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**Autor:** Smith, Ian / Hu, Lin J.  
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## Vibration Serviceability of Ribbed Plates by Modal Synthesis

Synthèse modale de l'aptitude au service sous vibrations des dalles nervurées

Schwingungseignung von Rippenplatten durch modale Synthese

### Ian SMITH

Dr. Eng.  
Univ. of New Brunswick  
Fredericton, NB, Canada



Ian Smith, born 1951, holds bachelors, masters and doctoral degrees in engineering from the Universities of Sunderland, Durham and South Bank respectively. He has spent 15 years researching structural use of wood products. Dr. Smith is professor of Forest Engineering and Director of the Wood Science and Technology Centre.

### Lin J. HU

Dr. Eng.  
Univ. of New Brunswick  
Fredericton, NB, Canada



Lin. J. Hu holds bachelors and masters degrees in engineering from the University of Tsinghua, and a doctoral from the University of New Brunswick. She has spent the last 10 years researching structural vibration and computational mechanics.

### SUMMARY

A generalized ribbed plate vibration model is presented, based on the free-interface modal synthesis method. Discussion centres on whether a model can reliably estimate parameters used in design criteria. A conclusion is, for example, that peak velocity of a ribbed plate can be predicted reliably using the authors' model, but not using approximate formulae proposed elsewhere.

### RESUME

Les auteurs présentent un modèle généralisé, basé sur la synthèse modale des surfaces de contact, pouvant servir à étudier le comportement des dalles nervurées soumises aux vibrations. La discussion porte sur le fait de savoir si un modèle est vraiment en mesure de fournir des estimations fiables pouvant servir de critères paramétriques de dimensionnement. Le modèle proposé par les auteurs permet, par exemple, de pronostiquer avec une certaine fiabilité la vitesse de crête des vibrations d'une dalle nervurée, alors que d'autres formules approximatives proposées par ailleurs s'avèrent inapplicables.

### ZUSAMMENFASSUNG

Für das Schwingungsverhalten von Rippenplatten wird ein verallgemeinertes Modell vorgestellt, das auf modaler Synthese bei freien Grenzflächen basiert. Es wird erörtert, ob überhaupt ein Modell zuverlässige Schätzwerte für die Bemessungsgrößen zu liefern imstande ist. Es ergibt sich, dass zum Beispiel die Spitzenschwinggeschwindigkeit einer Rippenplatte durch das Modell der Autoren zuverlässig vorhergesagt werden kann, während anderswo vorgeschlagene Abschätzformeln versagen.



## 1. INTRODUCTION

This paper explores the feasibility of applying dynamic serviceability design criteria proposed in documents such as ISO 2631 [1] and Eurocode 5 [2]. The discussion is amplified using a recently developed numerical model for vibration analysis of ribbed plates. The model accounts for various complicating factors such as plate orthotropy, shear deformation and rotatory inertia in ribs, semi-rigid connections between plate and ribs, and intermediate supports. The model is based on the free interface modal synthesis method proposed by Hou [3]. Constraint conditions are enforced at any intermediate supports by means of Lagrange multipliers. The advantage of the new model over existing models is demonstrated in detail by Hu [4]. This model is used here to evaluate the reliability of more approximate models in estimation of parameters used in design criteria.

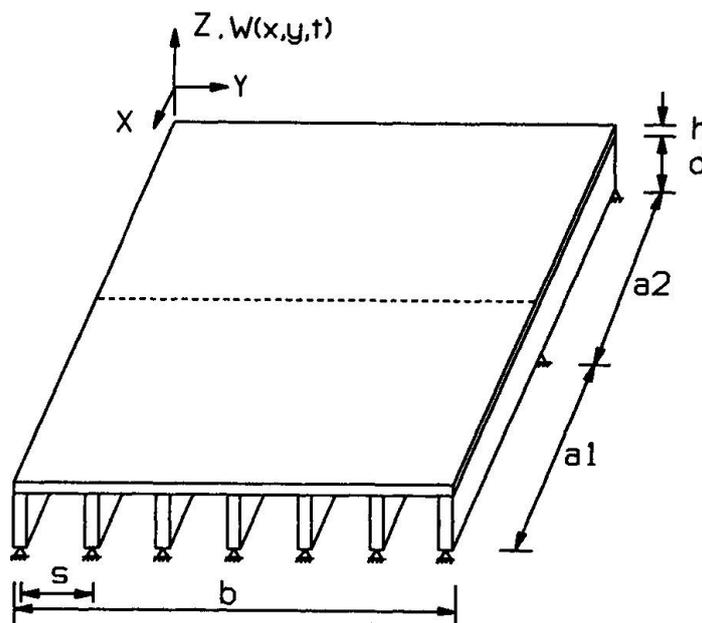


Fig. 1 A typical double-span ribbed plate arrangement

## 2. THEORY

A brief summary is given here of the theory on which the new model is based [4,5]. A representative rectangular double span ribbed plate is shown in Figure 1. It is assumed that transverse displacement ( $W$ ) dominates the displacement of the system; small deflection theory is valid; the ribs may be rigidly or semi-rigidly attached to the plate; and intermediate rigid line supports cross the plate perpendicular to the ribs.

