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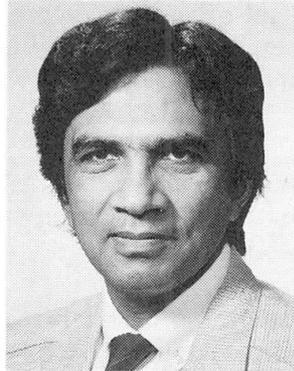
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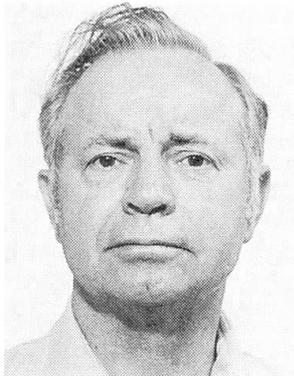
**Analytical and Observed Responses of Steel Girder Bridges**  
Comportement analytique et observé des ponts à poutres d'acier  
Berechnetes und beobachtetes Verhalten von Stahlträgerbrücken

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#### SUMMARY

The introduction over the past thirty years or so of ever more powerful methods of analysis of bridges has brought with it an increasing acceptance by designers of the position that "the results of the analysis must be right". This paper shows that even the most refined methods of analysis can lead to very serious errors in the prediction of bridge behaviour because of the presence of non-quantifiable behavioural factors, which by their nature cannot be included in the mathematical model on which the analysis is based.

#### RÉSUMÉ

Au cours des trente dernière années, l'introduction de méthodes d'analyse de plus en plus puissantes pour le dimensionnement des ponts a suscité l'acceptation de la part du concepteur que "les résultats de l'analyse doivent être corrects". Cette communication montre que même les méthodes les plus poussées peuvent conduire à de très graves erreurs de prédiction du comportement du pont, à cause de la présence de facteurs de comportement non-quantifiables, qui, par leur nature, ne peuvent pas être inclus dans un modèle mathématique sur lequel l'analyse est basée.

#### ZUSAMMENFASSUNG

Die Einführung immer wirksamerer Rechenmethoden im Brückenbau in etwa den letzten dreissig Jahren hat die Statiker vermehrt zu der Einstellung veranlasst, dass "die Ergebnisse von Berechnungen richtig sein müssen". In dieser Abhandlung wird gezeigt, dass selbst die genauesten Rechenmethoden zu schwerwiegenden Fehlern führen können, weil doch immer unfassbare Verhaltensweisen mitspielen, die ihrer Natur nach im mathematischen Modell der Berechnung nicht erfasst werden können.



## 1. INTRODUCTION

Very significant advances in analytical techniques during the past three decades have led to the development of extremely powerful and versatile methods of structural analysis that are generally computer-based. Especially within the linear elastic range, these methods are known to predict accurate and consistent sets of results. Even methods developed from different fundamental considerations predict virtually the same set of results, thereby lending credence to each other's accuracy. Because of their rigor and perceived capability to predict the actual behaviour of structures, these advanced methods of analysis are used extensively for both the design and strength evaluation of bridges.

Predictions from rigorous analyses are usually accepted with confidence as representing the actual behaviour of bridges, and reliance on the ability of analysis to predict the actual behaviour of a bridge increases with the rigor of the method.

Jointly between them, the authors have spent more than 60 years on research related to different aspects of bridge engineering; much of this research has been conducted in the analytical aspects of bridge engineering, e.g. [1], [2], [3], [4] and [5]; they also had the opportunity of being involved in a large number of field tests on short and medium span bridges, e.g. [6], [7], [8], [9] and [10]. Through such involvement, many comparisons have been made between the observed responses of bridges and those given by advanced methods analysis. It has frequently been found that significant discrepancies existed between the predicted and observed responses, even when the loading was within the linear elastic range of the structure.

As might have been foreseen, the discrepancies between the analytical and measured responses were subsequently found to be due not to inadequacies of the methods of analysis, but rather to the presence of behavioral factors which could not be included in the mathematical modelling because of difficulties in their quantification.

Without for a moment denying their usefulness, especially in the design office, the authors have come to believe that even highly rigorous methods of analysis cannot be relied upon unquestioningly to predict the actual response of a bridge. In support of this somewhat provocative assertion, results from tests on five bridges with steel girders are presented. It is emphasized that the limitation of the examples to bridges with steel girders is due only to considerations of space availability for the paper. Similar examples, underlying the difficulties in realistic analysis, are also available for other structures. It may also be noted that results are discussed herein of only those tests in which the authors had a direct involvement. For this reason, the references cited are only those contributed by the authors.

