

**Zeitschrift:** IABSE reports = Rapports AIPC = IVBH Berichte  
**Band:** 67 (1993)

**Artikel:** Definition of load spectra  
**Autor:** Jacob, Bernard  
**DOI:** <https://doi.org/10.5169/seals-51359>

### **Nutzungsbedingungen**

Die ETH-Bibliothek ist die Anbieterin der digitalisierten Zeitschriften auf E-Periodica. Sie besitzt keine Urheberrechte an den Zeitschriften und ist nicht verantwortlich für deren Inhalte. Die Rechte liegen in der Regel bei den Herausgebern beziehungsweise den externen Rechteinhabern. Das Veröffentlichen von Bildern in Print- und Online-Publikationen sowie auf Social Media-Kanälen oder Webseiten ist nur mit vorheriger Genehmigung der Rechteinhaber erlaubt. [Mehr erfahren](#)

### **Conditions d'utilisation**

L'ETH Library est le fournisseur des revues numérisées. Elle ne détient aucun droit d'auteur sur les revues et n'est pas responsable de leur contenu. En règle générale, les droits sont détenus par les éditeurs ou les détenteurs de droits externes. La reproduction d'images dans des publications imprimées ou en ligne ainsi que sur des canaux de médias sociaux ou des sites web n'est autorisée qu'avec l'accord préalable des détenteurs des droits. [En savoir plus](#)

### **Terms of use**

The ETH Library is the provider of the digitised journals. It does not own any copyrights to the journals and is not responsible for their content. The rights usually lie with the publishers or the external rights holders. Publishing images in print and online publications, as well as on social media channels or websites, is only permitted with the prior consent of the rights holders. [Find out more](#)

**Download PDF:** 16.03.2026

**ETH-Bibliothek Zürich, E-Periodica, <https://www.e-periodica.ch>**

**Definition of Load Spectra**  
**Définition de spectres de charge**  
**Definition von Einwirkungsspektren**

**Bernard JACOB**  
Civil Eng.  
Lab. Central Ponts et Chaussées  
Paris, France



Bernard Jacob, born in 1954, graduated of Ecole Polytechnique and Ecole Nationale des Ponts et Chaussées. He has been in charge of research works on probabilistic actions on structures and bridge behaviour, at SETRA and LCPC. He is an expert for the Eurocode on highway bridge loading and the national WIM-project manager.

#### **SUMMARY**

Knowledge of actions is essential for structural safety assessment. Most of the actions are random and require probabilistic models and definition of load spectra for the structural design and reliability evaluation. A review of the most important actions is presented, with information about the available data and probabilistic models now sufficiently tractable for engineering purposes. As an example, the traffic loads on bridges defined in Eurocode 1 are briefly presented. Finally some indications are given on the combination of random actions such as temperature and traffic loads on bridges.

#### **RÉSUMÉ**

La connaissance des actions est essentielle pour l'évaluation de la sécurité des structures. La plupart des actions sont aléatoires et nécessitent des modèles probabilistes de spectres de charge pour le dimensionnement et l'évaluation de la fiabilité des structures. Un panorama des principales actions présente données et modèles probabilistes actuellement accessibles aux ingénieurs. Les charges de trafic sur les ponts, traitées dans l'Eurocode 1, fournissent un exemple. Enfin quelques indications sont fournies sur les combinaisons d'actions comme le vent ou la température avec les charges de trafic.

#### **ZUSAMMENFASSUNG**

Die Kenntnis der Einwirkungen ist entscheidend für die Bewertung der Tragsicherheit. Die meisten Einwirkungen sind zufallsverteilt und erfordern Modelle der Wahrscheinlichkeitsrechnung und die Definition von Belastungsspektren für die Bemessung und Bewertung der Zuverlässigkeit der Tragwerke. Ein Überblick über die wichtigsten Einwirkungen stellt Daten und Wahrscheinlichkeitsmodelle vor, die zur Zeit den Ingenieuren zugänglich sind. Die Verkehrslasten auf Brücken, die im Eurocode 1 behandelt werden, geben ein Beispiel dafür. Und schliesslich werden einige Angaben zu kombinierten Einwirkungen wie Wind Zoder Temperatur und Verkehrslasten mitgeteilt.



