Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte

Band: 70 (1993)

Artikel: Modelling and monitoring the structure of Hagia Sophia in Istanbul

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DOI: https://doi.org/10.5169/seals-53293

Modelling and Monitoring the Structure of Hagia Sophia in Istanbul

Modélisation et surveillance de la structure de Hagia Sophia à Istanbul

Modellierung und Überwachung des Tragwerks der Hagia Sophia in Istanbul

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SUMMARY

Research is being conducted for assessing the seismic capacity of one of the most important historic buildings of Western civilization, the almost 15-century-old Hagia Sophia, by integrating data from on-site archaeology and measurements of both ambient and earthquake response into an interactive, numerical model. In addition to providing greater understanding of the behaviour of the building's complex structure, the study will also ascertain possible needs for seismic strengthening.

RÉSUMÉ

Une recherche est entreprise pour déterminer la résistance séismique d'un des monuments historiques des plus importants de la civilisation occidentale, Hagia Sophia, qui a près de 15 siècles. Celle-ci est réalisée par l'intégration de données résultant d'études archéologiques sur place et de mesures du comportement aux tremblements de terre dans un modèle numérique interactif. Cette étude permet d'avoir une meilleure compréhension du comportement de cet ensemble architectural complexe et permet de déterminer les besoins possibles d'un renforcement séismique.

ZUSAMMENFASSUNG

Gegenwärtig wird die seismische Tragfähigkeit eines der wichtigsten historischen Bauwerke der westlichen Zivilisation, der fast 15 Jahrhunderte alten Hagia Sophia bestimmt. Dazu werden die aus örtlichen archeologischen Studien mit Feldmessungen unter Umwelt- und Erdbebenerschütterungen gewonnenen Daten in ein interaktives numerisches Modell integriert. Die Forschungsarbeiten dienen dem besseren Verständnis des Verhaltens der komplexen Gebäudestruktur und klären den Bedarf nach gezielten Verstärkungsmassnahmen.

Historical Background

Because of the dual role that Justinian's great church of Hagia Sophia (Holy Wisdom) in Constantinople was to assume in both ecclesiastical and imperial liturgies, the architects, Anthemius of Tralles and Isidorus of Miletus, combined the traditional longitudinal basilican plan (a large rectangular hall having a high central space flanked by lower side aisles) with a great Roman central dome. Considering the close correspondence in scale between the original dome of Hagia Sophia and that of the Pantheon (ca. 118-128) it is likely that the earlier building provided the principal structural model for Justinian and his architects as they translated Roman concrete into Byzantine masonry [1]. Yet Hagia Sophia is a precedent-setting building: where the vast dome of the Pantheon rests on continuous, massive walls, four great arches and a like number of pendentives direct the weight of Hagia Sophia's superstructure to huge supporting piers (Fig. 1). The combination of large, glazed tympanum surfaces with a dome of monumental scale stands as one of the greatest architectural achievements of all times.