## 2. BRIDGE WITH TIMBER DECKING

The first example presented is that of the rolled steel girder Lord's bridge with nail-laminated timber decking in which the wood laminates are laid transversely. As described in [11], the bridge is 6.25 m wide and has a single span that is apparently simply-supported. The girders are 10.2 m long with a bearing length of 0.53 m at each end, and rest directly on timber crib abutments. There are no mechanical devices to transfer interface shear between the girders and the timber decking although there are 100 x 200 mm nailing strips bolted to the top flanges of the girders; the decking is nailed to these strips. The Lord's bridge was tested with a test vehicle under several load levels and different longitudinal and transverse positions. Even up to the highest load level, the girders responded in a linear elastic manner. For two of the load cases, the longitudinal position of the vehicle was the same but the eccentric transverse positions were the mirror images of each other.

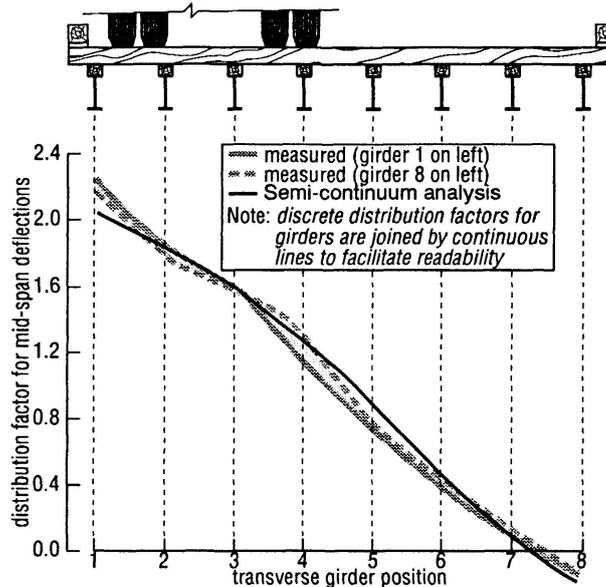


Figure 1/ Distribution factors for mid-span deflections in the Lord's bridge

For these two load cases at the highest test load level, the distribution factors for mid-span deflections are plotted in Fig. 1 by viewing the cross-section of the bridge from two different ends so that the two transverse distribution profiles overlap each other for easy comparison. It is noted that the distribution factor for deflection is the ratio of the actual and average girder deflections at the transverse section under consideration.

If the geometrically-symmetrical bridge was also symmetrical with respect to its structural response, the distribution factors for the two mirror-image load cases, noted above, would have led to transverse distribution profiles that lie exactly on top of each other. As can be seen in Fig. 1, the two profiles are fairly close to each other but not exactly the same, thus pointing out that the two transverse halves of the bridge do not respond in exactly similar manner to corresponding loads. The two sets of distribution factors obtained from measured deflections, are also compared in Fig. 1 with those obtained from deflections given by the semicontinuum method of analysis [4]. It can be seen that the analytical values of the non-dimensionalized deflections are not any more different from the two sets of observed values than the latter are from each other. This confirms that for the bridge under consideration, the semi-continuum method used for analysis is able to predict the pattern of transverse distribution of loads fairly accurately.

The same accuracy of prediction, however, cannot be claimed in the case of the absolute values of girder deflections. This is because of uncertainty in quantifying the parameters discussed below.

As noted earlier, the girders for the Lord's bridge are 10.2 m long with an unusually long bearing length of 0.53 m at each end. It is customary to assume that the nominal point-support for a girder lies midway along the bearing length, in which case the nominal span of each girder would be 9.67 m. It can be demonstrated, however, that for the case under consideration, the vertical pressure under the supported length of a girder, should have its peak away from the midway point and towards the free edge of the abutment. Determination of the exact location of this peak requires detailed knowledge of the modulus of subgrade reaction of the timber crib abutment. Clearly, this factor is not easily quantifiable thus making the task of determining the effective span very difficult. It can be appreciated readily that the clear span of the girder, being 9.14 m, is the lower-bound of the effective span of the girder.

