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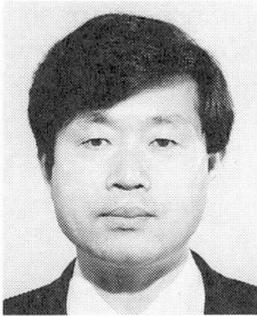
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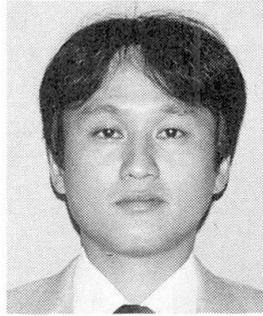
**Long-term Prediction of Behaviour of Cable-Stayed Bridges**  
**Prédiction du comportement à long terme de ponts à haubans**  
**Voraussage des langfristigen Verhaltens von Schrägseilbrücken**

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### SUMMARY

Presented herein is a method to predict the long-term change of cable forces and slip of cables out of sockets in cable-stayed bridges. Employed are full-scale long-term tension tests of cables to determine the visco-elastic constants of the cables and the sockets. Based on the experimental results, the analytical prediction of bridges were made through the finite element visco-elastic analysis together with the Laplace transform and the results were compared to the site measurement values.

### RÉSUMÉ

Cet article présente une méthode pour évaluer le comportement à long terme des contraintes des câbles d'un pont à haubans, ainsi que le comportement au glissement de ses ancrages. Des essais du câble en vraie grandeur ont été effectués afin de déterminer les propriétés visco-élastiques des câbles et des ancrages. La méthode des éléments finis combinée avec la transformation de Laplace est appliquée à l'analyse et les résultats analytiques sont comparés avec les mesures effectuées sur le chantier.

### ZUSAMMENFASSUNG

Es wird eine Methode vorgestellt, mit der langfristige Veränderungen in den Seilkräften, einschliesslich Schlupf in den Verankerungen, vorausberechnet werden können. Dazu waren im Massstab 1:1 Versuche zur Bestimmung der viskoelastischen Eigenschaften von Seilen und Ankerköpfen nötig. Mit diesen Daten wurden Finite-Element-Berechnungen für Brücken unter Verwendung der Laplace-Transformation durchgeführt und die Ergebnisse mit In-Situ-Messungen verglichen.



## 1. INTRODUCTION

Presented herein is a method to predict the long-term change in the cable forces and cables slip-out from their sockets in cable-stayed bridges. Based on long-term tension tests on full-scale cables of 5m length, a very simple analytical model is proposed and an effort is made to determine the visco-elastic constants of the cables and sockets, taking into account the scale effect of the length of the cables, by extrapolating the results of the measurements carried out in cables with limited length to actual cables with arbitrary length. In addition, visco-elastic F.E.M. analysis using the experimental results was carried out to predict the long-term behavior of several existing bridges and these results were compared to the measured values at the site.

Due to the fact that the erection of bridges is usually completed within a period of one year or one year and a half at the site, the visco-elastic constants of the cables and sockets were determined emphasizing first, the initial relatively short period of the erection stages and, secondly, focusing on the control of the cable forces for the much longer period of service life.

## 2. LONG-TERM TEST OF PROTOTYPE CABLES

### 2.1 Experimental Method

When investigating the time-dependent behavior of materials, there are two types of tests, namely, creep and relaxation tests. The former being carried out under constant loading with increasing deformation, and the latter under constant deformation with decreasing stress. The type of test carried out in the present study is a combination of both types [1].

The measurement system is presented in Fig.1, where the load is measured by a load cell and the relative displacements between the cable and the steel frame, by displacement transducers. To investigate the visco-elastic characteristics of the cables due to the difference in cable strength, two types of cables (Specimen types 1 and 2) were tested. In addition, four different combinations of cables and sockets (Specimen types 3 to 6) were tested, in a total of 6 specimens. Table 1 shows the specimen dimensions and characteristics.