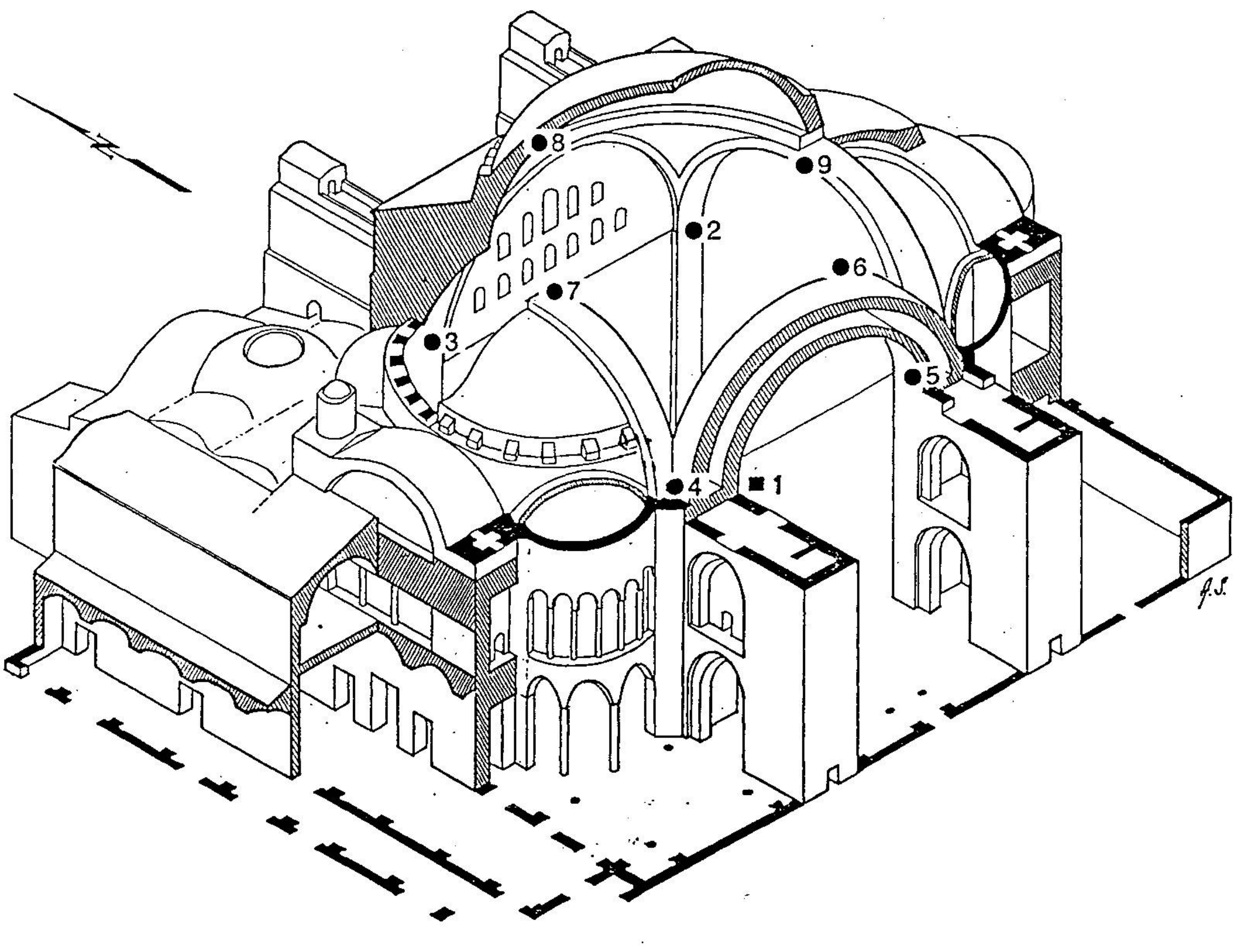

Figure 1. Analytical drawing of the Hagia Sophia structure (numbers indicate accelerometer locations).

In addition to the host of constructional challenges normally associated with such a major building project, political instability within the Empire required that Hagia Sophia, as the most visible symbol of Justinian's prestige in the capital, be completed as swiftly as possible. Begun in 532, construction proceeded in more or less horizontal layers until the erection in ca. mid 535, of the main arches, 31 meters in span and springing some 25 meters above the floor, to support the dome. Flying centering was probably used for assembly of these arches. And in all likelihood, this centering would not have been adequately tied to prevent enormous horizontal forces from impinging upon the upper portions of the main piers which then proceeded to tilt outward (the average, outward defection of the piers at the level of the springing is 45 cm). Before continuing with the construction, the exterior pier buttresses were reinforced and enlarged to their present height. The structure must have then seemed secure because the dome was raised in time to allow the vast building project to be completed in 537. Nonetheless, the great central dome fell in 558 after being subjected to two major earthquakes: the first one in August 553, and the second in December 557. A second dome having a higher profile than its predecessor was then erected in 558-562. Despite two partial collapses after earthquakes in the tenth century, and again in the fourteenth, the general form of the dome today remains unchanged from that of 562. But structural repairs associated with these incidents, as well as other adversities, have involved the placement of additional buttressing around the entire structure.

