Nonlinear Analysis of an Existing Prestressed Shell
Analyse non linéaire d'une coque précontrainte existante
Nichtlineare Berechnung einer bestehenden vorgespannten Schale

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SUMMARY

In this paper, a model developed for the nonlinear analysis of prestressed concrete shell structures is briefly described together with its application to the study of an existing shell constructed in Puerto Rico in 1971. Different types of analysis have been carried out in order to determine the influence of the material and geometric nonlinear effects on the structural behaviour until failure. This simulated behaviour is discussed and related to original design aspects.

RÉSUMÉ

Un modèle développé pour l'analyse non linéaire de structures de coques précontraintes est brièvement décrit ainsi que son application à une coque réalisée en 1971 à Puerto Rico. Différents types d'analyses ont été effectuées pour visualiser l'influence du matériau et des effets géométriques non linéaires sur le comportement de la structure jusqu'à rupture. Ce comportement simulé est décrit et relié aux aspects du projet original.

ZUSAMMENFASSUNG

Der Artikel beschreibt ein entwickeltes Berechnungsmodel zur nichtlinearen Analyse von Schalenstrukturen aus Spannbeton und seine Anwendung bei der Berechnung eines existierenden Tragwerks in Puerto Rico. Verschiedene Berechnungen wurden ausgeführt, um den Einfluss von Material- und nichtlinearen Geometrieeffekten auf die Struktur bis zum Versagen aufzuzeigen. Das simulierte Tragwerksverhalten wird diskutiert und mit den ursprünglichen Entwurfsaspekten in Verbindung gebracht.

1.- INTRODUCTION

Although powerful numerical tools for structural analysis have been developed during the past twenty years to simulate the nonlinear behavior of concrete structures, very few estudies have been effectivelly carried out on their use for the nonlinear analysis of large span shell existing structures. However, a knowledge of the complete structural response of shell concrete structures through their elastic, cracking, inelastic and ultimate ranges, is usually difficult to achieve by means other than a nonlinear analysis which takes into account both the geometric second order effects and the nonlinear aspects of the true behavior of the materials, including concrete cracking and crushing, yielding of steel, and the time-dependent effects of concrete creep and shrinkage.

The present paper describes the nonlinear finite element analysis of an existing prestressed concrete shell structure: the Ponce Coliseum roof, constructed in Puerto Rico in 1971. Through this analysis, the different causes of nonlinear behavior are characterized and their partial influence on the service and ultimate behavior is measured. The simulated behavior as obtained by the analysis is discussed in relation to the resulting reliability of the structure under instantaneous vertical loading.

The original structural design of the Project was carried out during the period 1968 to 1970 by a Joint Venture of T.Y. Lin International, San Francisco, California, and R. Watson, Engineer, and Sanchez, Davila and Suarez, Engineers of San Juan, Puerto Rico.

2.- DESCRIPTION OF THE SHELL

Ponce Coliseum consists of a 82.6 m span prestressed HP shell roof built in Ponce, Puerto Rico, in 1971 for the 1974 Pan American basketball chanpionship games. The complete roof is made up of four similar 10 cm thick saddle type shells connected to interior and cantilever edge beams to form a structure supported by four piers at the low points located at the centers of the four exterior panels. The high points of the shell, which rise 12.2 m above the low points, are at the four corner tips and at the center of the shell. The clear spans between opposite support piers in the two directions are 82.6 m and 69.2 m (Fig. 1). The cantilever edge beams are supported only at the piers and have a constant width of 76 cm throughout their entire length. These depth decreases gradually from the abutment to the corner tips, as may be seen in Fig. 2.

Reinforcing for the shell consists of reinforcing bars in the top and bottom surfaces throughout the shell with additional bar reinforcement added in the zones adjacent to the beams and near the tips. Prestressing is provided as well by a two-way set of tendons parallel to the straight line generators. The cantilever edge beams contain both normal reinforcing steel and prestressing steel, while the interior beams contain only normal reinforcing steel and are not prestressed.