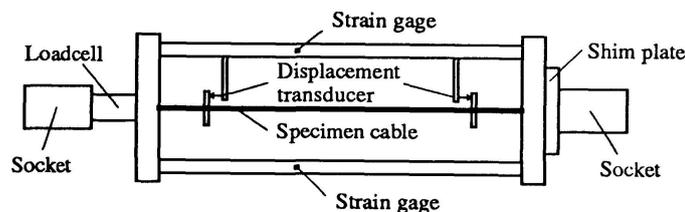


Fig.1 Measurement System

Table 1 Dimensions and Characteristics of the Test Specimen

Specimen Name	Type 1	Type 2	Type 3	Type 4	Type 5	Type 6
Cable Length (m)	5.9	5.9	5.4	5.4	5.4	5.4
Diameter of Wire (mm)	5.0	5.1	7.0	7.0	7.0	7.0
Number of Wires	127	127	19	19	19	19
Cable Type	PWS	PWS	PWS	PWS slightly twisted	PWS	PWS slightly twisted
Anchorage Length(cm)	44.0	44.0	16.3	16.3	30.7	30.7
Anchorage Type	Zinc-poured	Zinc-poured	HiAm	HiAm	Zinc-poured (Incl. 2% Cu)	Zinc-poured (Incl. 2% Cu)
Cross Sectional Area (cm <sup>2</sup> )	24.94	25.94	7.31	7.31	7.31	7.31
Breaking Force (kN)	4400	3910	1137	1137	1137	1137
Initial Cable Force (kN)	1400	1310	380	377	368	371

## 2.2 Test Results

Fig.2 shows the time variation of the tensile force in the cables. As it can be observed, the forces in specimen types 3 and 4 tend to stabilize in a relatively short time (about 20 days), whereas in the other cables, continue to decrease even after one year's measurement.

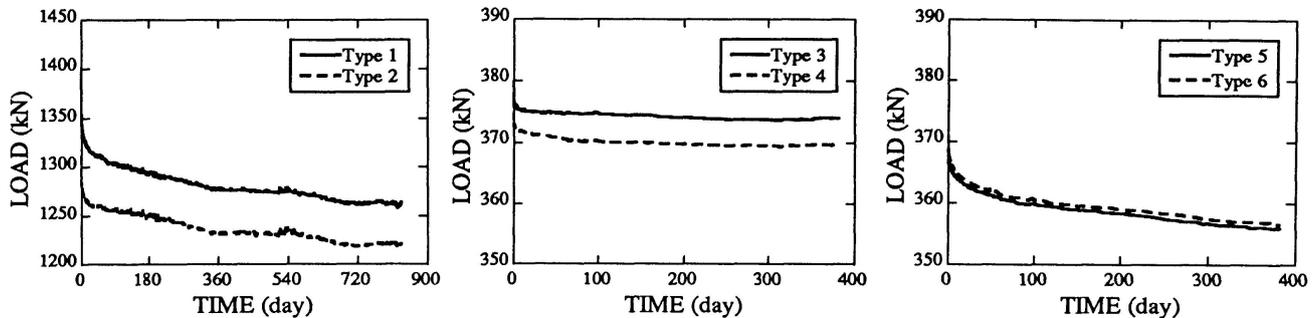


Fig.2 Time Variation of Cable Tensile Force

Although the tested cables and sockets were the same as the ones used in actual bridges, the cable length of the specimens differed from the actual cables, thus, it was decided herein to consider the cables and sockets separately. Fig. 3 illustrates the time-dependent strain of each cable type and Fig.4, the slip-out behavior of the different types of sockets.

