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Optimal Fatigue Testing - a Reassessment Tool

Essais de fatigue optimalisés - une nouvelle estimation

Optimale Ermüdungsversuche als Hilfsmittel zur Überprüfung

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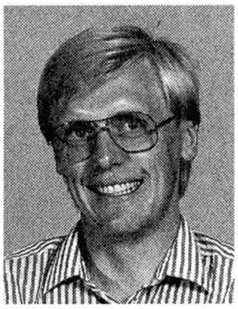

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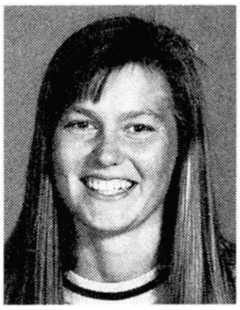

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SUMMARY

This paper considers the reassessment of the reliability of tubular joints subjected to fatigue load. The reassessment is considered in two parts namely the task of utilizing new experimental data on fatigue life to update the reliability of the tubular joint ant the task of planning new fatigue life experiments for the same purpose. The methodology is based on modern probabilistic concepts amd classical decision theory. The special case where the fatigue life experiments are given in terms of SN curves is considered in particular. The proposed techniques are illustrated by an example.

RÉSUMÉ

L'étude traite d'une nouvelle estimation de la fiabilité des jonctions tubulaires sous l'effet de charges de fatigue. La nouvelle estimation est présentée en deux parties: utilisation de nouveaux résultats expérimentaux de résistance à la fatigue pour une mise à jour de la fiabilité des jonctions tubulaires, et organisation de nouveaux essais dans le même but. La méthode est fondée sur la théorie des probabilités et la théorie classique de décision. L'étude traite spécialement les cas où des expériences sont présentées sous forme de courbes SN. Les techniques sont illustrées par un exemple.

ZUSAMMENFASSUNG

Der Beitrag behandelt die Überprüfung der Zuverlässigkeit ermüdungsbelasteter Rohrverbindungen. Dazu sind zwei Teilaufgaben zu lösen: Die Verwendung neuen Datenmaterials zu ihrer Ermüdungsdauer und die Planung neuer Ermüdungsversuche zu eben diesem Zweck. Die Methoden basieren auf modernen Konzepten der Wahrscheinlichkeitstheorie und auf klassischer Entscheidungstheorie. Der Fall, dass Versuchsdaten alks S-N-Kurven vorliegen, wird gesondert behandelt. Die vorgeschlagenen Verfahren werden an einem Beispiel erläutert.

1. Introduction

Engineering structures subjected to environmental conditions such as time varying loading and corrosion will fail when the accumulated damage of the structure reaches a certain critical level. When a structure is designed and its design is adjusted such that the target safety of the structure is maintained throughout its design lifetime. This is obtained either through classical code based design or using modern probabilistic concepts. Typically, however, for engineering structures the original use of the structure or the initial design conditions is changed several times before it is taken out of service. Such changes are e.g. a prolongation of the design lifetime, changes in the loading conditions, but also imposed accidental damage conditions have similar effects. In such cases it may be necessary to justify that the structure is capable of fulfilling its requirements in terms of safety, i.e. to reassess the structural safety. For this purpose information about the actual state of the structure is collected. Such information obviously includes the damage state of the structure, but also information about other important characteristics of the failure modes of the structure. Important examples hereof are material parameters, loading characteristics and geometry. The collection of such information can be rather expensive and cumbersome as is e.g. the case of inspection planning for offshore structures or in the case of material fatigue life testing. Therfore, it is mandatory to have access to a methodology which provides a rational decision basis on how to collect such additional information taking into account the economic aspects. The framework of modern reliability theory, see e.g. Madsen et al. [1] and classical decision theory, see e.g. Raiffa & Schlaifer [2] provides such a tool. The scope of the present paper is to present this tool and to illustrate its application in the case where the safety of a structure subjected to fatigue failure is reassessed using additional fatigue life experiment data. Two different cases of reassessment are considered, namely the case where the reliability of a structural component subject to fatigue failure is updated using new fatigue data and the case where a new experiment is planned for reassessment.

