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Upgrading Reliability Assessment of Degraded Structures
Meilleure évaluation de la fiabilité de structures endommagées
Neue Sicherheitsbewertung geschädigter Tragwerke

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SUMMARY
The paper sets forth the possibilities provided by the use of probabilistic methods in assessing the safety of existing structures, and in interpreting the results of tests made in situ. The methodology is based on coupling proven methods with the techniques of Bayesian Inference. The goals of this procedure are manifold: to optimize the inspection programmes on existing buildings; to interpret the results of these inspections; to carry out sensitivity analyses. One example of application is discussed in detail, concerning a reinforced concrete building, erected in 1916-1920, now subjected to a detail inspection programme in order to assess its actual reliability.

RÉSUMÉ
La contribution met en évidence les possibilités dues à l'emploi des méthodes probabilistes pour l'évaluation de la sécurité des construction existantes. La méthodologie proposée est basée sur la combinaison de certaines méthodes connues avec les principes de l'inférence Bayésienne. Les buts envisagés sont multiples: l'optimisation des programmes d'inspection des constructions; l'interprétation des résultats des essais expérimentaux; le développement des analyses de sensibilité. Les auteurs examinent successivement en détail un exemple d'application concernant un bâtiment réalisé en 1916-1920 soumis actuellement à une série approfondie de recherches afin d'évaluer son niveau réel de sécurité.

ZUSAMMENFASSUNG
In diesem Artikel wird erklärt, welche Möglichkeiten sich durch die Anwendung der Wahrscheinlichkeitsmethode beim Bewerten bestehender Tragwerke ergeben. In der Interpretation der Ergebnisse von In-Situ-Messungen werden die eingesetzten Verfahren mit der Technik Bayesscher Schlussfolgerungen gekoppelt. Folgende Ziele werden durch dieses Verfahren erreicht: Optimierung der Überprüfungsprogramme für Gebäude; Interpretation der experimentellen Prüfungsergebnisse; Durchführung der Sensibilitätsanalyse. Es wird dann die Anwendung an einem Stahlbetongebäude im Detail beschrieben, das in den Jahren 1916-1920 erbaut und jetzt einer Reihe von gründlichen Untersuchungen zur Bewertung des wirklichen Sicherheitsgrades unterworfen wurde.

1. INTRODUCTION

The reliability assessment of an existing structure and, eventually, the design of upgrading operations are processes demanding in-depth knowledge of the effective response of the structure under realistic action scenarios. Moreover, the structural behaviour has to be determined taking into account the level of deterioration of the structural elements.
In achieving this objective one must always deal with the considerable uncertainty that arises in defining both the action scenarios, and the structural model and the materials' mechanical properties, which are closely tied to construction quality and generally deteriorate in random fashion over time. Without considering the actions that actually involve the structure in its future operation, the other sources of uncertainty arise out of the variability in space and time of the geometric and mechanical characteristics of the structural elements, and out of the need to adopt an analytical model of their behaviour. They also arise when the available information on the basic variable is incomplete or not wholly significant.
In most cases then, to deepen understanding of the structural behaviour, it becomes necessary to work up information got from quality control, from proof testing, from experimental tests, and from periodic inspection or continuous monitoring, this information being all that is available when the original design is missing. By means of it structural safety can be assessed more reliably: in fact, the additional information, if accurate and consistent, provides substance to the models assumed for deriving the analytical reliability evaluations, which are made on the basis of estimates of the materials' mechanical properties and of their deterioration, and on the basis of forecasts of collapse mechanisms, which would otherwise be devoid of objective support.
A probabilistic approach to evaluating structural safety is made natural by the need to establish stochastic models for each of the several sources of uncertainty. Therefore, the possibilities provided by probabilistic methods in assessing the safety of existing buildings, and in particular in interpreting the results of tests made directly in situ, are set forth in the following, with special reference to an example case of notable importance.

