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# Diagnosis of Masonry Towers by Dynamic Identification

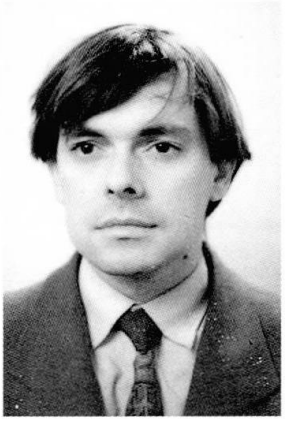
## Détermination de l'état de tours en maçonnerie par identification dynamique

## Diagnose von Mauerwerktürmen mittels dynamischer Identifikation

**Michele FANELLI**
Professor
ENEL-CRIS
Milan, Italy

M. Fanelli, born in 1931, received his civil engineering degree at the Univ. of Bologna. He became Professor in Construction Techniques in 1970. He is director of the Centre for hydraulic and structural research of ENEL.

**Alberto PAVESE**
Civil Engineer
Univ. of Pavia
Pavia, Italy



A. Pavese, born 1962, received his PhD from the Politecnico of Milan. His research activities are concentrated on dynamic tests and linear structural identification of masonry structures. He is now at the Univ. of Pavia with post-doctor fellowship.

## SUMMARY

The behaviour of monument structures can be assessed by using experimental tests in the dynamic field. Structural characteristics and pathologies can thus be investigated. Two parametric techniques are presented to obtain an optimal numerical model. The first one is an iterative procedure, based on eigenparameter sensitivity detected by parametric studies and model-real behaviour comparison; and secondly, the test modelling is performed through special formulation of the model-real behaviour discrepancy function.

## RÉSUMÉ

Le comportement des structures de monuments peut être déterminé par des essais dynamiques, ce qui permet de connaître les caractéristiques et l'état de la construction. Deux techniques paramétriques sont présentées qui permettent d'obtenir des modèles numériques optimaux: Une procédure itérative, basée sur la sensitivité des paramètres propres, détectés par des comparaisons entre des études paramétriques et du modèle réel ainsi qu'un modèle expérimental, réalisé à l'aide d'une formulation spéciale du comportement du modèle réel, basé sur une fonction de discrépances.

## ZUSAMMENFASSUNG

Das Verhalten von historischen Bauwerken kann durch experimentelle dynamische Versuche bestimmt werden, die Rückschlüsse auf Eigenschaften und pathologischen Zustand der Gebäudestrukturen erlauben Zwei parametrische Techniken werden vorgestellt, die optimale numerische Modelle ermöglichen: zum einen ein iteratives Verfahren, bei dem für die Abstimmung zwischen Modell- und Prototypverhalten die Empfindlichkeit der Eigenschwingungsparameter aufgrund von Parameterstudien eingesetzt wird; zum anderen eine Optimierung mittels einer speziell formulierten Diskrepanzfunktion.

## 1. INTRODUCTION

The knowledge of the behaviour of ancient masonry buildings is totally inadequate for attempting to construct any kind of reasonably representative numerical model. The main problems which arise stem from the identification of the composition ratios of the material properties.
The research basically aims at evaluating the possibility of identifying a numerical model of the towers through dynamic experiments, and at studying the sensitivity of the techniques applied in identifying any local defects in order to diagnose the presence of local defects (i.e. voids, cracks, material degradation). The experiments were preceded by numerical studies with simple models aimed at evaluating the degree of significance of the various parameters involved and at defining their probable ranges of variation.
Scope of this paper is to illustrate the results obtained from vibration tests performed on medieval towers and then to propose one method for identifying stiffness matrix based both on numerical-experimental comparison and sentivity analysis. An alternative identification technique named "PLECTRON" is illustrates at the end of the paper and an example of application will be presented in order to demonstrate its limits and capabilities in real cases.

