

**Zeitschrift:** IABSE reports = Rapports AIPC = IVBH Berichte  
**Band:** 67 (1993)  
  
**Artikel:** Reliability analysis of steel railway bridges under fatigue loading  
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**DOI:** <https://doi.org/10.5169/seals-51361>

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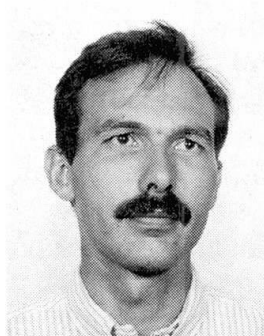
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# **Reliability Analysis of Steel Railway Bridges under Fatigue Loading** **Analyse de fiabilité des ponts rail en acier sous charges de fatigue** **Zuverlässigkeitsanalyse von Bahnbrücken aus Stahl unter Ermüdungslasten**

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## **SUMMARY**

Many railway bridges constructed in the last century are still in service today, some having been repeatedly strengthened to meet to new needs. Regular assessment is an important part of the management of their future service. An important question concerning these bridges is fatigue. This paper shows how the probability of failure of construction details can be calculated taking into account the statistical scatter of certain evaluation parameters. The remaining fatigue life is essentially dependent on stress range, the structural model adopted for the assessment and the uncertainties of the traffic models.

## **RÉSUMÉ**

Beaucoup de ponts en acier, construits au siècle passé, ont été renforcés plusieurs fois afin de satisfaire aux nouvelles exigences. Pour les maintenir en service, ils doivent être évalués et contrôlés périodiquement. Une question importante est le comportement à la fatigue. Cet article présente une méthode pour calculer la probabilité à la rupture des détails de construction à l'aide d'hypothèses probabilistes. La durée de service restante dépend essentiellement de la différence de contraintes, de la précision du modèle statique et des hypothèses sur les incertitudes liées aux charges de trafic.

## **ZUSAMMENFASSUNG**

Viele der heute noch im Betrieb stehenden Bahnbrücken wurden im letzten Jahrhundert erbaut und teilweise in der Zwischenzeit mehrfach verstärkt, um sie den neuen Bedürfnissen anzupassen. Im Hinblick auf eine weitere Nutzung müssen sie periodisch beurteilt und kontrolliert werden. Eine wichtige Frage bildet dabei die Ermüdung. In diesem Beitrag wird gezeigt, wie die Versagenswahrscheinlichkeit von Konstruktionsdetails mit Hilfe von probabilistischen Ansätzen ermittelt werden kann. Die Restnutzungsdauer ist im wesentlichen abhängig von der Spannungsdifferenz, der Genauigkeit des statistischen Modells und von den Annahmen über die Modellunschärfe bei den Verkehrslasten.

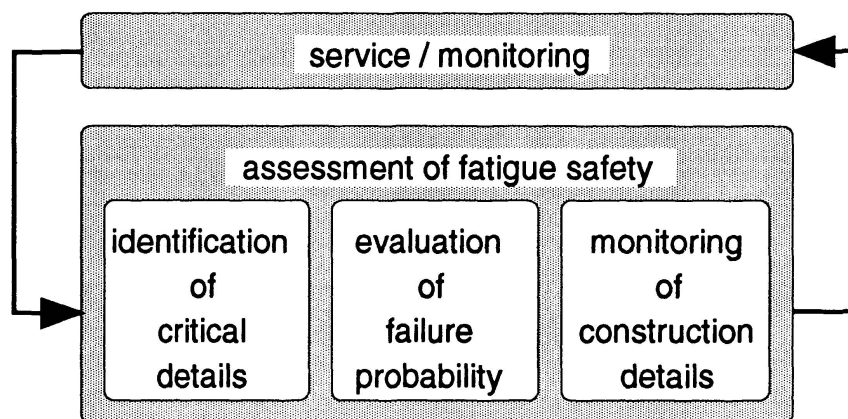


## 1 INTRODUCTION

Many railway bridges built at the end of the last or at the beginning of the present century are still in service today. The defined service lives of these structures have been reached or even exceeded [1]. Owners and controlling authorities must decide how these bridges can be used in the future. This involves a re-assessment of structural safety including fatigue. An example of the evaluation of a riveted bridge built in 1875 is presented in [2]. Other reasons for an assessment of the remaining service life are new service conditions or changed structural behaviour under service loads.