The final original design used for the structure in 1970 was based on a detailed computer analysis using a linear elastic finite element model which coupled a system of one dimensional beam type elements with another system of two dimensional triangular plane stress finite elements. This analysis showed the high degree of interaction that occurs between the shell and the beams when subjected to their dead load, so that the resulting stress states can not be completely understood using membrane theory, and coupled states of axial force and bending moments appear both in the beams and the shell.
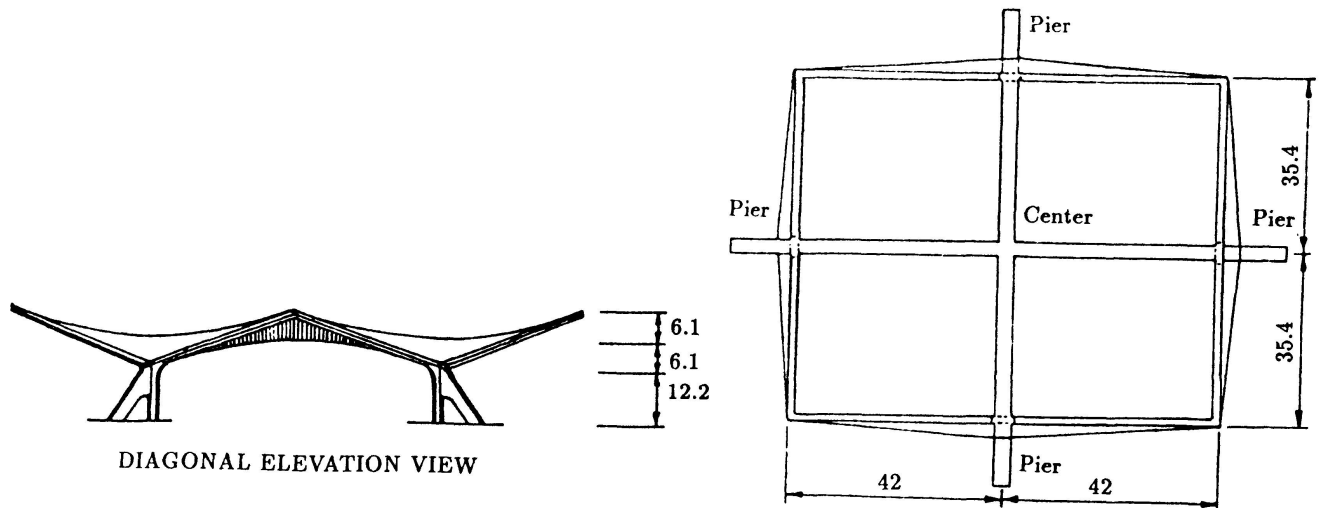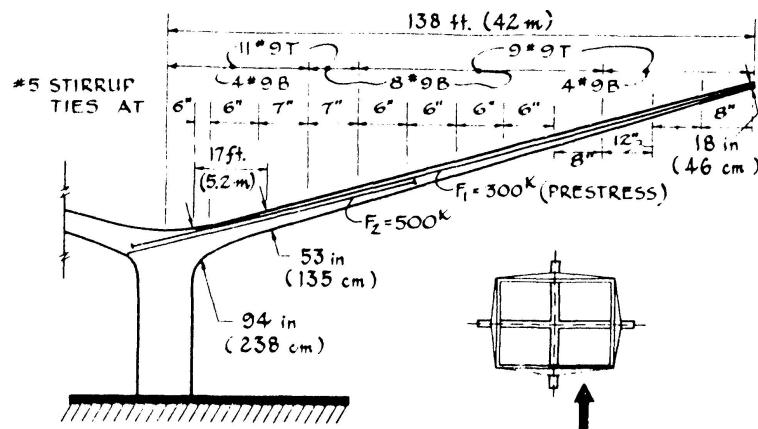
Fig. 1. General dimensions of the structure

Fig. 2. 42 m cantilever edge beam

Prestressing in the edge beam was used to partially balance its dead load bending moments and deflections while increasing its axial forces, thus resulting a set of stresses found acceptable for design. In the shell, the remaining tensile forces after the introduction of the prestressing were resisted by providing normal steel reinforcement. Also, to minimize cracks, prestressing shell tendons were used in combination with reinforcement. For bending moments in the shell adjacent to the edge beam and the tips, additional reinforcement was used.