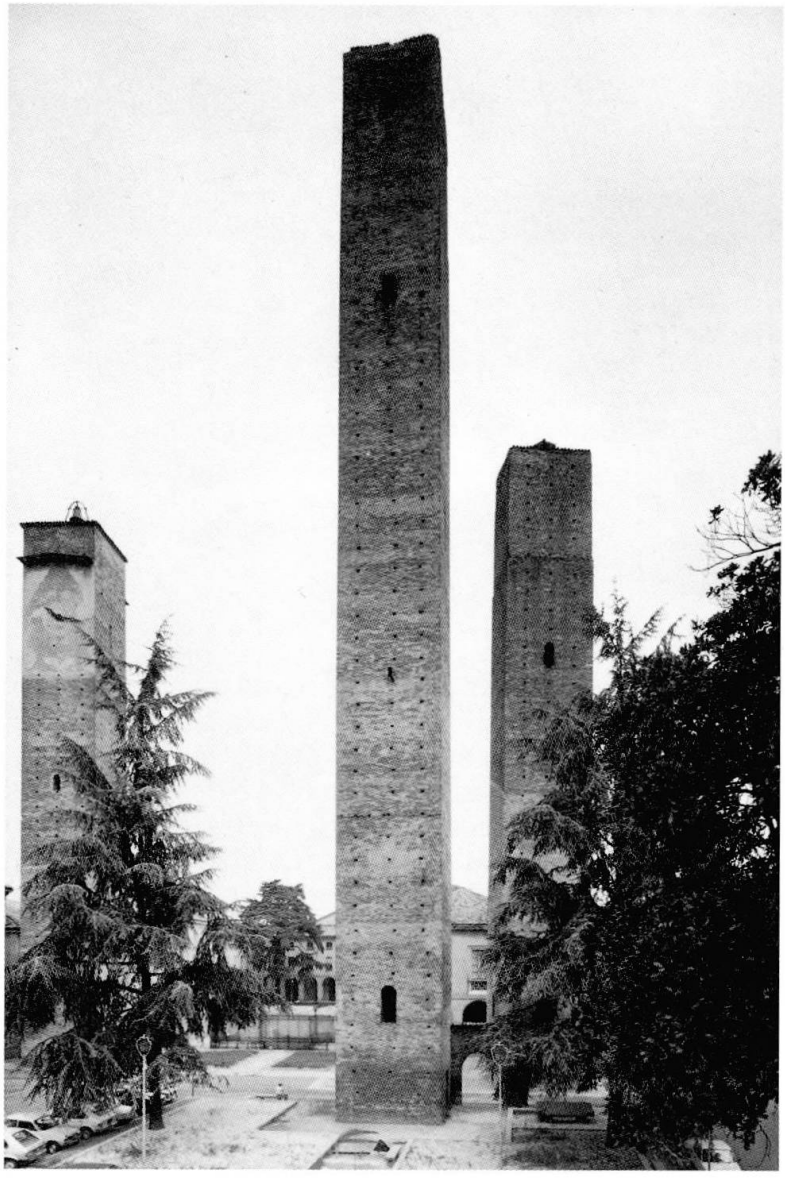


Figure 1 Orologio's, Majno's, and Fraccaro towers

The dynamic identification study presented here refers to University tower in Pavia, also known as the Fraccaro tower (Fig. 1 the right one). Fraccaro tower extend vertically for approximately 40 m above ground, while has openings variously arranged along the height and in plan, and rests on the ground by means of a foundation block whose dimensions are similar to the parts above ground and whose height is approximately 5 m. The masonry of the tower consists of two brickwork faces, about 15 cm thick, the intervening void being filled with mortar mix, bricks and stonework, whose thickness varies according to the height. The composition features of the foundations are similar to elevation walls. As far as the construction date is concerned, some historians date the towers in the early years of our millenium. Laboratory tests performed on specimens taken from blocks of masonry of the Civica tower, which collapsed in March 1989, have provided information on the mechanical properties of the material [1,2]. Main results of these tests are summarized in table 1

| | | | | |
|---|---|---|---|---|
| $f_u$ | 3.1 | 0.17 | 4.1 | 0.08 |
| $f_{fessur}$ | 2.5 | 0.20 | 3.1 | 0.13 |
| $E_t$ | 3314 | 0.31 | 2906 | 0.09 |
| $E_s$ | 2670 | 0.34 | 2654 | 0.22 |
| $\nu_t$ | 0.16 | 0.27 | 0.22 | 0.45 |
| $\nu_s$ | 0.32 | 0.05 | 0.35 | 0.02 |

Table 1 Results tests performed on specimens taken from blocks of masonry of the Civica tower

Further studies are perfomed on others two towers in figure 1, Majno's tower and Orologio's towers bat the results will be object of a successive work.

