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Serviceability of Beams with Slender Webs

Aptitude au service de poutrelles avec des âmes élancées

Gebrauchstauglichkeit von Trägern mit schlanken Stegen

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SUMMARY

Normally static serviceability criteria for beams are given as non-dimensional deflections. In this paper the relations between the non-dimensional deflection and slope, curvature and strain have been derived. One of the results is, that in the case of unsymmetrical end restraints and inner spans, the limitation of the non-dimensional deflection may be insufficient.

RESUME

L'aptitude au service des poutres est généralement exprimée par des flèches rapportées aux portées. Dans cette contribution, les relations de l'inclinaison, courbure et allongement sont présentées. La résistance au cisaillement des poutres est prise en considération. Dans le cas de conditions aux limites asymétriques et de travées centrales, la limitation de la flèche rapportée à la portée peut être insuffisante.

ZUSAMMENFASSUNG

Für Biegeträger gibt es praktisch nur Gebrauchstauglichkeitskriterien in Form von spannweitenbezogenen Durchbiegungen. In diesem Beitrag werden die Beziehungen zu den Verformungsgrössen Neigung, Krümmung und Dehnung unter Berücksichtigung der Schubsteifigkeit dargestellt. Es zeigt sich, dass bei Trägern mit unsymmetrischen Randbedingungen und bei Innenfeldern die pauschale Begrenzung der bezogenen Durchbiegung nicht ausreichend sein kann.

1. INTRODUCTION

For the check of serviceability of beams it is usually requested to limit the relative deflections (δ/L). The limiting values specified in codes are very general values. Therefore they are assumed to be independent from static systems. They should be understood as maximum deflections.

Currently an increasing use of slender beams with plane and profiled webs may be observed. While calculating the deformations of such beams, it is necessary to consider not only the bending stiffness but also the shear stiffness, otherwise the deformations would be underestimated.

It was pointed out very early, that the influence of the shear stiffness has to be taken into consideration while calculating the deformations of beams. Therefore the shear coefficient K was established. The different influences (cross section, static system, type of loading, loading point) on this coefficient K has been discussed in several papers [1 - 4].

Nowerdays in the german literature, the shear deformation is introduced into the differential equation of the elastic curve by means of the coefficient RHO. RHO describes the ratio of the bending stiffness to the shear stiffness and the length of the beam:

$$RHO = \frac{EI}{l^2 G A_Q} \qquad (1)$$

where $A_Q = A/K$.

Limit values for RHO have been estimated in [5]. If these values are exceeded, the shear stiffness should be introduced into the calculation of the deflections. These values are valid only for beams with symmetrical end restraints.

In this paper, all relevant characteristics of deformations are investigated as functions of the shear stiffness.

2. (NON DIMENSIONAL) DEFORMATIONS OF BEAMS

2.1 General

In the following, for different static systems, the relationship between the maximum values for the deflection, the slope of the elastic curve and the curvature will be shown. In addition to [6] the shear stiffness will be taken into consideration. Elastic material behaviour is assumed, web buckling is neglected. The considered static systems are shown in Table 1.

2.2 Deflection, Slope, Curvature and Strain

For beams with uniform load wich are handled here, it can be generally stated:

$$\max \frac{\delta}{l} = s_\delta \cdot \frac{q \cdot l^3}{EI}, \quad \max \alpha = s_\alpha \cdot \frac{q \cdot l^3}{EI} = \bar{s}_\alpha \cdot \frac{\delta}{l}, \quad \max \ae = s_\ae \cdot \frac{q \cdot l^2}{EI} = \bar{s}_\ae \cdot \frac{\delta}{l} \cdot \frac{1}{l} \qquad (2a,b,c)$$

where s_δ is the deflection coefficient, s_α and $\bar{s}_\alpha$ are slope coefficients and $s_\ae$ and $\bar{s}_\ae$ are curvature coefficients for the influence of the static system. The related coefficients $\bar{s}_\alpha$ and $\bar{s}_\ae$ are given to

