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# Initial and Long-Term Deformations of Prestressed Concrete Members

## Déformations initiales et à long terme d'éléments en béton précontraint

## Sofortige und Langzeitverformung von Spannbetonelementen

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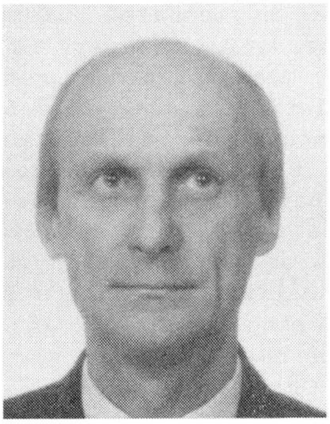

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**SUMMARY**

A method for the analysis of pre-tensioned and post-tensioned prestressed concrete sections subjected to long-term loads is presented. Assuming for concrete an algebraic linear viscoelastic constitutive law, the relationships giving the time-evolution of the cable force and of the sectional curvature are deduced. Design formulas for the evaluation of the amount of prestressing reinforcement necessary to satisfy prescribed stress bounds in concrete in the service stage are then stated.

**RESUME**

Une méthode pour l'analyse des sections en béton précontraint soumises à l'action de charges de longue durée est présentée. Dans l'hypothèse de béton obéissant à une loi viscoélastique algébrique linéaire les expressions qui donnent l'évolution dans le temps de la force dans le câble de précontrainte et de la courbure de la section sont tirées. L'article se termine avec la détermination de l'armature de précontrainte nécessaire pour assurer que, dans les conditions de service, les efforts de traction dans le béton ne dépassent pas leur valeurs admissibles.

**ZUSAMMENFASSUNG**

Es wird ein Verfahren zur Berechnung des Langzeitverhaltens von Spannbetonelementen mit sofortigem oder nachträglichem Verbund unter Dauerlast vorgestellt. Aus der Annahme eines algebraischen, linear-viskoelastischen Materialgesetzes für den Beton lassen sich die Beziehungen für die zeitliche Entwicklung der Spannkräfte und der Querschnittskrümmungen herleiten. Abschliessend werden Bemessungsformeln genannt, mit denen die zur Einhaltung vorgegebener Betongrenzspannungen erforderliche Spannbewehrung ermittelt werden kann.

## 1. INTRODUCTION

The analysis of the state of stress and deformation under long-term loads represents one of the basic aspects connected to the assessment of structural safety in the service stage of P.C. elements. At the present time the Codes, [1], [2], give some general guidelines for the evaluation of long term stresses and deformations in P.C. members, but they do not develop practical procedures for the structural analysis. In particular a linear viscoelastic constitutive law is prescribed for concrete while steel is considered in the elastic range, so that if the behaviour of concrete is described by means of refined formulations the sectional analysis of P.C. elements requires the solution of a system of two Volterra integral equations by means of numerical algorithms which are not customary for the practitioners. As the service analysis of P.C. elements has in general to be performed by means of procedures exhibiting good precision levels, the development of an analytical method able to accomplish this goal without requiring too cumbersome computations and suitably utilizable by designers assumes significant interest. For this reason many simplified methods of analysis have been proposed, but they have not solved the problem in an exhaustive way, in particular they can not be immediately reduced to analytical forms in agreement with Codes Specifications or to unified hypotheses regarding the constitutive laws or the structural idealization. In the present paper a contribute to this subject is given by developing an analytical formulation for the long term sectional analysis, based on the algebraic stress-strain linear viscoelastic law proposed for concrete by Trost, [3], widely used in other author's works, [4], [5] and recommended by the Codes. Regarding the sectional geometrical and mechanical description we shall adopt the hypothesis that the prestressing cables or strands will be grouped in order to form a virtual cable having area equal to the sum of the areas of the single cables or strands and passing through the point of application of the total prestressing force. In this way it is possible to proceed by means of the force method assuming as unknown the force in the resultant cable, reaching without great difficulty simple solutions of good approximation and suitable to discuss the basic properties of the service behaviour of P.C. elements taking also into account shrinkage deformations. In particular the differences in the long term statical interaction between the cable and the section when pre-tensioned or post-tensioned elements are considered, will be pointed out and simple formulas, suitable for evaluating the amount of prestressing reinforcement will be stated, together with the analytical expressions for the control of long-term deformations. At this subject suitable moment-curvature diagrams of P.C. sections, clearly describing the interaction between the prestressing and external loads and their effect on the deformational behaviour of P.C. members will be discussed. A simple numerical example, referred to an actual P.C. element will be finally performed in order to clearly show the basic practical aspects of the proposed procedure.