Additional detailed information regarding concrete dimensions, reinforcement and prestressing, as well as the original desing and construction of the Ponce Coliseum may be found in [1-4].

3.- NUMERICAL MODEL USED IN ANALYSIS

Chan's [5-7] previous numerical model for the nonlinear analysis of reinforced concrete shells, extended by Roca [7-8] to include both beam and shell internal prestressing tendons, has been adopted for the present study. This model includes both shell and beam elements to account for shell systems with edge beams, internal ribs or supports. The nine node Lagrangian

isoparametric element shown in Fig. 3 was adopted to model two dimensional curved shells, while beams are simulated by adding two straight one dimensional beam elements to the side of a curved shell element. Beam elements are prismatic but may have an arbitrary cross-section made up of discrete numbers of concrete and reinforcing steel filaments.
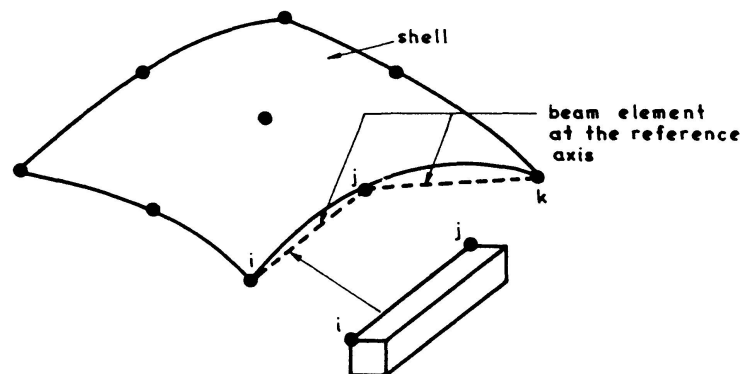

Fig. 3. Adopted L7 shell and straight beam elements

The shell element is regarded as a multilayered system where each layer is assumed to be under a biaxial state of stresses. The stresses and the state of the materials vary independently at each layer to account for the mechanical changes of the materials throughout the loading process. An hypoelastic biaxial concrete model is adopted together with a biaxial strength envelope [5] to reproduce the compressive behavior of concrete and crushing. In tension, concrete is assumed to behave as a linear elastic- perfect brittle material, accounting for tension stiffening in the reinforcement. Smeared cracking is initiated once the tensile stregth is reached. A second crack is permitted to appear normal to the first. Reinforcement strength is included as a set of additional layers of uniaxial behavior characterized by an equivalent thickness. A bilinear diagram is used to model the elasto-plastic behavior of reinforcing steel.

Shell prestressing tendons are individually defined as arbitrary spatial curves contained in the shell thickness. The geometric treatment of tendons is based upon a method where analytical parametric expressions are used for the definition of the mid surface of the shell as well as the tendon curves. Thus, an authomatic calculation of their geometric properties, as well as the further updating of their whole geometry and forces in the nonlinear geometric analysis, are possible. Prestressing in beams is treated consistently with the adopted beam straight finite element, by dividing the tendon into a number of straight segments, each of which spans a single beam element and is assumed to have a constant force. A multilinear stress-strain curve is adopted to model the stress-strain relationship for prestressing steel. In addition, the usual empirical formulae are used for stress relaxation and friction properties.

Nonlinear geometric effects are caused by the consideration of finite movements and finite rotations. First, the compatibility equations are used with their quadratic terms to obtain the strains from the displacement field. Furthermore, the geometry of the structure is continuosly updated by adding the displacement increments to the current nodal coordinates, according to the Updated Lagrangian Description. This requires an interactive procedure until convergence is obtained meaning that the equilibrium condition has been finally satisfied on the deformed geometry of the structure.