By modal expansion, the transverse motions of the uncoupled substructures, i.e. plate and  $j$ th rib, are

$$\text{plate: } W_p(x,y,t) = \sum_{m=1}^M \sum_{n=1}^N q_{m,n}^p(t) X_m^p(x) Y_n^p(y) \quad (1)$$



$$\text{jth rib: } W_r^j(x, t) = \sum_{k=1}^K q_k^j(t) X_k^j(x) \quad (2)$$

where the subscripts  $m, n$  define the modes selected for modelling motion of the plate;  $k$  defines the modes selected for modelling motion of the  $j$ th rib; and

$$q_k^j, q_{m,n}^p = \text{modal coordinate functions,}$$

$$X_k^j, X_m^p, Y_n^p = \text{mode shape functions.}$$

The kinetic and potential energies (in plate modal coordinates) are for a ribbed plate are

Kinetic energy:

$$T = \frac{1}{2} \sum_{m=1}^M \sum_{n=1}^N (\dot{q}_{m,n}^p)^2 M_{m,n} + \sum_{j=1}^J \frac{1}{2} \sum_{k=1}^K (\dot{q}_k^j)^2 M_k^j \quad (3)$$

Potential energy:

$$U = \frac{1}{2} \sum_{m=1}^M \sum_{n=1}^N (q_{m,n}^p)^2 \omega_{m,n}^2 M_{m,n}$$

$$+ \sum_{j=1}^J \frac{1}{2} \sum_{k=1}^K (q_k^j)^2 (\omega_k^j)^2 M_k^j \quad (4)$$

where

$$M_k^j, M_{m,n} = \text{modal mass terms,}$$

$$\dot{q}_k^j, \dot{q}_{m,n}^p = \text{modal velocity terms,}$$

$$\omega_k^j, \omega_{m,n} = \text{angular natural frequency terms.}$$

In the analysis the modal characteristics of substructures are obtained first. Two-dimensional theory of flexural vibration of an orthotropic elastic plate is used to determine characteristics of the plate. One-dimensional Timoshenko beam theory is used for the ribs, allowing inclusion of the effects of transverse shear deformation and rotatory inertia. To simulate the semi-rigid nature of connections between plate and ribs, an effective flexural rib rigidity is used, Smith [6].

Two sets of constraint equations are needed. One set is applied to synthesize the plate and rib substructures (to form the ribbed plate) and may be expressed as

$$W_p(x, y^j, t) = W_r^j(x, t) \quad (5)$$



This is transformed into the modal coordinate system, by substituting equations (1) and (2) in equation (5). Another set of constraint equations is needed to satisfy the zero transverse displacement requirements at intermediate rigid line supports. Let the location of the line support be at  $x = c$ . The constraint equations in modal coordinates are

$$\sum_{m=1}^M q_{m,1}^p X_m^p(c) = \sum_{m=1}^M q_{m,2}^p X_m^p(c) = \dots \sum_{m=1}^M q_{m,N}^p X_m^p(c) = 0 \quad (6)$$

To form the generalized forced vibration equations, it is assumed that a transverse force of arbitrary form acts normal to the surface of the plate. The force may be expressed as

$$P(x,y,t) = p(x,y)f(t) \quad (7)$$

The elements of the modal force vector  $Q$  are

$$Q_i = \int_0^b \int_0^a X_m^p(x) Y_n^p(y) p(x,y) f(t) dx dy \quad (8)$$

Applying Lagrange's equations in conjunction with the constraint equations, and adding a damping matrix, forced vibration equations of a ribbed plate are of the form

$$M \ddot{q} + C \dot{q} + K q = Q \quad (9)$$

The modal damping matrix  $C$  can be calibrated from experimental estimates of viscous damping ratios, Warburton [7]. A computer program implements the above under selected combinations of plate support conditions.