2. SOME REMARKS ON THE RELIABILITY ASSESSMENT PROCEDURE

The methodology examined, proposed in [1], and applied in [2], [3], [4], [5], [6], [7], is based on coupling the FORM (or SORM) methods with the techniques of Bayesian Inference.
This theory appears the most useful approach for quantifying uncertainties in structural engineering problems, especially when coupled with decision analysis. According to Bayesian Inference in fact, and proceeding in a consistent and explicit manner, design can deal with information on events and propositions (qualitative information that is, or estimates based on expert judgment). The Bayesian approach also provides a satisfactory way of explicitly introducing assumptions about prior knowledge, the relevant experience being quantified by the prior distributions. Moreover it does not break down where large amounts of data are absent, it providing a mechanism for using experience, intuition and judgment productively and in a scientifically responsible fashion. Finally, it is also compatible with first-order reliability methods, and is therefore suited to the problem's numerical treatment within a unitary formal context.
The method's goals are manifold: to optimize inspection programmes for existing buildings, with a view to more efficient repair or upgrading interventions, and according to the more likely deterioration factors; to correctly interpret the results of these inspections; and to carry on parametric sensitivity analyses. As concerns the methods for using Bayesian Inference, a "direct use" may be distinguished, aimed at updating the probability density functions of the basic random variables, as may be an "extended" use, coming out of the coupling of the criteria lying at its base with the techniques of reliability analysis of structural systems peculiar to the advanced first-order second-moment methods.

2.1 "Direct" use

Bayesian Inference can be directly applied to problems involving parameter estimation, that is, problems in which additional information are available about the parameters of the probability

density functions of the basic r.v.s, these parameters being considered as r.v.s having a prior distribution that expresses the designer's prior belief in (or knowledge of) their values. The method makes it possible to derive updated pdf's on the basis of all kind of additional information, as, e.g., those derived by experimental tests.
The prior density can be fitted empirically to observations in past experience: occasionally, subjective assignment has to be made. Depending on the probability density functions assumed for the "a priori" and "a posteriori" models, the problem's solution can be carried out in closed form or numerically. In the example case, the updating of the pdf's of the material properties has been carried out assuming that both the mean value and the variance are unknown.
Therefore, denoting by X a material property, by μ the mean value of X and by σ^2 its variance, the joint prior density of μ and σ^2 is expressed as the product of a conditional log-normal density LN ($[\mu],\tau\sigma^2$) and an inverted gamma density IG (α,β). The calibration of the parameters of the posterior pdf of X, conditional on the results of the experimental tests (represented by a vector $\underline{r}$ of actual observations upon X) can be performed by means of the updating procedure illustrated in [6] or [8].

2.2 "Extended" use

The Bayesian approach can also be useful to deal with the results of experimental tests furnishing more general information on structural behaviour, then calling into play a number of stochastic parameters. In fact those results which form one or more conditions on the vector of the basic r.v.s $\underline{X}$, may be interpreted by defining "artificial" events corresponding to functional relationships between the X_i. Such relations derive from the analytical model of structural response that is utilized to interpret the particular kind of test performed. The "artificial" events are expressed in the form:

(a) $H_r(\underline{y}) \leq 0$ $\qquad r = 1,2, \ldots n$

(b) $H_s(\underline{y}) = 0$ $\qquad s = 1,2, \ldots. m$

where $\underline{Y}$ is the vector of the basic r.v.s $\underline{X}$ plus others variables. These others are called into play by the particular type of test, or directly included in the analytical model to explicitly characterize the uncertainty attributed to the experimental results and to the analytical model itself.
Examples of type (a) artificial events are represented by proof loading results, where it has been ascertained that the strength of the structure is larger than the applied load; examples of type (b) artificial events are represented by direct measurements of derived quantities, that must be expressed by means of an analytical model as a function of the basic r.v.s. It is obvious that more complex experimental tests may provide information which can be represented with several events of type (a) and (b).
When put in this form, the additional information can be applied directly to the updating of the failure probabilities estimated a priori. In fact, updating the estimate of the structural reliability with respect to a given limit state by means of additional information is fairly simple if the analysis is performed by means of the advanced First-Order Second-Moment methods. The updating procedure requires the evaluation of the conditional probability expressed by the relation:

$$P_f = P\{H \leq 0 | H_1 \leq 0 \cap \ldots\ldots \cap H_n \leq 0 \cap H_{n+1} = 0 \cap \ldots\ldots \cap H_{n+m} = 0\}$$

where: $\{H(\underline{x})\}$ is the limit state function corresponding to the limit state considered, and the experimental tests furnish data that can be interpreted by means of n type (a) conditions and m type (b) conditions. The methods for evaluating the conditional probability are set forth in [2] and [9].