4.- DISCRETIZATION OF THE STRUCTURE

Advantage is taken of the structural symmetry, so that only one of the four connected HP quadrants is numerically modelled. Two meshes with a different degree of refinement were defined: Firts, a coarse mesh having 15 shell elements and 24 beam elements, and, second, a fine mesh with 55 shell elements and 52 beam elements. Although satisfactory results were obtained using the coarse mesh for linear elastic analyses, it was necessary to use the fine mesh to accurately reproduce the strongly local effects and local type of failure finally observed in the ultimate behavior. The results that are presented below were thus obtained using the fine mesh (Fig. 4).
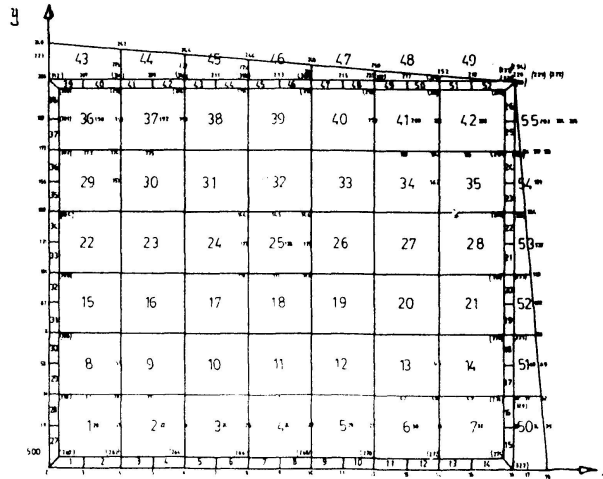

Fig. 4. Fine mesh including 50 shell and 52 beam elements

An automatic procedure for the generation of the shell finite element mesh and the courses of the tendons has been used which needs as input data the parametric description of the geometry of the choosen HP quadrant and the axial curves of the tendons included in it.

All mesh reinforcement keeps the direction of the HP generatrices parallel to the X,Y global axes. Untensioned reinforcement in the shell is defined by a set of layers of equivalent thickness and provided at each integration point in the mesh.

All shell tendons are defined with a cross steel area of $A_{pi} = 3.13 \ cm^2$, and stressed to $1158 N/mm^2$. Their distribution is designed to fit the expected average compression throughout the shell membrane.

Each straight beam element was defined with its cross sectional dimensions, flexural reinforcement and torsional reinforcement, which are assumed to be constant throughout the element. Each flexural reinforcement pattern consists of a set of discrete reinforcing bars defined by their position in the cross section and their individual amount of steel. Torsional patterns are determined by the core dimensions in x' and z' local directions, the area and spacing of hoop reinforcing bars, and the top and bottom longitudinal perimeter steel.

The four beam tendons included in the discretized quadrant of the structure produce a total prestressing force of 3550 kN and 2440 kN respectively for the 42.10 m and the 35.35 m edge beams. In both cases, the minimum cover between tendon axis and beam top surface is about 11.5 cm. Based on the available design data, the following mechanical properties were considered in the analysis: Concrete: $E_0 = 26320 \ N/mm^2$, $f'_c = 28 \ N/mm^2$, $f'_t = 3.0 \ N/mm^2$, $\varepsilon_c = 0.002$, $\nu = 0.15$, and $\gamma_c = 24 \ kN/m^3$. Reinforcing steel: $E_s = 210000 \ N/mm^2$, $E_{sh} = 0 \ N/mm^2$, $f_{sy} = 270 \ N/mm^2$, and $\varepsilon_{su} = 0.10$. A prestressing

steel with $f_{pu} = 1680$ N/mm^2 and $f_{py} = 0.7f_{pu}$. is considered. A penta-linear stress-strain diagram has been generated based on standard shape fitting these properties.

5.- PERFORMED ANALYSES AND RESULTS

5.1- Linear Elastic Analysis

The study in [9] (1991) by Molins of several loading cases in a linear elastic analysis showed a very good agreement with the similar results first obtained by Lo and Scordelis [1] (1969), thus illustrating the suitability of the geometric modelling adopted for the present study. The discussion of the linear elastic analysis, together with more details relative to the nonlinear analyses which are presented below, may be found in [9].

5.2.- Nonlinear Geometric Analysis (NLG) with Prestressing

In this case, the design live load Q_0 (1.437 kN/m^2) is increased indefinitely until a maximum load is obtained for the structure. The materials are considered to be linear elastic, so that no limiting tensile or compressive strengths are defined for the concrete or the steel, while the nonlinear effects caused by finite displacements are considered.

