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Stress Assessment of Reinforced Concrete Structures with Cracks

Estimation des contraintes des constructions en béton armé fissurées

Spannungsermittlung in gerissenen Stahlbetontragwerken

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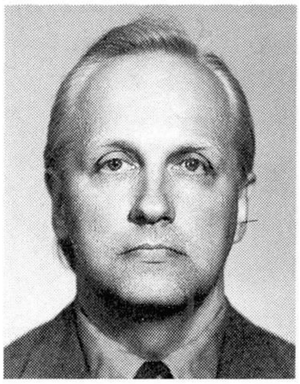

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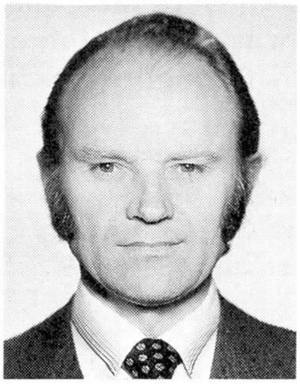

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SUMMARY

A new method to evaluate stress in tensile reinforcement and compressed concrete of reinforced concrete members is proposed. The method is based on principles of crack propagation theory in brittle bodies. The stresses are estimated by taking into account external force action and the parameters of cracks.

RÉSUMÉ

L'article propose une méthode précise de calcul des contraintes d'armature longitudinale tendue et du béton comprimé dans des éléments en béton armé. On détermine les contraintes sur la base des efforts externes et des paramètres de la fissure normale. La méthode de calcul se base sur la théorie linéaire de rupture mécanique dans un corps fragile.

ZUSAMMENFASSUNG

Es wird eine relativ genaue Methode vorgeschlagen, um bei bestehenden Stahlbetontragwerken die Zugs- und Druckspannungen in gerissenen Querschnitten zu ermitteln. Basierend auf der linearen Bruchmechanik für spröde Körper werden die einwirkenden Kräfte und Kenngrössen der Biegerisse berücksichtigt.

1. INTRODUCTION

Evaluation of existing reinforced concrete structures is very important for maintenance, renovation and reconstruction of industrial and other in-service buildings. There are several methods to evaluate stress and strains in structures using their examination data. In the case of reinforced concrete structures with normal cracks in tensile zone it is possible to evaluate stress by the crack parameters measured during the examination of these structures. The degree of crack development is considered as a result of existing stress - strain state in the member [1]. In this paper a posibility to employ classical theory of crack propagation in a brittle body for calculation of tensile reinforcement stresses at the cross section through crack is investigated.

2. REINFORCEMENT AND CONCRETE STRESSES IN CROSS SECTION THROUGH CRACK

Within the limits of assumed model of a brittle body in the case of macroscopic cracks the following equation is valid [2]:

$$\lim_{s\to 0} \{\sqrt{s}\ \sigma_{y(s,p)}\} = \frac{K}{\pi} \tag{1}$$

where s is the distance of body points situated in the crack plane from the top of the crack; $\sigma_{y(s,p)}$ is breaking stress for a member with macroscopic crack of $2l_0$ length and K is modulus of bond. It is shown in [2] that stresses in body outside the crack $(x>l)$ due to any normal pressure acting in the banks of this crack may be expressed by formula, see Fig.1(a):

$$\sigma_{y(x,0)} = \frac{1}{\pi\sqrt{x^2-l^2}} \int_{-l}^{l} \frac{p_n(\xi)\sqrt{l^2-\xi^2}}{x-\xi}\, d\xi \tag{2}$$

Due to limitation of stress $\sigma_{y(x,0)}$ in the body for the case of $x\to l$ the following equation must be observed:

$$\lim_{x\to l} \int_{-l}^{l} \frac{p_n(\xi)\sqrt{l^2-\xi^2}}{x-\xi}\, d\xi = 0 \tag{3}$$

Boundary conditions for the function of the normal pressure $p_n(\xi)$ in the tensile zone of a reinforced concrete member, see Fig.1(a), may be written as follows:

$$p_n(\xi) = \left| \begin{array}{ll} p_0(\xi) & |\xi| \le c \\ 0 & |\xi| \le l_0 - c \\ -f_{ct} & l_0 < -x \le l \\ -\dfrac{\sigma_s \beta}{\alpha} & l_0 < x \le l \end{array} \right| \tag{4}$$

where $p_0(\xi)$ is arbitrary function of pressure applied on banks of the crack and estimated as a resultant of stresses acting in