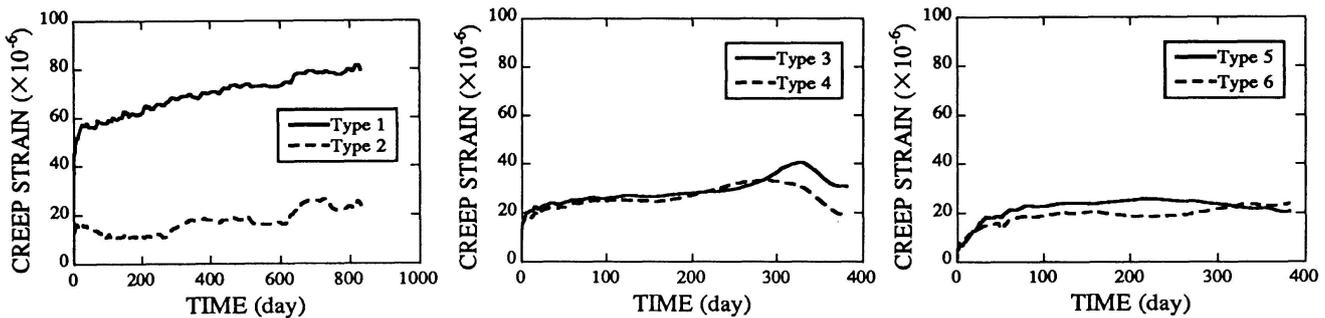


Fig.3 Time-dependent Behavior of Cables

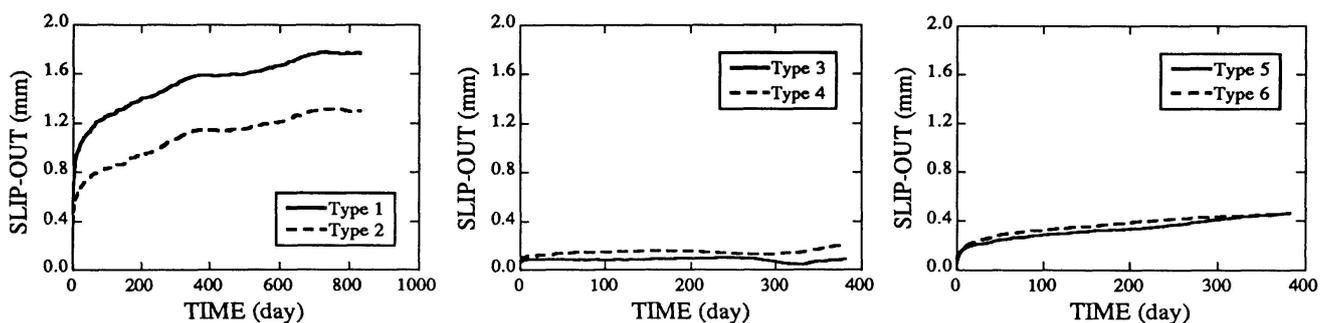


Fig.4 Time-dependent Behavior of Sockets

Specimen types 1 and 2 presented large values for the amount of slip-out from the sockets, compared to the values of cable creep. Specimen type 3 and 4, 5 and 6 have respectively the same sockets. In the formers, the forces, as well as the amount of slip-out stabilized in relatively short time (20 days); whereas in the latters, the amount of slip-out continued to increase.

Cable material of the specimen types 3 and 5, 4 and 6, being respectively the same, presented similar values for the final creep. However, time variation between cables of the same type were different, suggesting the influence of their sockets.



### 3. MODEL TO EVALUATE TIME-DEPENDENT CONSTANTS

#### 3.1 Mechanical Model

Due to its simplicity, the analytical model adopted is the three-element model shown in Fig.5 [2]. The total strain of the model  $\epsilon$ , can be expressed as the sum of the elastic ( $\epsilon_e$ ) and viscous ( $\epsilon_v$ ) strain:

$$\epsilon = \epsilon_e + \epsilon_v \tag{1}$$

For the total stress  $\sigma$ , the strain-stress relationship can be expressed as follows:

$$\sigma = \sigma_e = E_1 \epsilon_e \tag{2}$$

$$\sigma = \sigma_v = E_2 \epsilon_v + \eta \dot{\epsilon}_v \tag{3}$$

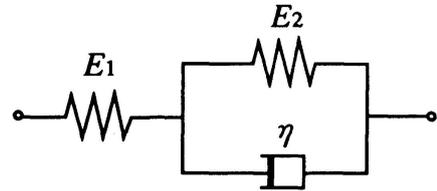


Fig.5 Three-element Visco-elastic Model

where  $E_1$ ,  $E_2$  are the elastic coefficients and  $\eta$  the viscosity coefficient of the three-element model. Differentiating Eq.1 and Eq.2 in relation to time  $t$  and introducing them in Eq.3, it yields the following equation, after some arrangement.

$$\dot{\sigma} + \frac{E_1 + E_2}{\eta} \sigma = E_1 \left( \dot{\epsilon} + \frac{E_2}{\eta} \epsilon \right) \tag{4}$$

#### 3.2 Evaluation of Time-dependency

The visco-elastic constants of the model were evaluated according to the three different methods described below.

##### 3.2.1 Method 1

Focusing on the viscous part of the model in Fig.5 (Eq.3) the following approach curve for the strain due to the viscosity was assumed.

$$\epsilon_v = \bar{\epsilon}_v (1 - e^{-\lambda t}) \tag{5}$$

The coefficient  $\lambda$  can be obtained through the least square method. The results of the evaluation for one of the cables is shown in Fig.6, with the corresponding slip-out from the sockets shown in Fig.7 and the viscosity constants thus evaluated are presented in Table 2.

This method is effective for cables in which the phenomena of creep and slip-out from the sockets stabilize in a short time, however, in cases in which the time dependent curves are not steep and long-term variation is observed, the curves tend to diverge from the predicted values.

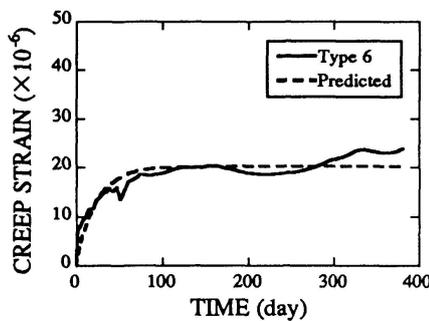


Fig.6 Predicted Time-dependent Behavior of Cables (Method 1)

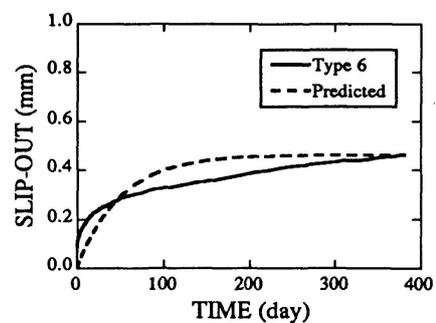


Fig.7 Predicted Time-dependent Behavior of Sockets (Method 1)

**Table 2** Evaluated Visco-elastic Constants (Method 1)

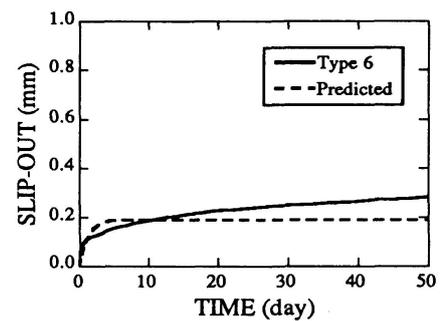
Specimen	Cable				Socket			
	1/λ (day)	E <sub>1</sub> (GPa)	E <sub>2</sub> (GPa)	η (year GPa)	1/λ (day)	K <sub>1</sub> (MN/m)	K <sub>2</sub> (MN/m)	η (year MN/m)
Type 1	2.97	217	6272	51.0	80.0	381	704	154
Type 2	0.26	196	30184	21.5	113.6	543	948	295
Type 3	1.168	202	19306	61.8	0.153	210	4192	1.76
Type 4	1.043	201	20690	59.1	1.188	228	2454	7.99
Type 5	25.5	191	20482	1428.4	75.0	785	769	158
Type 6	23.0	198	24304	1533.1	50.3	582	769	106

