Zeitschrift: IABSE reports = Rapports AIPC = IVBH Berichte

Band: 66 (1992)

Artikel: Modèles statistiques pour l'analyse en fatigue d'éléments de grande longueur

Autor: Castillo, Enrique / Fernandez Canteli, Alfonso

DOI: https://doi.org/10.5169/seals-50684

Modèles statistiques pour l'analyse en fatigue d'éléments de grande longueur

Statistische Modelle für die Ermüdungsanalyse von langen Elementen

Enrique CASTILLO
Prof. Dr.
University of Cantabria
Santander, Spain

Alfonso FERNANDEZ CANTELI
Prof. Dr.
University of Oviedo
Gijón, Spain

Enrique Castillo, born 1946, obtained his Civil Engineering degree at the Polytechnical University Madrid, his mathematical degree from the Complutensis University Madrid and his Ph. D. from the Northwestern University. He is a specialist in Applied Statistics, Numerical Methods, Functional Equations and Programming.

Alfonso Fernández Canteli, born 1945, obtained his Mechanical Engineering degree at the University of Basque Country and his Ph. D. from the Polytechnical University Madrid. He has followed postgraduate studies at the ETH Zurich. He has been working on fatigue problems since 1979.

SUMMARY

From the mechanical, statistical, modelling, design and testing aspects of fatigue a Weibull model for the product $(N-N_0)(\Delta\sigma-\Delta\sigma_0)$, with scale and shape parameters depending on length is suggested. The problem of extrapolation of fatigue strengths from lab specimens to actual structures is then analyzed. The design value is shown to be close to the endurance limit if the latter exists, and can be approximately calculated by means of the zero-percentile curve for long lengths. Finally, the implications of all the above in testing strategies is discussed.

RÉSUMÉ

On propose un modèle de Weibull pour le produit $(N-N_0)(\Delta\sigma-\Delta\sigma_0)$, comprenant un paramètre d'échelle et un paramètre de forme dépendant tous deux de la longueur. Ce modèle prend en compte les aspects mécanique, statistique et expérimental de la fatigue, ainsi que ceux relatifs à la modélisation, au calcul et aux essais. On analyse le problème de l'extrapolation à des structures réelles de résultats de résistance à la fatigue d'essais de laboratoire. On démontre que la valeur de calcul est proche de la limite d'endurance quand celle-ci existe et peut être approchée au moyen de la courbe correspondant au fractile zéro pour les grandes longueurs. Enfin, on discute les conséquences de tout ce qui précède sur les stratégies d'essais.

ZUSAMMENFASSUNG

In Hinsicht auf mechanische, statistische und experimentelle Untersuchungen, sowie auf Aspekte der Ermüdung in Beziehung auf das Modellieren, Bemessen und Prüfen wird ein Weibull-Modell für das Produkt $(N-N_0)(\Delta\sigma-\Delta\sigma_0)$ vorgeschlagen, das jeweils die von der Länge abhängigen Maßstabparameter und Formparameter einschliesst. Die Problematik der Extrapolation von den Ermüdungsfestigkeiten der Prüfkörper bis zu denjenigen der reellen Strukturen wird analysiert. Der Bemessungswert, soweit vorhanden, liegt der Ermüdungsgrenze nahe und kann mit Hilfe der Nullquantilkurve für grosse Längen berechnet werden. Schließlich werden die Folgen des Vorhergehenden auf die Prüfungsstrategie diskutiert.

1. INTRODUCTION

The problem of fatigue life of reinforcing bars, prestressing wires and strands has called the attention of researchers for many years.
According to the ASTM [1],"*fatigue is the process of progressive localized permanent structural change occurring in a material subjected to conditions which produce fluctuating stresses and strains at some point or points and which may culminate in cracks or complete fracture after a sufficient number of fluctuations*".
Even though a lot of effort has been made in the past in order to understand this process and the factors influencing it, some important aspects remain to be clarified. For instance, the random distribution of flaws or cracks, which plays an important role in fatigue and seems to be determinant in the statistical properties of fatigue, is not well known.
The experience gained over this period of time allows a researcher to have some feeling for the main qualitative features of the statistical behaviour of the fatigue strength (endurance limit, scatter, shape of the $S-N$ curves, size effect, independence of the fatigue lifetimes of different pieces, etc.). However, a precise statistical description of the fatigue phenomena is not available and the problem of model selection and design criteria has not been definitely solved.
Design engineers also need to extrapolate from small laboratory specimens to the actual length of structures such as cable-stayed or suspended bridges. In order for this extrapolation to be made with reasonable reliability, extra knowledge is required. Thus, we can conclude that further work, research and discussion is needed in order to achieve an adequate design of reliable and economic prestressed structures, bridges and similar structures.
As primary aims for the statistical analysis of fatigue data, the ASTM proposes:

- to estimate certain fatigue properties of a material or a component (together with measures of the reliability) from a given set of data.
- to give objective procedures for comparing two or more sets of fatigue data
- to provide information on the most efficient use of a limited number of test specimens and on the number of test specimens required to give a specific degree of confidence in the test results.

However, these objectives, which are mainly stated by statistical experts seem to correspond to the task of the testing engineer, but do not exactly coincide with those of practitioners. An engineer is generally faced with the problem of obtaining *design values*.
Though several aspects of fatigue could be considered for discussion, we shall concentrate primarily on the problem of the determination of design values and their main associated aspects (testing strategies and so on). We also assume that we start from laboratory tests and that we wish to extrapolate these results to actual structures. Thus, the problem of *extrapolation* will be one of our main concerns.
In this paper, more than stating particular problems and trying to give solutions to them, our intention is to present an overview of the existing open questions in order to motivate discussion during the workshop and promote a minimal consensus about some existing controversial problems.
In sections 2. to 6. we shall analyze single elements, that is wires, and we shall postpone the study of composed elements, such as cables or strands, till section 9.. However, it is worthwhile mentioning that the latter can always be considered as single elements if the analysis is based on adequate testing data.