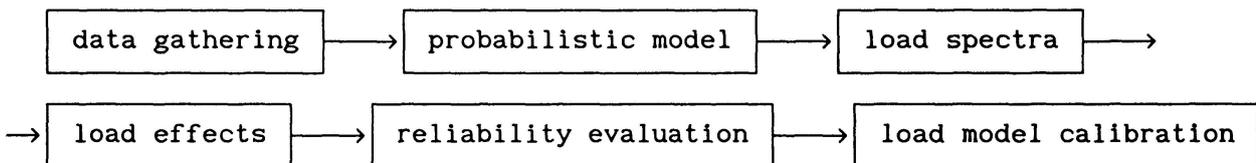
## 1. INTRODUCTION

Knowledge and modelling of the actions on structures are a basic step in the structural safety and reliability assessment. Most of the actions on structures, either natural or due to human activities, are random and impossible to forecast. For about 20 years, many research works were undertaken on probabilistic models of actions, in order to describe and quantify their occurrences and intensities or to compute the load effects, stresses and strains induced in the structures. Many papers were published by the ASCE, WCSE, OMAE, IFIP, Structural Safety, CEB, ICOSSAR and ICASP, IABSE etc..., and some state of the art reports are prepared by the CIB, Commission W81 "Actions on Structures" [1.a to 1.j].

Then engineers discovered the great advantages of using such probabilistic models for the design and reliability evaluation of bridges, off-shore structures, buildings, nuclear power plants, and other structures. Also the experts who are writing or rewriting the new semi-probabilistic codes, such as the Eurocodes in the EC countries or the Ontario bridge code, are using such advanced methods for the calibration of the load models and the partial loads safety factors. As an example, an accurate definition of the load spectra is very important for fatigue assessment of bridges under variable vehicular loads or of off-shore jackets under wave loads.

But if the advanced probabilistic load models, based on the use of random variables or stochastic processes, are useful for accurate reliability calculations, such as those made by the FORM-SORM methods to evaluate  $\beta$ -indices and probabilities of failure, they are not always easy for engineers, companies, or consultants to use. Common structural design and checking are by application of codes, which mainly contains conventional load models. These load models must be well calibrated, with the help of convenient data and random models, and are often presented as load cases or load spectra.

For a given action and type of structure, the definition of load spectra generally follows the diagram:



## 2. DEFINITIONS - ACTION CLASSIFICATION AND MODELLING

### 2.1 Definitions

An action is an external phenomenon that induces stresses or strains in a structure. It is often expressed by a set of concentrated or distributed loads which produce load effects. An action is defined by its occurrences, the time periods of application, its spatial and directional properties, and its intensities. There are normal, abnormal or accidental actions. The correlations between several actions must be known.

The JCSS and ISO consider, depending on their occurrences:

- **permanent actions:** the period of application is the whole structure lifetime and the intensity variations are small or rare, and mainly due to changes in the structure use or affectation (e.g.: dead load).



- **variable actions:** the occurrences are discrete, the durations, intensities and other characteristics vary with time. There are some "high-frequency" variable actions (wind, wave or traffic loads), and some "low-frequency" or semi-permanent actions with slow variations (floor live loads, temperature and snow loads).
- **accidental loads:** the occurrences are exceptional, unpredictable and almost unavoidable, and they have short periods of application (e.g.: shocks, fire, explosions).

From the viewpoint dynamic load effect or structural response, there are:

- **static actions:** which do not induce significant accelerations in the structure (snow, temperature, furniture loads),
- **dynamic actions:** which produce large accelerations, vibrations, or impact forces in the structure (vehicle loads, earthquakes, wind).

## 2.2 Classification

Actions on structures may be classified, according to their origin, into:

- **environmental actions:** these result from natural phenomena, climatic or not, and are generally very little controlled if at all. The main climatic actions are: wind, snow, ice, rain and water currents, temperature and wave loads. The others are earthquakes and other ground motions, avalanches, etc.
- **actions due to human activities:** these result from the use of the structure and are generally at least partially controlled. In this category are: traffic loads on bridges and pavements, live loads in buildings, fluid pressures in tanks, etc.
- **accidental loads:** these result from human error, misuse of the structure or very exceptional and unpredictable climatic events. The vehicle collisions on parapets, ship collisions on piers, falling aircraft, explosions, fire, tornados, typhoons and tidal waves are accidental actions.