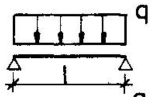
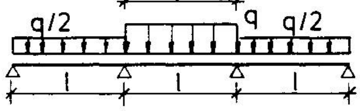
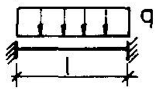
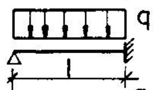
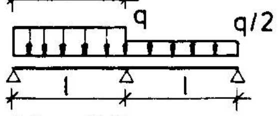
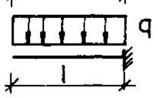
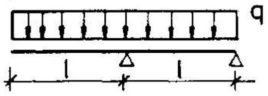
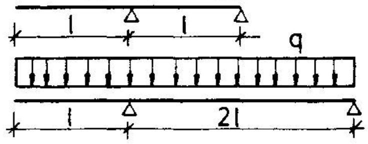
$$\bar{s}_\alpha = s_\alpha / s_\delta \quad \text{and} \quad \bar{s}_\ae = s_\ae / s_\delta . \qquad (3a,b)$$

With these related coefficients it is possible to show the influence of the non-dimensional deflection on the other deformations.

For simply supported beams (system 1) they are given as:

$$s_\delta = \frac{5 + 48 \cdot RHO}{384}$$
$$\bar{s}_\alpha = 3.2 \cdot \frac{1 + 12 \cdot RHO}{1 + 9.6 \cdot RHO} \qquad (4a,b,c)$$
$$\bar{s}_\ae = \frac{9.6}{1 + 9.6 \cdot RHO}$$

In Table 1, these three coefficients are given for RHO = 0.0, i.e. without consideration of the shear stiffness (index 0).

static system		$s_{\delta,0}$	$\bar{s}_{\alpha,0}$	$\bar{s}_{\ae,0}$
1		5/384	3.20	9.60
2		1.4/384	2.89	20.6
3		1/384	3.08	32.0
4		2.08/384	3.85	23.1
5		2.79/384	3.59	12.9
6		48/384	1.333	4.0
7		96/384	1.167	2.0
8		48/384	1.333	4.0

<u>Table 1</u> System coefficients for some static systems

It should be noted, that the ratio EI/GA_Q has to be constant for the regarded system. For beams with cantilevers, l is the length of the cantilever.