2. Experiment Planning as a Decision Tool

The use of experimental data for the purpose of modelling is recognized as one of the most important tools in the design of engineering structures, see e.g. Ditlevsen [3]. Typical examples hereof are the estimation of material characteristics such as yield stresses and modulus of elasticity, but experimental results are also used for the estimation of parameters in parametric equations such as fatigue crack growth models. In the past experiments have normally been performed such that the uncertainty associated with the measured quantity is adjusted to some specified acceptable range, see Viertl [4]. These methods disregard economic aspects and the actual engineering application where the statistics of the considered quantity are used. Experiment planning was i.e. seen as an isolated problem.

The increasingly accepted application of modern probabilistic methods such as FORM/SORM methods in structural engineering allows for a more refined formulation of experimental planning. This is due to realistic probabilistic modelling of loading, consistent representation of experimental results together with efficient tools for the estimation of probabilities. Using these tools it is possible to perform experiment planning from a more rational basis namely to reduce the total expected costs for the considered engineering structure. This approach is fundamentally different from the classical approach mentioned above as it allows to perform experiment planning in a cost optimal fashion. Following results from classical decision theory, see e.g. Ang & Tang [5] the optimal experiment plan is the experiment plan which minimizes the expected total cost $E[C_T]$ of the considered engineering structure. Here, total expected costs include all costs associated with the planned experiments $E[C_e]$, the expected costs of the structural design $E[C_d]$, the expected costs of maintenance $E[C_m]$ together with the expected costs of failure of the structure $E[C_f]$. Hence, the expected total costs for an engineering structure can be written as

$$E[C_T] = E[C_e] + E[C_d] + E[C_f] + E[C_m] \tag{1}$$

Experiment planning can in a wide sense be understood as the planning of any action revealing information which has impact on the predicted performance of the structure. Therefore, an experiment can be the action of performing experiments for the estimation of the structural material parameters but it can also be the action of measuring unknown (and uncertain) quantities such as structural damage, structural dimensions and characteristics of the loading environment. With this interpretation of experiment planning it is seen that experiment planning becomes an essential tool in decision making for engineering structures not only in the design phase of the structure but also in the situation of a reassessment of the structural integrity.

3. Experiment Planning in Fatigue Testing

In the fatigue life assessment of engineering structures such as steel bridges and offshore steel structures parameters of importance are among others the crack growth law material parameters, the stress concentration factors, the actual geometry of the considered structural detail and the damage state of the structure.

In the reassessment situation the reliability of the structure is updated through experiments revealing information about any of the above-mentioned quantities.

Assume that the failure probability can be estimated from

$$P_f = P(g(\mathbf{X}, N) \leq 0) \tag{2}$$

where $g(\star)$ is the limit state function, $\mathbf{X}$ is a vector including the basic uncertain variables such as geometric parameters and stress concentration factors, see e.g. Dover et al. [6], and N is the random fatigue lifetime. Then the failure probability can be updated through experiments revealing the realizations of the basic uncertain variables and/or through experiments revealing realizations of functional relationships of the basic uncertain variables.

In the following it is assumed that it is possible to perform fatigue experiments using a material which is representative for the material used in the structure under consideration. The specific problem of making a cost optimal experiment plan is treated.

Typically, fatigue life experiments are performed in order to estimate the generally uncertain parameters $\mathbf{P}$ of the lifetime distribution $F_N(n|\mathbf{P})$ for a given material. Given a distribution assumption of the fatigue lifetime in terms of the number of constants or equivalent stress range load cycles to failure N the distribution parameters $\mathbf{P}$ are estimated using standard tools from the statistics, such as the maximum likelihood method, the method of moments or Bayesian statistics. Fatigue life experiments can also be used to estimate parameters in distribution free material lifetime models using e.g. regression analysis in the statistical analysis of SN data.

The SN curve and the data points illustrated in figure 1 are taken from Dover et al. [6]. The curves represent the mean value and the two standard deviation fractile of fatigue life tests of offshore tubular joints obtained in the study [7]. The data points in the figure are used later in an example as new information in a reassessment situation.