## 2.3 Data and models

### 2.3.1 General remarks

Knowledge of any action and the definition of load spectra must be based on both data and models. The data obtained by measurements of samples are necessary to choose and calibrate the models and to check their hypothesis. But they are not sufficient in most cases to fully describe an action and its characteristic values. For a time-variable random action, a set of data only gives a specified sample time history but does not represent its scattering and randomness. For example, the famous record of the El Centro earthquake provides an accelerogram which is very often used for structural design or checking, but it always reproduces the same frequency spectrum and the same magnitude and intensity.

In most problems, except for fatigue calculations, the maxima of the loads are relevant, and these maxima have to be considered over long time periods, close to the expected structure lifetimes (e.g. 50 to 100 years). It is rare to get measurements of an action time history over such long periods at the right location and under the same conditions. And even in such a case it would be not enough to derive reliable extreme value distributions or far fractiles. Then it is necessary to build good models based on verifiable assumptions with parameters which can be fitted on the available data. These data may also be used to check some of the model assumptions, e.g. by statistical tests.



### 2.3.2 Probabilistic models

Most of the common actions and loads on structures are random or may be considered as random because of lack of knowledge. The probabilistic models give an account for the statistical uncertainties and make possible the calculation of mean values, standard deviations, fractiles, probability of level crossing, extreme value distributions and finally probability of failure or safety indices. A "good" model must be simple enough to be tractable for calculations, compatible with the available data - and less informative, i.e. not create any additional and unverifiable information not included in the data -, but nevertheless detailed enough to give an account of the physical properties of the action. Special attention must be paid to the correlations between various loads, such as wind and waves or temperature and snow, in order to avoid gross errors in load combinations. In particular a set of  $n$  correlated random variables is only well described by an  $n$ -dimensional density called the cross-correlation probability distribution. The convenient assumption of independence must always be carefully checked.

Random time varying loads are modelled by stochastic processes, generally stationary, and some of them by random fields if there are significant space variations to be taken into account. The time-independent loads and some other loads are represented by random variables, if the time influence may be removed, above all in static or quasi-static analysis. In this case the load effects are more easily calculated, with no need for stochastic differential equations. In this common case, the notion of return period is very useful in choosing the characteristic load values:

if  $X_1, X_2, \dots, X_n$  are independent random variables with the same law as  $X$  (i.e. are the random samples of  $X$  at times 1, 2, ...,  $n$ ), the return period  $T(x)$  of the value  $x$  of  $X$  is the mean time interval between two level crossings of  $x$  by the  $X_i$ . If  $F(x)$  is the cumulative distribution function of  $X$ , we get:

$T(x) = [1 - F(x)]^{-1}$ . If  $P(X \geq x) = p$  (small) during  $n$  time units, we have:

$T(x) \approx -n / [\ln(1-p)]$  for any  $X$ . One must take care not to mistake the return period  $T(x)$  for the reference period  $n$ : in this way a 5% upper fractile over a reference period of 100 years has a return period of about 2000 years!

For most of the actions the characteristic values generally adopted are those having a return period between 50 and 200 years (climatic actions) and up to 1000 to 2000 years (traffic loads on bridges).

The models of extreme values commonly require the three types of asymptotic distributions [2]: type I (Gumbel) for exponentially decreasing probability distribution functions (PDF), type II (Fréchet) for geometrically decreasing PDFs and the type III (Weibull) for upper-bounded PDFs (maxima). One must be careful with multiple-peak PDF representing a mixture of several distributions, as in the case of snow loads in a maritime climate.

### 2.3.3 Data gathering and use

Some data are necessary to choose the PDF type of any random variable and its parameters. They may provide the basis for any statistical test of hypothesis, such as normality (Shapiro-Wilk), the choice of some fitted parameters (chi-square test) or independence.

For time independent loads, a large enough sample of data may be sufficient to derive reliable estimators of the characteristic parameters or a PDF. But for random loads varying with time, stationarity must be first checked; if not, the seasonal or periodic variations as well as the trends must be removed before any stationary model fitting. In addition the process must be ergodic

(i.e. the means on time history samples converge to the statistical mean) in order to make the statistical use of the data meaningful. For periodic actions, the sampling frequency  $f$  must be adapted to the smallest period  $T$ , according to Shannon's rule:  $f \geq 2/T$ . A spectral analysis is often useful for variable actions (wind, waves, earthquakes, traffic loads), in order to identify the stationarity time periods and the frequency peaks of energy. A spectral representation (by the power spectral density) of the stationary processes is used in dynamic stochastic analysis (structures under seismic loads or off-shore jackets in random waves).