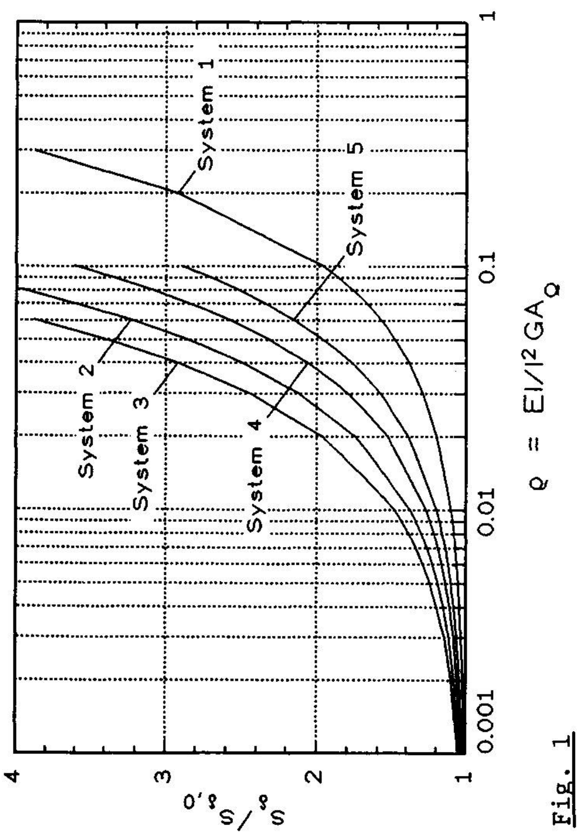


Fig. 1

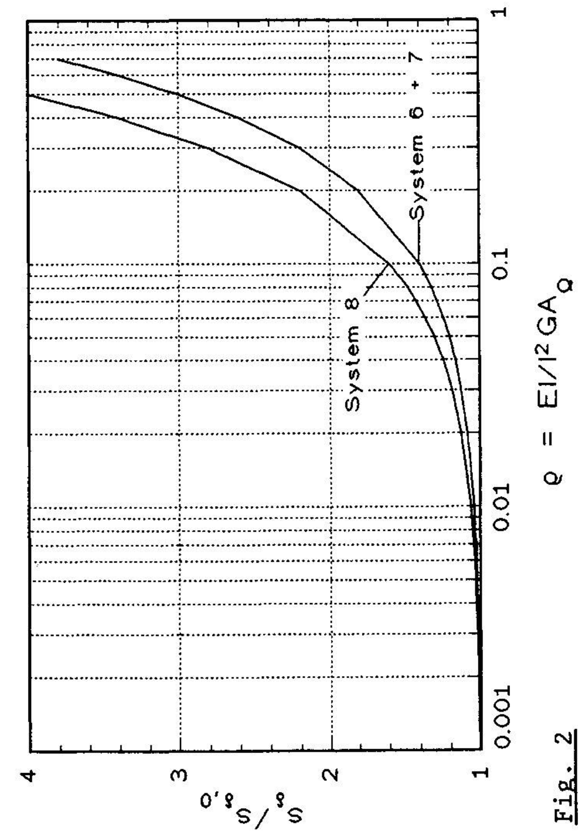


Fig. 2

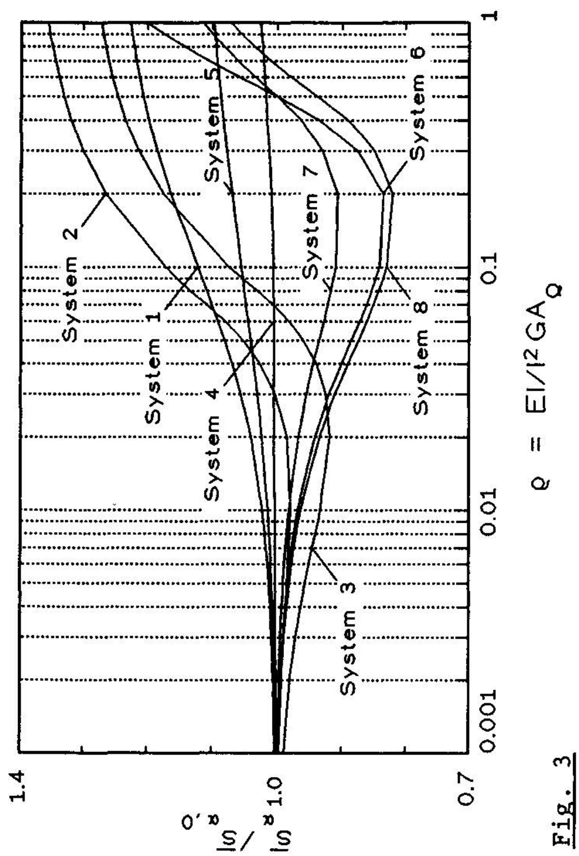


Fig. 3

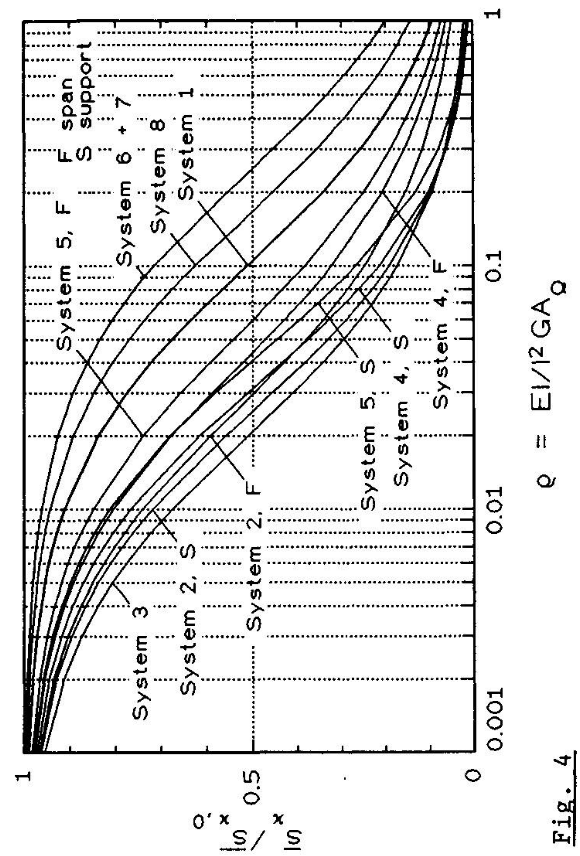


Fig. 4

From Table 1 follows: The advantage of continuous respective fixed beams is due more to their small deflection than to their greater load carrying capacity. The deflection coefficients (under consideration of the shear stiffness) are shown in Fig. 1 and 2. They are presented as related coefficients $s_\delta / s_{\delta,0}$. As expected, the advantage of statically undetermined systems gets lost, in case of large RHO values. The stronger the end restraints of the analysed span, the larger the deflection due to the shear deformation. In case of cantilever beams, this increase is relatively small because of the weak boundary conditions (Fig 2). Normally cantilevers are shorter than simply supported beams. This is the reason for larger RHO values of cantilever beams.

The same is true for the maximum slopes. Regarding the related slope coefficients $\bar{s}_{\alpha,0}$ in Table 1 follows: With beams fixed at one end and simply supported at the other, the slope of the elastic curve with a given non-dimensional deflection is great, whereby it is irrelevant at cantilever beams.

The related slope coefficients (under consideration of the shear stiffness) are shown in Fig. 3. They are presented as related coefficients $\bar{s}_\alpha / \bar{s}_{\alpha,0}$ ($\bar{s}_{\alpha,0}$ from Table 1). Therefrom follows that the advantage of systems with symmetrical end restraints gets lost not before large RHO values are reached. In case of cantilevers this influence may be neglected.

On principle, the advantage of continuous beams with more than two spans are smaller curvatures in comparison with the other systems. From Table 1 follows: Assuming the same non-dimensional deflection, systems with higher stiffness lead to larger curvatures.