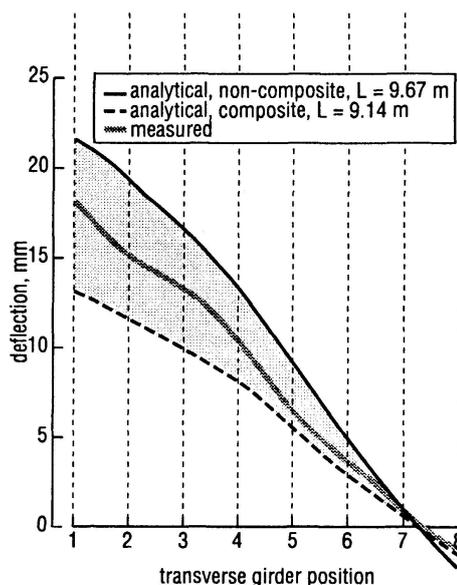


Figure 2/ Girder deflections at mid-span in the Lord's bridge

The transverse modulus of elasticity of wood, which is operative in the longitudinal direction of the bridge, is extremely small compared to the longitudinal modulus. Even if the transverse laminated deck were made composite with the girders, the contribution of the deck to the strength and stiffness of the composite section would usually be expected to be so small as to be negligible. Consequently, no attempt is usually made to provide shear connectors in such bridges. There are some holding down devices, however, to connect the deck to the girders through the nailing strips; these devices, by transferring some interface shear, do make the girders partially composite with the nailing strips and the decking. From measured girder strains, it was discovered that despite the absence of shear connectors, the decking and the nailing strips of the Lord's bridge were partially composite with the girders. The degree of composite action was found to vary from girder to girder, and clearly was not quantifiable.

The Lord's bridge was analyzed using two different sets of idealizations. In one idealization, the girders were assumed to be non-composite and with a simply-supported span of 9.67 m. In the other idealization, full composite action was assumed between the girders and the timber components, being the nailing strips and the decking; the girders were assumed to have the lower-bound span of 9.14 m. As can be seen in Fig. 2, the measured deflections for the same load case for which the distribution factors are plotted in Fig. 1, are bracketed entirely with very large margins by the analytical results corresponding to the two idealizations. It is tempting to believe that the actual condition of the bridge lies somewhere between the two sets of conditions assumed in these idealizations and consequently, errors in analysis are related only to the uncertainties of span length and degree of composite action. However, there is at least one other complicating factor, namely bearing restraint, which was not accounted for in these idealizations and which can have a significant influence on the bridge response; this factor is discussed below.

Observed bottom strains of the girders near the two abutments were generally found to be compressive, indicating the presence of significant bearing restraint forces which varied almost randomly between the girders. It was found that there was no consistent pattern in the bottom flange strains at the mid-span, these being smaller or larger than the corresponding top flange strains. This observation points towards the random, and hence deterministically unquantifiable, nature of both the bearing restraint and the degree of composite action. Because of the presence of

these factors and the difficulty in the estimation of the effective span, the analysis for the bridge under consideration cannot be expected to replicate the actual behaviour of the bridge.

### 3. TWO-GIRDER BRIDGE

The Adair bridge is a single span, single lane structure with a clear span of 12.8 m, as shown in Fig. 3. As is also shown in this figure, the bridge comprises a concrete deck slab supported by two outer steel girders and five inner steel stringers, with the latter spanning between the abutments but also supported within the span by two transverse floor beams that frame into the two girders. A proof test on this bridge is described in [12].

Mid-span strains in the top and bottom flanges of the two girders due to two load cases are plotted in Fig. 4 against the longitudinal position of the test vehicle. It can be seen in this figure that the strains in the top flanges are always much higher than the corresponding strains in the bottom flanges. This observation confirms the presence of fairly large bearing restraint forces. Large compressive strains in the bottom flanges of the girders near their supports also confirmed the presence of significant bearing restraint which again cannot be practically quantified for inclusion in the mathematical model for analysis.

Much larger magnitudes of strains in the top flanges of the girders also point to the lack of composite action between the girders and the deck slab, this bridge not having any mechanical shear connection with the girders. Because of the lack of composite action, the top flanges of the girders getting little relief from bearing restraint at the bottom flanges, govern the load carrying capacity of the girders.

It is interesting to note that, unlike the case in the Lord's bridge and other bridges discussed later, bearing restraint does not provide any significant reserve of strength in the Adair bridge.