The present study is aimed at deriving a better understanding of the structural history of Hagia Sophia over its one-and-a-half-millennium life, including the strategies employed for its design and construction, and to determine the monument's current earthquake worthiness (and if necessary, recommend possible structural improvement). To accomplish these ends, several concurrent efforts are being undertaken which include: 1) creation of numerical models to account for both short- and long- term non-linear material behavior, including the consequences of cracking and effects of component deformation during the initial sequence of construction and subsequent structural modification; 2) determination, from physical and chemical tests, of the properties of the building materials, particularly the time-dependent behavior of early mortars; and 3) monitoring of measurements from accelerometers placed on the actual building structure under the action of vibrations produced by earthquakes.

Modeling

Two parallel types of numerical models are being formulated. The first, based on SAP 90 software, provides a purely linear elastic representation of the structure. The second, using the program FENDAC being developed by Colby Swan at Princeton, can account for the non-linear elasto-plastic behavior of masonry. Linear elastic models (including elastic models rendered non-linear by allowing cracking or weakening at specified tensile stress levels), although insensitive to values of elastic moduli, can provide essential information about overall stress distributions where the prototype is essentially composed of a single material (for example, to highlight regions of tension where cracking is likely to occur). The structure of Hagia Sophia, on the other hand, incorporates at least three major classes of materials: stone, brick, and mortar, the later containing brick dust and fragments that impart to it pozzolanic characteristics, but with a relatively long curing time (see below). In this case, criteria for modeling integrity are based on matching deformations; i.e., 1) predicted static deformations should agree in both form and magnitude with those observed in the prototype; and 2) natural frequencies and mode shapes computed by the models should match those determined from the on-site measurements. Because of the long curing time of the Byzantine mortar, however, both criteria cannot be simultaneously satisfied using the same material characteristics as is illustrated in the following.

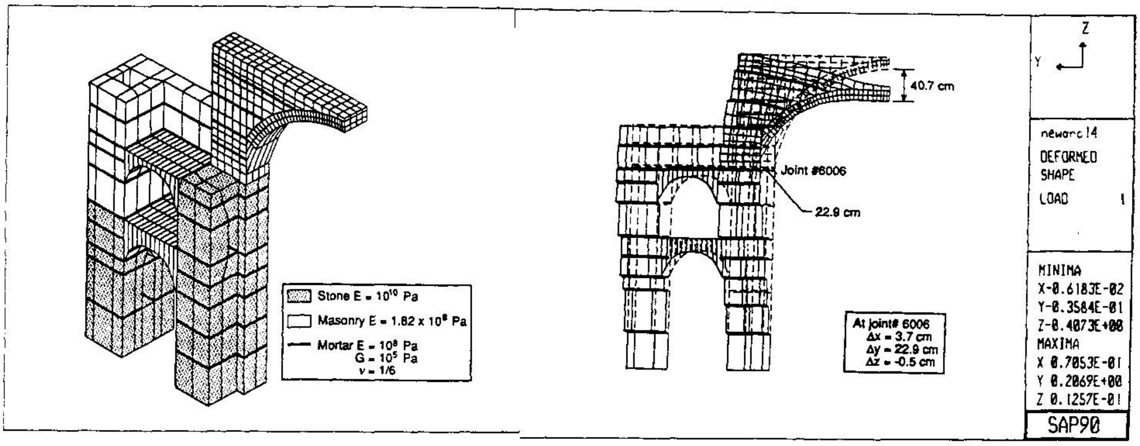

Figure 2. NEWARC 14: Partial finite element model of the Hagia Sophia structure under dead-weight loading.