The method for the **assessment of fatigue safety** comprises three stages :

1. Identification of critical construction details.
2. Evaluation of fatigue safety by calculation of failure probability.
3. Monitoring of construction details by regular inspection.



**Figure 1** Method for assessing the fatigue safety of critical construction details

This paper presents a method for assessing fatigue safety, in particular the procedure for the calculation of failure probability is discussed. The relationship between the calculations of crack propagation and of damage accumulation is emphasized.

## 2 METHOD FOR ASSESSING THE FATIGUE SAFETY

The condition of a structure should be monitored by regular inspection throughout its service life. An assessment of fatigue safety may be required as a result of observations (displacements, vibration characteristics, corrosion, cracks) or because of changes in service conditions (increase of axle or uniformly distributed loads) or for legal reasons when a defined service life is reached. Fatigue safety is mainly dependent on the following three parameters :

- **Applied stress ranges** : The applied stress ranges are a function of the service loads and the structural behaviour of the bridge.
- **Geometry of construction details** : The stress concentrations caused by the geometry of construction details and the crack shape may lead to an acceleration of crack propagation and a decrease in fatigue category.
- **Number of stress cycles** : The number of stress cycles applied in the past directly influences the remaining service life of a structure.

In the method for the **assessment of fatigue safety** (Figure 1) [1], based on the three main parameters mentioned above, the following stages are identified assuming that the bridge to be evaluated has been defined.

**STAGE 1 :** For the structure to be assessed, the first task is the **identification of critical construction details**. In order to identify critical construction details, it is often sufficient to carry out an expert inspection and an intensive study of the available documents. Inspection and maintenance reports can be particularly helpful. The critical construction details can be identified using a deterministic calculation appropriate to current design methods. In this way, one can derive a list of priorities for later investigations.

**STAGE 2 :** In this paper a probabilistic method to **evaluate the failure probability** will be presented. This approach enables an assessment of fatigue safety that is more sophisticated than was previously possible.

**STAGE 3 :** The **monitoring of construction details** makes it possible to maintain the structure. To this end, the inspection intervals and techniques need to be selected.

In the following chapter, the discussion is focused on stage 2 relating to the evaluation of failure probability.

### 3 EVALUATION OF FAILURE PROBABILITY

#### 3.1 Relationship between crack propagation and damage accumulation calculations

The calculation of remaining fatigue life is improved by combining the advantages of both damage accumulation and crack propagation calculations.

The advantages of the **damage accumulation calculation**, based on fatigue strength curves (S-N curves, Wöhler curves), are : this procedure is widely accepted, many test results are available and the probability density functions of the fatigue strength of the construction details are known. In addition, a classification system [3] is available, which is applicable to frequently used construction details.

The advantage of a **crack propagation calculation**, based on fracture mechanics, is that crack propagation for each individual stress range can be calculated. Using this approach the influence on crack propagation of each stress range can be considered. The important disadvantage is that the fracture mechanics parameters for the individual construction details are generally not well known.

Stable crack propagation can be calculated by using a fracture mechanics model (equation 1), shown in Figure 2. Crack propagation depends on the difference between stress intensity factors  $\Delta K$ , also called stress intensity factor, defined by

$$\Delta K = Y(a) \cdot \Delta \sigma \cdot \sqrt{\pi \cdot a} \quad (1)$$

$\Delta K$  : difference between stress intensity factors (stress intensity factor)