In many cases the data must be filtered when recorded to eliminate noise. The passband of the filter must be matched to the frequencies of the action. The sensitivity of the recording device must be sufficient and any bias must be eliminated or corrected. In some cases the samples are truncated and the upper tails of the PDF may be underestimated!

### 3. ACTION LOAD SPECTRA

#### 3.1 Definition of a load spectrum

The improper but commonly used term of "load spectrum" means a load distribution, either derived from a sample or measurements by a statistical analysis (histogram, fig. 1) or defined by the theoretical or fitted PDF of the load parameter (e.g. intensity, fig. 2). In both cases, the load spectrum may be used to simulate randomly a time history of the load (Monte-Carlo simulation) or to calculate a load effect distribution. It is especially useful for fatigue calculations (steel bridges under traffic loads, jackets in waves, tall buildings under wind load, etc), because the whole distribution of stresses may contribute to the damage increase. A load spectrum may also be conventional, instead of being the result of a statistical or probabilistic analysis. This is the case of the code load models used for structural design or checking. Some such load models contain a set of load cases, with various intensities, each modified by a frequency or a probability.

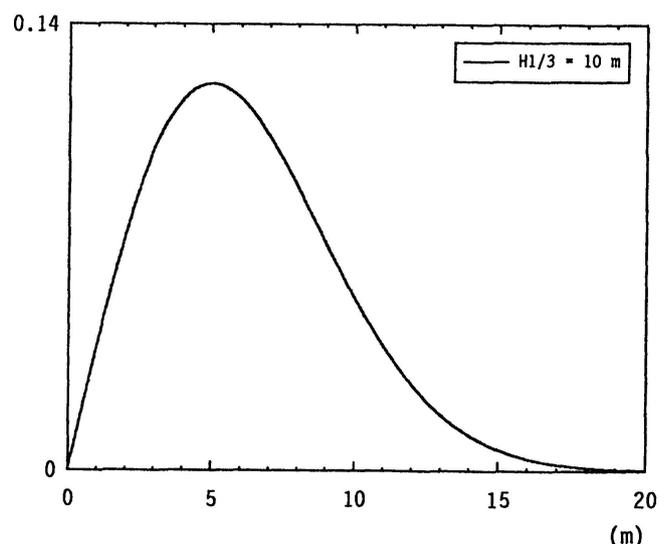
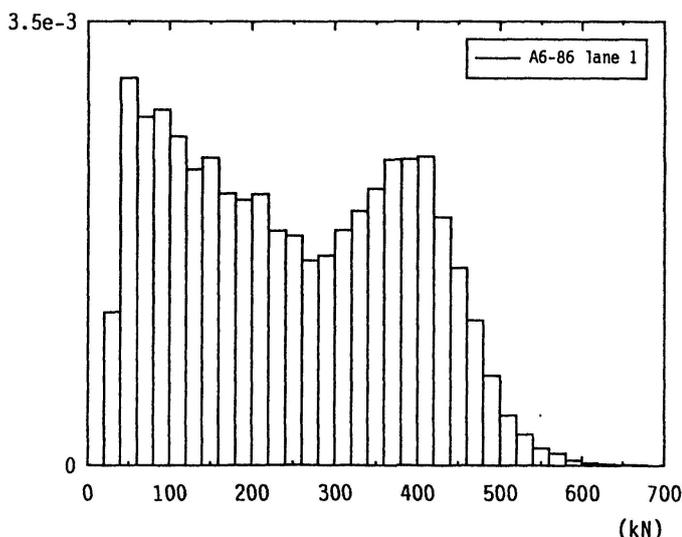