The uncertain nature of the composite action in slab-on-girder bridges without mechanical shear connection is underlined by the observation that, in the same Adair bridge, the inner stringers are able to develop full composite action with the deck slab despite the lack of mechanical shear connectors.

Because of the composite action, the stringers had become considerably stiffer thus relieving the non-composite girders of a much greater share of the applied loading than would have been the case if they were also non-composite. It can be appreciated that analysis cannot be very effective without the knowledge of the degree of composite action in the various beams; such knowledge is practically impossible to obtain without a test.

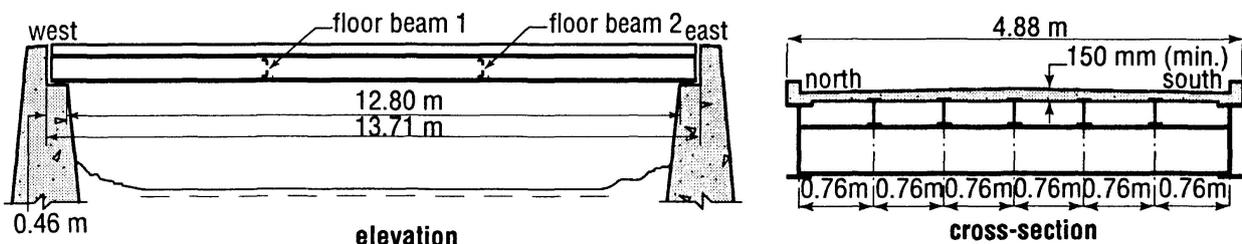


Figure 3/ Details of the Adair bridge

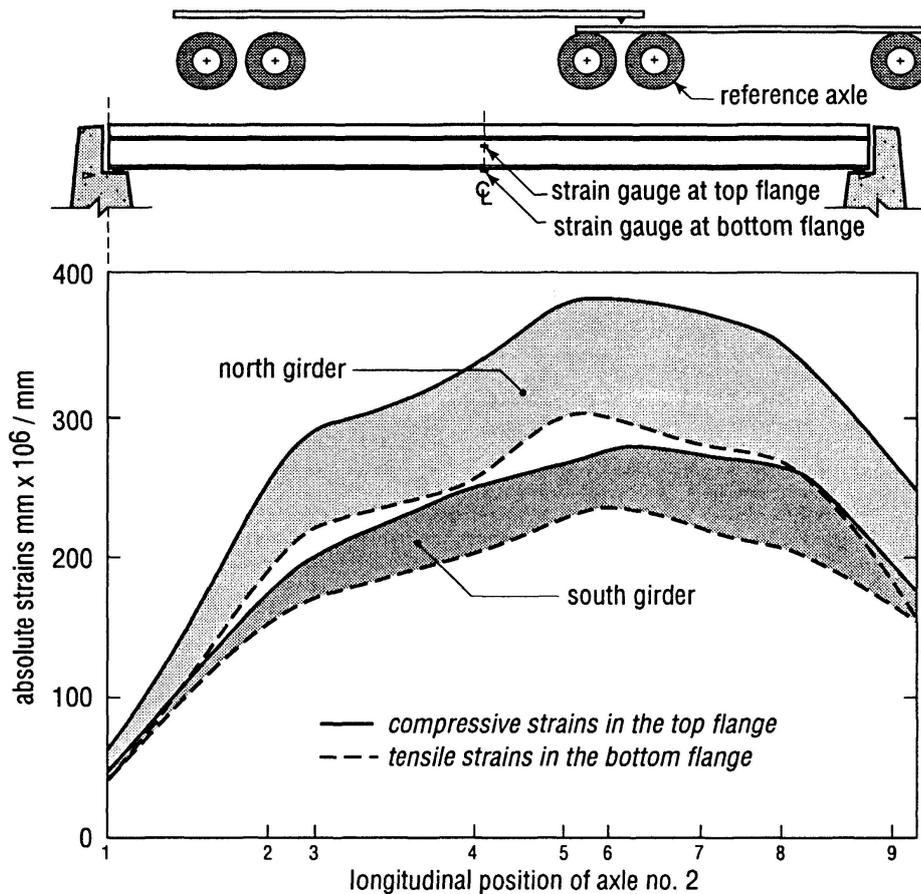


Figure 4/ Girder strains at mid-span in the Adair bridge

#### 4. ULTIMATE LOAD TEST ON A BRIDGE

An ultimate load test on a single span, right, i.e. skewless, slab-on-girder bridge, called the Stoney Creek bridge, is described in [10]. The bridge, which had a clear span of 13.26 m, was loaded to failure in 1978 by loading it with concrete blocks piled in six layers. Girder strains at the mid-span were recorded after each layer of blocks had been placed on the bridge.