### 3.2.2 Method 2

The prediction of the time dependent behavior of bridges during their construction stages requires more accurate values for the initial steep part of the time variation curves. Thus, in the second method, the equations used in Method 1 were applied to a relatively short time interval corresponding to the average interval of time between the prestress of one of the cables and the prestress of the cable of the succeeding stage. This method converges for the initial part of the time variation curves and leads to reliable values for the initial 40 days. Fig.8 shows one of the curves evaluated by this method with the respective experimental curve. Table 3 presents the visco-elastic constants evaluated for specimen types 1,2,5 and 6.

**Table 3** Evaluated Visco-elastic Constants for Sockets (Method 2)

Specimen	1/λ (day)	K <sub>1</sub> (MN/m)	K <sub>2</sub> (MN/m)	η (year MN/m)
Type 1	0.921	381	1491	3.76
Type 2	1.043	543	2154	6.16
Type 5	2.48	785	2030	13.8
Type 6	1.319	582	1910	6.90


**Fig.8** Predicted Time-dependent Behavior of Sockets (Method 2)

### 3.2.3 Method 3

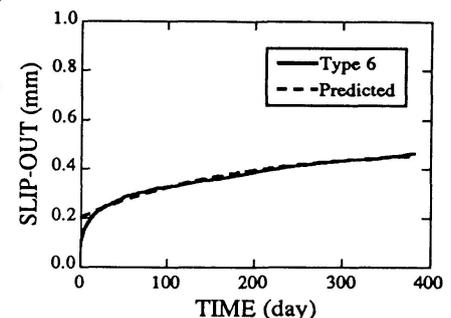
For the maintenance of the bridge during its service life, a long-term prediction is necessary and the strain variation for the time period succeeding the one considered in Method 2, shall be assumed as it follows.

$$\varepsilon_v = \bar{\varepsilon}_v (1 - \alpha e^{-\lambda t}), \text{ where } \lambda = E_2 / \eta \quad (6)$$

Considering  $\bar{\varepsilon}_v$  as a determined parameter,  $\alpha$  and  $\lambda$  can be determined by the least square method. Fig.9 illustrates the results of the analysis for one of the specimens and Table 4 shows the evaluated visco-elastic constants for specimen types 1, 2, 5 and 6.

**Table 4** Evaluated Visco-elastic Constants for Sockets (Method 3)

Specimen	α	1/λ (day)	K <sub>1</sub> (MN/m)	K <sub>2</sub> (MN/m)	η (year MN/m)
Type 1	0.450	351	344	684	658
Type 2	0.541	413	491	866	980
Type 5	0.748	607	505	479	797
Type 6	0.588	178	406	731	356


**Fig.9** Predicted Time-dependent Behavior of Sockets (Method 3)



#### 4. PREDICTION OF TIME-DEPENDENT BEHAVIOR OF A CABLE-STAYED BRIDGE

##### 4.1 Finite Element Formulation

The equilibrium equations for the cable, tower and girder elements after applying Laplace transform leads to a linear system of equations, whose stiffness matrices  $K_{ij}$  are as presented bellow.

$$\bar{K}_{ij}(s) = \sum_{i=1}^m \sum_{j=1}^n \int_V B_{im} \bar{E}_{mn}(s) B_{nj} dV \quad (7)$$

where  $B_{im}$  and  $B_{nj}$  are strain matrices and  $E_{mn}(s)$  is the elastic modulus corresponding to the Laplace space:

$$\bar{E}_{mn}(s) = \frac{s + \frac{E_2}{\eta}}{s + \frac{E_1 + E_2}{\eta}} E_1 \quad \text{for visco-elastic elements} \quad (8-a)$$

$$\bar{E}_{mn}(s) = E_1 \quad \text{for elastic elements} \quad (8-b)$$

The F.E.M. analysis as above described provides solutions in the Laplace space which has to be converted into the real time domain, so as to give the final solution. Therefore, an inverse transformation has to be carried out [2].

