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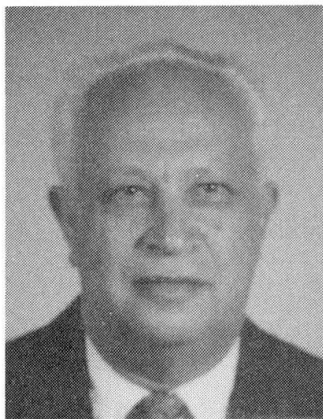
## Structural Serviceability Depending on Multiple Parameters

Aptitude au service de constructions dépendant de plusieurs paramètres

Gebrauchstauglichkeit in Abhängigkeit von mehrfachen Parametern

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### SUMMARY

A probabilistic cracking analysis of prestressed slabs based on an experimental model and on the variation characteristics of all essential parameters, is presented. Traditional estimations of serviceability in terms of cracking yields significantly different reliabilities for structures with different variable parameters. The situation may be improved with the aid of multiple analysis, whereby some of parameters can be taken as constants.

### RESUME

Une analyse probabilistique de la fissuration des dalles précontraintes basée sur un modèle expérimental est mise en évidence; elle tient compte de la variation des caractéristiques de tous les paramètres essentiels. L'approche traditionnelle de l'aptitude au service basée sur la fissuration de constructions avec plusieurs paramètres variables donne des sécurités différentes. Cette situation peut et doit être améliorée à l'aide d'une analyse de plusieurs paramètres, certains paramètres pouvant être considérés comme constants.

### ZUSAMMENFASSUNG

Aufgrund von Experimenten und der Variationseigenschaften aller wichtigen Parameter wird eine probabilistische Berechnung der Rissbildung in vorgespannten Platten entwickelt. Der herkömmliche Nachweis der Rissebeschränkung ergibt deutlich abweichende Tragwerkszuverlässigkeiten mit anderen variablen Parametern. Der Unterschied verringert sich durch Einsatz mehrfacher Parameteranalyse, wobei einige Parameter konstant gehalten werden können.



## 1. INTRODUCTION

For many concrete structures, mainly prestressed members, the deciding factor has become serviceability in terms of cracking. As a rule, crack appearance depends on one, two or more variable parameters, such as loads, tensile strength of the concrete, residual prestress, topping weight and its shrinkage, etc.

The universally accepted method of verification of structure serviceability in terms of cracking is based on the characteristic values of most of the above-mentioned variable parameters [1-4]. It is known that structures differ from one another by the number of variable parameters, as well as by the degree of their variation. Consequently, different structures should have considerably differing service reliability [5-8]. At the same time not all variable parameters have a pronounced effect on the serviceability of a structure.

In this paper structural serviceability is analysed under allowance for all essential variable parameters: tensile strength of the concrete (including its partial variation over the length of each member), prestress at the extreme fibres, weight of topping, shrinkage effects, as well as live loads. The cracking probability analysis, proposed by the author, is based on consideration of a structural model with given spacing of potential crack sites, predetermined by experiments, and on evaluation of the overall cracking probability of the considered structure as function of the above multiple parameters. It is shown that only part of the variable parameters should be taken into consideration as variable; the others may be treated as constants through their mean values.

## 2. STRUCTURAL MODEL FOR ANALYSIS

The model of a member under the weight of structure and topping and live load -  $q$ , is presented in Fig. 1 [5].

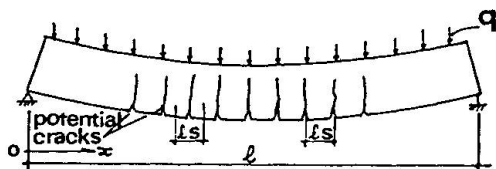


Fig. 1: Model of member with potential cracks.

The probability of crack appearance in a member with a particular mean concrete strength -  $f_{cm}$  under the considered loads, according to the accepted model, is:

$$P_q = 1 - \prod_0^{l/l_s} (1 - P_x) \quad (1)$$

where :

$\ell$  - member span,

$\ell_s$  - spacing of potential crack sites, predetermined by experiments [5],  $\ell_s = 40$  mm.

$P_x$  -probability of crack appearance in the x-section, for a Gaussian distribution:

$$P_x = \sum_{j=-\infty}^{j_0} [\exp(-j^2 / 2)] S_c / \sqrt{2\pi} \quad (2)$$

$S_c$  - summation step .

