

**Zeitschrift:** IABSE reports = Rapports AIPC = IVBH Berichte  
**Band:** 69 (1993)  
  
**Artikel:** Serviceability analysis of structures including creep effects  
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**DOI:** <https://doi.org/10.5169/seals-52576>

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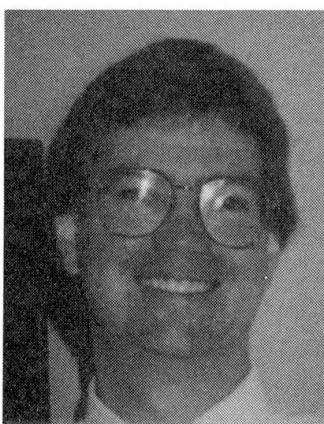
## Serviceability Analysis of Structures Including Creep Effects

Aptitude au service de bâtiments, tenant compte du fluage

Gebrauchstauglichkeit unter Berücksichtigung von Kriecherscheinungen

### Kenneth J. FRIDLEY

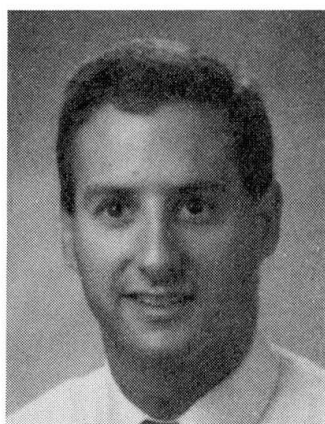
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### SUMMARY

Certain current and possible future building materials (e.g. wood, concrete, structural plastics, etc.) experience time-dependent deflections when subjected to sustained load. The incorporation of the viscoelastic response of materials into the reliability analysis of structures and the effect of viscoelastic material behaviour on the serviceability performance and reliability of structural elements are discussed.

### RESUME

Certains matériaux de construction présents et futurs (bois, béton, plastiques structurels, etc.) souffrent de déformations qui dépendent du temps lorsqu'ils sont soumis à des charges continues. L'incorporation du comportement viscoélastique des matériaux à la sécurité de l'analyse des bâtiments et l'effet de propriétés du matériau viscoélastique à la performance de fonctionnement et à la sécurité des éléments structurels sont discutés.

### ZUSAMMENFASSUNG

Manche Baustoffe, die derzeit oder möglicherweise in Zukunft verwendet werden (z.B. Holz, Beton, Kunststoffe etc.) unterliegen zeitabhängigen Verformungen, wenn eine ständige Last auf sie einwirkt. Dieser Artikel befasst sich zum einen damit, wie viskoelastische Reaktionen von Baustoffen in die entsprechenden Zuverlässigkeitsanalysen eingearbeitet werden können, zum anderen, wie das viskoelastische Verhalten dieser Baustoffe die Funktionstüchtigkeit und Zuverlässigkeit von Tragelementen beeinflusst.



## 1.0 INTRODUCTION

Certain current and possible future building materials (e.g., wood, concrete, structural plastics, etc.) experience time-dependent deflections when subjected to sustained load. Depending on the material and as illustrated in Fig. 1 for structural lumber, loads of duration as short as one day may result in an appreciable increase in deflection [5]. Current deflection serviceability design checks are based on an elastic analysis and an essentially time-independent approach. Typically, the calculated elastic deflection resulting from a maximum assumed service load is multiplied by a creep factor to account for creep effects (e.g., [1, 7]). This approach does not address previous load history or even duration of the load under consideration. Owing to advances in time-dependent reliability analysis, the availability of large-scale computing for simulation, and stochastic creep models developed for construction materials, the effects of creep can be included in reliability analyses of structural building members and systems.

## 2.0 VISCOELASTIC MATERIALS

The characteristics of a viscoelastic material include (1) elastic deformation, (2) primary (decelerating) creep, (3) secondary (steady-state) creep, (4) zero stress creep recovery, and (5) stress relaxation. Various models are available for predicting the stress-strain-time relationship for a material, including empirical, semi-empirical, and phenomenological models. A commonly used phenomenological model which, with a single expression, predicts the five characteristics listed above is the four-element Burger model consisting of a Maxwell element added serially to a Kelvin element. The relationship between stress and strain prescribed by the Burger model is as follows:

$$\sigma(t) = K_e \varepsilon_e(t) + \mu_v \frac{d\varepsilon_v(t)}{dt} + \mu_k \frac{d\varepsilon_k(t)}{dt} + K_k \varepsilon_k(t) \quad (1)$$

where  $\sigma(t)$  is the stress in the material,  $K_e$  is the Maxwell element spring parameter,  $\mu_v$  is the Maxwell element viscous damper parameter,  $K_k$  is the Kelvin element elastic spring parameter, and  $\mu_k$  is the Kelvin element viscous parameter. The strains  $\varepsilon_e(t)$ ,  $\varepsilon_v(t)$ , and  $\varepsilon_k(t)$  are those portions of the total strain owing to the Maxwell element elastic spring (elastic strain), the Maxwell element viscous damper (visco-plastic strain), and the Kelvin element (visco-elastic strain), respectively.

