maçonnerie, et elle est passée en revue.

ZUSAMMENFASSUNG

Mauerwerk aus natürlichen und künstlichen Steinen gehört zu jenen klassischen Baumaterialien, deren Erhaltung heute grosses Augenmerk geschenkt werden muss. Dennoch mangelt es an Kenntnissen, wie grundlegendes Materialverhalten von Mauerwerk zu modellieren oder analysieren ist. Ein möglicher Weg besteht in der Anwendung von Distinkten Elementen. Dabei wird ein 'Kontinuum' in Blöcke unterteilt, die grossen Translationen und Rotationen unterworfen werden können. Diese neuere numerische Methode wird für Mauerwerksprobleme angewandt und kritisch beleuchtet.

1. INTRODUCTION

Preservation of the Architectural Heritage demands first an assessment of the structural serviceability of, in most cases, blocky materials. Brick or block arches, stone bridges and masonry walls in general, they all exhibit more or less a discontinuous material behavior due to their jointed nature. Modeling techniques which do not consider this discontinuous behavior of blocky structures can only model the over-all behavior. For a closer and more detailed look at the deformation and load bearing behavior, one has to use numerical schemes which account for the jointed behavior of e.g. masonry.

It is doubtful that new and advanced techniques would have saved the classic Tower of Babel (Fig. 1), but, at least we might have gained a deeper insight why, if not for a biblical spell, the structure failed.
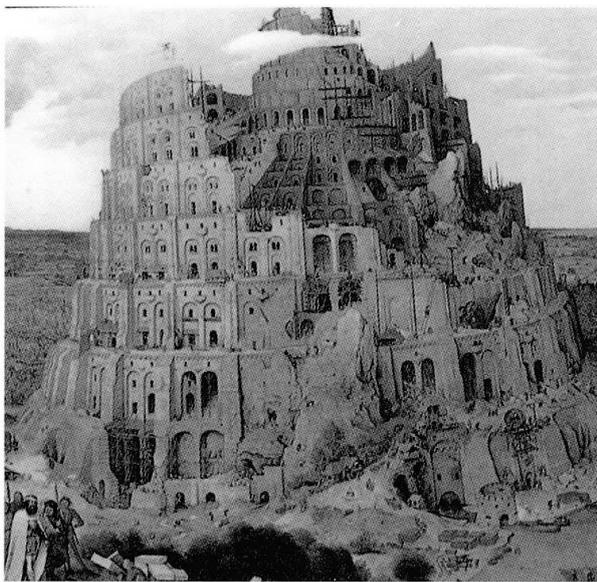

Fig. 1 The Tower of Babel by Peter Brueghel (1525/30-1569), cut

Thus, this paper introduces the Distinct Element Method (DEM) as proposed by Cundall [1] as an alternative modeling technique. The code was developed to model the behavior of discontinuous media, such as jointed rock masses and for other geotechnical applications (e.g. dam engineering). However, as certain restrictions are kept in mind, this method is also well suited to model masonry.

Tests on shear walls from a previous investigation by the author [2,3] at the Technical University of Munich were used to model those results applying the program UDEC [6].

2. CLASSIC MASONRY

Dealing with historical masonry, one has to consider the big difference between classic masonry and e.g. North-American Masonry made of concrete blocks. In the former, there is no reinforcement and no grouting. The mortar joints act as a real plane of weakness. Therefore, unreinforced ungrouted plain masonry has to be treated as a real discontinuum.

3. EXPERIMENTAL RESULTS

3.1 Test set-up

Experimental results following a test set-up as described in Figure 2 yielded the equations 1 to 3. A more detailed description of these tests and derivations is given in [2,3].