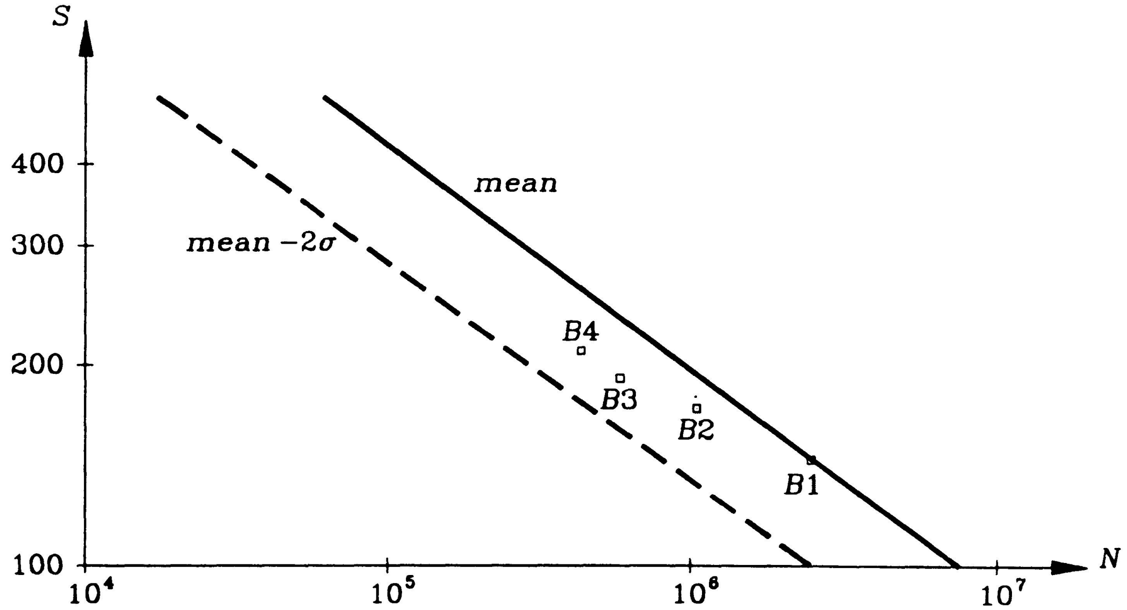

Figure 1. Illustration of typical representation of fatigue lifetime in terms of an SN diagram, Dover et al. [6].

To define an experiment plan the number of experiments, the stress range levels for the individual experiments and the maximum number of load cycles until termination are most frequently used as decision parameters. When the number of experiments is increased the uncertainty structure associated with the model parameters **P** is changed. The uncertainty will in general decrease if the number of experiments is increased. Therefore, the expected failure costs for the mechanical component considered are also changed.

The experimental costs due to additional experiments are obviously dependent on the stress range levels for which the experiments are to be performed. Therefore, when deciding if additional experiments should be performed the relevant failure criteria and all the information about the uncertain variables involved in the problem have to be taken into consideration.

As stated above the optimal experiment plan is the plan which minimizes the total expected experiment and failure costs caused by additional experiments at a given stress range level and a given maximum number of load cycles before termination. As the design costs cannot be changed in a reassessment situation their part in the expected total costs can be omitted in the present context. For simplicity, maintenance costs are not considered even though they can play an important role in the case of experimental planning for future reassessments. It is assumed that some prior information exists, for example in the form of existing experimental results and the problem is to determine an optimal plan for additional experiments. The existing experiments are assumed to be performed at M stress range levels $s_1, s_2, \ldots, s_M$. The number of additional experiments are $\mathbf{n} = (n_1, n_2, \ldots, n_M)^T$ at the M levels $s_1, s_2, \ldots, s_M$. As decision variables $\mathbf{n}$ and the number of load cycles to termination N_{ter} can be used.

Because the number of load cycles to failure in an additional experiment is random the change in total expected costs has to be integrated over all possible outcomes of load cycles to failure weighted by their likelihood. This corresponds to a pre-posteriori analysis from the classical decision theory see e.g. Raiffa & Schlaifer [2].