## 2. DYNAMIC MEASUREMENTS IN SITU

For the purposes of comparison on the Fraccaro tower, the techniques of ambient vibration and forced excitation were used in the present study. To this end velocity transducers, seismometers and Ladirs (which are coherent light sensors based on the Michelson's interferometer) are used in order to study oscillation induced by the exciting source.
The signals were stored in a digital memory and processed by a spectrum analyser using windowing tecnique in order to obtain better definition of the spectrum amplitudes (Flat-top window), resonance frequencies and damping ratios (Hanning and uniform windows). Table 2 reports the results of the spectrum analysis of the signals, recorded during two differents tests on Fraccaro tower in terms of resonance frequencies and damping ratios determined by the half-power method.
The forced vibration tests with a vibrodyne are currently one of the most effective methods for dynamically caracterizing a structure. In the case in question a rotating vector generating vibrodyne was cantilever-mounted on the north wall of the Fraccaro Tower at a height of 27.25 m.

| $\omega_i$ | Ambient tests | | Forced tests July 90 | | | | Ambient tests | | Forced tests July 91 | | | |
|---|---|---|---|---|---|---|---|---|---|---|---|---|
| | July 90 (E-W) | | Direct. E-W | | Direct. N-S | | July 91 (E-W) | | Direct. E-W | | Direct. N-S | |
| | Hz | ξ | Hz | ξ | Hz | ξ | Hz | ξ | Hz | ξ | Hz | ξ |
| 1° flex | 0.75 | 2.70 | 0.73 | 2.70 | 0.76 | 2.60 | 0.74 | 2.7 | 0.72 | 2.4 | 0.740 | 2.30 |
| 2° flex | 3.56 | 1.70 | 3.46 | 1.70 | 3.58 | 1.50 | 3.54 | 1.2 | 3.44 | 1.0 | 3.520 | 1.00 |
| 1° tors | 4.25 | 0.80 | 4.18 | 1.0 | 4.18 | 1.0 | 4.36 | 0.7 | 4.24 | 1.1 | 4.240 | 1.10 |
| 1° axial | - | - | - | - | - | - | 6.30 | 5.5 | - | - | - | - |
| 3° flex | 8.00 | 1.10 | 7.86 | 2.50 | 7.76 | 1.50 | 8.18 | 2.3 | 7.84 | 2.5 | 7.960 | 2.70 |
| 2° tors | 11.44 | 0.60 | - | - | - | - | 11.70 | 0.3 | 11.60 | 1.0 | 11.60 | 1.00 |
| 4° flex | - | - | - | - | - | - | 13.90 | 3.5 | 13.72 | 4.0 | 13.80 | 3.50 |

Table 2 Ambient and forced vibrationts tests - Frequencies and damping ratios

The results are also reported in table 2; it is important to note that by comparison of the results obtained by forced tests, only in W-E direction, with the corresponding values obtained through ambient vibration tests it is possible to verify a good correspondence up to the third frequency of vibration (torsional shape); beyond this limits the tower behaviour becomes dependent on the intensity of exciting force. This can determine differente values both in frequencies and damping, probably due to the fact that different mechanisms are involved in different levels of forces. Same conclusion is also possible to do it on damping ratios.

## 3. NUMERICAL MODELLING

The Fraccaro tower was modelled with linear elastic solid elements. The use of a linear elastic model to represent a structure which by its very nature is non-linear can be justified by the fact that the level of force with which the dynamic tests were performed limited, generally not greater than 5 kN. The mechanical properties of the materials used in modelling are those obtained from the laboratory tests and in situ [1,2]. Table 3 give the results obtained from the numerical analysis compared with corresponding experimental values.

| Natural | Direction N-S | | | Direction E-W | | |
|---|---|---|---|---|---|---|
| Frequencies | Experim. | Numerical | Δ | Experim. | Numerical | Δ |
| | Hz | Hz | % | Hz | Hz | % |
| 1°flex | 0.73 | 0.709 | -2.87 | 0.76 | 0.705 | -7.2 |
| 2°flex | 3.46 | 3.834 | +10.8 | 3.58 | 3.867 | +8.0 |
| 1°tors | 4.18 | 5.059 | +21.0 | 4.18 | 5.059 | +21.0 |
| 3°flex | 7.86 | 9.395 | +19.5 | 7.76 | 9.406 | +21.0 |