Much of the effort to date has concentrated on the linear, SAP 90 models. A static example is afforded by the model designated as NEWARC 14, illustrated in Fig. 2. Mechanical properties of the constituent model materials are here determined from an inverse analysis that focused on the northeast main pier -- whose outward deflection at the point of springing was estimated to be 18 cm during initial construction, just prior to erection of the first great dome [2] -- the associated pier buttress, and adjacent great arches. The main piers are formed of stone, either limestone or a local granite, of up to about a meter in length and 45 cm thick. Mortar layers between stones are relatively thin, probably no more than several centimeters. For modeling, the stones are represented by elements whose thickness averages about 3 meters, interspersed with 25 cm of mortar. The pier buttress is assumed to incorporate similar stone and mortar layers up to the level of the first connecting arch, above which it is composed of brick masonry containing a large cavity for the existing stairwell. As shown in Figure 2, good results were achieved from the model by adopting the indicated values of Poisson's ratio and elastic and shear moduli. Using these properties for early material behavior, a full model was then constructed in three stages, allowing those portions of the structure in each stage to deform and weaken (at prescribed levels of tension) before the portions of subsequent stages were added. This model, while accounting for geometric non-linearity, is still elastic; nevertheless, it helps to reveal some of the characteristic behavior of the prototype that will influence future non-linear modeling.

Because Byzantine mortar today exhibits vastly altered physical properties from those displayed during the early stages of the building's construction, the indicated values need to be modified in order to allow the elastic model to meet the second, dynamic modeling criterion. This is exemplified by the eigen-value-analysis of the linear-elastic model designated as DYN26. With mechanical properties represented by: E (stone) = 10^{10} Pa; E (surcharge) = 2.5 $(10)^9$ Pa; E (tension areas) = 10^9 Pa; E (all else) = 5 $(10)^9$ Pa; and the density of the pendentives taken as three-fourths of the nominal density of masonry, the model indicated the first three vibrational modes described in Table 1 which also presents a comparison with measured ambient natural frequencies (from a test described below). In this context, it is interesting to note that the application of the material properties from the (static) NEWARK 14 model to the DYN26A model not only results in frequencies that are too low by a factor of seven, but mode shapes that are incorrect as well.

TABLE 1: Hagia Sophia Ambient Vibrational Modes

	Mode	Mean Measured Frequency	Calculated Frequency
1st:	east-west	1.8 Hz	1.97 Hz
2nd:	north-south	2.1 Hz	2.09 Hz
3rd;	rotational	2.4 Hz	2.38 Hz

Mortar Analysis

The critical role of mortar in the structure of the Hagia Sophia has led to a collaboration with researchers at the National Institute of Standards and Technology, Gaithersburg, MD. This investigation has the main objective of determining the composition of the material from which strength and deformation properties may be inferred [3]. Obtaining samples of mortar for such tests, however, presented some difficulty. Most interior surfaces of Hagia Sophia are covered by a thick layer of plaster on top of which are frescoes or mosaics. Brick and mortar are accessible in the interior passages of buttresses, but it is not always clear whether materials date from the original construction or from later restoration; nor is large-core drilling permitted. Fortunately, there is a significant collection of thoroughly documented samples from Hagia Sophia in the Dumbarton Oaks Museum, collected by Robert Van Nice during his decades long study of the building, and some of these have been kindly made available to us for testing.

The analysis, to derive both chemical composition and physical properties, involves a set of instrumental methods including quantitative X-ray diffraction, thermal analysis, and automated image analysis of polished sections under scanning electron microscopy. These conventional methods are supplemented by neutron diffraction and neutron and X-ray small angle scattering. Test results to date are consistent with pozzolanic mortar, the pozzolan being provided by the crushed brick. Pozzolan mortars offer far higher tensile strengths than mortars of pure lime; yet strengths for such mortars develop relatively slowly compared with modern Portland cement. Lea has reported tensile strength data for modern lime-pozzolan cements as 0.7-1.4 MPa after 7 days, 1.4-2.6 MPa after 28 days, 2.4-3.5 after 90 days, and 3.6-3.9 MPa after one year of curing [4].