σ_I, σ_{II} applied principal stresses

σ_x, σ_y average normal stresses

τ_{xy}, τ_{yx} average shear stresses

$$X = \frac{\sigma_x}{\sigma_y} \quad , \quad Y = \frac{\tau_{xy}}{\sigma_y}$$

μ coefficient of friction in the joints

c   cohesion

$\beta_{t,b}$ tensile strength of bricks

β_c compressive strength

$$v = \frac{2\Delta y}{\Delta x} \text{ see Fig. 3}$$
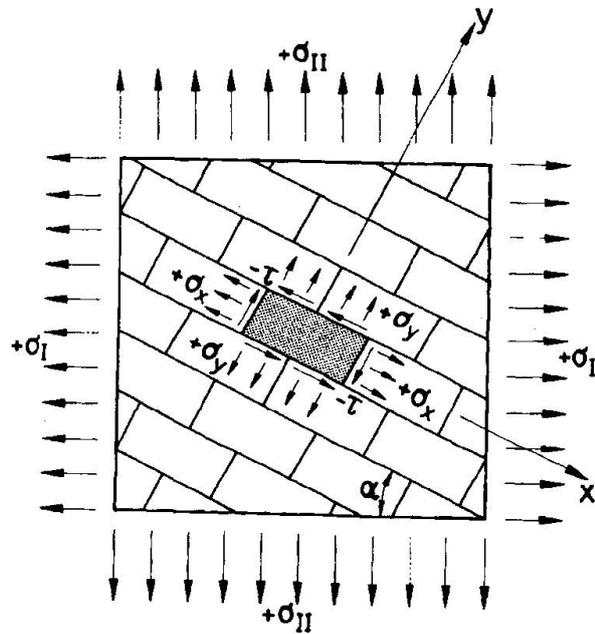
$$w = \frac{1}{v} \quad \text{see Fig. 3}$$

Fig. 2 Test set-up, assumption of a homogeneous state of stress

3.2 Model for ultimate strength

Figure 3 shows a model proposed by [5] and expanded by [2] for a non-uniform stress distribution as a result of the unbalanced shear stresses in the head and bed joints.
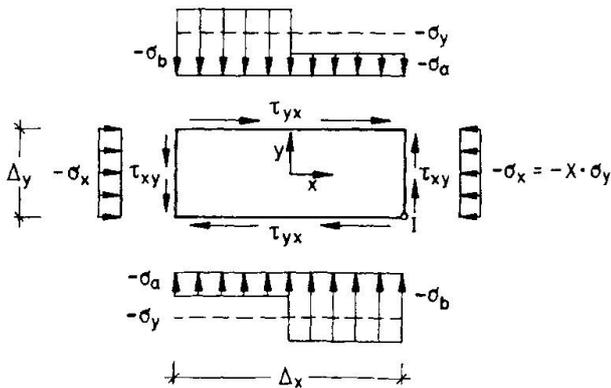
Fig. 3 Model

Three different failure modes can be distinguished. An opening of the bed joints was not observed, as tensile stresses in the bed joints have not been investigated in these tests. Furthermore, considering $c_{xy} \neq c_{yx}$ and $\mu_{xy} \neq \mu_{yx}$ leads to:

Shear failure of the bed joints:
$$\tau_{yx} = \frac{c_{yx} - \mu_{yx}\sigma_y + v\mu_{yx}(c_{xy} - X\mu_{xy}\sigma_y)}{(1 + v\mu_{yx})} \tag{1}$$

Tensile failure of the brick:
$$\tau_{yx} = \frac{1}{2}(c_{xy} - X\mu_{xy}\sigma_y) + \frac{\beta_{t,b}}{2.3}\sqrt{1 - \sigma_y\frac{(1+X)}{\beta_{t,b}} + \frac{X\sigma_y^2}{\beta_{t,b}^2}} \tag{2}$$

Compression failure of the panel: $\tau_{yx} = c_{xy} + w\beta_c + \sigma_y(w - X\mu_{xy})$ (3)

4. DISTINCT ELEMENT METHOD

4.1 Introduction

There are a couple of ways to analyze a structure: experimentally, analytically or using a numerical scheme. Analyzing large and more complex structures, one normally is forced to use numerical approaches. As there are three major groups of numerical models, one should be precise in using terminology. The different methods in use for discretizing a structure are:

Finite Elements   (FEM)
Boundary Elements (BEM)   Discrete Elements
Distinct Elements (DEM)
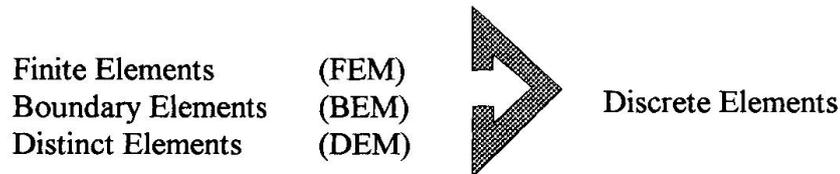
All these three methods together should be called Discrete Elements to avoid confusing terms.