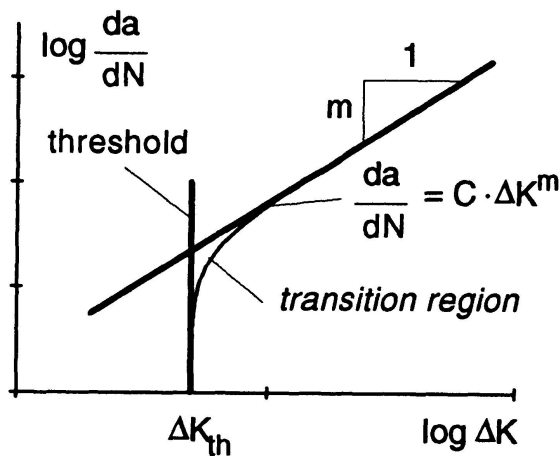
- $\Delta K_{th}$  : threshold value of stress intensity  
 $Y(a)$  : stress concentration factor  
 $\Delta\sigma$  : applied stress range  
 $a$  : crack size

For a constant stress range  $\Delta\sigma$ , the stress intensity factor  $\Delta K$  will increase with increasing crack size  $a$ . In addition, a threshold value can be observed, i.e. a limit below which no crack propagation will occur. For an applied spectrum of different stress ranges, the number of the stress ranges which are greater than the threshold limit will increase as the crack size increases.

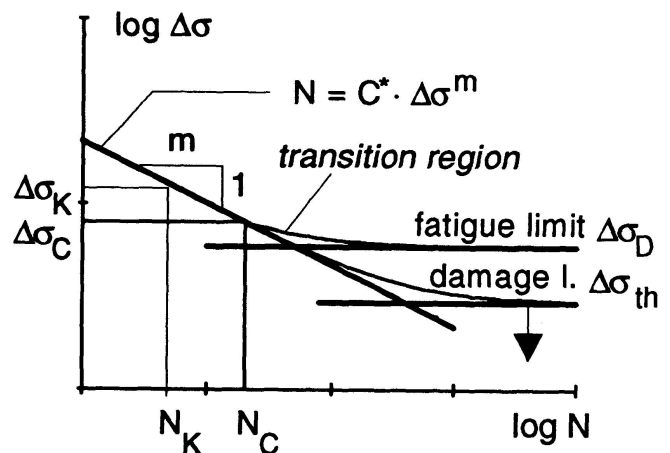
On fatigue strength curves (Figure 3) for constant amplitude stress ranges, a fatigue limit  $\Delta\sigma_D$  can be observed, i.e. a limit below which no crack propagation will occur. In the case of variable amplitude stress ranges, damage will accumulate if a part of the spectrum is above the fatigue limit. Therefore, the limit below which no crack propagation occurs is no longer constant but decreases with increasing crack size. This limit is now called the **damage limit**  $\Delta\sigma_{th}$ , defined by

$$\Delta\sigma_{th} = \Delta\sigma_D \cdot (1 - D) \quad (2)$$

- $\Delta\sigma_D$  : fatigue limit  
 $\Delta\sigma_{th}$  : damage limit  
 $D$  : existing damage



**Figure 2** Crack propagation as a function of the stress intensity factor



**Figure 3** Fatigue strength curve

The transition region from the threshold to the Paris law in the crack propagation curve can be described as follows :

$$\frac{da}{dN} = C \cdot (\Delta K^m - \Delta K_{th}^m) \quad (3)$$

- $da/dN$  : crack propagation per stress cycle  
 $C$  : crack propagation constant  
 $m$  : slope of the crack propagation curve (Paris law)

By analogy a transition region can also be considered in a damage accumulation calculation. The point  $(N_K, \Delta\sigma_K)$  is a reference point on the S-N curve having a constant

slope  $m$  (Figure 3). The position of the fatigue strength curve is defined by the fatigue strength  $\Delta\sigma_C$  at  $N_C$  cycles; this allows the correlation with reference to the detail category in the tables of the ECCS recommendations [3]. The damage increase per cycle as a function of the applied stress range  $\Delta\sigma_i$  is [4] :

$$d_i = \frac{\Delta\sigma_i^m - \Delta\sigma_D^m \cdot (1-D)^m}{\Delta\sigma_K^m - \Delta\sigma_D^m \cdot (1-D)^m} \cdot \frac{1}{N_K} \quad (4)$$