The corresponding optimization problem is written

$$\min_{N_{ter},\mathbf{n}} E[C_T(N_{ter}, \mathbf{n})] \tag{3}$$

Constraints related to the failure probability can easily be incorporated into (3). The total expected costs $E[C_T(N_{ter}, \mathbf{n})]$ associated with additional experiments are then

$$E[C_T(N_{ter}, \mathbf{n})] = C_f E_{\mathbf{N}^U}[P_f{}^U(\mathbf{N}^U, N_{ter}, \mathbf{n})] + E_{\mathbf{N}^U}[C_e(\mathbf{N}^U, N_{ter}, \mathbf{n})] \quad (4)$$

where C_f is the cost of failure, $P_f^U(\mathbf{N}^U, N_{ter}, \mathbf{n})$ is the 'updated' probability of failure given the additional unknown experimental results modelled by the number of load cycles $\mathbf{N}^U$ to failure at the corresponding stress range levels. How to determine this probability is described in the next section. $\mathbf{N}^U$ are modelled as random variables and $E_{\mathbf{N}^U}[\star]$ denotes the expectation operation with respect to $\mathbf{N}^U$. C_e is the experimental costs. The expectation operations in (4) can be estimated by nested FORM/SORM, see e.g. Guers & Rackwitz [8].

The above-mentioned technique can without theoretical difficulties be generalized to the situation where no experimental results are available at the time where the test planning is made. In this case subjective prior information can be used.

4. Probabilistic Reassessment

When new information becomes available the estimates of the probability of failure (and the reliability) of structures can be reassessed. The information considered in this paper is divided into two types

- information of functions of basic stochastic variables
- sample information of basic stochastic variables

The first type of information is related to information about events involving more than one basic stochastic variable. Examples of this type of information are proof load tests, non-failure observations, measurements of response quantities and inspection results related to damage quantities such as fatigue crack sizes.

The information is generally modelled using a stochastic variable Y which is a function of the basic stochastic variables, i.e. $Y = h(\mathbf{X}_1, \mathbf{X}_2, ..., \mathbf{X}_n, N)$. The actual measurements are thus realisations (samples) of Y. The observations can be modelled as equality events $E = \{H = 0\} = \{Y = y_m\}$ or inequality events $I = \{H \leq 0\} = \{Y \leq y_m\}$ where y_m can be some observed quantity.

The probability of failure of a single element with safety margin $M_F = g(\mathbf{X}, N) \leq 0$ can then be updated, see e.g. Madsen [9] and Rackwitz & Schrupp [10].

$$P_f^U = P(M_F \leq 0 | H \leq 0) = \frac{P(M_F \leq 0 \cap H \leq 0)}{P(H \leq 0)} \quad (5)$$

or in the case of observations modelled by equality events

$$P_f^U = P(M_F \leq 0 | H = 0) = \frac{P(M_F \leq 0 \cap H = 0)}{P(H = 0)} \quad (6)$$

These conditional probabilities can be evaluated by standard FORM, see e.g. Madsen [9].

The second type of information is related to situations where samples of one or more basic stochastic variables are obtained. Examples of this type of information are measurements of the geometrical quantities and test results for the fatigue life of a component. Bayesian statistical methods can be used to obtain updated (predictive) distribution functions of the stochastic variables, see Lindley [11] and Aitchison & Dunsmore [12].

Based on prior information (subjective and/or test data) a density function $f_N(n|\mathbf{P})$ for a single basic stochastic variable N is established. $\mathbf{P}$ are parameters defining the distribution function for N. The initial (prior) density function of $\mathbf{P}$ is denoted $f'_{\mathbf{P}}(\mathbf{p})$.