### 3. VERIFICATION

Verification testes were carried out on a series of single and double span ribbed plates built with wood I-joists [4,5]. Comparison has also been made with test results reported by Ohmart [8] who studied forced vibration response of concrete slabs with steel I-joist ribs. Figure 2 illustrates how predictions of maximum displacement amplitudes from the authors' model compare with test observations, and Ohmart's own model predictions. Systems 1 and 2 had spans of 4.47m and variable widths [8]. It can be seen that predictions with the model reported here are relatively good. As the new model has been found to predict natural frequencies and peak displacements under harmonic forces with reasonable accuracy (over a range of systems), it is concluded that velocity or acceleration predictions will be reliable.



Section 4 of this paper contains further indications of the level of agreement attained between the authors' model and test observations.

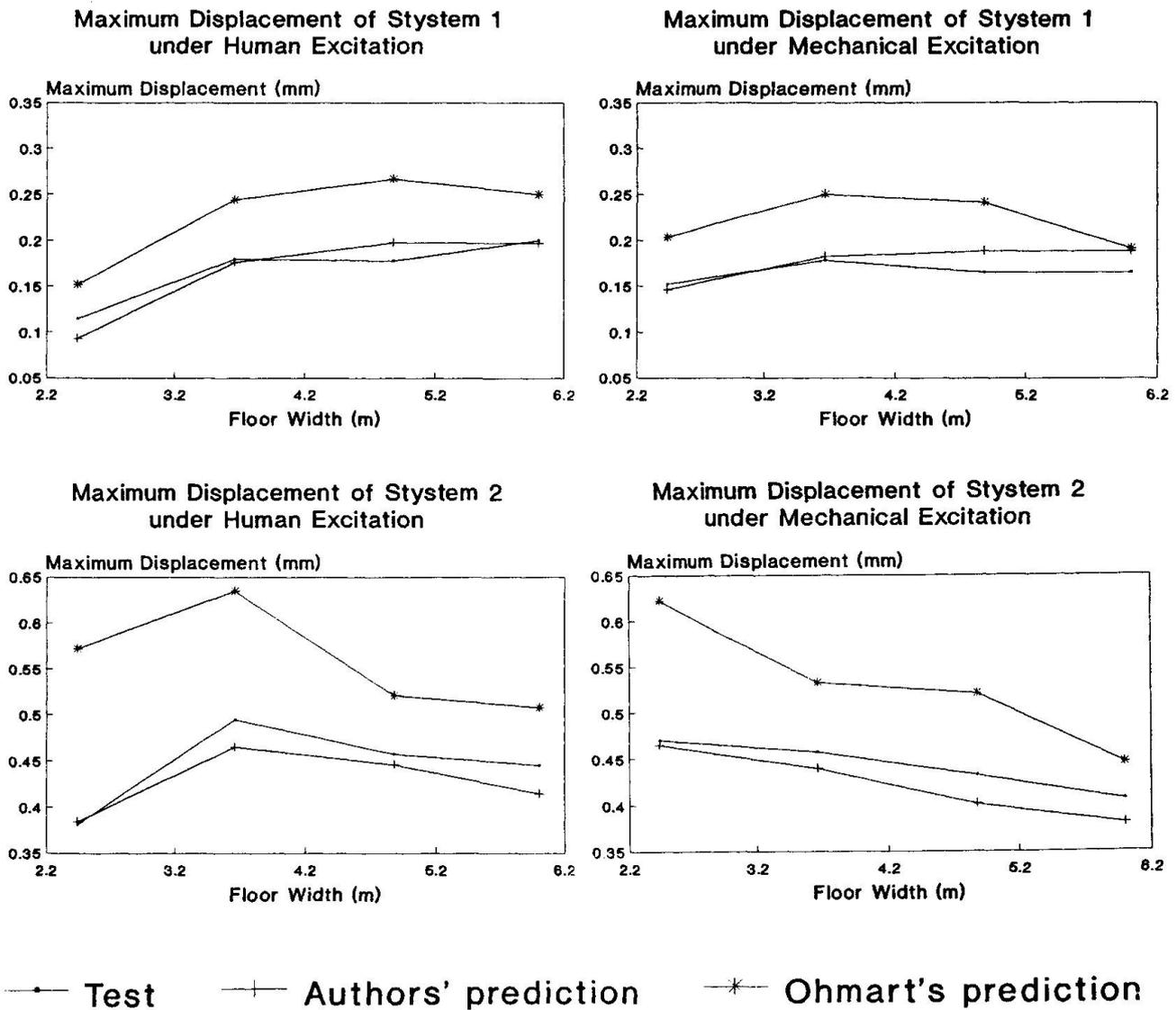


Fig. 2 Predicted vs. test maximum displacement of steel-concrete floors



#### 4. PREDICTING VIBRATIONAL SERVICEABILITY OF FLOORS

Feasibility of applying the authors' ribbed plate model, or other more approximate models, within serviceability design of floors was investigated. Contemporary criteria require that models used in design be capable of predicting parameters such as fundamental natural frequency,  $N_{40}$  the number of natural frequencies less than 40 Hz and initial peak velocity [1,2]. Three types of systems have been analyzed: lumber joist system [9], wood I-joist system [4] and concrete slabs with steel I-joists [8]. Over 100 systems have been considered, but due to lack of space only an illustrative sub-set will be discussed here.

Ohlsson presents approximate design formulae for predicting fundamental natural frequency,  $N_{40}$  and peak velocity under a 1 Ns impulsive force [2]. These approximations are intended for predicting behaviour of systems simply supported on all edges (single span) and having natural frequencies greater than 8 Hz. Because the formulae are incorporated in draft Eurocode 5, the predictions are labelled 'Eurocode 5'. Table 1 compares test measurements with predictions by the authors' model, models developed by Ohmart [8] and Chui [9] and Eurocode 5 method. It should be noted that Ohmart only provided test and predicted frequencies for the fundamental mode. The comparison in Table 1, and other comparisons not reported here, suggests