## 2. SECTIONAL ANALYSIS OF P.C. MEMBERS

With reference to fig. 1, let us consider a P.C. pre-tensioned, (fig. 1a) or post-tensioned section (fig. 1b). Indicating by $N_p$ the prestressing force imposed to the reinforcement, by $\delta_c$, $\delta_s$, the elastic influence coefficients of concrete and steel, by $\epsilon_s$ $\epsilon_c$ the corresponding deformations at time t, we can write the subsequent compatibility equation

$$\epsilon_c+\epsilon_s=N_p(a\delta_c+\delta_s)+\epsilon_{sh} \tag{1}$$

where $\epsilon_{sh}<0$ is the shrinkage deformation. In eq. (1) the coefficient a vanishes for pre-tensioned sections (a=0) and it is equal to unity (a=1) for post-tensioned sections as in the first case the prestressing initial force $N_p$ is applied only to steel while in the second it affects also the concrete section. According to the previously mentioned hypotheses about the materials behaviour we can write [3], [4]

$$\epsilon_c=\sigma_{co}(1+\phi)/E_{co}+(\sigma_c-\sigma_{co})(1+\mu\phi)/E_{co} \qquad (2)$$
$$\epsilon_s=\sigma_s/E_s$$

where the quantities with index o are referred to initial time, $\phi$ is the creep coefficient and $\mu$ is the concrete aging coefficient. Expressing the stresses by means of internal actions N, $M_g$ and introducing eq. (2) in eq. (1) we obtain

$$N=N_R\{[\alpha(1+\Omega a^*)+\Omega b-1](1-\Omega c)+1-\beta k_{sh}\Omega c r^2/(\phi e^2)\}=N_R k_N \qquad (3)$$

where

$$N_R=\beta M_g/e;\ \beta=e^2/r^2/(1+e^2/r^2);\ \alpha=N_p/N_R;\ \Omega=(1+e^2/r^2)/(1+e^2/r^2+1/mn_s)$$
$$m=E_s/E_{co};\ n_s=A_s/A_c;\ a^*=a-1;\ c=\phi/(1+\Omega\mu\phi);\ k_{sh}=|\epsilon_{sh}|E_{co}A_c/N_R \qquad (4)$$

with $A_c$, r, respectively area and radius of gyration of concrete section. In eq. (3) the parameter b specifies pre-tensioned sections for b=1 and post-tensioned sections for b=0. This distinction is necessary as the external moment $M_g$ is active on the reinforcement when it has been connected with concrete For this reason it has no effect at initial time on the reinforcement of post-tensioned elements and only successively interacts with it as a consequence of concrete creep deformations and of steel to concrete solidarisation obtained by means of the sheets grouting. Eq. (3) allows to evaluate the force in the cable at any time so that remembering the constitutive law (2) we reach the subsequent expression for the long-term sectional curvature

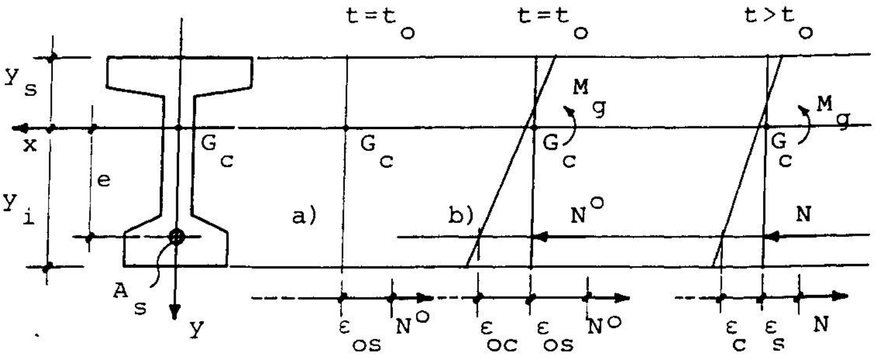

Fig.1 General features of a P.C. section

$$\Phi/\Phi^o=(1+\phi)\{1-\beta\{[\alpha(1+\Omega a^*)+\Omega b-1](1-\Omega cd)-\beta k_{sh}r^2\Omega cd/(\phi e^2)+1\}\} \qquad (5)$$