Model	Reference	$S-N$ curves
1	[17]	$\ln(N) = A + B\Delta\sigma$
2	[17]	$\ln(N) = A + B\ln(\Delta\sigma)$
3	[15]	$\ln\left(\frac{N}{N_0}\right) = A\ln\left(\frac{\Delta\sigma}{\Delta\sigma_0}\right) + B\left\{\ln\left(\frac{\Delta\sigma}{\Delta\sigma_0}\right) + \frac{1}{\alpha}\ln\left[1+\left(\frac{\Delta\sigma}{\Delta\sigma_0}\right)^{-2\alpha}\right]\right\}$
4	[2]	$(N-N_0)(\Delta\sigma - \Delta\sigma_0) = Aexp\left[-C(\Delta\sigma - \Delta\sigma_0)\right]$
5	[5]	$\ln\left(\frac{N}{N_0}\right)\ln\left(\frac{\Delta\sigma}{\Delta\sigma_0}\right) = A$

Table 1: Different models for the $S-N$ curves.

In the following, we shall analyze different aspects of fatigue including the problem of fatigue modelling.

It shoulkd be noted that in the present terminology N refers to the logarithm of the number of cycles to failure, while $\Delta\sigma$ can represent either the stress range or its logarithm.

2. MECHANICAL ASPECTS OF FATIGUE

As mechanical aspects of fatigue we understand those directly influencing the mechanical fatigue resistance of the piece. Two main factors have been considered in the past as being determinant in design:

- stress level and
- size effect.

2.1 Stress level

It is well known from testing that the higher the stress level, $\Delta\sigma$, the lower the fatigue lifetime, N. Figure 1 shows a typical lifetime-stress level curve for a given piece. Different elements, due to their random selection, show different behaviour (see Figure 2). Knowledge of the precise shape and properties of these curves might be important for design values. Several models have been proposed in the past for these curves. Some of them are shown in Table 1.

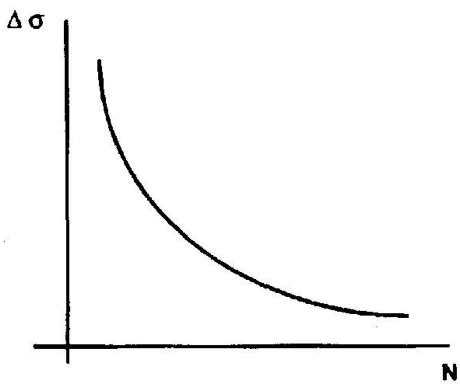

Figure 1: Lifetime-stress level failure curve for a given piece

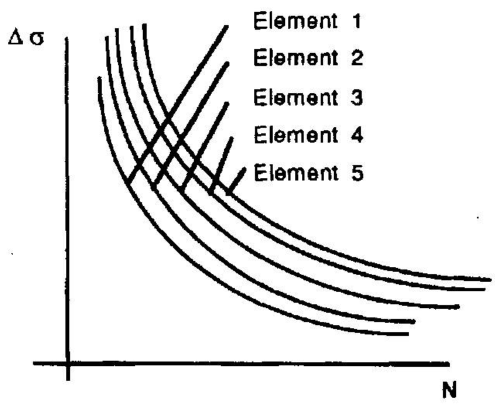

Figure 2: Lifetime-stress level failure curves for different elements or percentile curves.

Models 1 and 2 lead to families of parallel straight lines when using $ln(N)$ and either $\Delta\sigma$ or $ln(\Delta\sigma)$, respectively. Model 5 and some special cases of models 3 and 4 lead to families of hyperbolas on a logarithmic scale (see figure 3). Other variants of models 3 and 4 lead to more complex families.

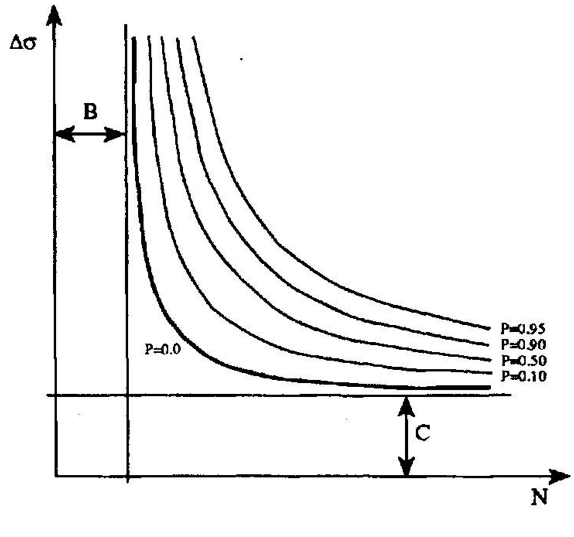

Figure 3: Whöler curves (percentiles)

The shape of the $S-N$ curves depends markedly on the material and testing conditions. Thus, models for a given material and testing condition cannot be used for other materials and/or conditions.

The lifetime-stress level curves can be non-intersecting (monotonic) as in figure 2 or they can intersect as shown in figure 4. Those defending non-monotonicity explain that different types of flaws or cracks can have different associated failure rates. Others argue that, because failure is governed by the stress intensity factor, no intersection occurs. Nevertheless, even in the first case an equivalent non-intersecting family (the percentile family) exists that leads to the same statistical distribution of N for given $\Delta\sigma$ or $\Delta\sigma$ given N. Thus, we can always think of a (fictitious) set of non-intersecting curves associated with elements of increasing strength. This is an interesting physical interpretation of percentile curves.

Perhaps the most commonly accepted and relevant property of these curves is the existence

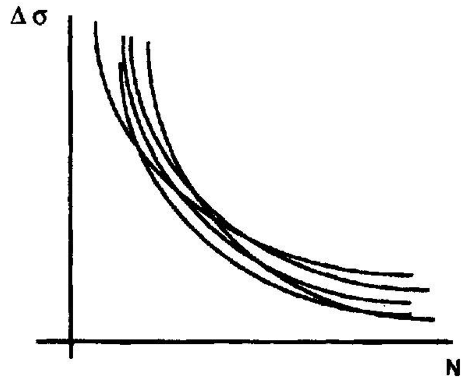

Figure 4: Intersecting lifetime-stress level failure curves

of an *endurance limit* (a stress level below which fatigue failure does not occur). A discussion on this existence remains to be clarified. Some important questions that arise are: does an endurance limit exist?, what are the practical implications if we make an erroneous assumption about this existence?

2.2 Size effect

Size effect, that is, the influence of length on the fatigue life of longitudinal elements, is another factor to be considered. Due to the fact that one element of length ks can be considered as divided into k small pieces of length s (see Figure 5), and that any of these pieces is subjected to the same fluctuating stresses as the whole piece, then the *weakest link principle* states that the strength of the element is that of the weakest piece, i.e.

$$N = min(N_1, N_2, \ldots, N_k) \tag{1}$$

This extreme property of N will be the key for its statistical behaviour.