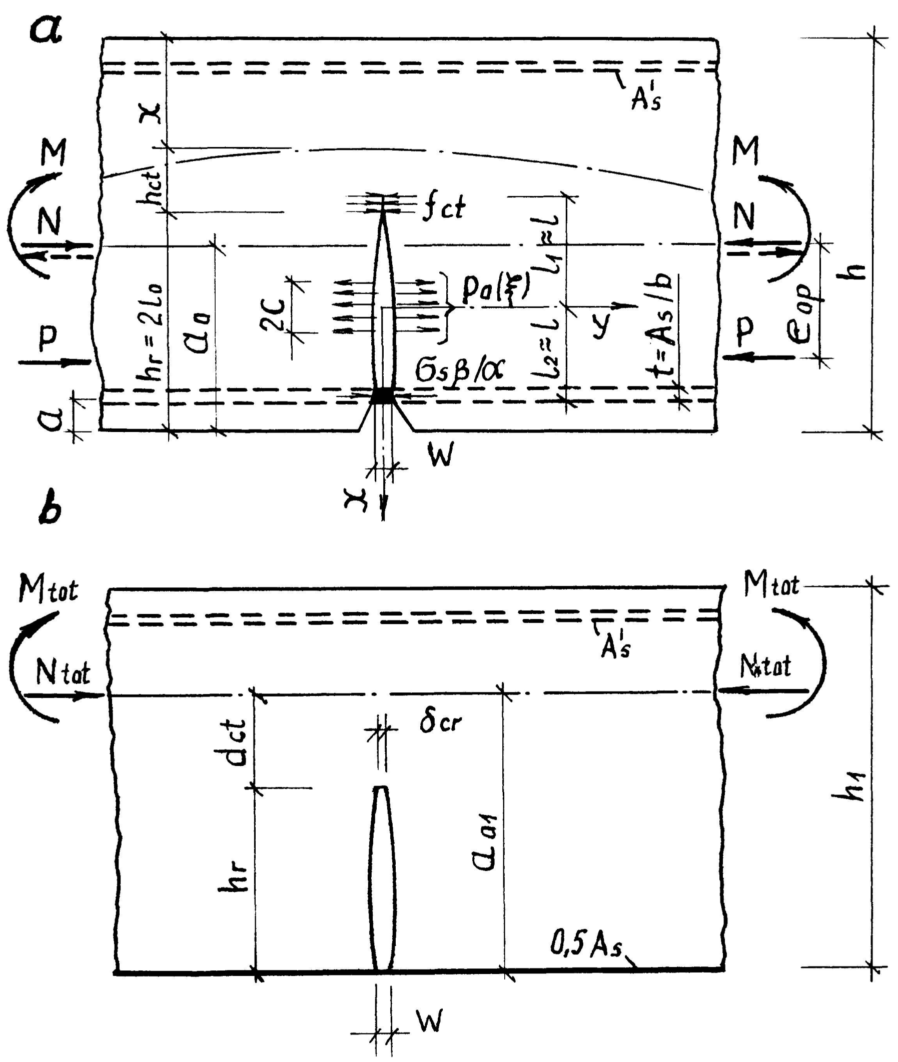

Fig. 1 Models of normal crack in tensile zone of reinforced concrete member: a - for evaluation of reinforcement stress intensity coefficient; b - for assesment of tensile reinforcement stress in crack

continuous concrete of the crack zone $|x| \le l$; f_{ct} is tensile strength of concrete; $\sigma_s\beta/\alpha$ is stress at the level of tensile reinforcement; $\alpha = E_s/E_c$; $\beta = 125\,\rho$ and is used when longitudinal reinforcement ratio $\rho = A_s/(bd)$ is small when $\rho \ge 0.008$, $\beta = 1$.

Using equation (4) and considering that $s = x - l_0$, $\sqrt{x^2 - l_0^2} = \sqrt{s}\,\sqrt{l_0 + s}$, equation (3) may be rearranged into the following form

$$\lim_{s\to 0}\{\sqrt{s}\;\sigma_{y(s,p)}\} \approx \frac{2\sigma_s\beta}{\alpha\pi}\sqrt{t} \qquad (5)$$

From equations (1) and (5) tensile reinforcement stress

$$\sigma_s = \frac{K\alpha}{2\beta\sqrt{t}} \qquad (6)$$

and critical steel stress intensity factor

$$K_{cr} = 1.6\ f_s \sqrt{t}\,\frac{\beta}{\alpha} \tag{7}$$

where f_s is strenght (yield limit) of reinforcement steel. Values greater than critical K_{cr} values leads to the fracture of not overreinforced concrete member with tensile steel.