where $\Phi^o=M_g/E_{co}J_c$; $d=(1+\mu\phi)/(1+\phi)$, with $J_c$ centroidal moment of inertia of concrete section. By means of expressions (3), (4), (5), it is possible to evaluate the sectional state of stress and deformation, assuming for the coefficient $\mu$ the expression $\mu=-k_R^{-1}-\phi^{-1}$ where $k_R<0$, $\phi>0$ are respectively the relaxation and the creep coefficient of concrete. These coefficients depend from the model assumed to describe concrete creep, in particular, for the model exposed in [5], they are reported, in graphical form, in [6]. When no data are available for $\mu$ we can however assume the subsequent approximate relation connecting creep and relaxation coefficient: $k_R=-\phi/(0.8\phi+1)$, so that for $\mu$ we deduce the approximate constant value $\mu=0.8$. As indicated in [1], this simplification can be adopted without great error in sectional P.C. members analysis.In fig. 2, referring to prescribed values of $\beta$, $\phi$ and for $\epsilon_{sh}=0$ the expressions (3), (5) are reported for $t=t_o$ and $t>t_o$. The corresponding diagrams are straight lines and describe the variability in time of the force in the resultant cable when varying the prestressing force. The diagrams allow to draw some interesting observations. As regards the cable force, represented at initial time $t_o$ by the two dotted lines $b_1$, $b_2$ respectively related to pre-tensioned and post-tensioned sections, and by the two corresponding lines $a_1$, $a_2$ at time t we see that the force is greater in post-tensioned cables as they are subjected at the initial time to the total prestressing force $N_o=N_p=\alpha N_R$, (eq.(3) with $a^*=b=c=0$), while in pre-tensioned cables this initial force assumes the reduced value $N_o=N_R(\alpha(1-\Omega)+\Omega)$, (eq.(3) with $a^*=-1$, b=1, c=0). When $\alpha=0$, $t>t_o$ the line $b_2$, initially passing through the origin is trasformed in the line $a_2$ intersecting the vertical axis with a positive ordinate as a consequence of the delayed statical effect exerted by the permanent moment $M_g$ on the cable after its solidarisation to concrete section by means of the sheet grouting. Finally we can observe that for $\alpha=1$, we reach at any time $N=N_R$, independently from the type of prestressing, so that the straight lines $a_1$, $a_2$, $b_1$, $b_2$ have a common centre. This can be effective as for $\alpha=1$ the cable assumes the force $N_R$, coinciding with that produced by moment $M_g$ on a rigid cable. In this circumstance the hypotheses of the first theorem of linear viscoelasticity become valid so that no variations in the cable force can take place. This fact evidentiates also that for $0<\alpha<1$ the force in the cable grows in time as we can state observing that lines $a_1$, $a_2$ are disposed upwards the initial lines $b_1$, $b_2$. On the contrary for $\alpha>1$ the force in the cable always decreases in time. This is the customary case of P.C. elements as values of $\alpha<1$ are connected to rather small values of the prestressing force which are not effective to guarantee a sufficient prestressing level suitable to be emploied in practical applications. Nevertheless levels of prestressing with $\alpha<1$ are often adopted for complex structures as cable-stayed bridges so that in these structures we can have an increasing of the cable force in time. The dotted lines $c_1$, $c_2$ refer to pre-tensioned and post-tensioned section adimensional curvatures at time $t_o$ while the corresponding lines at time t are represented by lines $d_1$, $d_2$. The curvatures vary in time assuming increments which have generally the same sign of the initial curvature but in the intersecting zone of the straight lines, about for $4\leq\alpha\leq5.15$, in the present case, initial curvature and its time increment exhibit opposite signs. In other words for small prestressing forces the initial curvatures and their increments are controlled by the moment $M_g$ and the element exhibiting downwards transverse displacements increases them in the same direction owing to concrete creep deformations. For intermediate $\alpha$ values the curvatures are initially negative as prestressing prevails on external load, but creep effects produced by loads are more significant so that the upwards initial displacements, exhibit downwards variations which can produce downwards, upwards or even vanishing final displacements. As a particular case we have the singular situations connected to a final value of the displacements equal to the initial ones. Finally, for high $\alpha$ values prestressing is always prevailing on the loads so that the upwards initial displacements increase in time in the same direction. In order to evaluate the importance connected to the choise of a reliable analytical procedure in reaching good results, in fig. 2 the two limiting lines $e_1$, $e_2$, represented for post-tensioned sections and related respectively to the simplifying hypothesis of considering constant in time the cable force with its initial or final value are reported. It is immediate to see that the first hypothesis produces too marked errors so that the cable force variability has to be accounted for in order to reach reliable results. On the other hand the second hypothesis gives very good results, but it can be applied only when the time-variability of N has be evaluated so that no substantial simplifications in computational works are introduced, and for this reason it is more convenient to