In the Distinct Element code a medium is simulated as an assemblage of blocks which interact through corner and edge contacts. At these contacts, either rigid or fully deformable blocks are connected by spring like joint-normal and joint-shear stiffness, k_n and k_s, respectively (Fig. 4).
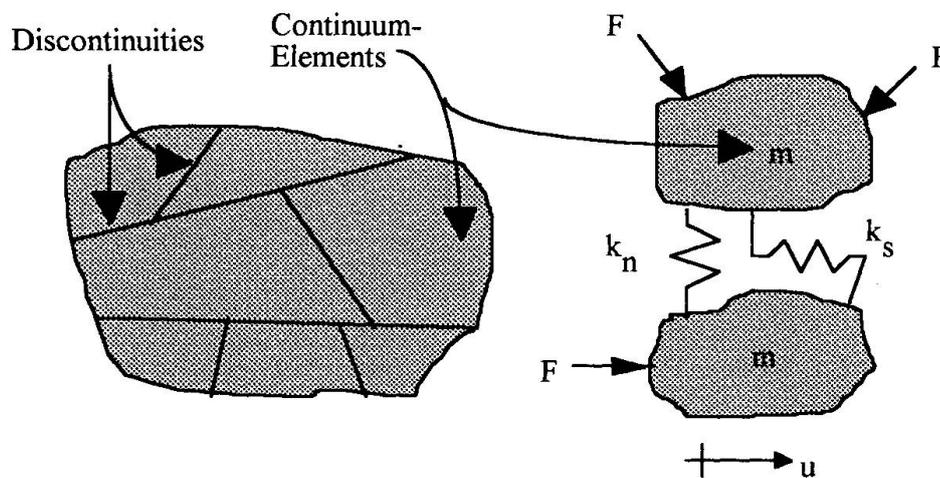
Fig. 4 Basic separation in a Distinct Element formulation

4.2 Theory and Background of the Code

Alike in FEM, the unknowns in DEM are the displacements and rotations of the blocks. However, unlike FEM, these unknowns are solved explicitly by solving the dynamic equilibrium of motion. For a single mass, avoiding matrix notation, this can be written in a general form (Newton's 2nd Law):

$$m\ddot{u}^t + c\dot{u}^t + ku^t = F^t \qquad (4)$$

with

m	mass
$u, \dot{u}, \ddot{u}$	displacements and their derivations, respectively
c	damping
k	joint stiffness
F	loads
t	time

A similar equation can be written for rotations. A damping must be provided, otherwise, an elastic system oscillates forever. The static solution is interpreted as the end result of a long time oscillation (dynamic relaxation scheme).

The main task of the scheme is to determine a set of displacements by iteration that brings all elements into a state of equilibrium, or, if not possible, will indicate a failure mode.

Thus, the blocks in the system can develop large deformations and rotations, while compatibility is always satisfied. No stiffness matrix has to be formed and non-linearities can be implemented in an incremental form.

5. APPLICATIONS

5.1 Calibration tests

First computational runs on small two-brick specimen were carried out to investigate the correct stress distribution in joints and to check the overall behavior.