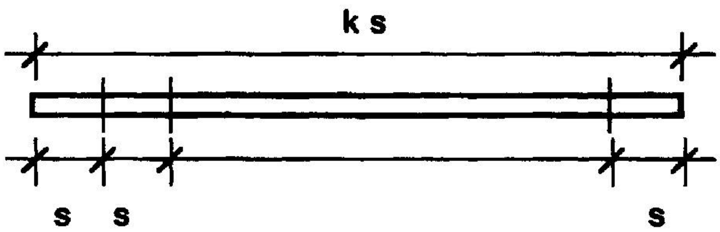

Figure 5: Longitudinal element and constituting pieces

An important aspect to be considered is the source of flaws or cracks and their spatial (longitudinal) occurrence. Among other questions, the following need to be answered: does the existence of a flaw or the occurrence of a crack in a given piece influence the occurrences in other pieces or do they behave independently? This mechanical behaviour, which is related to the fabrication process, plays an important role in the fatigue strength.

3. STATISTICAL ASPECTS OF FATIGUE

Fatigue life shows a markedly random character which implies that statistical analysis cannot be obviated. In real structures many factors influence fatigue life, such as stress levels, tem-

peratures, corrosion, quality of the elements, etc. In this study, we shall consider only stress level and the quality of the elements. In addition the random variation of stress levels is not considered. On the contrary, we assume a constant deterministic stress level. In other words, what is studied is what happens to an element under constant stress fatigue, which is previous to the problem of random stresses.
In the following paragraphs we shall illustrate some statistical aspects that can definitely help in the modelling of fatigue problems.

3.1 Stress level

Once the element (wire) is given, that is, selected at random from a given population (the factory production), the flaws or cracks are given. That is, we can assume that its lifetime is deterministic, equal to the strength of its weakest flaw and follows a law similar to that shown in figure 1, as is indicated by Fracture Mechanics Theory. This implies that every element has a unique and fixed associated failure curve $S-N$.
Consequently, we can state that the random character of lifetime comes from the fact that in longitudinal elements chosen at random from a given population (factory), some elements have larger cracks or weaker flaws than others (see figure 2).
The statistical influence of the stress level has been analyzed by means of the well known Whöler field, that is, the isopercentile lines associated with stress level and lifetime. Several models have been accepted for a log linear scale of lifetimes: some are linear (a family of straight lines) and some, non-linear.
In the Whöler field, we can think of two different random variables:

1. The lifetime N for a given stress range, $\Delta\sigma$.

2. The stress range $\Delta\sigma$ leading to a given lifetime, N.

Figure 6 shows one example of the probability density functions of these two random variables.

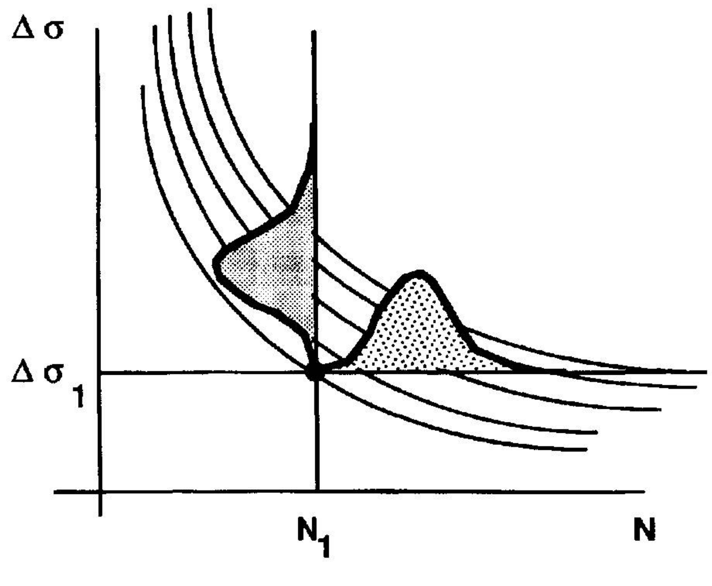

Figure 6: Two random variables: N for given $\Delta\sigma$ and $\Delta\sigma$ for given N

They lead to what has been called the *compatibility condition* (see reference [5]) that states that their survival functions (the survival function (s.f.), $S(N)$, is a function that gives the probability of surviving a period of duration N) must coincide, i.e.

$$S_1(N,\Delta\sigma,x) = S_2(\Delta\sigma,N,x) \tag{2}$$

where $S_1(N,\Delta\sigma,x)$ is the s.f. of N given $\Delta\sigma$ and the length x, and $S_2(\Delta\sigma,N,x)$ is the s.f. of $\Delta\sigma$ given N and the length x. This condition will be assumed in the following by using a single function $S(N,\Delta\sigma,x)$.

3.2 Size effect

From a statistical point of view, the analysis of the size effect can be made equivalent to the analysis of the influence of length on the reliabiliy function. Several models have been given in the past to solve this problem (see reference [6]). Unfortunately, most of them are based on the assumption of independence of the lifetime of non-overlapping pieces.
If independence holds, and based on the weakest link principle, we state that the s.f. of the lifetime of one element of length x is given by

$$S(N,\Delta\sigma,x) = S(N,\Delta\sigma,x_0)^{x/x_0} \tag{3}$$

This means that the graph of the survival function moves to the left, becomes steeper with increasing length x (see figure 7) and degenerates to a step function for infinite length.

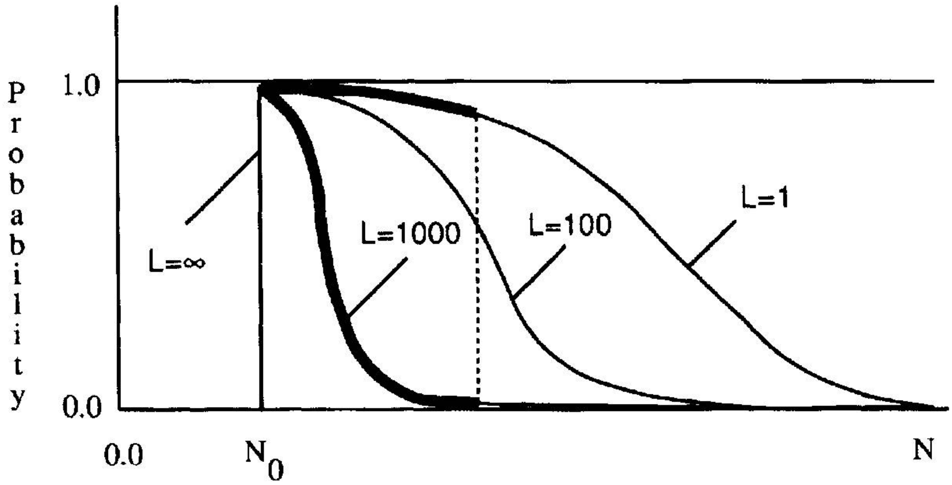

Figure 7: Survival function for different lengths 1, 100, 1000 and ∞.