Solving (1) for the case of constant applied stress (i.e.,  $\sigma(t) = \sigma$ ) yields

$$\varepsilon(t) = \frac{\sigma}{K_e} + \frac{\sigma t}{\mu_v} + \frac{\sigma}{K_k} \left[ 1 - \exp\left(-\frac{K_k t}{\mu_k}\right) \right] \quad (2)$$

where  $\varepsilon(t)$  is the total, time-dependent strain at time  $t$  beyond the application of the constant stress  $\sigma$ .

Depending on the material under consideration, the characteristic viscoelastic responses may differ; that is, the relative contribution of elastic, viscoelastic, and visco-plastic strain to the total strain may vary depending on the material. Regardless, the apparent time-dependent stiffness of the material,  $K(t)$ , may be expressed in terms of its elastic stiffness,  $K_e$ , or

$$K(t) = \frac{K_e}{\kappa(t)} \quad (3)$$

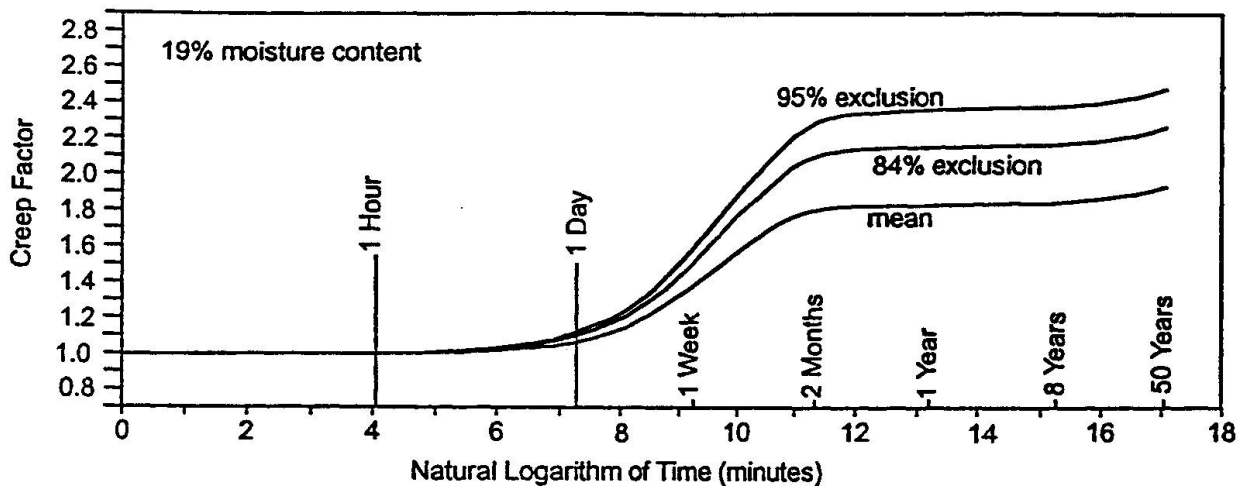


Fig. 1. Relative Creep Response of Structural Lumber [5].

where  $K(t)$  is often referred to as the relaxation modulus and  $\kappa(t)$  is a time-dependent creep factor. The creep factor  $\kappa(t)$  is defined by the particular viscoelastic constitutive model assumed in the analysis and is a function of the load pulse form and duration. For example, using the Burger model with a constant applied stress,  $\kappa(t)$  is derived from (2) as

$$\kappa(t) = 1 + \frac{K_e t}{\mu_v} + \frac{K_e}{K_k} \left[ 1 - \exp\left(-\frac{K_k t}{\mu_k}\right) \right] \quad (4)$$

The general relationship between the creep factor and the duration of a constant applied load for structural lumber is illustrated in Fig. 1. The four Burger model parameters used to develop Fig. 1 were assumed to be independent random variables following lognormal distributions with mean of COV values as defined in Table 1.