Fig.1 Histogram of the lorry weights

Fig.2 Rayleigh's PDF of wave heights

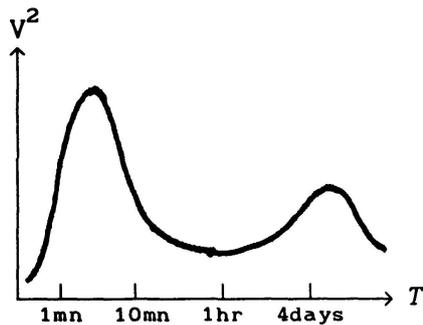
An example of a conventional load spectrum is given by the so-called "fatigue convoys" used for fatigue calculations of steel bridges. A "fatigue convoy" is a set of a few lorries or concentrated loads, with given gross weights, axle loads, axle and vehicle spacings and number of passages for each vehicle. For any sub-structure of the bridge, such as a cross-beam or a stiffener, the



application of the load spectrum means the passage of the convoy on the bridge deck, and provides a stress variation time history. The stress cycle amplitudes may then be counted by the "rain-flow" method and the fatigue damage or the expected lifetime of the structure may be computed.

A load spectrum is generally defined by using a large sample of data (histogram) with a number of classes adapted to the sensitivity of the load effect considered. But in some cases it is derived from a theoretical model, in order to simplify the calculations.

### 3.2 Load spectra of climatic actions



Wind loads are applied as pressures on structures. The pressure is proportional to the square of the wind velocity. The conventional wind velocity is measured at 10 m above the ground level and increases with altitude. The peaks of the frequency spectrum are between 3 s and 10 mn and around 4 days. Because of the nonstationarity of the wind, the velocity is described by its mean values or maxima over 10 mn to 1 hr periods. These stationary periods correspond to the lower

parts of the frequency spectrum (fig. 3). The velocity PDF is often described by a Weibull distribution [1.h], and the maxima over 1 to 100 years follow the type I (Gumbel) extreme distribution. The successive mean values are independent.

The wave loads on an off-shore structure or a sea wall are linked to the wave heights, which are themselves correlated with the period or wavelength. Because of the uncertainty on the wave top due to breaking, the upper 1/3 height is considered. For a given period range, a Rayleigh function is generally adopted as the PDF of this height. During short time periods of about 20 mn, the sea state is considered as stationary, giving a specified parameter  $a$  to the Rayleigh PDF, and the successive wave heights are independent.

### 3.3 Traffic loads on bridges

The most important design loads for bridges are the traffic loads. They are quite complex and the definition of load spectra is difficult because of the randomness of many parameters: vehicle occurrences and spacing, axle loads and gross weights. The traffic processes are generally nonstationary, correlated from one traffic lane to the next, and traffic jams or platoons of lorries occur randomly and often govern the structural design. Theoretical and numerical approaches and models have been proposed by various authors (CIB, ASCE, Eurocode) [1.f], [3].

The simplest and most common vehicular load models are built with a set of random variables; e.g. for a traffic lane: type of vehicle (from a given silhouette classification), gross weight, axle loads, spacing between vehicles and traffic flow. Vehicle length, speed, duration of jam, number of lorries in a platoon, and spacing in congested traffic may also be considered, as well as the distribution of the vehicles among lane. Correlations between some of these variables are introduced (gross weight/axle loads/silhouette/vehicle length; speed/spacing/flow, etc). Some authors have proposed more sophisticated traffic load models using random processes (Poisson and marked Poisson [4], Markov, renewal Markov [5]) in order to better describe the space distribution of the loads.



Most of these models are based on detailed measured traffic data recorded by Weigh-In-Motion (WIM) techniques. Large samples of traffic are now available [6] providing sufficient information for model definition and calibration. Moreover these data are often rich enough (more than 30,000 lorries continuously recorded on one location) to use them directly for load effect calculations [7]. The results are then much more accurate than by Monte-Carlo simulation, especially for short and medium spans.

| Load   | length L | Eurocode [8] | TNO [3] |
|--------|----------|--------------|---------|
| axle   | -        | 290 kN       | 260 kN  |
| tandem | -        | 440          | 410     |
| triple | -        | 530          | 580     |
| lorry  | -        | 925          | 940     |
| EUDL   | 20 m     | 47-56 kN/m   | 62 kN/m |
| EUDL   | 50       | 38-43        | 36      |
| EUDL   | 100      | 33-38        | 28      |
| EUDL   | 200      | 25-36        | 20      |

Table 1 Characteristic loads with a return period of 1000 years, slow lane.