Next it is assumed that an experiment or inspection is performed. m realisations of the stochastic variable N are obtained and are denoted $\mathbf{n}^* = (n_1^*, n_2^*, ..., n_m^*)$. The measurements are assumed to be independent. The updated (posterior) density function $f''_{\mathbf{P}}(\mathbf{p}|\mathbf{n}^*)$ of the uncertain parameters $\mathbf{P}$ taking into account the realisations is

$$f''_{\mathbf{P}}(\mathbf{p}|\mathbf{n}^*) = \frac{f_m(\mathbf{n}^*|\mathbf{p})f'_{\mathbf{P}}(\mathbf{p})}{\int f_m(\mathbf{n}^*|\mathbf{p})f'_{\mathbf{P}}(\mathbf{p})d\mathbf{p}} \tag{7}$$

where $f_m(\mathbf{n}^*|\mathbf{p}) = \prod_{i=1}^m f_N(n_i|\mathbf{p})$.

The predictive density function (i.e. the updated density function) of the stochastic variables N taking into account the realisation $\mathbf{n}^*$ is obtained by

$$f_N(n|\mathbf{n}^*) = \int f_N(n|\mathbf{p})f''_{\mathbf{P}}(\mathbf{p}|\mathbf{n}^*)\, d\mathbf{p} \tag{8}$$

An updated estimate of the probability of failure $P_f^U(\mathbf{n}^*) = P(g(\mathbf{X}, N) \leq 0)$ can then be determined using the updated (predictive) density function $f_N(n|\mathbf{n}^*)$ as density function for N.

An updated estimate of P_f can also be obtained using the posterior density function of $\mathbf{P}$.

$$P_f^U(\mathbf{n}^*) = P(g(\mathbf{X}, N(\mathbf{P})) \geq 0) \tag{9}$$

In (9) $N, \mathbf{X}$ and $\mathbf{P}$ are stochastic variables. The density function for N is $f_N(n|\mathbf{P})$ and the density function for $\mathbf{P}$ can be the posterior density function $f''_{\mathbf{P}}(\mathbf{p}|\mathbf{n}^*)$.

Instead of using the posterior density an updated stochastic model for $\mathbf{P}$ can also be obtained using classical statistical methods, e.g. the maximum likelihood method. In this case the parameters $\mathbf{P}$ are treated as stochastic variables and the distribution parameters in the joint distribution function $f_{\mathbf{P}}$ are determined by e.g. the maximum likelihood method.

5. Example

In the following example a reassessment situation is considered for an offshore tubular joint subjected to fatigue crack growth. The joint considered in particular is the joint also considered in Dover et al. [6] where the fatigue life has been experimentally determined. It is assumed that the prior information about the fatigue life of the considered joint is given through the SN curve in figure 1. Two problems are considered here. First the problem of updating the reliability of the joint for reassessment by introduction of the four new data points in figure 1 is considered. Thereafter the problem of planning an additional fatigue experiment for the purpose of reassessment is considered.

In order to model the prior information of the fatigue life of the joint the model from Madsen et al. [1] is used with the modification that the slope of the SN curve m is assumed to be a deterministic variable $m = -\beta$. Thereby the fatigue lifetime of the offshore joint can be given as

$$\log N = \overline{y} + \sqrt{\frac{D^2 + S_{xx}(b+\beta)^2}{T_3}}(I + \frac{T_1}{r}) - \beta(\log S - \overline{x}) \tag{10}$$

where r is the total number of experiments, $\overline{x} = \frac{1}{r}\sum_{i=1}^r \log s_i$ and $\overline{y} = \frac{1}{r}\sum_{i=1}^r \log n_i^*$. The parameters S_{xx}, b and D are different combinations of first and second moments of the r experiments as defined in [1]. T_1 and I are standardized normal stochastic variables. T_3 has a $\chi^2(r-1)$ distribution. The stochastic variables are assumed to be independent.

As prior information the curves in figure 1 are used. It is assumed that the SN curve in figure 1 is based on $r = 20$ experiments, four experiments at five different levels of effective stress ranges. The logarithm to the fatigue lifetime N given S is assumed to be normal distributed with mean value equal to $29.69 - 3.0 \log S$ and standard deviation equal to $0.6/\sqrt{1+1/r}$. Based on this assumption the sample moments defining the parameters in (10) are estimated using simulation.