In case of cable-stayed bridges, the cables, towers and girders have different visco-elastic constants and therefore, different intervals of convergence, which makes it difficult to perform a numeric inverse transform considering, simultaneously, all the structural elements visco-elastic. Thus, the bridge analysis was carried out considering each element, separately, visco-elastic (case 1, ..., case n) and the final solution was assumed to be a linear combination of all the n cases [3]. The contribution factor for each of the terms of the linear combination is determined by means of the least square scheme in the Laplace image space and the use of the Regula-Falsi method.

##### 4.2 Model Bridge

The model bridge, as presented in Fig.10, is a cable-stayed bridge with a central span of 238.0m and the side spans supported by a PC rigid frame bridge. The structural analysis was carried out for half the bridge, considering its structural symmetry and the cables actually used in this bridge were of type 4 and type 6.

Thus, the following cases were considered for the analysis:

- case 1: only the cables are linearly visco-elastic;
- case 2: only the concrete members of the PC rigid frame bridge are linearly visco-elastic, and
- case 3: final solution assumed as a linear combination of the former cases.

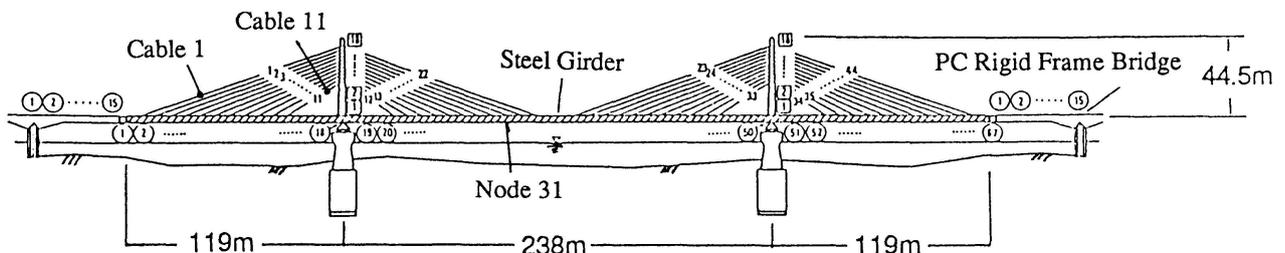
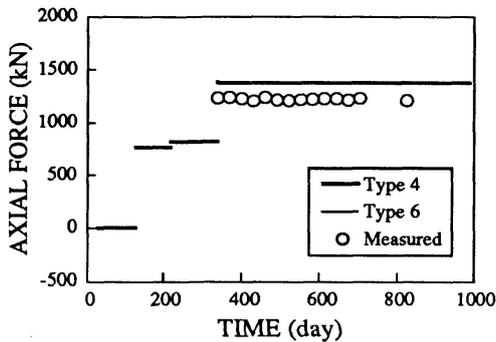


Fig.10 Model Bridge

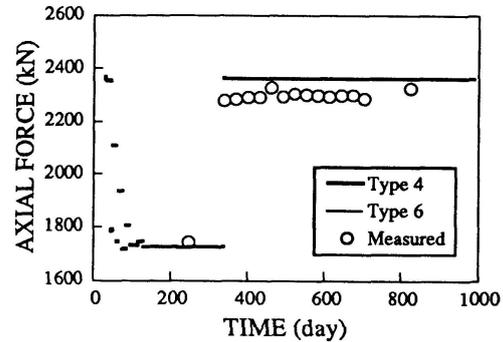
##### 4.3 Analytical Results

The structural analysis was performed using the experimental values of the specimen types 4 and 6, which, although having the same cable material, had different types of sockets. Fig.11 presents the time variation of the axial force in cable No.1 (one of the longest cable) and Fig.12, the time variation

of the axial force in cable No.11 (one of the shortest cables). The forces stabilized in a short time, presenting similar values for both types of cables.