Table 1. Creep Model Paramter Distributions for Structural Lumber (from [5]).

Parameter	15% MC		19% MC		28% MC	
	Mean	COV	Mean	COV	Mean	COV
$K_e$ (GPa)	11.6	0.16	10.7	0.16	9.0	0.16
$K_k$ (GPa)	19.1	0.41	15.0	0.41	11.2	0.41
$\mu_k$ (GPa-min)	31.4	0.42	23.2	0.42	12.3	0.42
$\mu_v$ (GPa-min)	38.3	0.39	30.6	0.39	22.3	0.39

### 3.0 LOAD MODELS

The probabilistic modeling of loads, including dead and live loads, has been conducted by various researchers (e.g., [2, 3, 4]). For use in deflection serviceability reliability analyses including creep behavior, the stochastic load models must contain information on the load duration and arrival rate in addition to load magnitude. A pulse process model can be used to generate a complete load history for the reference period under consideration (e.g., one year, eight year, etc.). A separate load pulse process can be generated for each load type such as dead load, occupancy live load, and



snow load. The generated load histories can be superimposed to allow consideration of various load combinations. In this paper, the dead plus occupancy live load combination is considered.

### 3.1 Dead Load Modeling

Although the self weight of a structure is well defined, the dead load is usually assumed to be underestimated by a small amount due to uncertainty in other dead load components such as permanent equipment, partitions, floor coverings, etc. [4]. Dead load statistics are summarized in Table 2.

### 3.2 Occupancy Live Load Modeling

Occupancy live load is typically modeled as having two components: a sustained component and an extraordinary component [2]. The sustained component includes items normally associated with the intended use of the structure. For example, furnishings and occupants in typical office and residential space are included as sustained live load. The extraordinary live component accounts for atypical use of a structure such as crowding of people during special events or the temporary use of the space for storage during renovation [3]. Statistics for both components of occupancy live load are summarized in Table 2 for two coverage areas.

Table 2. Dead and Occupancy Live Load Process Statistics.

Load Component	Intensity			Arrival	
	Mean	COV	CDF	Mean rate/year	Duration
Dead	$1.05D_n$	0.10	LN	n/a	50 years
Sustained Live					
20 sq. m	$0.24L_n$	0.90	Gamma	0.125	8 years
75 sq. m	$0.30L_n$	0.60	Gamma	0.125	8 years
Extrodinary Live					
20 sq. m	$0.16L_n$	0.90	Gamma	1	1 week
75 sq. m	$0.19L_n$	0.60	Gamma	1	1 week

## 4.0 LIMIT STATE FUNCTION INCLUDING CREEP

A limit state function can be derived to include creep effects from a specific design equation. Considering a deflection serviceability check where the actual deflection of a member,  $\delta_{actual}$ , must be less than some allowable value,  $\delta_{allowable}$ , the limit state function including creep effects will take the form

$$g(x) = \delta_{allowable} - \delta_{actual} \quad (5)$$

The actual deflection is typically assumed as the elastic deflection. For a viscoelastic material, however, the actual deflection must include the elastic *and* the creep deflection components. Using (3), the actual deflection of a viscoelastic member can be written as

$$\delta_{actual} = \kappa(t) \cdot \delta_e \quad (6)$$

where  $\delta_e$  is the elastic deflection. For a load pulse process, both  $\kappa(t)$  and  $\delta_e$  vary with each pulse. In a reliability analysis, the maximum product of  $\kappa(t)$  and  $\delta_e$  which occurs during serviceability reference period, or

$$\delta_{actual} = \max\{\kappa(t) \cdot \delta_e\} \quad (7)$$

must be determined and compared with the allowable deflection. The elastic deflection is a function of the actual modulus of elasticity, actual loading, and the creep factor, each of which is assumed to be a random variable, and the design span and beam moment of inertia, both of which are assumed to be deterministic.

By setting the nominal beam deflection equal to the deflection limit, the allowable deflection can be written as a function of the nominal loading and the nominal elastic modulus. By substituting this information into (5) and reducing, the following limit state equation is obtained for the deflection limit state including creep effects:

$$g(x) = \frac{E}{\phi_E E_n} \left( \frac{\gamma_D}{L_n / D_n} + \gamma_L \right) - \max \left\{ \kappa_i \cdot \left( \frac{D + L}{L_n} \right) \right\} \quad (8)$$

in which  $\phi_E$  is the resistance factor,  $E$  and  $E_n$  are the random and nominal elastic moduli,  $\gamma_D$  and  $\gamma_L$  are dead and live load factors,  $D$  and  $L$  are the random dead and live loads,  $D_n$  and  $L_n$  are the nominal dead and live loads, and  $\kappa_i$  is the creep factor for each load  $i$  in the reference period. Note that (8) is not dependent on the assumed allowable deflection, beam support, or loading conditions.