Figure 8 shows the Whöler field for two different lengths. Note that the 1 probability of survival curve (or zero-percentile) coincides.
Experience shows that for small lengths the independence assumption can be inadequate and, when used for extrapolation purposes, can lead to either unsafe or very costly designs. This important problem has not yet been solved. Nevertheless we can state that the problem of dependence is crucial and the most critical factor to be considered in the analysis of size effect. In addition to the above problems there are other statistical concepts that need some clarification because of their influence on design. One important fact to be taken into consideration is that only the tail behaviour influences the survival function for design purposes. Thus, mean values are not adequate for fatigue analysis. Initially, and only with the purpose of illustrating this property, we shall assume that the hypothesis of independence of the lifetimes of neighbouring pieces holds. According to expression (3), the value of the s.f. for a given N and length

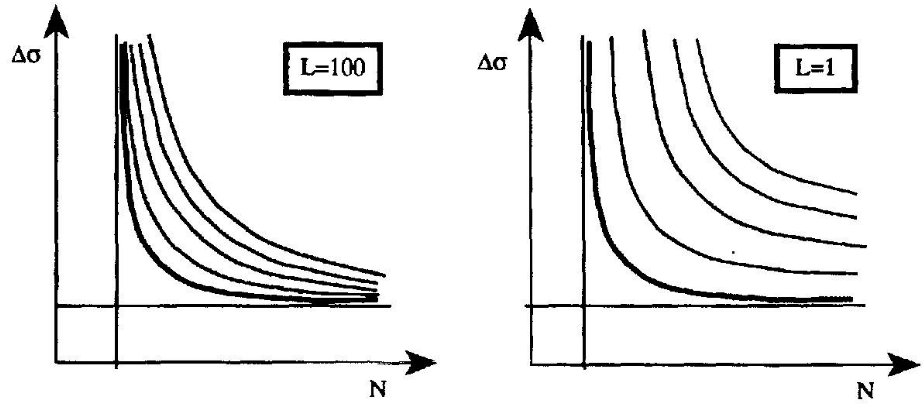

Figure 8: Whöler fields for two different lengths.

x coincides with the value of the s.f. for the same N and length x_0 but raised to a power of (x/x_0). In other words, expression (3) shows that the value of $S(N, \Delta\sigma, x)$ for any interval of N can be calculated from the value of of $S(N, \Delta\sigma, x_0)$ in the same interval. Thus, the main part of the s.f. for a length x comes from the tail of the s.f. for length x_0 (see the thick lines in figure 7). If two survival functions should hypothetically share the same left tail for $L = 1$ (see Figure 9), then, the associated survival functions for $L = 100$, would coincide in a wide range of the probability scale and would differ only by small differences in the rest. Some designers are unaware of this fact, which determines the modelling and testing aspects of fatigue as will be shown. If in addition to this, we take into account the fact that design values are associated with very small percentiles (in the left tail), then we arrive at the conclusion that we need to know much smaller percentiles of the cdf associated with length x_0. This illustrates the statistical difficulties of the problem.

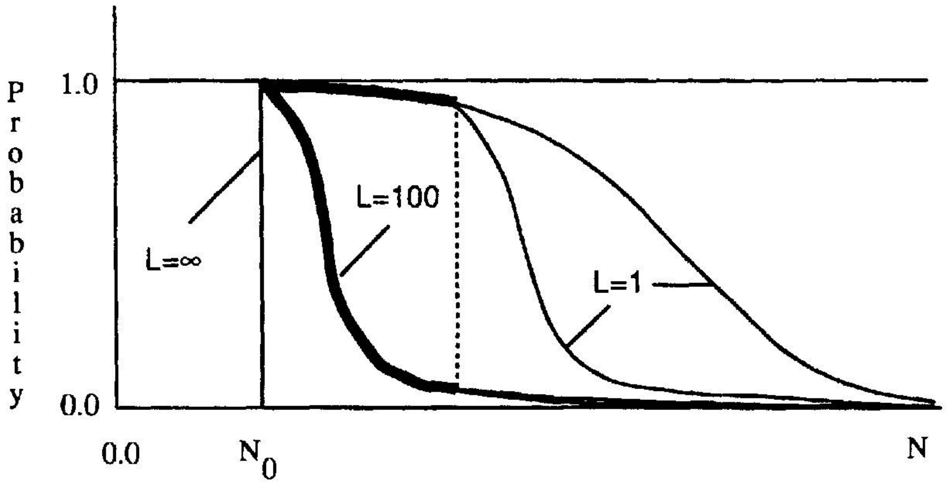

Figure 9: Two survival functions for L=1 leading to practically the same design curve for L=100.

Figure 10 shows a typical sample of lifetimes in Weibull paper obtained from lab specimens of length $L = 10$ cm. and the part of the cdf that could be determined for a length of 10 m. in the case of independence. Note the sample zone (the zone covered by the sample data), that corresponds to the part of the cumulative distribution function that can be calculated for length $100L$ (thick line in the figure), the extrapolation zone, that is, the zone where the cdf must be extrapolated based on the assumption of a Weibull model (linearity) and the design zone, which is the zone where the design values are calculated.