3. AN EXAMPLE CASE

The basic concepts outlined in the preceding paragraphs have been applied to the reliability assessment of an important R.C. building in Turin, designed by Italian engineer Giacomo Mattè Trucco to serve as an industrial plant for the production of cars and industrial vehicles, and now re-analyzed in view of a change in its usage assignment. The main body of the complex, built between 1916 and 1920, consists of two parallel identical 5 storey buildings, connected to each

other by means of transversal elements located every 120 m; the total length of the complex is about 556 m. At the ends of the main body are two ramps, built in 1925-1926 and of helical form, which allow vehicles to reach the flat roof, where there is a test track with banked curves.
In order to assess the structure reliability, an extensive campaign of investigation was planned and developed, this comprising, besides the search for and the analysis of the original drawings, and the survey made of the effective shape and dimensions of the structural elements:

- compression tests on concrete samples, which were cored from the main columns (a total number of 49 samples were tested);
- the measurement of ultrasonic pulse velocity in the column cores. These tests were performed on 5638 columns, the final result of each test being taken equal to the mean of two measurements, made near the bottom and near the top of each column;
- the measurement of the rebound Schmidt hammer index (in 3082 different positions);
- the measurement of the electrical potential in order to evaluate any corrosion of the reinforcement (a total number of 238 tests were developed);
- the evaluation of the depth of carbonation, by means of phenolphthalein tests;
- tensile tests on reinforcing bars cut off from the structure (22 specimens);
- compression tests on entire columns, cut out of the structure where some demolition was required by the architectural restructuring design (3 tests);
- load tests on beams and decks.

The preliminary structural analysis, made using the results of a first series of tests, led to the conclusion that the horizontal elements (beams and decks) should have a satisfactory bearing capacity, therefore requiring only the repair of local damage; but all columns located on the first and second levels seemed to be critical, and some of those on the third level too.
A more refined analysis was then performed in order to evaluate the failure probability of the columns. Taking into account the possibility that different contractors worked at the same time in different parts of the structure, the safety check was performed independently for each building portion delimited by two adjacent construction joints.
The main steps of the analysis carried out in order to verify the need for any upgrading were:

- the evaluation of the mechanical properties of the materials (i.e., of the probability density functions of the concrete compression strength and of the steel tensile strength);
- the safety check of the columns;
- the updating of the failure probabilities derived in the previous step, and according to the results of the direct compression test made on a column sample.

The main results of the analysis are summarized in what follows, with special reference to two different zones: the northern ramp and a zone of the main building, called Zone 1. The northern ramp was erected in 1925. It is helical in form, and is supported by columns located along the internal and external ramp perimeter, and the structure of the main building is very regular, and consists of span equal to 6 m in both directions.

3.1 Evaluation of the probability density functions of the strengths of concrete and steel

3.1.1 Concrete

A preliminary sensitivity analysis has shown that concrete compression strength is of major importance to the safety check of the columns. Therefore, to obtain the most accurate evaluation of the pdf of this variable, it is mandatory that proper account be taken of a number of available information items.