By similar method as in [2] assuming bodies as isotripic expression of the modulus of bond K for reinforced concrete member has been deduced. A fictitious cross section of the depth h_1 was assumed, see Fig.1(b), the centroid of which coincides with the central axis of the member. The depth h_1 is obtained from the following expression:

$$h_r + h_{ct} = \frac{S_1}{A_1} \tag{8}$$

where S_1 is statical moment of ficticious cross - sectional area A_1 in respect to the centroid of longitudinal tensile reinforcement.

Modulus

$$K = \left(\frac{M_{tot} - \Delta M}{W_1} - \frac{N_{tot}}{A_1}\right) \sqrt{h_r}\ \frac{\pi}{2} \tag{9}$$

where $M_{tot} = M - Pe_{op}$ and is the total moment in relation to the centroid of the real cross section of the member, see Fig.1; P is longitudinal reinforcement prestressing force; e_{op} is the eccentricity of P; $\Delta M = N_{tot}\ (a_{01}\text{-}a_0\ )$ and is the increase in bending moment due to deviation of centroid of fictitious cross section from the centroid of the real cross section of the member; $N_{tot} = P \pm N$ and is the total longitudinal force acting in the centroid of the real cross section. Compressive N force is positive; $W_1 = I_1/(0.75h_r + h_{ct}\ )$ and is modulus of cross section ; I_1 is inertia moment of fictious cross section in relation to the neutral axis situated at distance $h_r + h_{ct}$ from the tensile face of fictitious cross section; $h_{ct} = \delta_{cr} h_r / w$ and is the depth of concrete tensile zone above the crack, see [1].

The critical width of the crack end

$$\delta_{cr} = 0.00012\ a\ \sqrt[3]{\phi}\ \eta\ \frac{3.5 - 100\ \rho_1}{3.5 - 100\ \rho_2} \tag{10}$$

where ϕ is bar diameter of tensile reinforcement in mm; η is factor for consideration of bond between concrete and reinforcing steel; $\rho_1 = A_s\ /\ [bh + (b_f - b)h_f\,] \leq 0.02$ and $\rho_2 = A_s\ /\ (bh) \leq 0.02$ and they are longitudinal reinforcing factors; b_f and h_f are width and the thickness of the flange in the tensile zone of the member.

Substitution of modulus K value by (9) into formule (6) gives the final expresion of tensile reinforcement stress value

$$\sigma_s = \left[\frac{(M_{tot} + \Delta M)(0.75h_r + h_{ct})}{I_1} - \frac{N_{tot}}{A_1}\right]\frac{\pi\alpha}{4\beta}\sqrt{\frac{h_r}{t}} \tag{11}$$

If the neutral axis is located extreme compressed concrete fiber stress σ_c may be calculated by the equation:

$$\sigma_c = \frac{M\,(h - h_r - h_{ct})}{I_{red}} \tag{12}$$

where I_{red} is inertia moment about neutral axis of transformed cross section by reducing its area by $A_{ct} = h_r b$.

For concrete coefficient

$$K_{cr} = \sqrt{\delta_{cr} E_c f_{ct}} \tag{13}$$

where E_c and f_{ct} are modulus at elasticity and tensile strength of concrete respectively.

3. RESULTS OF EXPERIMENTAL INVESTIGATION

Special test specimens, beams and eccentrically compressed reinforced concrete members, were investigated. Prestressed concrete beams of rectangular cross section $b \times h = 100 \times 180$ *mm*, span $l = 1800$ *mm*, longitudinal prestressing steel ratio $\rho = 0.8 - 0.95$ *%*, prestressing force $P = 41.2 - 132.5$ *kN* were subjected to two concentrated forces at 1/3 distances from supports. Electrical resistance gauges were used to measure steel strain in the cross section through crack. The parameters h_r and w of crack in pure bending zone of beams were measured by 24 times magnifying microscope and concroled by electrical resistance strain gauges closely spaced on beam face along its height.

Results of special tests on beams reported in [3] also are employed to evaluate theoretical proposotions of this paper. Reinforced concrete beams of rectangular crosis section $b \times h = 120 \times 300$ *mm*, span length $l = 2000$ *mm*, longitudinal tensile reinforcement ratio $\rho = 0.48 - 3.83\%$, without prestress were loaded by two concentrated forces at 1/3 distances from supports. Special notches were formed in the test beams to locate the main cracks. Steel strain in the main crack and parameters h_r and w of this crack were measured.