$d_i$  : damage increase per cycle  
 $\Delta\sigma_i$  : applied stress range  
 $\Delta\sigma_K$  : reference stress range  
 $N_K$  : number of cycles to failure for the reference stress range

The damage accumulation calculation with equation (4) can be used to derive the remaining service life by simulations with an accuracy of 5 to 7 % (according to the spectrum) with respect to a crack propagation calculation using fracture mechanics. This corresponds to a significant improvement compared to the fatigue strength curves used for design. The ECCS [3] and Eurocode classification systems may be used in this calculation, bringing with them the possibility of calculating the design life on the basis of the statistics of a large number of fatigue strength tests.

### 3.2 Evaluation of reliability

The remaining service life depends on the traffic loads used for the simulations. The traffic load model used accounts for the fatigue effect of service loads on the structure. Traffic in the past can be represented approximately by the standard UIC (International Union of Railways) trains [5]. A corresponding model has also been developed for future traffic. Dynamic coefficients are used for the evaluation of stress ranges for standard trains, thereby taking into account the dynamic characteristics of the structure and the vehicles.

A simplified standard load, called the fatigue load  $Q_{fat}$ , multiplied by a dynamic coefficient  $\Phi$ , is used as a reference value. With the fatigue correction factor  $\alpha$ , the relationship between the stress range due to the fatigue model  $\Delta\sigma(\Phi \cdot Q_{fat})$  [6] and the damage due to the stress ranges of the traffic model can be established.

Based on simulation calculations, in which the statistical scatter for each parameter is considered, the statistical distribution of the correction factor  $p(\alpha)$  as a function of a given number of future trains  $N_{fut}$  can be calculated. The distribution of the required fatigue strength for a construction detail in an existing bridge can be calculated as the product of the fatigue load stress range and the distribution of the fatigue correction factor [4] :

$$p(\Delta\sigma_{req}) = p(\alpha) \cdot \Delta\sigma(\Phi \cdot Q_{fat}) \quad (6)$$

$\Delta\sigma(\Phi \cdot Q_{fat})$  : stress range due to the fatigue load multiplied by the dynamic coefficient  
 $p(\Delta\sigma_{req})$  : probability distribution of the required fatigue strength  
 $p(\alpha)$  : probability distribution of the fatigue correction factor for a given number of future trains  $N_{fut}$



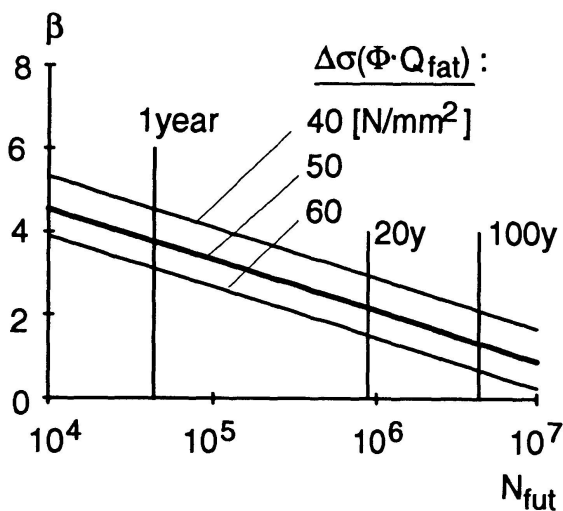
Knowing the required fatigue strength, the available strength of a given detail has to be determined. Probability distributions of the fatigue strength  $p(\Delta\sigma_R)$  of typical construction details have been obtained from tests [7, 8]. By comparison of the two distributions, the probability of failure can be calculated and expressed in terms of the reliability index  $\beta$  :

$$\beta = \frac{R - S}{\sqrt{s_R^2 + s_S^2}} \quad (8)$$

$\beta$  : reliability index  
 $R, S$  : mean values of the resistance and the load effect, respectively  
 $s_R, s_S$  : standard deviations of the resistance and the load effect, respectively

Figure 4 shows the relationship between the reliability index  $\beta$  and the number of future trains  $N_{fut}$ . For an increasing number of trains, the reliability index  $\beta$  can be seen to decrease. For a semi-logarithmic scale and using analytically derived expressions for  $R$  and  $S$ , this results in a straight line. This simple relationship is very convenient for a sensitivity analysis of the main parameters.