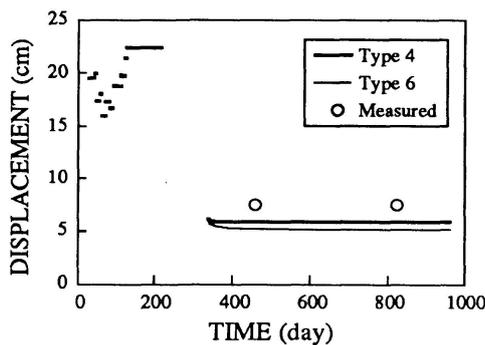


**Fig.11** Variation of the Cable Force Cable No.1 (Types 4 and 6)

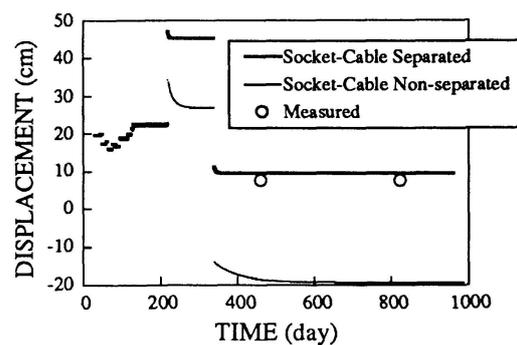


**Fig.12** Variation of the Cable Force Cable No.11 (Types 4 and 6)

On the other hand, as it can be noticed in Fig.13, the time-dependent behavior of the nodal displacements were more remarkable in type 6, whose socket is of a more sensitive type. The effectiveness of considering the cables and sockets separately in the evaluation of the visco-elastic parameters is shown in Fig. 14, where the curves for the case in which cable and sockets are considered separately provides values closer to that of the data measured in situ.

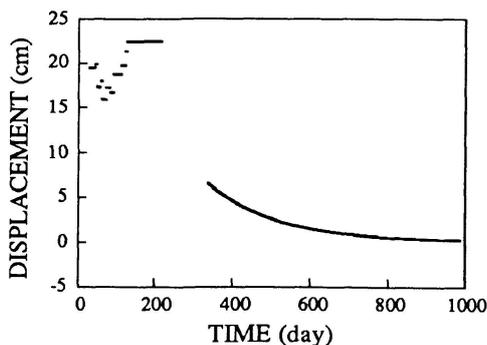


**Fig.13** Displacement of Node No.31 (Types 4 and 6)

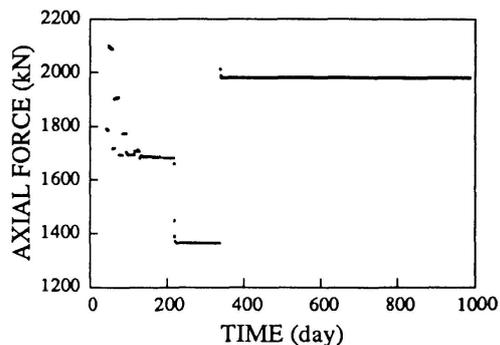


**Fig.14** Displacement of Node No.31 Comparison between (i) Socket-Cable Separated and (ii) Not Separated

A simulation was also performed using a fictitious type of cable, assuming elastic constants similar to that of type 1 ( $E_1=200\text{GPa}$ ,  $E_2=4000\text{GPa}$ ) and the delay-time ( $T=\eta/E_2$ ) of 50 days, which correspond to the values of locked coiled rope [1]. Fig.15 illustrates the time-dependent behavior of the nodal displacement when using the fictitious cable.



**Fig.15** Displacement of Node No.31 (Fictitious Cable)



**Fig.16** Variation of the Cable Force Cable No.11 (Fictitious Cable)