Assumption of the prior probability density function

The evaluation of the parameters of the prior pdf of the concrete strength is very difficult because no indications were found in the original design documents, nor were Italian Standards for reinforced concrete buildings available at the time of the construction. Therefore, the prior pdf must be derived only on the basis of the experimental results got from similar buildings of the same age. In this case, the measurements made on an industrial building erected in the same years in Venice were available [7], giving a mean value of the strength of 16 MPa. For safety, a mean value of 15 Mpa and a coefficient of variation of 0.5 were assumed. The prior pdf is thus (in Mpa):

$$f_c = LN(15;56.25)$$

Updating on the basis of core tests

Compression strength tests were performed on 49 cores, taken from the buildings at different levels. From the tests results, a mean value of 15.73 Mpa was obtained, together with a standard deviation of 5.82 MPa, corresponding to a coefficient of variation (c.o.v.) of 37%. The large value of the c.o.v. corroborates the assumption of poor homogeneity of concrete in different zones. Applying the updating procedure illustrated in Sec. 2.1, the posterior density function is:

$$f_c' = LN(15.72;35.58)$$

Correlation between the ultrasonic pulse velocity and concrete strength

From the 5638 ultrasonic pulse velocity tests, the 49 values obtained on the columns from which the cores were taken are considered. In Fig. 1 the measurements on cores of pulse velocity and of the compression strength are compared: the scatter appears to be quite large. Nevertheless, the data was fit to a correlation function between the ultrasonic pulse velocity and the concrete compression strength in the form:

$$f_c = c_1 \cdot \exp(c_2 \cdot V) \cdot \varepsilon$$

where: V is the ultrasonic pulse velocity; c_1 and c_2 are constants to be determined in order to fit experimental data; ε is a r.v. measuring the model uncertainty associated with the form of the correlation curve and with the scatter around the mean of the results obtained by the correlation.
The values of c_1 and c_2 corresponding to the best fit of experimental data, derived by means of a non-linear regression procedure, are equal to: 2.6792 and 0.0005, respectively. The above relationship is represented by the solid line in Fig. 1. The standard error of the correlation is about 0.29, so that the pdf of the r.v. ε can be taken equal to:

$$\varepsilon = LN\ (1;0.45)$$

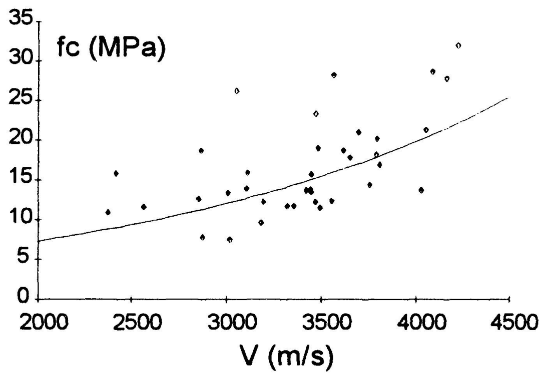

Fig. 1 Correlation between V and f_c

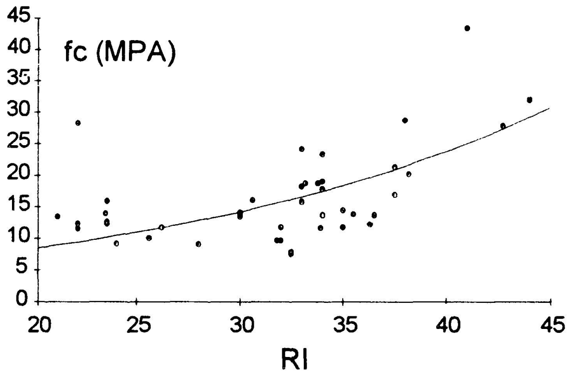

Fig. 2 Correlation between RI and f_c

Correlation between the rebound Schmidt hammer index and the concrete compression strength

The rebound Schmidt hammer test derives the concrete compression strength from the amount of rebound at the surface of the structural element. Many calibration tests are available to validate the results of this non destructive technique; however its considerable uncertainty owes mainly to the need to establish a relation between the Young's modulus of elasticity (conditioning the amount of rebound) and the concrete compression strength. Unfortunately, in the case of very old concretes, another source of uncertainty must be considered: in fact, the test refers exclusively to the surface of the structural element, where carbonation produces a local increase in strength. Consequently, the use of the correlation diagram accompanying the instrument would produce a serious overestimation of the strength. Therefore, the original correlation diagram was disregarded, and a new one was sought directly using the results on cores, as was done in the case of the ultrasonic pulse velocity tests.