The number of trains for 1, 20 and 100 years are also identified in Figure 4. For the assessment of fatigue safety of an existing bridge, the region between  $10^5$  and  $10^6$  trains is of most interest. This corresponds to a future service period of between 5 and 25 years.



Assumptions :

- number of trains per day : 120
- year of construction : 1900
- influence length : 20 m
- fatigue limit at :  $7 \cdot 10^6$  cycles
- coefficient of variation of the parameters with scatter : 10 %

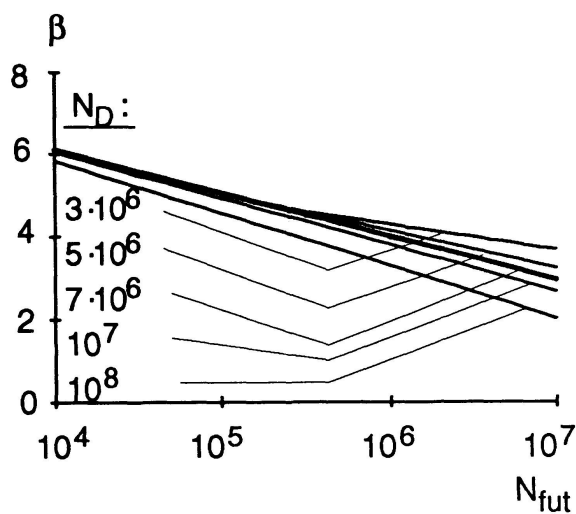
**Figure 4** Variation of reliability index with number of future trains

### 3.3 Sensitivity analysis

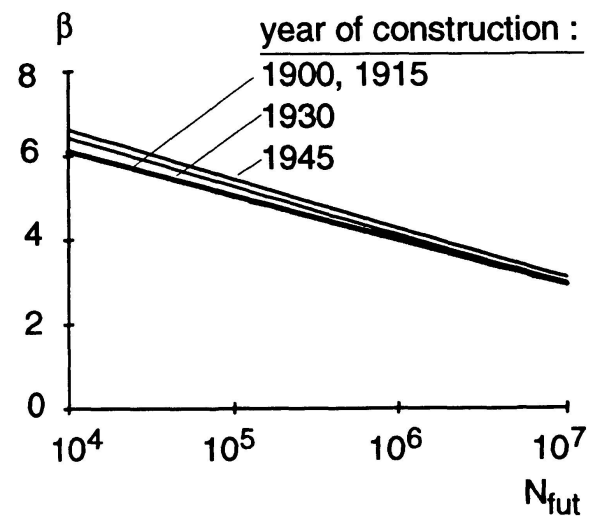
Figure 4 shows the influence of the stress range due to fatigue load. Improving the accuracy of the structural model directly improves the value of the stress range and thus increases the calculated reliability. Site measurement of bridge behaviour is recommended for calibration of the structural model. The assumptions of Figure 4 using a stress range of  $50 \text{ N/mm}^2$  will be used for the following figures.

Figure 5 shows the effect of the fatigue limit. The position of the fatigue limit  $\Delta\sigma_D$  is characterised by the corresponding number of cycles  $N_D$ . The longer a bridge will remain in service, the more important the determination of  $N_D$  will be. To this end, the fatigue limit should be defined for each category of construction detail.