The limit of summation  $j_0$  in (2) is evaluated by :

$$j_0 = [(M_{xp} / W_1 + M_{xq} / W + f_{p1} - f_p) / f_{cm} - 1] C_{v1} \quad (3)$$

where :

$C_{v1}$  - variation coefficient of concrete strength in a member over its length,

$f_{p1}$  - tensile stress due to topping concrete shrinkage,

$f_p$  - residual prestress in most stressed fibre,

$W_1$  - section modulus of considered prestressed member,

$W$  - section modulus of member with topping,

$M_{xp}$  - moment in the x-section due to dead loads,

$$M_{xp} = M_{xg} + M_{xt} \quad (4)$$

$M_{xg}$  - moment due to dead weight (g) of prestressed part of structure,

$$M_{xg} = g/2 (\ell x - x^2) ,$$

$M_{xt}$  - moment due to dead weight of topping ( $g_t$ ),  $M_{xt} = g_t/2 (\ell x - x^2) ,$

$M_{xq}$  - moment due to live load q,  $M_{xq} = q/2 (\ell x - x^2) ;$

$$M_{xp} = (g + g_t) \cdot (\ell x - x^2) / 2 \quad (5)$$

q - sum of given live loads,

$f_{cm}$  - mean strength of concrete in considered member.

### 3. ESTIMATION OF CRACK APPEARANCE IN A MEMBER IN THE STRUCTURE POPULATION

The probability of crack appearance in a member (Fig. 1) with a given grade of concrete under a particular load is evaluated by :

$$\sum_c (P_q \cdot P_1) \quad (6)$$



where

c - number of summation steps for given concrete grade ( $f_{cmm}$  - mean strength for this grade),

$$f_{cm} = f_{cmm}(1 + i_o \cdot C_{vo}) \quad (7)$$

$P_1$  - probability of occurrence of the particular mean concrete strength in the considered member, namely

$$P_1 = S_1 [\exp(i_o^2 / 2)] \sqrt{2\pi} \quad (8)$$

where

$S_1$  - summation step,  $i_o$  - independent parameter,  $C_{vo}$  - variation coefficient of the mean concrete strength in members. The overall variation coefficient of tensile strength :

$$C_v = \sqrt{C_{v1}^2 + C_{vo}^2} ,$$

by [1] :  $C_v = 18.3\%$  .

The overall cracking probability of the considered structure in the general case, with all variables taken into account, may be computed by :

$$P = \Sigma(\Sigma(\Sigma(\Sigma(\Sigma(P_q \cdot P_1) \cdot P_2) \cdot P_3) \cdot P_4) \cdot P_5) \cdot P_6 \quad (9)$$

where the subscripts 1-6 of the partial probabilities refer specifically to : (1) mean concrete strength, (2) residual prestress, (3) member weight, (4) topping weight, (5) topping shrinkage, (6) live loads, respectively.

#### 4. NUMERICAL ANALYSIS OF CRACKING IN PRESTRESSED SLABS

The different floor slabs (Fig. 2) are analysed for the characteristic live load ( $q_k$ ) determined by the traditional approach as the live load causing crack appearance in the middle section of the considered member :

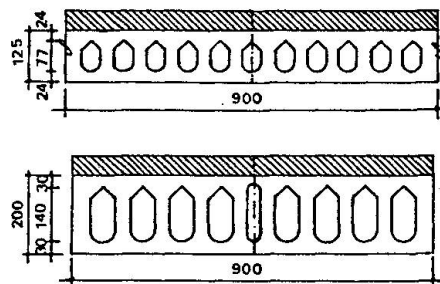


Fig. 2: Sections of analysed floor slabs.

$$q_k = q_{kt} - (g_m + g_{lm}) W / W_1 \quad (10)$$

where

$q_{kt}$  - total load causing crack appearance in the member :

$$q_{kt} = 8 (f_{pk} + f_{cmk} - f_{p1}) \times W / \ell^2 \quad (11)$$

$f_m$  and  $g_{lm}$  - dead weights of prestressed element and of topping, respectively.

$f_{pk}$  - design residual prestress in the most stressed fibre.