Considering the rebound Schmidt hammer results on the same columns from which the cores were taken, the results of Fig. 2 are obtained. The same figure shows the best fit obtained with the relationship:

$$f_c = c_3 \cdot \exp(c_4 \cdot N) \cdot \varepsilon$$

where: N is the rebound Schmidt hammer index; c_3 and c_4 are constant coefficients, whose values, derived by means of a non-linear regression procedure, are equal to: 3.023 and 0.0516, respectively. The above relationship is represented by the solid line in Fig. 2. The standard error of the correlation is about 0.29, so that the pdf of the r.v. ε can be taken as:

$$\varepsilon = LN\,(1;0.45)$$

Correlation between the pdf of the ultrasonic pulse velocity and the rebound Schmidt hammer index and the pdf of the concrete strength

Considering the local values of the ultrasonic pulse velocity, the parameters of the concrete compression strength pdf are deduced as follows.
For the northern ramp, 38 measurements are avaliable, giving:

$$E[V] = 3872 m/s \qquad Var[V] = 44100 (m/s)^2$$

therefore, applying the procedure outlined in Sec. 2.1:

$$E[f_c] = 20.49 MPa \qquad Var[f_c] = 90.56 (Mpa)^2$$

For the 1st level of Zone 1, 176 measurements are available, giving analogously:

$$E[V] = 3507 m/s \qquad Var[V] = 71829 (m/s)^2$$

$$E[f_c] = 18.22 MPa \qquad Var[f_c] = 127.7 (MPa)^2$$

Considering the local values of the rebound Schmidt hammer index, the parameters of the concrete compression strength pdf are derived as follows.
For the northern ramp, 38 measurements are avaliable, giving:

$$E[RI] = 37.45 \qquad Var[RI] = 28.51$$

$$E[f_c] = 23.78 MPa \qquad Var[f_c] = 168.57 (Mpa)^2$$

For the 1st level of Zone 1, 38 measurements are available, giving:

$$E[RI] = 29.04 \qquad Var[RI] = 24.38 (m/s)^2$$

$$E[f_c] = 15.49 MPa \qquad Var[f_c] = 68.23 (MPa)^2$$

Combination of the results derived from ultrasonic pulse velocity and rebound hammer tests

The two resulting densities are then combined, a weighting being attributed to each of them, whose value is subjectively set on the basis of the degree of confidence given to the various tests. In this case, the parameter λ, representing the relative weight attributed to the first test method [(1 - λ) being the weigthing factor for the second one], is assumed equal to 0.6, in order to take into account the greater uncertainty associated to the rebound Schmidt hammer test, due to the effects of carbonation.
Consequently, the posterior pdf of the concrete compression strength is:

$$f_c = LN(21.11; 48.34)$$

for the northern ramp, and:

$$f_c = LN(16.56; 32.67)$$

for the 1st floor of Zone 1.

3.2 Steel

Assumption of the prior pdf

As already discussed for the concrete compression strength, no useful indications can be derived from the design documents or the Italian Standards. Therefore, and referring to the same industrial building in Venice, the following prior pdf is assumed for the steel tensile strength:

$$f_y = LN(320;6400)$$

Updating with results of tensile tests

Visual inspection of the reinforcing bars in the structural elements brought out the existence of three kinds of rebars: round section rebars; elliptical folded section rebars; small rectangular section rebars. To evaluate the mechanical properties of the reinforcing steel in the columns, 22 test were available, giving a mean strength value equal to 352.6 MPa and a standard deviation of 56.41 MPa. Applying the updating procedure, the posterior pdf is then:

$$f_y' = LN(347;19710)$$

3.3 Local verification of columns

In the reliability asessment of the main columns, the basic random variables are: the concrete compressive strength (f_c); the reinforcement yield strength (f_y); the cover thickness (c); the unintentional eccentricity of the live load (e); the section heigth (h) and width (b); the intensity of the permanent load (G); and the intensity of the live load (Q). The main characteristics of the input variables, evaluated for the most loaded columns, are reported in Table 1.