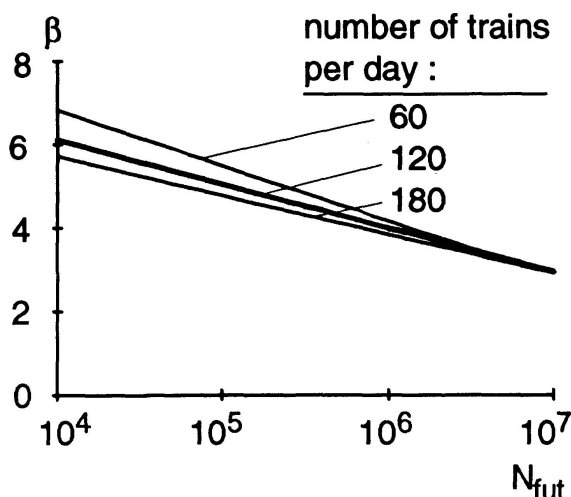
The influence of the year of construction is shown in Figure 6. This parameter is normally known. The small number of trains at the beginning of this century does not influence the reliability index. In addition, with increasing number of future trains this parameter is less important. This means that the same correction factor  $\alpha$  can be used for plus or minus 20 years, for the values assumed for Figure 4.



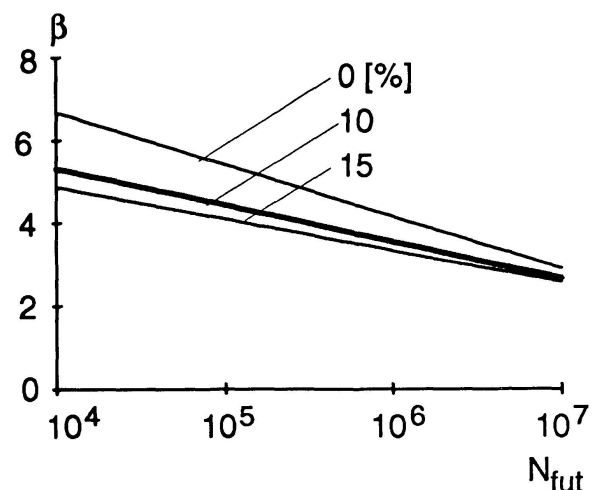
**Figure 5** Influence of the position of the fatigue limit  $N_D$



**Figure 6** Influence of the year of construction



**Figure 7** Influence of the number of trains per day



**Figure 8** Influence of model uncertainty

Figure 7 shows the influence of the number of trains. The number of trains in the past has been fixed according to a proposal by UIC and is based on the actual number of trains as a reference value. The figure shows that the number of future trains is significant for less than  $10^5$ ; for a greater number this parameter has little importance.



The influence of model uncertainty is shown in Figure 8. The scatter has a significant effect on the reliability index. When the coefficient of variation increases, the reliability index decreases. Results are not conservative when the model uncertainty is neglected (higher values of  $\beta$ ). However, it can be seen that the difference for  $\beta$  due to a change of uncertainty between 10 and 15 % is small. The influence of the model uncertainty decreases for increasing coefficients of variation and with increasing number of trains.

## 4 CONCLUSIONS

A method for the evaluation of fatigue safety for existing railway bridges has been presented. The following conclusions can be made :

- Based on the principles of fracture mechanics and especially on the concept of a threshold value for crack propagation, a new damage limit for the fatigue strength curves can be defined. This limit decreases with increasing damage. For calculation purposes, it is assumed that below this damage limit no crack propagation and therefore no damage will occur. Based on this assumption a good agreement between damage accumulation and the more complex crack propagation calculation is obtained.
- The probability of failure can be calculated by comparing the required and existing fatigue strength. The most important parameters are stress range due to the fatigue load and the position of the fatigue limit. Of lesser importance are the year of construction, the year in which service began and the influence length, if it is greater than 10 m.

The probability of failure can be related to the probability of crack detection [4]. A construction detail with a theoretical probability of failure below the target value can remain in service, when the probability of crack detection (based on a given inspection technique) and the inspection intervals are taken into account.

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