A failure was obtained for a vertical load of $1.0DL + 32.3Q_0$, followed by a descending post-failure branch (Fig. 5). The maximum deflection at failure is reached in the external X—edge beam at a distance of 11.0 m from the top (Fig. 6). The obtained geometric instabilty seems related to an excessive deflection of the X—edge beam, which tends to work like a simply supported beam rather than a cantilever, having a support at the tip of the Y—edge beam. Thus, the geometric failure is partly related to the geometric asymmetry of the structure.

5.3.- Nonlinear Geometric Analysis (NGL) with no Prestressing

A similar type of analysis was performed without any prestressing to study this effect. No significant change was observed during the loading process for live load, as illustrated by Fig. 5. As known, prestressing should not affect the global nonlinear geometric behavior of the structure since the set of equivalent forces caused by prestressing consitute a self-balanced system.

5.4.- Nonlinear Material Analysis (NLM) with Prestressing

In this case, the limiting material strengths defined in Sec. 4 are considered, together with the nonlinear constitutive equations included in the numerical method of analysis. Displacements and rotations are assumed to be small.

An ultimate load of $1.0DL + 4.23Q_0$ is obtained (Fig. 7). The maximum vertical deflection at failure appears at the tip of the HP quadrant and reaches 22.1 cm. Almost no post-failure branch is obtained, so that a rather brittle behavior is detected. An inspection of the analytically obtained state of the materials shows that the failure is mainly due to exceeding the tensile strength at certain membrane zones near the HP tip which are under-reinforced while submitted to a state close to pure tension.

5.5.- Nonlinear Geometric and Material Analysis (NLGM) with Prestressing

The coupling of both types of nonlinearity, material and geometric, caused a reduction in the
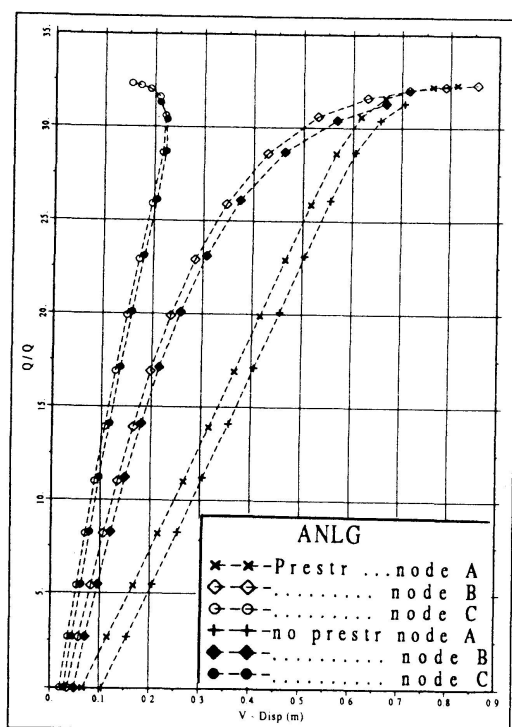
Fig. 5 Load vs. vertical deflection for
        NLG analysis, (a) with prestressing
        (b) without any prestressing.
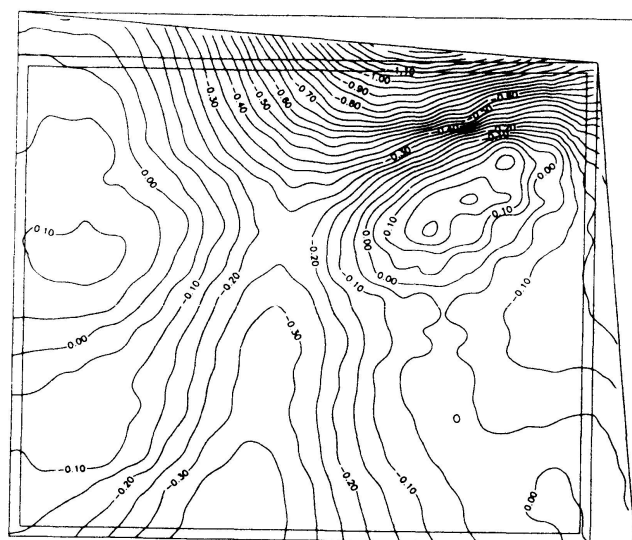