Table 3 experimental and numerical eigenvalues before identification procedure

The error in evaluating resonance frequencies is particularly evident in the torsional mode and in the 3rd flexural mode, in this case with approximately a 20% error is shown. The modal parameters of the structure depend on stiffness, mass distrbution, restraint conditions and dissipative mechanisms characteristics. Further on it is assumed that the terms of the mass matrix are determined with sufficient precision from the volumes calculated and from the specific density and that any cavities are such that they do not cause appreciable variation.

| Natural | Direction N-S | | | Direction E-W | | |
|---|---|---|---|---|---|---|
| Frequencies | Experim. | Numerical | Δ | Experim. | Numerical | Δ |
| | Hz | Hz | % | Hz | Hz | % |
| 1°flex | 0.73 | 0.727 | -0.40 | 0.76 | 0.741 | -2.5 |
| 2°flex | 3.46 | 3.621 | +4.65 | 3.58 | 3.732 | +4.24 |
| 1°tors | 4.18 | 4.242 | +1.48 | 4.18 | 4.242 | +1.48 |
| 3°flex | 7.86 | 8.494 | +8.07 | 7.76 | 8.550 | +10.2 |

Table 4 Experimental and numerical eigenvalues after identification procedure

The damping ratio being not greater than 2.7% it appears resonable to neglect in the calibration procedure the dissipative behaviour. In the model the tower was divided in 13 zones delimited by orizontal planes and moduli of elasticity of the portions were made to vary within a range of possible values, determining the ensuing variations in frequencies and modal forms. The variations in E are considered on individual layers of the mesh, maintaining the value constant on the rest of the tower and eliminating the combinations of variation in the various layers, which would otherwise increase the number of analyses required Fig. 2 and 3 shows the variations in modal and frequencies domain. It can be seen that the rotations of the torsional mode are very sensitive to the changes in modulus of elasticity over most of the height, whereas in the 2nd and 3rd bending modes the positions of the nodes undergo appreciable changes when the variations in E affect the zones of maximum bending of the modes. On observing the results obtained in the modal analysis of the mesh with uniform E, a tendency to overestimate the frequencies and ratios in relation to the experimental can be noted. By comparing then the experimental shapes, e.g. in direction E-W, is possible to note that position of node and maximum deflection of the modal shapes are localized in different heights with respect