To check the validity of the recorded data, the mid-span moments taken by the girders and the associated portions of the deck slab, computed from measured strains, were compared with the total applied moments. It is recalled that in a right, simply-supported bridge, the total moment across any transverse section is obtained by simple beam analysis, and is statically determinate. When it was found that the moments computed from measured strains were up to 30% smaller than the applied moments, the accuracy of the measured data was initially questioned. An example of the comparison of moments thus computed from measured strains and average applied moments is presented in Fig. 5 for load due to one layer of concrete blocks. It is noted that the girder strains under this loading were well within the limit of computed elastic strains.

The initial computations of moments from measured strains were made by assuming that the girders were free from any horizontal restraint at the bearings. The bearing restraint forces were not initially entertained as possible cause for the moment discrepancies mentioned above. This was because bearing restraint forces of the magnitude needed to reduce the applied moments by up to 30%, were believed to be unlikely to develop in practice.

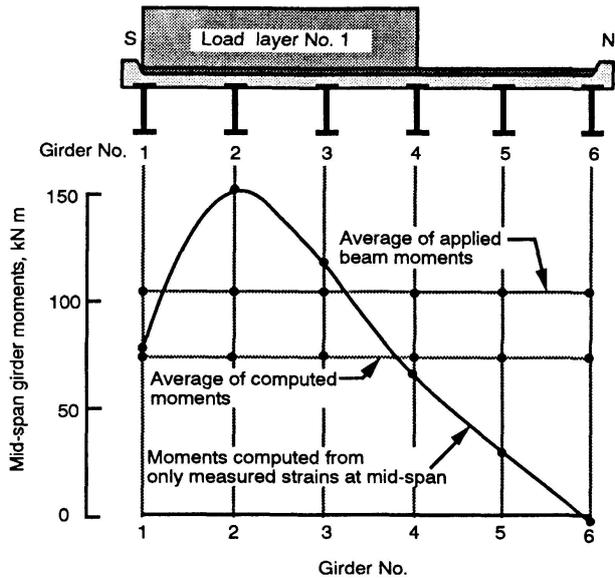


Figure 5/ Girder moments in the Stoney Creek bridge computed from observed data by ignoring bearing restraint

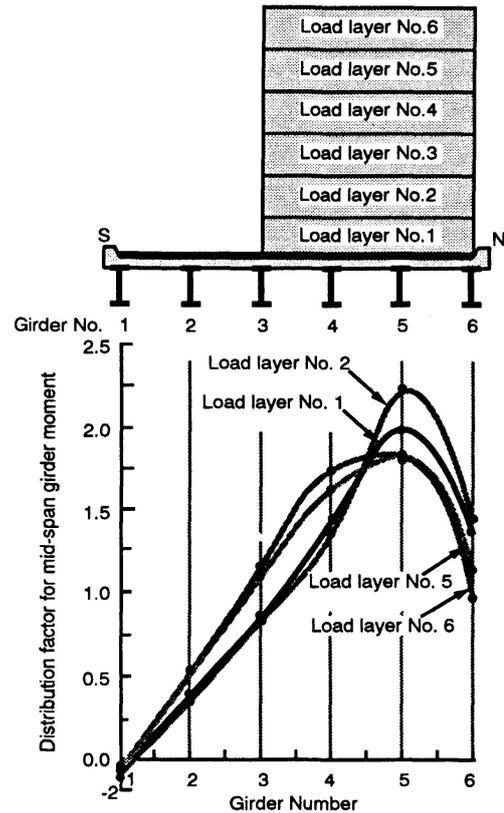


Figure 6/ Girder moments in the Stoney Creek bridge due to load at different levels

Subsequent tests, some of which are discussed herein, confirmed the presence of significant bearing restraint forces in similar slab-on-girder bridges in which girders rest upon steel bearing plates. The presence of these forces invalidates the assumption of simple supports and the computation of moments obtained from measured strains on the basis of no external forces. In light of the knowledge gained from the other tests, the data from the test on the Stoney Creek bridge were re-analyzed about ten years after the test by back-calculating the bearing restraint forces that may have occurred. From these revised computations, it was found that the bearing restraint reduced the applied moment by up to 18%, rather than the 30% range that had been wrongly deduced by previous calculations.