$f_{cmk}$  - characteristic concrete flexural-tensile strength.

$f_{p1}$  - stress in above-mentioned fibre due to concrete shrinkage of the topping.

$W$  - section modulus of composite slab.

$W_1$  - section modulus of prestressed element.

$\ell$  - slab span.

The essential characteristics of the analysed slabs are the following: concrete strengths - C50 - of prestress elements, C - 30 - of toppings; spans - 6 m and 9 m respectively for section depth 125 mm and 200 mm. The live loads were varied within the limits of the variation coefficient  $C_q$ , from .05 to .3 . The residual prestress was varied in limits of  $C_p$ , from .02 to .15 . The coefficients of variation of concrete stress were included in the analysis by [1]:

$C_{v0} = C_{v1} = .13$ , the weight variations by  $C_v = .1$  and topping shrinkage by  $C_{v,sh} = .15$  .

The essential computation results are shown in Figs. 3(a) and (b).

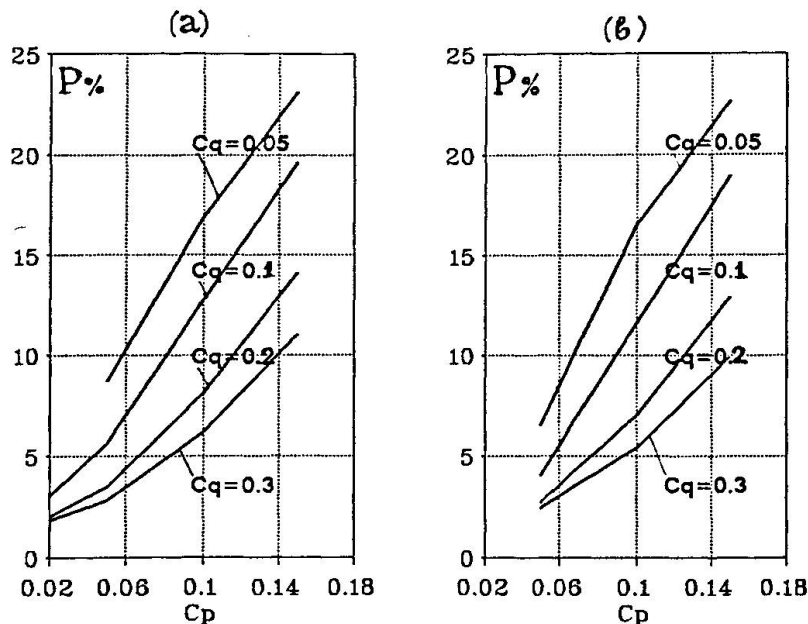


Fig. 3: Crack appearance probabilities in 6m-span slabs (a) and in 9m span slabs (b) vs. variation coefficient of prestress ( $C_p$ ) for different variations of live load.

It is seen that under the traditional approach we can have in practice a wide range of reliability in terms of cracking.



The variability of member- and topping weights as well as of topping shrinkage affects the crack appearance probabilities only slightly. Variation of the concrete strength, residual prestress and live load may significantly affect the service reliability in terms of cracking. In principle, if all parameter variations obey the codes, the structural reliability in terms of cracking should be very low. The situation, it seems, is improved through such favorable factors as : (1) superior real concrete strength with lower variability and (2) lower real live load.

## 5. CONCLUSIONS

Traditional estimation of concrete structure serviceability in terms of cracking depending on multiple parameters yields significantly differing reliabilities for structures with different variable parameters. The probability of crack appearance in analysed common prestressed multihollow slabs with given codified parameters may range from a few percent to 20% and even more. At the same time, in actuality, some favourable factors, such as higher concrete strength, lower strength variability and lower loads can increase the structural reliability.

The main problem is to arrive at equally reliable structures. The best solution is suitable probabilistic analysis based on probabilistic criteria and statistical initial data. Such initial parameters as concrete strength, prestress and live loads should be taken into account as variables. Other parameters can be taken as constants, by their mean values.

In any event, the traditional cracking estimate should be corrected by behaviour factors, taking into account the load- and prestress variations, as well as the variable characteristics of the concrete strength and real possible deviations of the essential structure parameters.

All the foregoing calls for supplementation of the codes by suitable probabilistic restrictions and by statistical data on loads and strength of materials.

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