Fig. 6 Vertical deflection in shell at
        ultimate load for NLG analysis
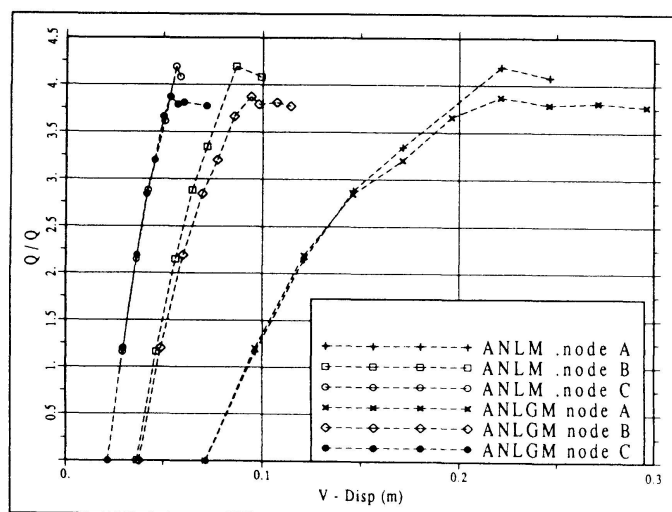   with prestressing.

Fig. 7 Load vs. vertical deflection for
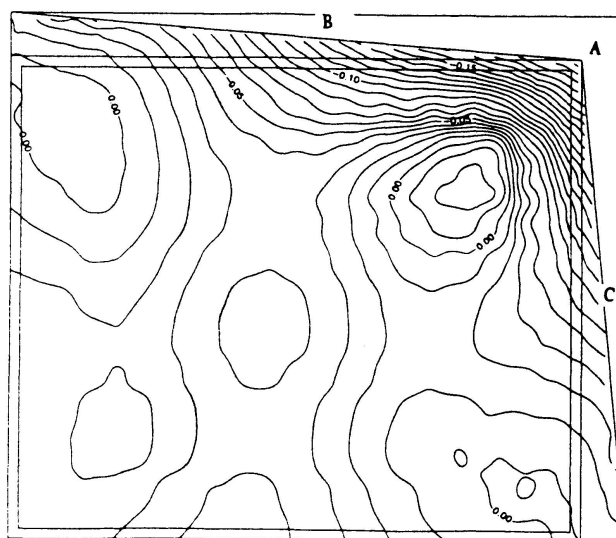   NLM and NLGM analyses

Fig. 8 Vertical deflection at ultimate
       load for NLGM analysis.

ultimate load of the structure with respect to the analysis in which only material nonlinearities
were considered. The obtained ultimate load is now $1.0DL + 3.87Q_0$, showing a drop in the
maximum live load Q of 8.5% from that for the nonlinear material alone analysis (Fig. 7). A
maximum deflection of 22.1 cm for the ultimate load, appears at the tip of HP quadrant. As
Fig. 8 shows, the deformed shape of the roof is qualitatively similar to that one obtained by
the nonlinear material analysis. The influence of the nonlinear geometric effects may be seen
in the slight reduction of the ultimate load and in the ductility acquired in the post-failure
behavior. However, failure is still due to excesive cracking of concrete near the tip of the
shell. Considering the live load capacity of $3.87Q_0$ found in this nonlinear analysis, in which
Q_0 was the design live load of $1.437 kN/m^2$, it can be said that a design level of safety for the

structure exists, and the adequacy of the original design process is shown for the structure subjected to instantaneous vertical loading. Shell and edge beam prestressing affect in a strong way the performance of the structure not only during service conditions, but also at the ultimate range.

6.- CONCLUSIONS

A numerical model for the nonlinear geometric and material analysis of prestressed concrete shells has been used to study an existing long span prestresed concrete shell structure: the Ponce Coliseum, in Puerto Rico. Using an accurate discretization which included both shell and beam finite elements, several different analyses were carried out under vertical loads to study the service and ultimate behavior of the structure, together with its failure mechanism. In an analysis considering both the geometric and the material nonlinear effects, the designed safety of the structure under short-term vertical loading was demonstrated.

Some aspects not yet included in the analysis, such as the influence of the actual construction sequence, as well as the time dependent effects of creep and shrinkage of the concrete and the long-term losses in the prestress are planned for future studies.

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