The conventional civil engineering approaches for determining mechanical properties of materials are essentially ruled out here because of the lack of large specimens. Another generic problem with strength testing of brittle materials, ancient or modern, concerns finding a suitable specimen configuration for tensile strength measurement. A scaled-down modulus of rupture (bending) test seems the only feasible solution for the specimens at hand. The (static) modulus of elasticity can also be estimated from modulus of rupture measurements on thin disks using a method developed by Wittman and Prim [5]. And of course, the dynamic modulus of elasticity can be determined non-destructively by measuring the speed of sound waves through a specimen.

Monitoring

Two sets of measurements have so far been taken at the building site. The global dynamic behavior of the structure at low amplitudes was established from ambient vibration measurements taken at different locations within the building in 15 separate tests using a set of four seismometers. From preliminary model results, it was expected that the structure would exhibit greatest motion in the east-west direction. This first mode of vibration was confirmed by the ambient measurements, as were also the second and third modes, as shown in Table 1 above. Of import too for the overall building study, the measured frequency spectrum falls within the frequency content of typical earthquakes observed on the North Anatolian fault.

The structure of Hagia Sophia is expected to behave non-linearly at higher, earthquake-level forces. To delineate the higher-amplitude behavior, a network of digital strong-motion accelerometers (Kinemetrics SSA-2 accelerometers) having 30 channels of readout was permanently mounted at the locations shown in Figure 1. These were chosen to capture the motion of the main structural elements supporting the central dome including the four main piers and adjoining great arches. Ground motion at the base of the main piers is measured by the accelerometer labeled 1. (For purposes of analysis, it has been assumed that all pier bases lie on bedrock so that the effect of any base variation on the dynamic behavior of the response above is negligible.) Behavior of the main piers at the level of the vault springing is measured by the array labeled 2-5; and the behavior at the crown of the great arches is measured by the array labeled 6-9. The system response is then characterized by the motion of the two arrays relative to one another and to the base.

The strong-motion accelerometer system was triggered on 22 March 1992 by a 4.8 Richter-scale magnitude earthquake whose epicenter was at Karacabey, Turkey, 120 km south of Istanbul. This earthquake induced the type of horizontal acceleration component time history represented in Figure 3. While the traces indicate a non-stationary process, it is possible to consider three approximately stationary periods or time windows: the first subtending the beginning of the record up to about 10 seconds; the second, from 15 to 25 seconds; and the third, from 30 to 40 seconds. Spectral analysis [6] of the data led to something of a revelation: the observed frequencies indicate non-linear behavior for the masonry structure even at these rather low response levels. The peak frequency at the springing level of a main pier varies from a high of 2.03 Hz in the first window to 1.74 in the second and back to 1.84 Hz in the third, a pattern corresponding to that of a non-linear system in which the system characteristics vary according to the intensity of loading.