Typically, serviceability analyses utilize unfactored loads, i.e.,  $\gamma_D = \gamma_L = 1$ ; however, some design specifications and building codes suggest other load combinations when considering creep. For example, the *National Design Specification of Wood Construction* [7] implies a  $1.5D_n + L_n$  load combination be used when considering creep effects in wood structures comprised of seasoned lumber by recommending deflections owing to "permanent" loads be increased by a factor of 50%.

## 5.0 RELIABILITY ANALYSIS

Reliability analyses including creep effects must consider the in-time behavior of the structure throughout the specified reference period. Therefore, first-order second-moment techniques are not applicable and Monte Carlo simulation procedures must be utilized. For serviceability, reduced reference periods, such as one year and eight years, are often considered in the reliability analysis. Obviously, the longer the reference period, the lower the associated reliability index. A target reliability index also must be established in order to propose appropriate creep factors for use in design. A value of  $\beta = 2$  has been identified in the past as an appropriate index for serviceability (e.g., [6]). This value is generally lower than the target reliability for a strength analysis owing to the reduced consequence of failure.

Using the general viscoelastic material response presented in Table 1 and Fig. 1 for structural lumber, the load statistics presented in Table 2, and a load combination of  $D_n + L_n$ , the relationship between the reliability index,  $\beta$ , and the resistance factor,  $\phi_E$ , was determined and is presented in Fig. 2 for one and eight year reference periods. From this example, a resistance factor,  $\phi_E$ , of 0.45 is required to attain the target reliability index for a one year reference period and 0.35 for an eight year reference period.

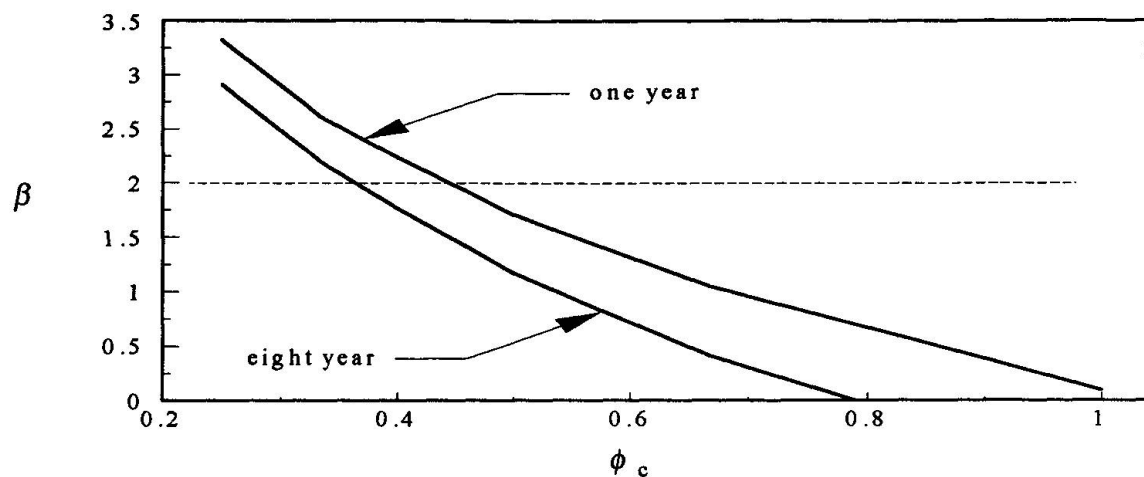


Fig. 2.  $\beta - \phi_c$  Relationship for One and Eight Year Reference Periods

## SUMMARY

The incorporation of the viscoelastic response of materials into the serviceability reliability analysis of structures and the effect of viscoelastic material behavior on the serviceability performance and reliability of members has been discussed. To include creep effects, the viscoelastic response of the material must be defined, and information on the load duration and arrival rate as well as load magnitude is required. The deflection limit state function was written as a function of a resistance factor, the random and nominal elastic moduli, the random dead and live loads, the nominal dead and live loads, and a creep factor for each load pulse in the reference period. The viscoelastic response of structural wood members was used to illustrate the serviceability analysis of structures including creep effects.

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