5.2 Masonry Shear Walls

Shear tests on biaxial loaded small scaled specimen have been carried out and have been reported in [3]. The panel in Figure 5 has been loaded in uniaxial compression with an inclination of the joints of α = 28.7 degree. Thus, producing an averaged inner stress ratio with $X = \sigma_x : \sigma_y = 0.30$ and $Y = \tau_{xy} : \sigma_y = - 0.55$ (Notations see Fig. 2). Figure 5 shows a typical crack pattern of the joints at shear limit as a result of a UDEC-simulation using a Mohr-Coulomb-failure criterion for the joints. The failure pattern correlates well with real test data.
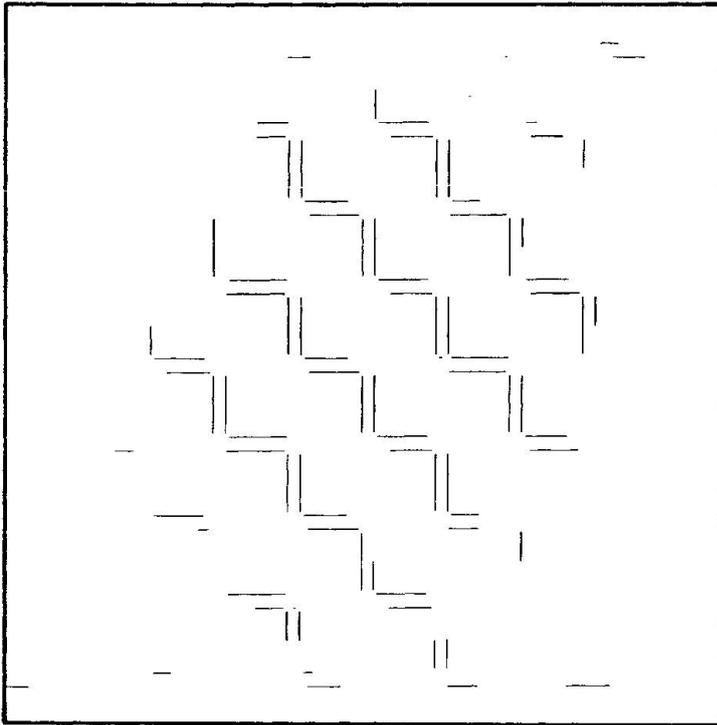
Fig. 5   Crack Pattern

The influence of the joints is remarkable for blocky structures. Furthermore, the qualities of joints in different directions are also different, being the basis of several shear failure criterions. This behavior was experimentally verified in [2], where it was shown that the bricks start to rotate in shear stressed masonry until a new state of equilibrium is reached.

To save calculation time, the mortar in the joints of the panels was now simulated using an equivalent joint stiffness, which was different for bed and head joints. Thus, for the same example as in Fig. 5, Fig. 6 shows the clockwise rotation of the bricks before failure. A calculation of the stresses is given in [4].
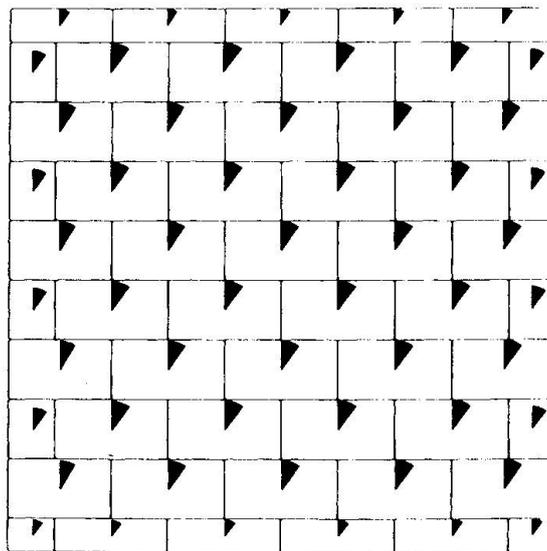