Distribution factors for mid-span moments taken by the girders and the associated portions of the deck slab are plotted in Fig. 6 for loads at different levels. It is interesting to note that the transverse distribution pattern of the bridge does not change very significantly as the load approaches the ultimate sixth layer. As the failure of the bridge approaches, the load gets redistributed only slightly among the most heavily loaded girders. The girder most remote from applied loading, receiving little load at early stages of loading, continues to receive low levels of load even when the total load approaches the failure load of the bridge.

An important outcome of the test was the observation that in the absence of mechanical shear connection, the composite action between a girder and the deck slab, that may exist at low levels of load, breaks down completely as the load approaches the failure load for the girder.



## 5. A NON-COMPOSITE SLAB-ON-GIRDER BRIDGE

The unquantifiable and random nature of the bearing restraint forces, and of the degree of composite action in the absence of mechanical shear connection, is illustrated by the results obtained from a test on the Belle River bridge [13]. The Belle River bridge is also a slab-on-girder bridge with steel girders and an apparently non-composite concrete deck slab. The nominal span of the bridge is 16.3 m and the width 9.1 m.

As indicated earlier, the transverse load distribution analysis of slab-on-girder bridges without mechanical shear connectors is made difficult, to the point of becoming impossible, by the uncertain degree of the composite action. One is tempted to believe that the actual load distribution pattern of such bridges could be bracketed by two sets of analyses: one corresponding to full composite action and the other to no composite action at all, with the former analysis always leading to safe-side estimates of the maximum load effects in the girders. In reality, a deterministic analysis, no matter how advanced, might fail completely to predict safely such maximum load effects. This assertion is illustrated below with the help of the results from the test on the Belle River bridge.

Transverse profiles of the distribution factors for mid-span girder moments in the bridge under consideration, are plotted in Fig. 7 for a transversely symmetrical load case. One of these profiles corresponds to moments computed from observed girder strains both at the mid-span and near the abutments, with the latter providing information regarding the bearing restraint forces. The other two transverse profiles are obtained from the results of the semicontinuum method of analysis [4] for the two bounds of the composite action. It is noted that no attempt was made to model the bearing restraint in these analyses.

It can be seen in Fig. 7 that the pattern of transverse distribution of actual moments is similar, but only in a general way, to the two analytical patterns. It is also quite random. Unlike the analytical patterns, the actual pattern is far from being symmetrical. In fact, the actual distribution factor for maximum girder moments is about 10% larger than the corresponding analytical factor for the fully composite bridge. It can be appreciated that the occurrence of the very high distribution factor and significant departure from symmetry are probably caused by the middle girder becoming accidentally much stiffer through composite action by bond than the adjoining girders. In light of the results plotted in Fig. 7, there can be little doubt that, for the kind of