Table 1 *Parameters of the basic variables*

	TYPE	Northern ramp	Zone 1 1st level	Zone 1 2nd level
Cover thickness (mm)	*N*	*50 / 0.40*	*50 / 0.40*	*50 / 0.40*
Eccentricity (mm)	*N*	*80 / 0.50*	*50 / 0.50*	*50 / 0.50*
Section heigth (mm)	*N*	*600 / 0.10*	*600 / 0.10*	*600 / 0.10*
Section width (mm)	*N*	*800 / 0.10*	*600 / 0.10*	*600 / 0.10*
Permanent load (kN)	*LN*	*1367 / 0.05*	*1928 / 0.05*	*1594 / 0.05*
Variable load (kN)	*LN*	*690 / 0.15*	*810 / 0.15*	*594 / 0.15*

The amount of reinforcement in the columns section is equal to 3768 mm^2 in the northern ramp, and 1848 mm^2 in the Zone 1.
The limit state function is derived considering the ultimate limit state of the base section of the columns subjected to bending and compression.
Assuming for the materials the prior pdf's, the values of the safety index β and of the probability of failure P_f reported in Table 2 are obtained. If, instead, the posterior pdf's corresponding to updating according to core tests for concrete and tensile tests for steel are used, the values β' and P_f' are obtained.
A more precise evaluation can be performed using for concrete the local pdf resulting from the combination of ultrasonic pulse velocity and rebound hammer tests with core tests: the corresponding values of the safety index β'' and of the probability of failure P_f'' are reported in the same Table 2. This level of safety is satisfactory for the northern ramp, while it is doubtful for the second level of Zone 1, and insufficient for the first level of the same Zone. A direct updating of P_f on the basis of the results of destructive tests was then decided. Taking advantage of the necessity of demolishing one span to erect a staircase, three full-scale samples of column were tested until collapse: the ultimate resistance for the columns of interest resulted equal to 6768 kN. Characterizing this result as a normal r.v. (with a c.o.v. equal to 0.20, to account for measurements uncertainty, and, mostly, for differences between the tested specimen and the other columns), the direct updating procedure of Sec. 2.2 has been applied.

The corresponding "artificial" event (i.e., the comparison between the theoretical ultimate N_u and the measured N_{proof} normal force) forms a type (a) condition on the entire set of the vector of the r.v.s. The results of this updating are reported in the last column of Table 2.
The reliability of second level of Zone 1 resulted completely satisfactory, while it was decided to upgrade the most loaded columns of the first level of Zone 1.

Tabel 2 *Values of the safety index β and of the probability of failure P_f for the various updating*

	β / P_f	β' / P_f'	β'' / P_f''	β''' / P_f'''
northern ramp	$2.33 / .99 \cdot 10^{-2}$	$3.31 / .47 \cdot 10^{-3}$	$4.92 / .43 \cdot 10^{-6}$	$5.72 / .54 \cdot 10^{-8}$
zone 1 - 1st level	$1.67 / .54 \cdot 10^{-1}$	$2.37 / .88 \cdot 10^{-2}$	$2.97 / .15 \cdot 10^{-2}$	$3.90 / .48 \cdot 10^{-4}$
zone 1 - 2nd level	$2.17 / .15 \cdot 10^{-1}$	$3.10 / .95 \cdot 10^{-3}$	$4.29 / .91 \cdot 10^{-5}$	$4.77 / .94 \cdot 10^{-6}$

4. CONCLUSIONS

Bayesian Inference has proved to be a powerful procedure for improving knowledge of the properties of materials in existing structures, and of the bearing capacity of the structural system. The coupling of Bayesian Inference with FORM or SORM methods provides a very straigthforward process for directly updating the failure probability of a structure, taking advantage of load tests. The cost of in situ testing a structure and the increased complexity of the calculations are usually more than compensated for by the saving made possible by the improved knowledge of the capability of the structure. In the example case presented here, a very accurate in situ investigation has been performed, producing a large amount of data. Owing to the statistical processing within the rational framework of Bayesian Inference, and using the FORM method, the amount of strengthening required has been notably reduced.

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