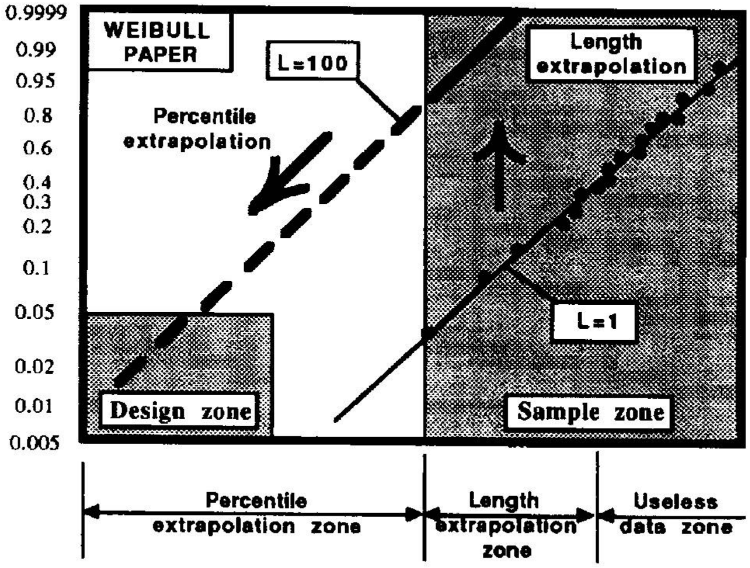

Figure 10: Extrapolation based on laboratory data

The hypothesis of independence, on which the previous approach is based, can be relaxed and replaced by asymptotical independence. By asymptotic independence we mean that extrapolation can be made using expresion (3) if x_0 is large enough. Being more precise we mean that the following relation holds

$$\lim_{x_0 \to \infty} \frac{S(N, \Delta\sigma, kx_0)}{S(N, \Delta\sigma, x_0)^k} = 1 \tag{4}$$

Experience demonstrates that for short lengths independence does not hold. However, for large lengths physical and theoretical reasons justify this assumption. Thus, it is extremely important to determine a threshold value of length above which we can use this assumption. On the other hand the asymptotic independence assumption is the only possibility to reasonably extrapolate far from laboratory data.
It is well known that:

- If independence or assymptotic independence hold, only three limit distributions (Gumbel, Weibull and Frechet for minima) are possible
- due to the non-negative character of lifetime, the Frechet distribution can be excluded
- any Gumbel distribution can be approximated as closely as desired by Weibull distributions.

Thus, the Weibull distribution with survival function

$$S(N) = \exp\left[-\left(\frac{N - N_0}{\delta}\right)^{\alpha}\right] \tag{5}$$

would be the right choice.

4. MODELLING ASPECTS OF FATIGUE

In this section we shall discuss some of the properties which are required in a mathematical or statistical model of fatigue failure.

4.1 Consistency

A primary condition is the *consistency* of the model, that is, its validity under changes in length. In order to illustrate this property we show two examples.
First, let us assume (see reference [1]) that the lifetime of lab specimens follows a normal distribution and that the independence assumption (expression (3)) holds. Then, if we select a lab specimen length $x_0 = 10$ *cm*, according to the model, the s.f. for length x becomes

$$S(N,x) = \left[\Phi\left(\frac{N-\mu_{10}}{\sigma_{10}}\right)\right]^{x/10} \tag{6}$$

where μ_{10} and σ_{10} are the mean and the standard deviation associated with $x_0 = 10cm.$, and $\Phi(N)$ is the s.f. of the standard $N(0,1)$ distribution.
Thus, the survival function (6) is not normal for $x \neq x_0$.
However, if we select a lab specimen of length $x_0 = 100$ cm. we obtain

$$S(N,x) = \left[\Phi\left(\frac{N-\mu_{100}}{\sigma_{100}}\right)\right]^{x/100} \tag{7}$$

It is clear that expressions (6) and (7) are not coincident for the same value of x. This is due to the fact that the normal family is not stable (closed) under minimum operations (see equation (1)). Thus, according to this model only one length can have a normal distribution. Consequently, this model is not consistent. A simple modification of the model (including the length as one more parameter) leads to consistency. This is called the extended normal model which includes the mean, the standard deviation and the length as parameters. This new model only states that lab specimens follow an extended normal model depending on three parameters. We should only obtain a normal model for lab specimens by coincidence.
Secondly, Bogdanoff and Kozin, [3], based on some experimental results, state that the s.f. of the lifetime is given by

$$S(N,x) = S(N,x_0)^{H(x,x_0)} \tag{8}$$

where $H(x,x_0)$ is an arbitrary positive function such that $H(x,x_0) > 1,\ \forall x > x_0$. This model is consistent if and only if we have (see reference [6]):

$$\begin{aligned} S(N,x_1) &= S(N,x_0)^{H(x_1,x_0)} \\ S(N,x_2) &= S(N,x_1)^{H(x_2,x_1)} \\ S(N,x_2) &= S(N,x_0)^{H(x_2,x_0)} \end{aligned} \tag{9}$$

which implies

$$H(x_2,x_0) = H(x_2,x_1)H(x_1,x_0) \Rightarrow H(x_2,x_0) = \frac{q(x_2)}{q(x_0)} \tag{10}$$

with $q(x)$ an arbitrary increasing function. Thus, our degrees of freedom are reduced to a function of one single variable and not to one of two variables as equation (8) seems to indicate. One interesting tool for obtaining consistent models is the use of functional equation techniques (see Castillo and Ruiz-Cobo, [8]). Note that equation (8) is a functional equation which is implicit in the function S, and whose solution leads to the general forms of the S and H functions satisfying (8).

Length (mm)	Shape parameter
140	2.99
1960	4.99
8540	10.2

Table 2: Experimental results reported by Castillo et al. for fatigue lifetime of wires.

4.2 Compatibility

The *compatibility* condition was already mentioned in section 3.. Castillo et al., [5], derived a Weibull model based on this property which leads to a functional equation. This model shows that for a given length, the s.f. becomes

$$S(N, \Delta\sigma) = \exp\left\{-\left[\frac{(N - N_0)(\Delta\sigma - \Delta\sigma_0) - \lambda}{\delta}\right]^{\alpha}\right\} \tag{11}$$

Castillo et al., [7], also gave a Gumbel model based on the same property. In this case they found

$$S(N, \Delta\sigma) = \exp\{-\exp[-AN - B\Delta\sigma + C]\} \tag{12}$$

Both equations (11) and (12) are derived from the original functional equations, and represent compatible solutions for the fatigue analysis.

4.3 Asymptotic behaviour

One outstanding property for a candidate model of fatigue is asymptotic stability. This means that models belong to that family not only for finite length but for lengths going to infinity. According to section 3., models (11) and (12) are the only ones having this property, that is, the limit models for $n \to \infty$ are also Weibull and Gumbel, respectively.
However, these models only take into account the influence of the stress level on lifetime. Thus, the natural extension of these models when dealing with the influence of length are the same Weibull and Gumbel models, but with parameters depending of length, with the only exception of the location parameter that, due to physical considerations, remains constant, that is, independent of length.