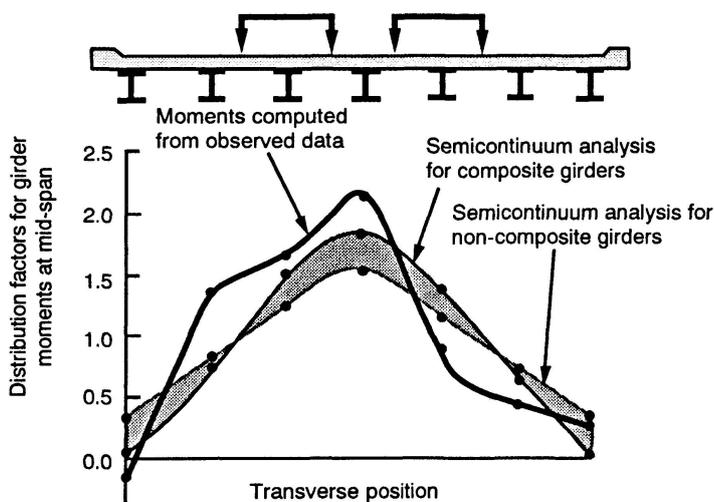


Figure 7/ Distribution factors for midspan moments in the Belle River bridge

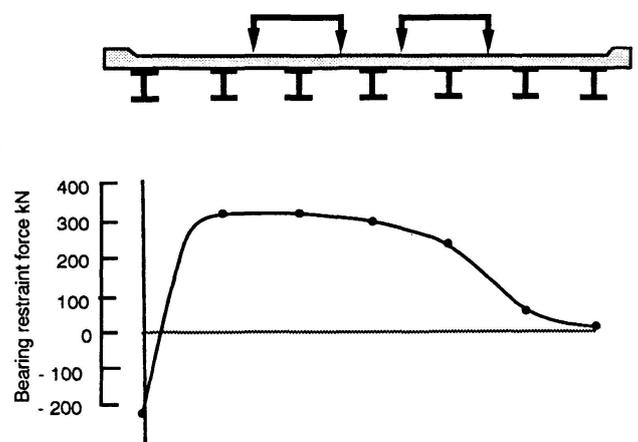


Figure 8/ Bearing restraint forces in the Belle River bridge





The cross-section of the single-span North Muskoka River bridge is shown in Fig. 9. This bridge comprises five steel girders and a composite deck slab; its span and width are 45.7 m and 14.6 m, respectively. Both ends of every girder rest on laminated elastomeric bearings each measuring 560 x 335 mm in plan and 64 mm in thickness. The design shear rate for each bearing is about 30 kN/mm.

A dynamic test showed the North Muskoka river bridge to be about 20% stiffer flexurally than could be rationalized by even a very detailed analysis in which all those components of the bridge were taken into account which could conceivably enhance the flexural rigidity of the bridge. To determine the cause for the apparent discrepancy, a diagnostic static test was conducted subsequently. For this latter test, all the girders were instrumented with strain measuring devices to measure longitudinal strains at three transverse sections of the bridge, one section being near the mid-span and each of the other two near each abutment [8].

If the elastomeric bearings had permitted a free longitudinal movement of the girders, then under live loads the strains in the bottom flanges near the bearings would have been tensile and very small. It was found that this was not the case. The test loads induced fairly large compressive strains in the bottom flanges near the elastomeric bearings. Bearing restraint forces computed approximately from observed strains are plotted in Fig. 10 for different load cases. It is interesting to note that under transversely symmetrical loads, the corresponding bearing restraint forces were not exactly the mirror image of each other, as should have been the case for an ideally symmetrical structure. Bearing restraint forces as high as 175 kN, which can be seen in Fig. 10, are considerably larger than a functioning elastomeric bearing would be expected to develop. Nevertheless, such large forces were really present despite the fact that the bearings were apparently in excellent and functioning condition.

A further proof of the presence of large bearing restraint forces in the North Muskoka river bridge was provided by comparisons of applied moments obtained from considerations of simple supports with those computed from girder strains. Figure 11 shows the comparison of mid-span girder moments computed from measured strains with those obtained by the familiar grillage analogy method. The bearing restraint forces were not accounted for in this analysis. It can be readily concluded from this figure that the total moment sustained by all the girders is noticeably less than the corresponding applied moment obtained on the basis of simple supports; this confirms that the applied moments were reduced by the effect of bearing restraint.

It was found that at the time of the test the bearing restraint in the North Muskoka river bridge reduced the mid-span deflections due to test loads by about 12%. This reduction is considerably smaller than the 20% reduction observed in the previous test on the same bridge. The first test was conducted on a relatively cool day in October and the second on a very hot day in June. It is hypothesized that the elastomeric bearings had become stiffer in the cold temperature when the first test was conducted thereby generating higher restraint forces which consequently caused the bridge to become effectively stiffer than it was at the time of the second test.

Results of tests on the North Muskoka river bridge demonstrate the significant influence of the restraining effects of elastomeric bearing which may change with load level and temperature. To be able to analyze bridges with these bearings more accurately, it is essential to include their effective shear stiffness in the mathematical model.