Fig. 6   Modeled brick rotations in shear stressed masonry

5.3 Infilled Frames

Structures made of infilled frames were already common hundreds of years ago. Although this example shows an infilled concrete structure, the basics remain the same. There is still little known about the fundamental load bearing behavior. Obviously, the influence of the joint between concrete and masonry is one of the driving factors for the bearing capacity. Fig. 7 shows the horizontal x-displacements of the frame and the infill as an example.
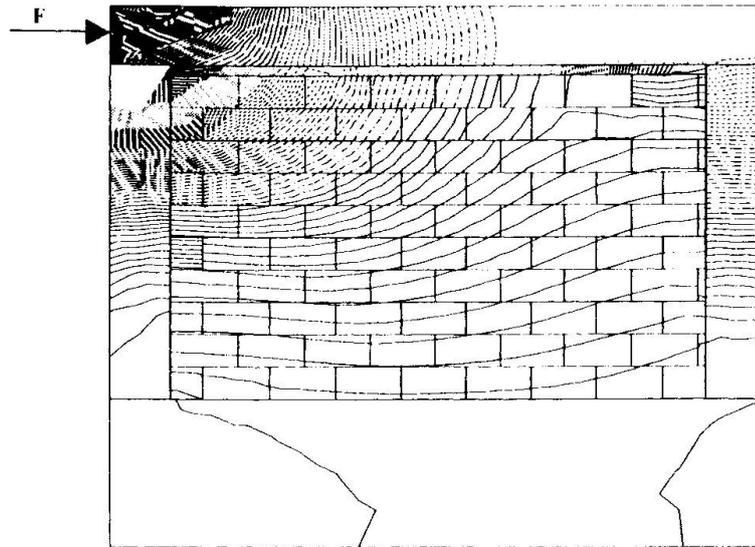

Fig. 7 Horizontal displacements of an infilled frame under a horizontal load.

6. CONCLUSIONS AND QUALIFIERS

The Distinct Element Method (DEM) solves the dynamic equilibrium for each body and boundary interaction forces. Each body communicates with surrounding bodies via boundary contacts which may change as a function of time. The forces which are generated between the contacts can obey various interaction laws.

Simulating different qualities of bed and head joints (cohesion and friction), the numerical results show brick rotations (Fig.6) and stress distributions along the joints which correlate very well with the test results. Furthermore, modeling techniques and a critical review of advantages and disadvantages for a further use of Distinct Elements in masonry are presented.

As the code was mainly developed for geotechnical applications where plane strain conditions are widely assumed, this state of strain is certainly not true for masonry and has to be considered.

For clay brick masonry, one typical failure mode is an internal split-off due to different Poisson's ratios of brick and mortar. Mortar, in general, has a Poisson's ratio which is twice as large as a brittle clay brick. Therefore, the mortar tears the brick apart. This out-of-plane characteristic, obviously, cannot be described with a plane program.

Furthermore, if one is interested in a more detailed deformation analysis, an orthotropic material law has to be derived and incorporated in the code. This would make the analysis for masonry more realistic.

Some of the driving code parameters which still need further investigation are:

- joint stiffnesses k_n, k_s
- damping factor c
- time step Δt.

These factors influence the solution process and, therefore, the state of equilibrium has to be checked by the user.

ACKNOWLEDGMENTS

REFERENCES

1.   Cundall P.A., A Computer Model for Simulating Progressive Large Scale Movements in Blocky Rock Systems, Proc. of the Symposium of the Int. Society of Rock Mechanics, Nancy, France, 1971, Vol. 1, Paper No. II-8

2.   Dialer C., Bruch- und Verformungsverhalten von schubbeanspruchten Mauerwerksscheiben, zweiachsige Versuche an verkleinertem Modellmauerwerk, Dissertation, Berichte aus dem Konstruktiven Ingenieurbau, Technische Universität München, 1990, Ph.D.-thesis in German

3.   Dialer C., Some Remarks on the Strength and Deformation Behavior of Shear Stressed Masonry Panels under Static Monotonic Loading, Proceedings of the 9th Int. Brick Masonry Conference (IBMaC), Berlin, Vol. 1, 1991 pp. 276 283

4.   Dialer C., A Distinct Element Approach for the Deformation Behaviour of Shear Stressed Masonry Panels, 6th Canadian Masonry Symposium, Saskatoon, Saskatchewan, 1992, pp. 765-776

5.   Mann W., Müller H., Versuche zur Bruchtheorie von querkraftbeanspruchtem Mauerwerk, Proc. of the 4th Int. Brick Masonry Conference, Brügge, 1976, p. 4.a.4

6.   UDEC: Uniform Distinct Element Code, Version IGG 1.6, Itasca, Minneapolis, Minnesota, 1991