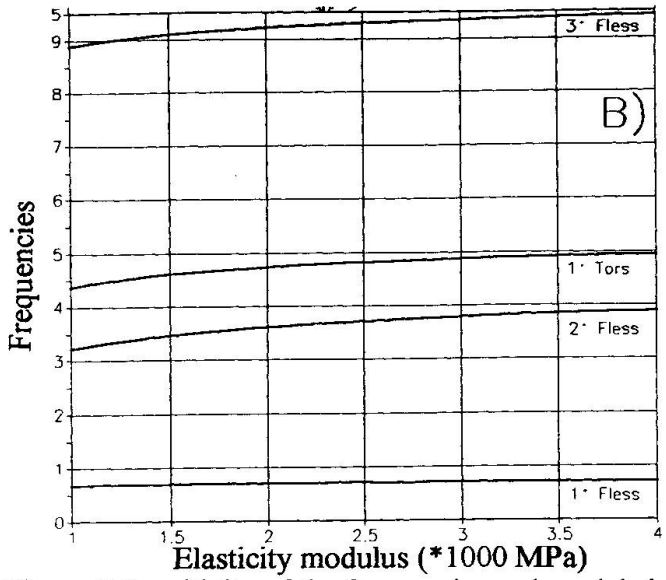



Figure 2 Sensitivity of the frequencies and modal shapes

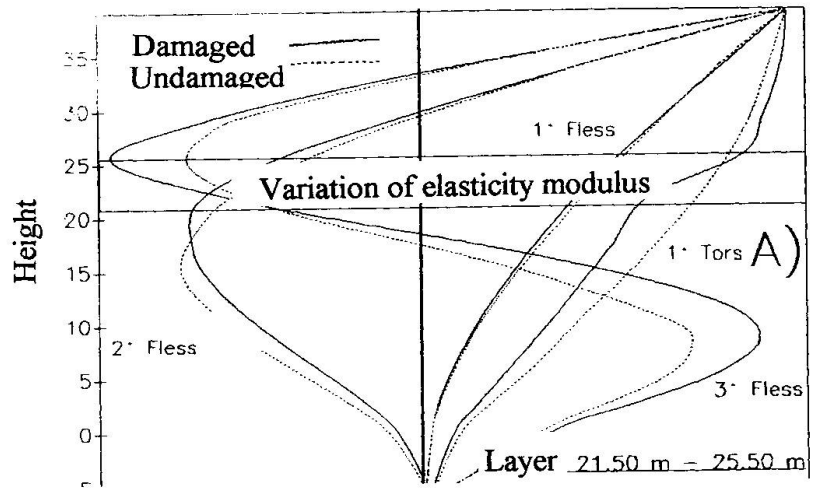



Figure 3 Sensitivity of the frequencies and modal shapes

to experimental observation. By adjusting the values of modulus of elasticity by successive approximations in the zones of maximum bending, a distribution is achieved which provides the frequency response given in Table 3. It can easily be seen that the frequencies have values closer to those of the experimental case. The result was obtained by modifying the modulus of elasticity in the zones of maximum bending of the second and third modal shapes.

## 5. IDENTIFICATION BY INVERSE PARAMETRIC TECHNIQUE

At the most basic level, it is common practice to "adjust" the modulus of elasticity value in order to bring into line the value of the first natural frequency calculated, by means of a mathematical model of the structure, with that obtained from field. It is however commonly found that, having made this adjustment, there is then no automatic agreement between the "theoretical" and experimental values of the successive frequencies (see previous section for example). It has recently been established that important information is contained not only and not so much in the natural frequency values, but also and possibly above all in the corresponding modal shapes obtained from measurements made on the

actual object. Matching the mathematical model and actual structure through differentiated adjustment by zones of the elastic constants is normally performed by trial and error, aiming at "matching" on the modified mathematical model the greatest number of natural frequencies and corresponding modal shapes. If it is true, as supposed, that the structure provides us -through test results- with a large copy of information revealing its modal shapes, it is worthwhile investigating whether it is possible to exploit this information directly in a zonal parameters identification.
A direct identification procedure of this kind can be produced in theory, apart from practical difficulties of performance and leaving aside momentarily the problems of precision and reliability of results. The last part of the paper intends to analyses the potentialities of this method in view of a future application at a reale structure like a the University's tower.
We shall consider a structure for which the modal shapes of the first natural P modes have been extracted from in situ tests. We then suppose that this structure can be discretized by means of a suitable linear elastic mathematical model; with reference to unforced, undamping vibration problems we can write the system equilibrium equations

$$\underline{\underline{K}}\,\underline{\Phi}_j = \omega_j^2\,\underline{\underline{M}}\,\underline{\Phi}_j \quad (1)$$

where $\underline{\underline{K}}$ and $\underline{\underline{M}}$ are stiffness and mass matrix of reference structure, $\omega_j$ is the jth eigenvalues whereas $\underline{\Phi}_j$ is the corresponding eigenvector obtained from experimental tests.
We can note how the matrix of rigidity K of the mathematical model may be written formally as follows:

$$\underline{\underline{K}} = \varepsilon_1\underline{\underline{K}}_1 + \varepsilon_2\underline{\underline{K}}_2 + \cdots + \varepsilon_n\underline{\underline{K}}_n \quad (2)$$

where $\underline{\underline{K}}_i$ are the matrices of rigidity constructed for the entire discretization on the basis of the incidence matrices of the individual zones defined previously to differentiate the elasticity constants, whereas $\varepsilon_i$ are the multiplicative coefficients of a reference elasticity modulus.
Similarly we can write for the matrix of mass M:

$$\underline{\underline{M}} = \mu_1\underline{\underline{K}}_1 + \mu_2\underline{\underline{M}}_2 + \cdots + \mu_n\underline{\underline{M}}_n \quad (3)$$