The related curvature coefficients (under consideration of the shear stiffness) are shown in Fig. 4. They are presented as related coefficients $\bar{s}_{æ} / \bar{s}_{æ,0}$ ($\bar{s}_{æ,0}$ from Table 1). Assuming equal non-dimensional deflections, it may be observed, that the curvature decreases with increasing RHO. This is caused by the strong increase of the deflection with increasing RHO. The curvature decreases more at supports than in the spans. In case of cantilevers, the curvature does not depend on RHO.

The strains may be obtained from:

$$\epsilon = æ \cdot e = \bar{s}_{æ} \cdot \frac{\delta}{l} \cdot \frac{e}{l} \, . \qquad (5)$$

Shear deformation and finally slopes of the elastic curve occur on both sides of the supports of continuous beams (Fig. 5). Adjacent members get curved additionally (important for brittle members!). Therefore it shouldn't be concluded from Fig. 4, that the problem of strains in coverings decreases with increasing RHO!

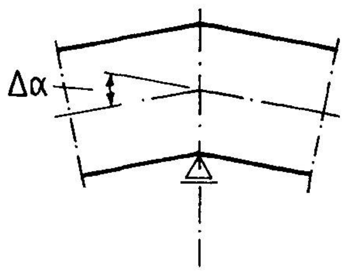


Fig. 5 Shear deformation at the supports of continuous beams

3. SERVICEABILITY CRITERIA

What about the importance of theoretical results for serviceability? According to the authors knowledge, static serviceability criteria for beams are given as absolute values (lim δ) or non-dimensional deflection (lim δ/l). In this paper the relations between the non-dimensional deflection and slope, curvature and strain have been derived. It can be shown that in cases of unsymmetrical end restraints and inner spans, the limitation of the non-dimensional deflection may be insufficient.

The derived relations are valid for all existing beam shapes and materials. If e.g. girders with trapezoidally corrugated webs are used, RHO should be completed by an additional term.

CONCLUSIONS

According to [5], the shear stiffness should be introduced into the estimation of the deflections of spans with symmetrical end restraints, if RHO > $8.5\cdot10^{-3}$ for statically determined systems resp. RHO > $2.5\cdot10^{-3}$ for statically undetermined systems. The presented investigations lead to the conclusion, that these limit values are not sufficient considering the other relevant characteristics of the deformations. More accurate investigations concerning different building materials are necessary.

The other characteristics of the deformations (slope, curvature and strain) shouldn't get lost.

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