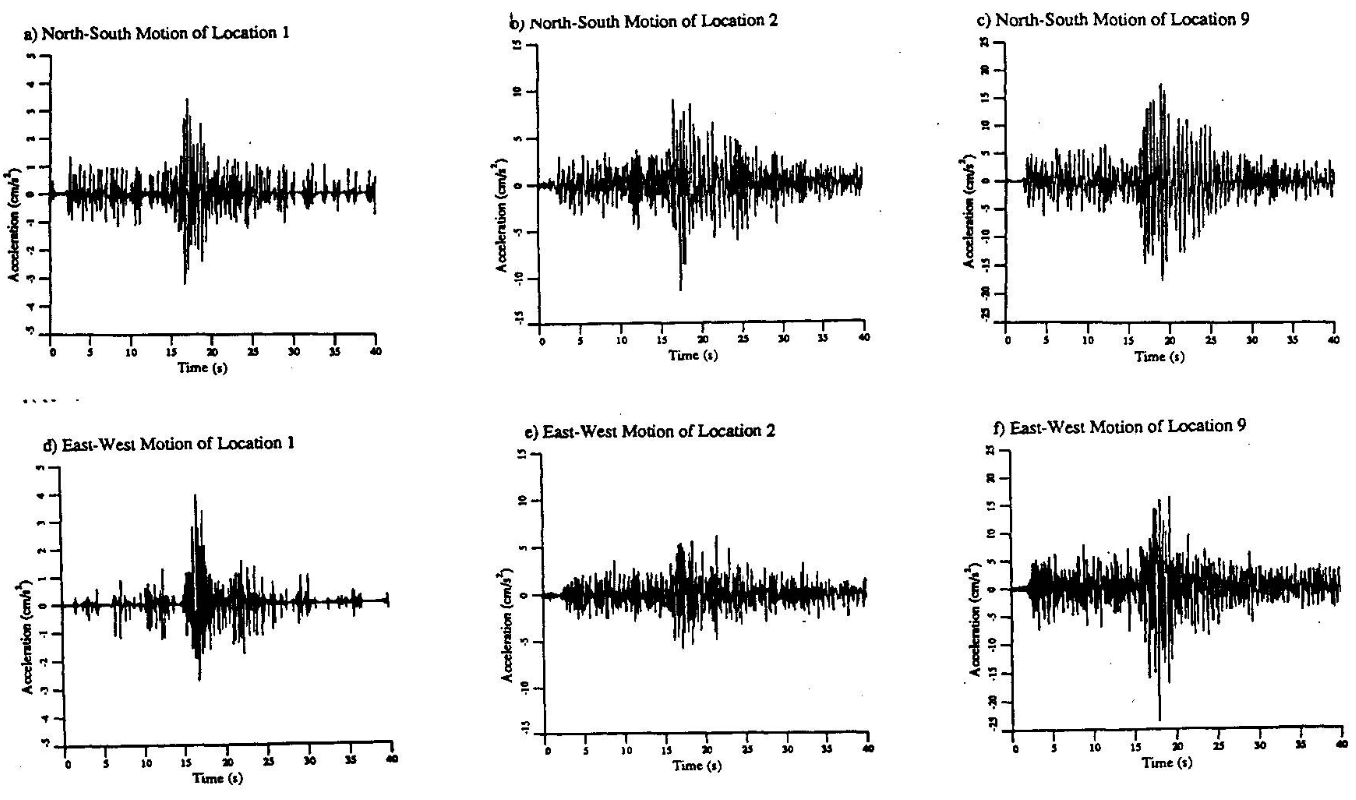

Figure 3. Horizontal acceleration component time histories in Hagia Sophia during earthquake of 22 March 1992.

If indicating somewhat lower frequencies, nominal vibration modes established from the earthquake data (Table 2) were similar to those established from the ambient tests (cf. Table 1):

TABLE 2: Earthquake-induced Vibrational Modes

Mode	Mean Measured Frequency
1st: east-west lateral	1.73 Hz
2nd: north-south lateral	1.85 Hz
3rd: rotational, about z-axis	2.3 Hz

Conclusion

Although much additional work on this project remains to be undertaken, some new perceptions about the structure of this magnificent monument are already coming to light. The first concerns the role of the slow-curing pozzolanic mortar during the initial construction process. The mortar allowed the development of early, large deformations, but its inherent plasticity also helped to reduce possible cracking. A second insight derives from the observed (in the numerical models) predisposition of the east and west great arches, that provide support to the central dome, to warp out-of-plane under gravity loading. With additional out-of-plane motion caused by an earthquake (in this regard note also that the lowest vibration mode is east-west), the basis for the collapse of adjacent portions of the central dome (in the east and west) at different times throughout the building's history becomes more clear.

Perhaps more important for historical interpretation of the Hagia Sophia structure is the finding that the changing of the first to the second dome configuration had only small effect on relieving the total outward thrusts on the main piers. This new understanding goes counter to almost every modern historical explication of the second dome form. Finally, data from the single, low intensity earthquake so far experience by the instrumented Hagia Sophia structure has revealed the non-linear response of its masonry.

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