5. EXPERIMENTAL ASPECTS OF FATIGUE

Some internal features of the lifetime models can be tested by experimentation. The results reported by Fernández-Canteli et al.,[9], Castillo et al., [5], Phoenix et al., [14], etc. confirm the dependence of the scale and shape parameters on length. All of them coincide in showing an increase of the scale parameter with increasing lengths. On the contrary, they show either an increase or a decrease of the shape parameter with increasing lengths (see Tables 2 and 3), depending on the material tested.
Experimental results, reported by Spindel and Haibach, [15], Nishijima, [13], Haibach et al., [11], Bastenaire, [2], Castillo et al., [5], etc. confirm that the families in table 1 could be adequate. In general, the available data suggest the existence of an endurance limit for the $S - N$ curves.

Length (mm)	Shape parameter
5	6.5
10	5.4
50	4.3
500	4.0

Table 3: Experimental results reported by Phoenix et al. for creep rupture of IM-6 wires.)

6. DESIGN ASPECTS OF FATIGUE

In this section we shall try to answer the question of what we really need for design purposes? The following four alternatives can be taken into consideration

- the whole Whöler field
- the endurance limit
- one or some percentile curves
- confidence intervals

During the past decades a lot of effort has been devoted to the study of the whole Whöler field. It is clear that this knowledge is sufficient for design purposes, but is such an amount of information reasonable? or can we solve the design problem with less information?

In fact what we are looking for is the design value of the stress level, that is, a value leading to a very small probability of fatigue failure for a given very large number of cycles (normally 2×10^6 or 10^7) (see the design zone in figure 10). In addition, this is for large lengths (think of suspended or cable-stayed bridges).

Using, just for the sake of reasoning, the Castillo et al. model (11), the $\Delta\sigma$ associated with a given probability of failure p and a given lifetime N, is:

$$\Delta\sigma = \Delta\sigma_0 + \frac{\lambda + \delta\left\{\frac{x_0}{x}\left[-\ln(1-p)\right]^{\frac{1}{\alpha}}\right\}}{N - N_0} \tag{13}$$

Normally, design values are associated with very small values of p and very large values of N. In addition, $x >> x_0$. Thus, we are interested in the limiting cases:

$$\lim_{p\to 0} \Delta\sigma = \lim_{x\to\infty} \Delta\sigma = \Delta\sigma_0 + \frac{\lambda}{N - N_0} \tag{14}$$

which is the zero-percentile curve. This, for $N \to \infty$ becomes

$$\lim_{N\to\infty, p\to 0} \Delta\sigma = \lim_{N\to\infty, x\to\infty} \Delta\sigma = \Delta\sigma_0 \tag{15}$$

that is, the endurance limit.

Consequently, the design value is expected to be close to the zero-percentile curve or to the endurance limit if the design number of cycles is large enough (see Figure 11).

This discovery has important consequences in the design of testing strategies as we shall see.

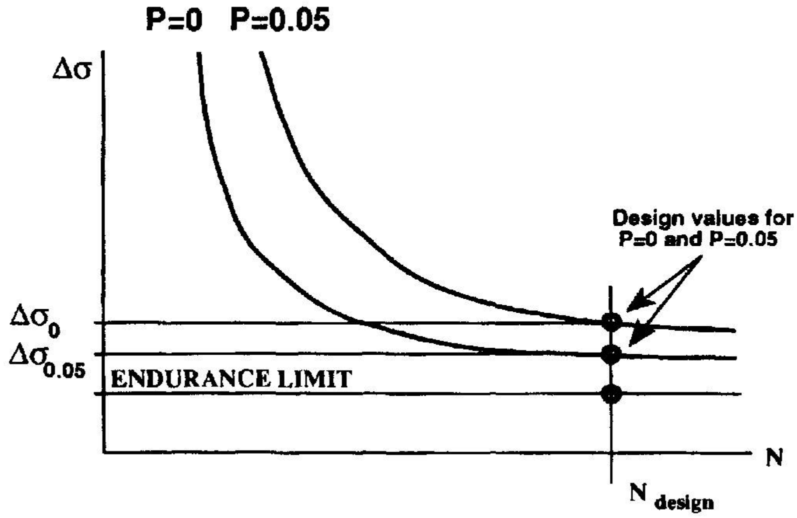

Figure 11: Illustration of the closeness of endurance limit and design values

For the general case of loading, the engineer needs a cumulative damage hypothesis in order to determine the design value (equivalent stress range). All the above also justifies the zero-percentile curve as the basis for this analysis.
Finally, confidence intervals are needed in order to give an idea of the precision and the reliability of the estimations.

7. THE PROBLEM OF EXTRAPOLATION

As we mentioned at the beginning of the paper, the engineer is faced with the problem of extrapolating fatigue strengths from lab results, normally obtained with very small specimens, to the large sizes of actual structures. In this step the independence or the weakened asymptotical independence assumptions are crucial and probably the only two objective alternatives we can make. Though the independence assumption has been proved not to be valid for small lengths, the asymptotical independence is sustained by the extreme value theory (see Galambos, [10], or Castillo, [4], that guaranties this behaviour even for cases of not too strong dependence of neighbouring pieces. Common sense and a bit of intuition allows also to suppose that for large lengths independence is a very reasonable assumption. Even though this is one of the topics for discussion, we shall indicate how the problem of extrapolation could be solved if the previous assumption holds.
An interesting model that takes care of dependence is the model in equation (8) together with the derived consistency condition (10). This model with the extra condition of a Weibull survival function becomes

$$S(N,\Delta\sigma,x)=\exp\left\{-\frac{q(x)}{q(x_0)}\left[\frac{(N-N_0)(\Delta\sigma-\Delta\sigma_0)-\lambda}{\delta}\right]^{\alpha}\right\} \qquad (16)$$

Castillo et al., [6], have shown that this is the most general model satisfying equation (8) and coincides with their non-stationary Poisson model in which flaws occur due to a nonstationary Poisson process with intensity $\lambda(x)$, changing with the position x. They have also shown that the Marshall Olkin model is of this type.