The reliability of the joint can now be estimated by considering the following limit state function

$$g(\mathbf{x}) = \log N - \log n_c \tag{11}$$

where it is assumed that the effective stress range S is log-normal distributed with expected value 200 MPa and standard deviation equal to 20 MPa, n_c is assumed equal to $5 \cdot 10^4$. A FORM analysis gives a reliability index $\beta = 3.772$. The reliability is next updated using the four new experiments from figure 1. The results of this updating is shown in table 1. It is seen that inclusion of experiment B1 and B2 gives an increased reliability index whereas B3 and B4 decreases the reliability. If all four experiments are used, then the reliability increases.

experiment	β
B1	3.857
B2	3.820
B3	3.765
B4	3.769
B1+B2+B3+B4	3.892

Table 1. Reassessed reliability indexes using the new experimental data from figure 1.

Finally the problem of planning an additional experiment for reassessment of the reliability is considered. Assuming that the expected cost of a fatigue life experiment is $E[C_e] = 1 \cdot 10^6 + E[N]$ and that the cost of failure of the offshore joint is $C_f = 1.1 \cdot 10^{11}$ the expected total costs $E[C_T]$ of the joint given one additional experiment are plotted in figure 2 as a function of the stress range where the additional experiment is performed. Also, the total costs corresponding to no experiment are plotted. It is seen that the largest utility is obtained by performing an experiment at $S = 340$MPa.

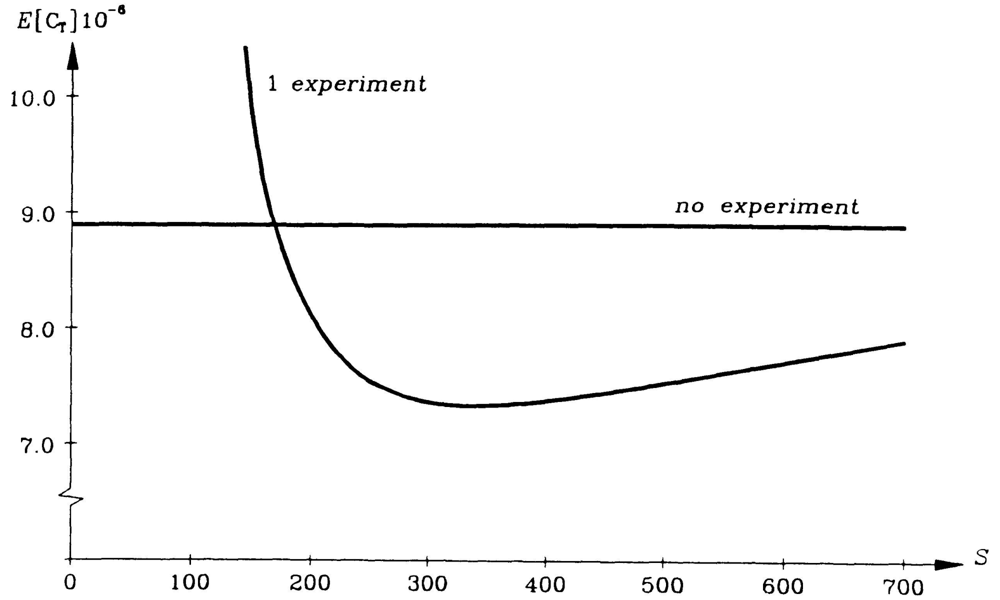

Figure 2. The total expected costs given 1 additional experiment and the total expected cost if no experiment is performed.

6. Conclusions

Based on the modern reliability theory and the classical decision theory a methodology has been proposed for the reassessment of the reliability of engineering structures subject to fatigue failure. Two situations are considered in particular, namely the situation where the reliability is updated using new information about the fatigue life and the situation where a fatigue life experiment is being planned taking economic aspects into account. The methodology is illustrated by an example where a tubular offshore joint is considered for which experimental data are available in terms of SN data. The example clearly shows the significance of additional experiments for the reliability of the joint. It is also shown that the proposed methodology for planning of future fatigue life experiments can be used to identify the most cost-effective stress ranges for additional SN experiments.

7. Acknowledgements

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