Eccentrically compressed members of rectantgular cross section $b \times h = 100 \times 150$ *mm*, length $l = 1000$ *mm*, tensile reinforcement rat io $\rho = 0.46 - 1.26\%$ were tested by the authors of this paper. Notches in the middle of the member length from tensile face up to longitudinal reinforcing bars deep were formed. Crack parameters were measured by a microscope magnifying 24 times and by a dial gauge at the level of tensile reinforcement.

The values of tensile reinforcement stresses obtained by tests $\sigma_{s,obs}$ and calculated by equation (11) σ_s are compared, see Fig.2. In the case of eccentrically compressed

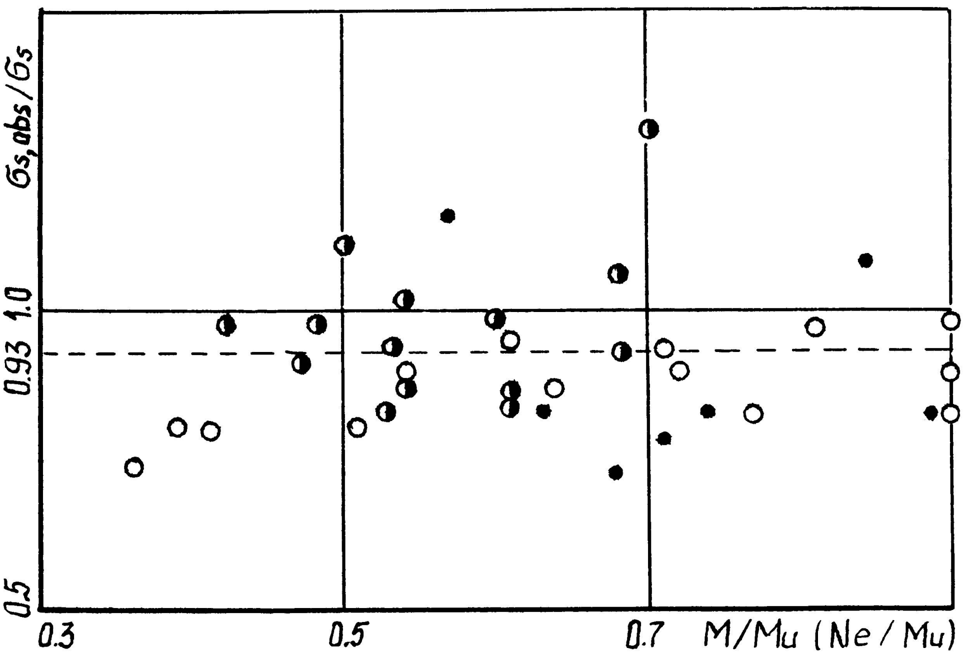

Fig.2 Comparison of measured in tests $\sigma_{s,obs}$ and theoretical σ_s calculated by formula (11) tensile reinforcement stress values: • - prestressed concrete beams; o - reinforced concrete beams; ◑ - eccentrically compressed members

members ratio $\sigma_{s,obs} / \sigma_s$ is plotted in respect to quantity Ne / M_u where N is external force acting at distance e from centroid of tensile reinforcement and M_u is the ultimate bending moment of the cross section.

Theoretical values σ_s calculated by (11) are on the average 7% higher than $\sigma_{s,obs}$ measured in tests. The variation coefficient of the ratio $\sigma_{s,obs} / \sigma_s$ is equal to 0.15.

Experimental values of concrete extreme fiber stress $\sigma_{c,obs}$ obtained from tests [3] were compared with theoretical values σ_c of this stress calculated by equation (12). Theoretical values σ_c on the average are 3% higher than experimental values $\sigma_{c,obs}$ of this stress. Coefficient of variation of the ratio $\sigma_{c,obs} / \sigma_c$ is equal to 0.11.

4.CONCLUSIONS

Contour of through normal crack in tension zone of a reinforced concrete member always is continguous to tensile concrete and to reinforcement. The latter has substantial influence on rupture strength of tensile concrete. Parameters of this strength K_{cr} and δ_{cr} expressed by (7), (10) and (13) define correctly character of stable crack propagation observed in tested samples.

Calculation of stresses in tensile reinforcement by (11) and in extreme fiber of compressed concrete by (12) makes it possible to assess stress state of reinforced concrete structures in service with sufficient accuracy.

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