where $\underline{\underline{M}}_j$ are the mass matrices constructed for the entire discretization, whereas $\mu_j$ are multiplicative coefficients of reference density.
We now wish to establish that the mathematical model with differentiated elasticity and inertia constants matches as perfectly as possible the frequencies and modal shapes determined in the experimental tests. If it were possible to achieve perfect agreement, for each mode j, clearly being $\omega_j^2$ and $\underline{\Phi}_j$ eigenparameters correlated to tested structure and $\underline{\underline{K}}$, $\underline{\underline{M}}$ matricies connected to reference structure wich behaviour we want to superimpose to real one, in general we can write

$$\underline{\underline{K}}\,\underline{\Phi}_j - \omega_j^2\,\underline{\underline{M}}\,\underline{\Phi}_j = \underline{D}_j \neq \underline{0} \quad (4)$$

Taking into account (2), (3) e (4) are able to construct a global error norm on the p modes:

$$N = \sum_{j=1}^{p} \underline{D}_j^t\,\underline{D}_j = \sum_{j=1}^{p} (\underline{\Phi}_j^t\,\underline{\underline{K}}^t - \omega_j^2\,\underline{\Phi}_j^t\,\underline{\underline{M}}^t)(\underline{\underline{K}}\,\underline{\Phi}_j - \omega_j^2\,\underline{\underline{M}}\,\underline{\Phi}_j) \quad (5)$$

where evidently

$$N = N(\varepsilon_1, \cdots, \varepsilon_n, \mu_1, \cdots, \mu_m) \quad (6)$$

The criterion of maximum likelihood of the mathematical model is now to be expressed in the conditions of minimising the global error norm N, in relation to the adjustment variables $\varepsilon_i$ and $\mu_j$:

$$\frac{\partial N}{\partial \varepsilon_1} = 0, \cdots\cdots, \frac{\partial N}{\partial \varepsilon_n} = 0$$
$$\frac{\partial N}{\partial \mu_1} = 0, \cdots\cdots, \frac{\partial N}{\partial \mu_m} = 0 \qquad (7)$$

wich is a linear system in n+m equation; in matrix form we can write

$$\underline{\underline{A}}(\varepsilon_i, \mu_j) = \underline{0} \qquad (8)$$

It is possible to verify that one of the equations in (8) is linearly dependent on the others. Therefore $\varepsilon_i$ and $\mu_j$ are determined, assuming one of them to be known. In general it will be possible to take one of the densities as known: it can immediately be seen that an error in the estimate of the density supposed to be known, expressed by a factor $\mu_j$ relation to the "true" value, involves an equal factor of error for all the densities, and factor likewise equal to μ for all the moduli of elasticity. That procedure directly and simultaneously takes into account experimental information ($\omega_j$ and $\underline{\Phi}_j$).

The question naturally arises whether the proposed procedure can actually be performed and whether it is effective. Without prejudice to the fact that only application to sufficiently complex real cases can demonstrate the effective potential and limits of the procedure, the intention was to test it on a very simple case, which we shall now illustrate. With reference to Fig. 4, we have taken the system with 20 degrees of freedom. Experimental test were simulated by a F.E. code.