For asymptotic independence we must have

$$\lim_{x_0 \to \infty} \frac{q(kx_0)}{kq(x_0)} = 1 \tag{17}$$

Since the present maximal lengths for actual wires in stayed cable bridges is larger than 250 m. and the usual length used in lab tests is in the range of 200 mm., then the value of k becomes close to 1250. This involves an extrapolation very far beyond lab results, which implies a high risk, specially if the independence assumption has not been validated.
Figure 12 shows an example, reported by Castillo et al., [6], of a $q(x)$ function which satisfies the above condition.

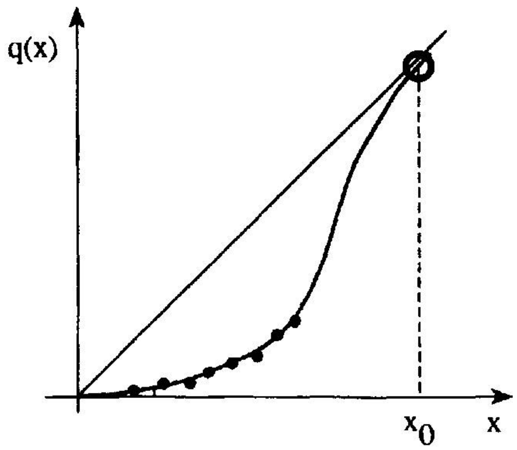

Figure 12: Example of asymptotic independence reported by Castillo et al., [6].

Following all the above we arrive at the model

$$S(N, \Delta\sigma, x) = \exp\left\{ - \left[\frac{(N - N_0)(\Delta\sigma - \Delta\sigma_0) - \lambda}{\delta(x)} \right]^{\alpha(x)} \right\} \tag{18}$$

that is, a Weibull model for the random variable $(N - N_0)(\Delta\sigma - \Delta\sigma_0)$ with constant location parameter and scale and shape parameters depending on length.
We shall assume that condition (4) holds and more precisely that we have

$$\lim_{x_0 \to \infty} \frac{\alpha(kx_0)}{\alpha(x_0)} = 1 \quad ; \quad \lim_{x_0 \to \infty} \frac{\left(\frac{1}{k}\right)^{\frac{1}{\alpha(x_0)}} \delta(x_0)}{\delta(kx_0)} = 1 \tag{19}$$

Note that equation (19) implies that the model (18) satisfies (4). This means that for large enough x_0 we have

$$S(N, \Delta\sigma, x) \cong \exp\left\{ - \left(\frac{x}{x_0}\right) \left[\frac{(N - N_0)(\Delta\sigma - \Delta\sigma_0) - \lambda}{\delta(x_0)} \right]^{\alpha(x_0)} \right\} \tag{20}$$

Consequently we must derive a threshold value x_0 above which equation (20) becomes a reasonable approximation.
Models (16) and (18), with restrictions (17) and (19) are equivalent for long lengths.

8. TESTING ASPECTS

Before starting this section, we remind the reader that the testing strategy should be oriented to the aim motivating the testing. It is not the same planning tests for determining the whole Whöler field as planning tests for determining the zero-percentile curve including the endurance limit.
In this section, some of the classical testing questions based on the discussion above and with the primary purpose of finding design values are discussed. In particular we address the following questions:

- What should be recommended: testing short or long elements?
- Are run-outs useful?

According to the above, testing should be addressed to the obtention of the zero-percentile curve above the N_{design} (number of cycles used in design) or the endurance limit.
As we have mentioned before (see figure 3), the zero-percentile curve is the only one that is independent of length. This facilitates things to a great degree and avoids the problem of dependence.
Let $N_0 = N_0(\Delta\sigma)$ be the zero-percentile curve. Then, we have

$$N_0(\Delta\sigma) \leq min(N_1, N_2, \ldots, N_p) \tag{21}$$

where, $N_1, N_2, \ldots, N_p$ are the fatigue lives of p specimens tested at the stress range $\Delta\sigma$.
Expression (21) gives only an upper bound for $N_0(\Delta\sigma)$. However, we can get a better estimate from the left tail of the distribution using the asymptotic Weibull model above.
Note that only left tail data should be used. This notably influences the testing strategies because tests can be stopped after the number of cycles associated with given percentiles. The problem of designing an optimal testing strategy is outside the aims of this paper.
In order to estimate the zero-percentile curve we shall divide the $S - N$ plane into two zones: the testing zone and the extrapolation zone, which are bounded by the limit number of cycles N_{limit} used in testing.
If the design number of cycles is smaller than N_{limit} then, extrapolation is not necessary and information beyond that number of cycles is irrelevant. In this case, testing, including at least three conveniently chosen levels of $\Delta\sigma$, allows the obtention of the zero-percentile curves.
On the contrary, if $N_{design} > N_{limit}$ then, the portion of the zero-percentile curve must be obtained by extrapolation (see Figure 13).
In any case, testing of the longest possible elements (10 m or more) is strongly recommended due to the fact that:

- Smaller values of k (real/lab length ratio) are obtained, which implies a weaker extrapolation.
- Closer sample values to the zero-percentile curve are obtained and hence, a better estimate follows.
- Shorter lifetimes and lower costs are involved, compensating the larger cost of ad hoc testing machines
- More information is gained because testing one long element, say of length ks, is equivalent to testing k elements of length s.

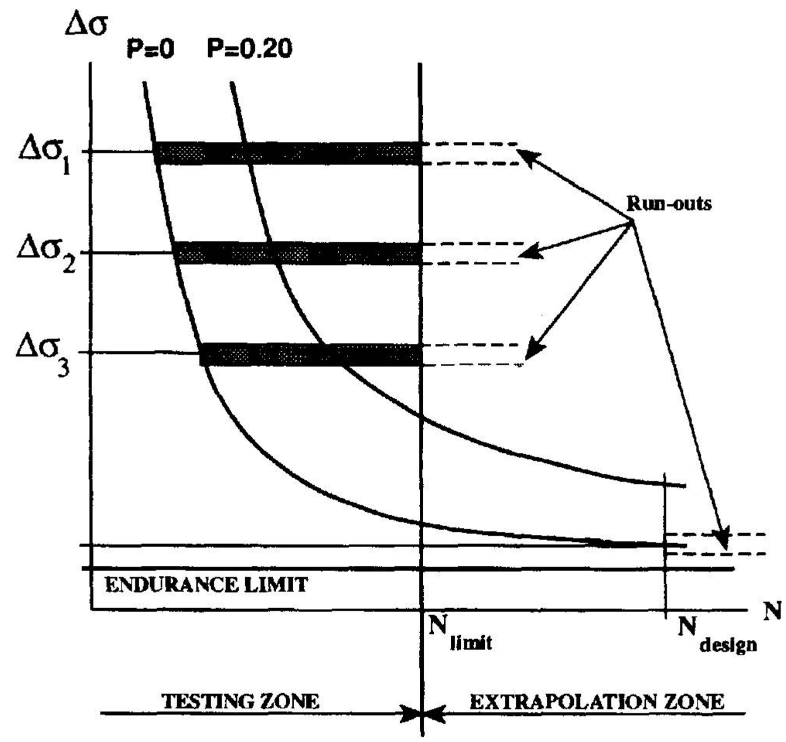

Figure 13: Illustration of some problems associated with the estimation of the zero-percentile curve.