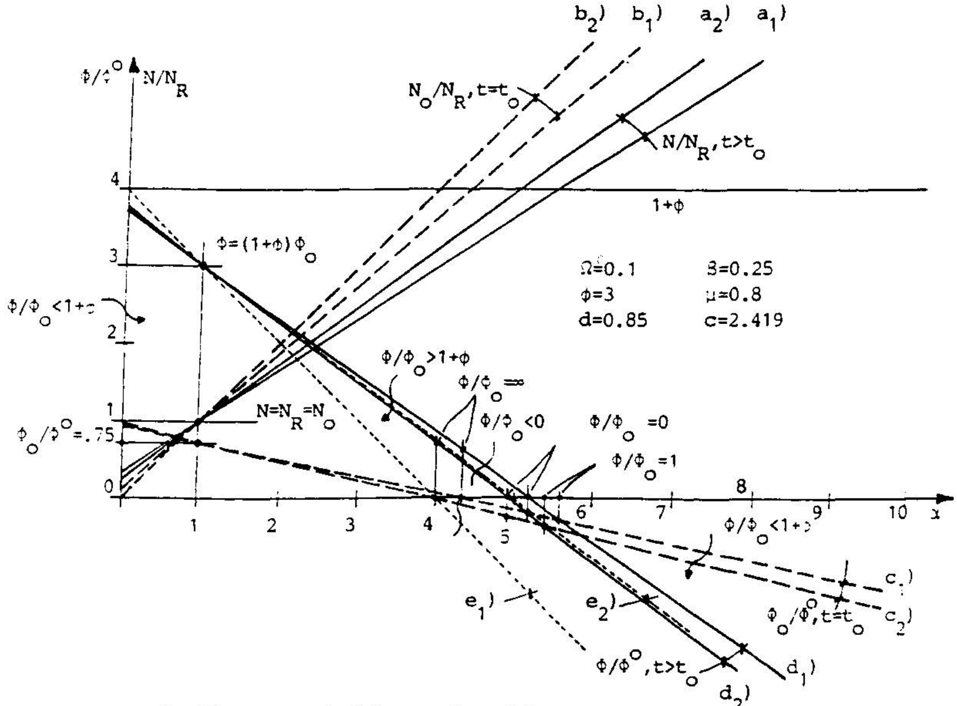


Fig.2 Time variation of cable force and curvature

proceed by means of the general expression (5).

## 3. DESIGN FORMULAS AND NUMERICAL EXAMPLE

Expression (3) can be emploied in evaluating the area of prestressing steel which allows to satisfy some limit state specifications in particular to get a prescribed stress level at the bottom sectional edge when permanent and variable loads are applied. Indicating by $M_q=sM_g$ the moment produced by variable loads, remembering eq. (3), the bottom edge stress produced by permanent and variable loads together with prestressing, assuming positive compressive stress, becomes

$$\sigma_i=|\sigma_g|[-(1+s)+(k_N+\Omega s)/z] \quad (6)$$

where:

$$|\sigma_g|=M_g/W_i; \quad z=(1+r^2/e^2)/(1+r^2/(ey_i))$$

Indicating by $\sigma_{ct}$ the maximum tensile stress allowed in concrete the safety inequality can be written $\sigma_i\geq\sigma_{ct}$, or according to eq. (6)

$$k_N\geq z[1+s+p]-\Omega s \quad (7)$$

with $p=\sigma_{ct}/|\sigma_g|$

Introducing the expression of $k_N$ deriving from eq. (3) in eq. (7), simple algebraic calculations drive to the subsequent inequality

$$\Omega\geq\pi(1-\Omega)\{z(1+s+p)-\Omega[b(1-\Omega c)+c(1-\beta k_{sh}r^2/(\phi e^2))+s]\}/[(1+\Omega a^*)(1-\Omega c)] \quad (8)$$

where

$$n_s=A_s/A_c=\beta r^2\Omega/[me^2(1-\Omega)]; \quad \pi=me|\sigma_g|/(\sigma_s y_i); \quad N_p=\sigma_s n_s A_c$$