that, generally, there are large discrepancies between peak velocities predicted by Eurocode 5 formulae and by more rigorous analytical models. In the case of the Eurocode 5 formulae,  $N_{40}$  is used as the basis of predictions of peak velocity. Part of the discrepancies can be attributed to the inability of the Eurocode 5 formulae to reliably predict  $N_{40}$ . It is acknowledged that for vibrational serviceability design of highly orthotropic systems, the concept of accounting for higher vibration modes is a reasonable proposition. However, the results presented here suggest that, almost inevitably, many simplifying assumptions made in the Eurocode 5 approach will be violated by practical floor systems. To ensure reliable predictions, more rigorous analysis must be employed.



Floor	Approach	Fundemamtal Frequency (Hz)	N40	Peak velocity (m/s/Ns × 10 <sup>-2</sup> )
Chui [9]: Floor 6	Test	22.3	3	
	Chui-model	21.1	2	
	Eurocode 5	18.5	3	2.28
	Authors-model	21.6	2	2.11
Hu [4]: Floor 2	Test	11.2	6	
	Eurocode 5	10.4	5	2.0
	Authors-model	10.9	6	1.12
Floor 4	Test	13.1	5	
	Eurocode 5	13.9	5	2.18
	Authors-model	14.1	5	1.22
Floor 6	Test	17.1	4	
	Eurocode 5	24.9	5	2.69
	Authors-model	22.0	4	1.39
Ohmart[8] 1-5B	Test	18.8		
	Ohmart-model	16.8		
	Eurocode 5	13.9	3	0.159
	Authors-model	17.6	2	0.070
1-4B	Test	20.8		
	Ohmart-model	18.6		
	Eurocode 5	14.8	2	0.141
	Authors-model	19.3	2	0.067
1-3B	Test	25.9		
	Ohmart-model	22.8		
	Eurocode 5	18.0	2	0.184
	Authors-model	23.7	1	0.086
1-2B	Test	35.7		
	Ohmart-model	36.1		
	Eurocode 5	30.5	1	0.166
	Authors-model	37.5	1	0.095
2-5B	Test	15.6		
	Ohmart-model	15.3		
	Eurocode 5	16.7	2	0.196
	Authors-model	16.3	3	0.140
2-4B	Test	16.7		
	Ohmart-model	15.9		
	Eurocode 5	18.2	2	0.237
	Authors-model	16.9	2	0.142
2-3B	Test	18.5		
	Ohmart-model	17.7		
	Eurocode 5	23.2	1	0.191
	Authors-model	18.8	2	0.149
2-2B	Test	23.8		
	Ohmart-model	24.2		
	Eurocode 5	41.7	1	0.269
	Authors-model	25.6	1	0.221

Table 1 Predicted vs. test natural frequencies, and initial peak velocities under 1 Ns impulse



## 5. CONCLUSION

The above shows that the responses of ribbed plates, such as displacement and velocity, can be predicted reasonably accurately for a standardized impulsive force. However parameters such as N40 used by Eurocode 5 may not always be predicted accurately. Approximate formulae such as those in Eurocode 5 introduce too many assumptions to give consistently reliable predictions of initial peak velocity.

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