| Real | Refer. | $\varepsilon_i, \eta_j$ | Identification results | | | | | |
|---|---|---|---|---|---|---|---|---|
| values | values | | 1° mode | | 2° mode | | 3° mode | |
| 5000 | 1000 | 5.0 | 2.7976 | 4.9745 | 4.9996 | 4.9999 | 5.0004 | 5.0000 |
| 4000 | 1000 | 4.0 | 2.2381 | 3.9796 | 3.9996 | 3.9999 | 4.0004 | 4.0000 |
| 3500 | 1000 | 3.5 | 1.9583 | 3.4821 | 3.4997 | 3.4999 | 3.5003 | 3.5000 |
| 3000 | 1000 | 3.0 | 1.6785 | 2.9847 | 2.9997 | 2.9999 | 3.0003 | 3.0000 |
| 2500 | 1000 | 2.5 | 1.3988 | 2.4872 | 2.4998 | 2.4999 | 2.5002 | 2.5000 |
| 2000 | 1000 | 2.0 | 1.1190 | 1.9898 | 1.9998 | 1.9999 | 2.0002 | 2.0000 |
| 2500 | 1000 | 2.5 | 1.3987 | 2.4872 | 2.4997 | 2.4999 | 2.5002 | 2.5000 |
| 3000 | 1000 | 3.0 | 1.6785 | 2.9846 | 2.9997 | 2.9999 | 3.0002 | 3.0000 |
| 3500 | 1000 | 3.5 | 1.9582 | 3.4821 | 3.4996 | 3.4999 | 3.5003 | 3.5000 |
| 4000 | 1000 | 4.0 | 2.2380 | 3.9816 | 3.9996 | 3.9999 | 4.0003 | 4.0000 |
| 26873 | 26873 | 1.0 | 1.0000 | 0.9949 | 1.0000 | 0.9999 | 1.0000 | 1.0000 |
| 25420 | 25420 | 1.0 | 0.5595 | 0.9949 | 0.9999 | 0.9999 | 1.0001 | 1.0000 |
| 29294 | 29294 | 1.0 | 0.5595 | 0.9949 | 0.9999 | 0.9999 | 1.0000 | 1.0000 |
| 25562 | 25562 | 1.0 | 0.5594 | 0.9948 | 0.9999 | 0.9999 | 1.0001 | 1.0000 |
| 23000 | 23000 | 1.0 | 0.5595 | 0.9949 | 0.9999 | 0.9999 | 1.0000 | 1.0000 |
| 17915 | 17915 | 1.0 | 0.5593 | 0.9946 | 0.9999 | 1.0000 | 1.0001 | 1.0000 |
| 15494 | 15494 | 1.0 | 0.5597 | 0.9952 | 0.9988 | 0.9989 | 1.0000 | 1.0000 |
| 14525 | 14525 | 1.0 | 0.5595 | 0.9948 | 0.9999 | 0.9989 | 1.0000 | 1.0000 |
| 14525 | 14525 | 1.0 | 0.5595 | 0.9946 | 0.9999 | 0.9999 | 1.0000 | 1.0000 |
| 7262 | 7262 | 1.0 | 0.5595 | 1.0000 | 0.9999 | 1.0000 | 1.0000 | 1.0000 |

Table 5 Identification of stiffness and mass constants using flexural shapes

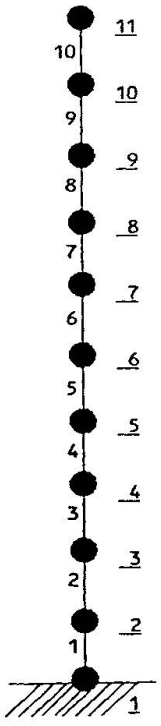



Fig.4 Example

In table 5 are shown the identification results; the first ten values are the coefficients $\varepsilon_i$, whereas the others values are related to mass concentrated to nodal points. Underlined values were imposed in the identification procedure. It should be noted that there is greater precision in the parameters estimation when mass was imposed at the top of the cantilever, according to the fact that in this zone the contribution to total kinetic energy takes on significant value. In general accuracy increase according to modal shapes with a large number of nodes.

## 4. CONCLUSIONS

This work illustrates two identification methods. The first method, useful in preliminary structural caracterization, is based on an iterative procedure of arrangement of elasticity modulus distribution using sensitivity analysis and numerical-experimental comparison. The second method basically consists of modifying the stiffness matrix of the structure on the basis of the frequencies and of the modal shapes measured experimentally. This method is translated in a calculation program obviously named "PLECTRON". This method have demonstrated good precision and accuracy in a simple case of application bat presently is waiting further applications to check its efficiency in practical problems Finally we will attempt to make some preliminary critical considerations on the method proposed. It is clearly that for an effective application to cases of a complexity corresponding to objects of actual importance, the information on the experimental modal shapes would have to be accurate and complete. Since in practice a completeness with the same detail as the discretization of the mathematical model is not feasible, the experimental information will have to be completed adequately before being able to develop the method at a numerical level. This could be performed for example using cubic splines for those components of the modal vectors some of which are in fact measured (e.g. the orizontal displacements due to the flexural deformations of a tower). The diagnostic aspect of the proposed method would also appear to be of importance, not only in a comparative sense, i.e. to reveal any variations in time between one in-situ test and another, but in an absolute one with regard to the space variation of variables. In concept, the method could alos be extended to achieve a differentiated estimates of damping coefficients of the various zones (e.g. introducing complex rather than real $\varepsilon_i$ adjustment factors in the case of Hysteretic damping).

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