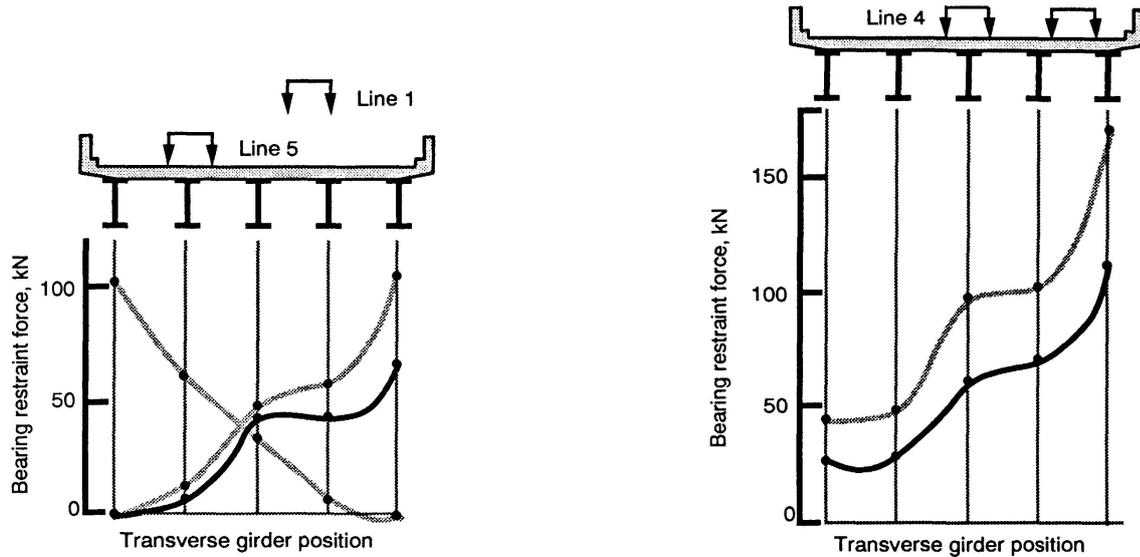


Figure 10/ Bearing restraint forces in the North Muskoka River bridge

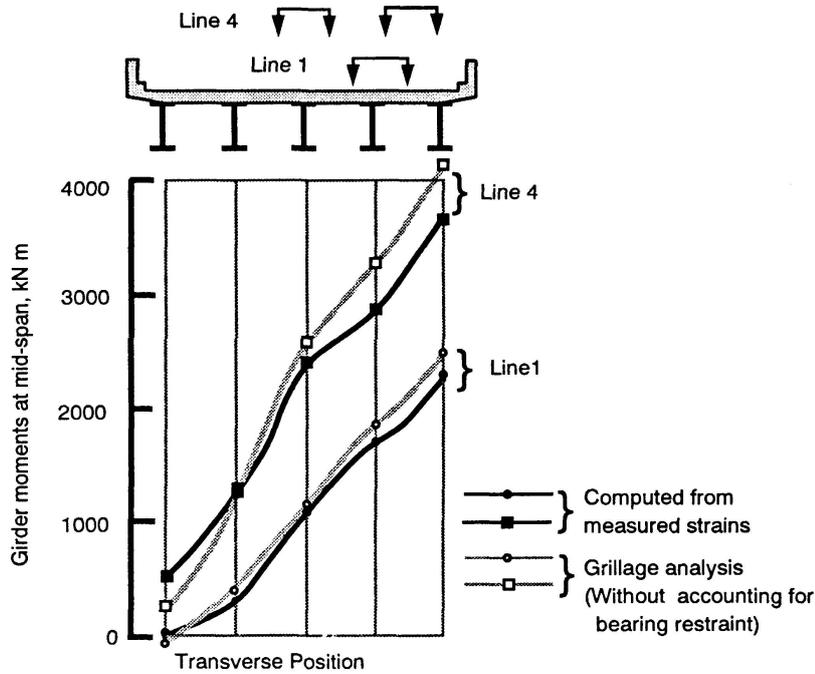


Figure 11/ Mid-span girder moments in the North Muskoka River bridge

### 7. CONCLUSIONS

The purpose of this paper is not to discredit the rigorous methods of analysis, but to note that there are certain unquantifiable behavioral aspects in bridges which cannot be accounted for realistically in a mathematical model. Because of this difficulty, the predictions of even highly rigorous and very accurate methods of analysis may not reflect reality. This contention has been illustrated with the help of results obtained from tests on five short or medium span bridges with steel girders. It is suggested that in some cases, a realistic evaluation of the load carrying capacity of an existing bridge can be conducted only through a field test.



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