and $\sigma_s$ represents the prestressing imposed stress to steel. Expression (8), written with equality sign, represents an implicit equation in the unknown $\Omega$ which can be solved by a trial and error method, so that we can successively evaluate by means of the expression of $n_s$ the necessary amount of prestressing steel in order to satisfy the safety inequality (7). As we can observe the result depends from the values assumed by the parameters $a^*$ and b, or that, by the type of prestressing imposed to the section. When the $\Omega$ coefficient has been determined, from eq. (3) we reach the value N of the cable force, so that according to the constitutive law (2) we obtain the subsequent expression for the curvature

$$\Phi/\Phi^o=(1+\phi)[1-\beta k^o_N\phi(1-\mu)/(1+\phi)-\beta k_N(1+\mu\phi)/(1+\phi)] \quad (9)$$

where $k^o_N$ is the initial value of the adimensional force $k_N$, which from eq. (3), written for $t=t_o$ becomes

$$k^o_N=\alpha(1+a^*\Omega)+\Omega b \quad (10)$$

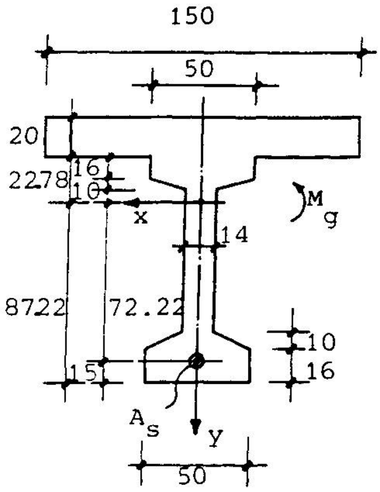


Fig.3 P.C. section of a bridge beam

In order to apply the discussed procedures to an actual case, let us consider the bridge section of fig. 3 for which we assume: $E_s=1.95\cdot10^5$ MPa; $E_{co}=3.6\cdot10^4$ MPa; m=5.417; $M_g$=1500 KNm; $|\sigma_g|$=11.81 MPa; $\sigma_s$=1350 $MP_a$; s=1; $\epsilon_{sh}=-50\cdot10^{-5}$, $\phi$=3. With reference to the assessment of structural safety for the decompression limit state, ($\sigma_{ct}$=0), assuming a pre-tensioned reinforcement, from eqs. (9), with $a^*=-1$, b=1 , we reach $\Omega$=0.09367; $n_s=0.4956\cdot10^{-2}$; $N_p$=4049 KN; N=3080 KN; $N/N_p$=0.7607. For the curvature, according to eq. (9), we obtain $\Phi_o/\Phi^o=-0.836$; $\Phi/\Phi^o=-2.136$; $\Phi/\Phi_o=2.555$. Assuming on the contrary a post-tensioned reinforcement, $a^*=-1$, b=0 the results are $\Omega$=0.08794; $n_s=0.4623\cdot10^{-2}$; $N_p$=3777 KN; N=3080 KN; $N/N_p$=0.8155; $\Phi_o/\Phi^o=-0.819$; $\Phi/\Phi^o=-2.139$; $\Phi/\Phi_o=2.612$. We observe that the two kinds of pre-stressing are practically equivalent with respect the deformations while show a not negligible difference regarding the amount of steel reinforcement which is about 7% greater for pre-tensioned strands. This fact allows to reduce the prestressing reinforcement in post-tensioned sections as the final force in the cable is of about 7% greater than that existing in pre-tensioned strands as they are affected by an elastic initial loss of prestressing force which is not on the contrary present, due to the prestressing technique, in post-tensioned cables.

## 4. CONCLUDING REMARKS

The viscoelastic analysis of P.C. sections performed assuming the simplifying hypothesis of considering the prestressing force concentrated in the resultant cable and assuming for concrete a viscoelastic algebraic law has allowed to point out significant aspects related to the long-term behaviour of these structures. Regarding this subject it has been proven that in the pre-tensioned sections the reduction of prestressing force is more marked as a consequence of the initial elastic loss which for the most usual prestressing situations, connected to high values of $\alpha$, is not adequately recovered by the increasing produced by external loads. Regarding this aspect the post-tensioned sections are more profitable, but a final judgement has to be drawn also considering the friction loss which specially for cables with significant curvatures can produce high reductions of the prestressing force. Regarding the deformational aspect the two way of prestressing lead to substantially similar results but it is noteworthy to emphasize the different character of the time evolutions of the sectional deformation which can have equal or opposite sign with respect the intial curvature depending from the level of the imposed prestressing force. It is so necessary to proceed with care and precision in the analysis of long term behaviour of R.C. sections and at this subject in author's mind the proposed method represents a suitable and simple tool to reach reliable results.

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