- The threshold value associated with the asymptotic independence assumption is more likely to be overcome.

9. COMPOSED ELEMENTS

When dealing with composed elements we usually work with two variables: the number of wires, m, and the length, usually denoted by n, i.e. the number of times the element contains an elementary piece of given length, s_0 (see figure 14).

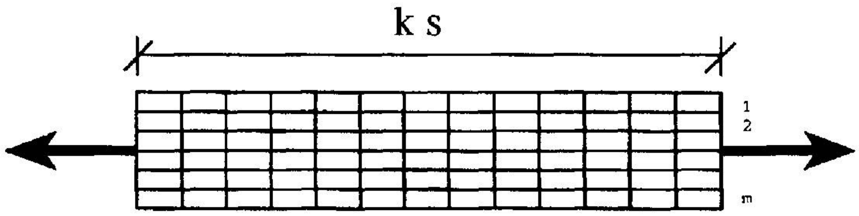

Figure 14: Composed element of m wires and length ks

Due to the presence of these two parameters, several limit distributions can be analyzed:

1. Constant n and very large m. This case is not useful for the fatigue problem.
2. Constant m and very large n. This case can be useful for fatigue analysis.
3. Very large n and m with m/n=k. This case has sense for fatigue analysis.
4. Very large n and m with m/n=0. This case has sense for fatigue analysis.

Cases (2) and (4) lead to Weibull or Gumbel models and model (3) leads to normal models. Thus, a discussion on the validity of the assumptions implied by models (2) to (4) is relevant for fatigue analysis.
The single element model is also applicable to the case of composed elements. We only need to test these elements and fit one of the above models. The results of the experiments will take into account the structure of the bundle of wires. If we have easy access to these kind of tests there is no sense in modelling a bundle based on the properties of single wires because many more sources of error are present in this approach, as for example, the effect of fretting.

ACKNOWLEDGEMENTS

We thank the Institut für Baustatik und Konstruktion, ETH Zürich and the Dirección General de Investigación Científica y Técnica (DGICYT) (proyect PB90-0339), for partial support of this work.

References

[1] Andrä W. and Saul R. Die Festigkeit, insbesondere Dauerfestigkeit langer Paralleldrahtbündel. *Die Bautechnik*, Vol. 4, pages 128–130, 1979.

[2] Bastenaire F.A. New method for the statistical evaluation of constant stress amplitude fatigue-test results. In *Probabilistic aspects of fatigue*, ASTM STP 511, pages 3–28, 1972.

[3] Bogdanoff J.L. and Kozin F. Effect of length on fatigue life of cables. *Journal of Engineering Mechanics*, Vol. 113(6), pages 925–940, 1987.

[4] Castillo E. *Extreme value theory in engineering* Academic Press, New York, 1988.

[5] Castillo E., Fernández- Canteli A., Esslinger V. and Thürlimann B. *Statistical model for fatigue analysis of wires, strands and cables*, In *IABSE Proceedings P-82/85, Zürich*, pages 1–40, 1985.

[6] Castillo E., Fernández-Canteli A., Ruiz-Tolosa J.R. and Sarabia J.M. *Statistical models for analysis of fatigue life of long elements.* In *Journal of Engineering Mechanics. ASCE, Vol. 116, No. 5*, pages 1036–1049, 1990.

[7] Castillo E., Luceño A., Montalbán A. and Fernández-Canteli A. *A Dependent Fatigue Lifetime Model.* In *Communications in Statistics. Theory and Methods*, pages 1181–1194, 1987.

[8] Castillo E. and Ruiz-Cobo R. *Functional equations and Modelling in Science and Engineering* Marcel Dekker,1992.

[9] Fernández-Canteli A., Esslinger V. and Thürlimann B. *Ermüdungsfestigkeit von Bewehrungs- und Spannstählen.* Bericht Nr. 8002-1, Institut für Baustatik und Konstruktion, ETH Zürich, 1984.

[10] Galambos J. *The asymptotic theory of extreme order statistics.* 2nd Edition, Krieger, Malabar, Florida, 1987.

[11] Haibach E., Olivier R. and Rinaldi F. Statistical design and analysis of an interlaboratory program on the fatigue properties of welded joints in structural steels. In *Statistical analysis of fatigue data*, ASTM STP 744, R.E. Little and J.C. Ekvall, Editors, American Society for Testing and Materials, pages 24–54, 1981.

[12] J.F.Lawless *Statistical models and methods for lifetime data.* Wiley Series in Probability and Mathematical Statistics, 1982.

[13] Nishijima S. Statistical fatigue properties of some heat-treated steels for machine structural use. In *Statistical analysis of fatigue data*, ASTM STP 744, R.E. Little and J.C. Ekvall, Editors, American Society for Testing and Materials, pages 75–88, 1981.

[14] Phoenix S.L., Schwartz P., and Robinson IV H.H. Statistics for the strength and lifetime in creep-rupture of model carbon/epoxy composites. *Composites Science and Technology*, Vol 32, No. 5 pages 81–120, 1988.

[15] Spindel J.E. and Haibach E. Some considerations in the statistical determination of the shape of S-N curves. In *Statistical analysis of fatigue data*, ASTM STP 744, R.E. Little and J.C. Ekvall, Editors, American Society for Testing and Materials, pages 89–113, 1981.

[16] *A Guide for fatigue testing and the statistical analysis of fatigue data*, American Society for Testing and Materials. ASTM Special Publication No. 91-A, 1963

[17] *Statistical analysis of fatigue data*, ASTM STP 744, R.E. Little and J.C. Ekvall, Editors, American Society for Testing and Materials, 1981.