The PDFs proposed for lorry gross weights are in most cases bimodal, with normal or Weibull modes. The speed PDFs are often normal while the spacing PDF is a gamma distribution. From these models and data, characteristic axle loads and gross weights have been derived, as well as distributed loads on a lane length. Table 1 gives some results from [4] [8]. Based on the extrapolations of the load effects having a return period of 1000 years, a conventional unified load model was built and calibrated for Eurocode 1 (Bridge loading) for the EC countries [9]. It will include also some fatigue loads.

### 3.4 Load combinations

The combination of random actions is a difficult problem because of the possible correlations, and in any cases because of the very low probability to observe simultaneous extreme values and the lack of cross-observations. The codes contain  $\psi$  coefficients to modify the characteristic values in order to make load combinations, but they are largely "heuristic". In any load combination it is important to know the respective variations of each component, and its frequencies. It is quite easy to combine a permanent load  $G$  with a variable one  $L$ :  $\text{Max}(G+L) = \bar{G} + \text{Max}(L)$ . If  $G$  is a quasi-permanent load or a slowly varying one, it is possible to write:  $\text{Max}(G+L) \approx G' + \text{Max}(L)$ , where  $G'$  is a frequent value of  $G$ .

The load combinations must be studied case by case; for prestressed concrete bridges the combination of temperature and traffic loads is relevant. The temperature load (gradient) may be modelled by a sinusoidal function with a period of 24 hrs and a random amplitude, with a distribution depending on the season. The combination with the lorry loads is made by adding the bending moments due to each load, because of non-linearities in the stress calculations. A procedure using simulation and traffic records was developed for a bridge in France [10]. Some correlation exists between these loads, due to the traffic density peaks which occur at specific times in the day, like the temperature maxima or minima.

Traffic loads must be combined with the wind load for long cable-stayed or suspension bridges. This problem becomes more complicated than the previous one because both loads vary considerably, at high frequencies. The (negative) correlation only concerns extreme wind velocities, when traffic restrictions are imposed for safety reasons.



## REFERENCES

1. CIB - W81 (Actions on Structures):
  - 1.a. General Principles. 5th draft, 1986.
  - 1.b. Self-Weight Loads. CIB report n° 115, 1991.
  - 1.c. Live Loads in Buildings. CIB report n° 116, 1991.
  - 1.d. Snow Loads. 6th draft, 1989.
  - 1.e. Loads in Car Parks. 5th draft, 1991.
  - 1.f. Traffic Loads on Bridges. 1st draft, 1990.
  - 1.g. Fire. 4th draft, 1992.
  - 1.h. Wind Loads. 2nd draft, 1987.
  - 1.i. Earthquake Loads. 1st draft, 1987.
  - 1.j. Combinations of Actions. 4th draft, 1992.
2. ANG A.H. and TANG W.H., Probability Concepts in Engineering Planning and Design. J. Wiley & Sons, New-York, 1984.
3. WAARTS P.H. and VROUWENVELDER A., Traffic Loads on Bridges. TNO Building and Construction Research report B92-011, February 1992.
4. JACOB B., MAILLARD J.B., GORSE J.F., Probabilistic Traffic Load Models and Extreme Loads on a Bridge. Structural Safety & Reliability, Proceedings of ICOSSAR'89, ASCE, San Francisco, August 7-11, 1989.
5. GHOSN M., MOSES F., Markov Renewal Model for Maximum Bridge Loading. ASCE, J. of the Engineering Mechanics, vol 111 n°9, September 1985.
6. BRULS A., JACOB B., SEDLACEK G., Traffic Data of the European Countries. Report of the WG 2, Eurocode on Actions, part 12, March 1989.
7. EYMARD R. and JACOB B., Programme CASTOR-LCPC pour le Calcul des Actions et Sollicitations dans les Ouvrages Routiers. Bull. de liaison des LPC, n°165, Novembre-Décembre 1989.
8. JACOB B. and al., Methods for the Prediction of Extreme Vehicular Loads and Load Effects on Bridges. Report of the WG 8, Eurocode on Actions, part 12, August 1991.
9. EUROCODE 1 (Basis of Design and Actions on Structures), Traffic Loads on Bridges. Part 3, draft 7, June 1992.
10. JACOB B., CARRACILLI J., GODART B., TROUILLET P., Investigation of Fatigue of a Prestressed Concrete Bridge under Combined Actions of Traffic and Temperature Gradient. 4th DBMC, Singapore, November 4-6, 1987.  
*in french in:* Bull. de liaison des LPC, n°152, Novembre-Décembre 1987.