Modèles statistiques pour l'analyse en fatigue d'éléments de grande longueur

Statistische Modelle für die Ermüdungsanalyse von langen Elementen

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SUMMARY
From the mechanical, statistical, modelling, design and testing aspects of fatigue a Weibull model for the product $(N-N_0)(\Delta\sigma-\Delta\sigma_0)$, with scale and shape parameters depending on length is suggested. The problem of extrapolation of fatigue strengths from lab specimens to actual structures is then analyzed. The design value is shown to be close to the endurance limit if the latter exists, and can be approximately calculated by means of the zero-percentile curve for long lengths. Finally, the implications of all the above in testing strategies is discussed.

RÉSUMÉ
On propose un modèle de Weibull pour le produit $(N-N_0)(\Delta\sigma-\Delta\sigma_0)$, comprenant un paramètre d'échelle et un paramètre de forme dépendant tous deux de la longueur. Ce modèle prend en compte les aspects mécanique, statistique et expérimental de la fatigue, ainsi que ceux relatifs à la modélisation, au calcul et aux essais. On analyse le problème de l'extrapolation à des structures réelles de résultats de résistance à la fatigue d'essais de laboratoire. On démontre que la valeur de calcul est proche de la limite d'endurance quand celle-ci existe et peut être approchée au moyen de la courbe correspondant au fractile zéro pour les grandes longueurs. Enfin, on discute les conséquences de tout ce qui précède sur les stratégies d'essais.

ZUSAMMENFASSUNG
In Hinsicht auf mechanische, statistische und experimentelle Untersuchungen, sowie auf Aspekte der Ermüdung in Beziehung auf das Modellieren, Bemessen und Prüfen wird ein Weibull-Modell für das Produkt $(N-N_0)(\Delta\sigma-\Delta\sigma_0)$ vorgeschlagen, das jeweils die von der Länge abhängigen Maßstabparameter und Formparameter einschliesst. Die Problematik der Extrapolation von den Ermüdungsfestigkeiten der Prüfkörper bis zu denjenigen der reellen Strukturen wird analysiert. Der Bemessungswert, soweit vorhanden, liegt der Ermüdungsgrenze nahe und kann mit Hilfe der Nullquantilkurve für grosse Längen berechnet werden. Schließlich werden die Folgen des Vorhergehenden auf die Prüfungsstrategie diskutiert.

1. INTRODUCTION

The problem of fatigue life of reinforcing bars, prestressing wires and strands has called the attention of researchers for many years.
According to the ASTM [1],*"fatigue is the process of progressive localized permanent structural change occurring in a material subjected to conditions which produce fluctuating stresses and strains at some point or points and which may culminate in cracks or complete fracture after a sufficient number of fluctuations"*.
Even though a lot of effort has been made in the past in order to understand this process and the factors influencing it, some important aspects remain to be clarified. For instance, the random distribution of flaws or cracks, which plays an important role in fatigue and seems to be determinant in the statistical properties of fatigue, is not well known.
The experience gained over this period of time allows a researcher to have some feeling for the main qualitative features of the statistical behaviour of the fatigue strength (endurance limit, scatter, shape of the $S-N$ curves, size effect, independence of the fatigue lifetimes of different pieces, etc.). However, a precise statistical description of the fatigue phenomena is not available and the problem of model selection and design criteria has not been definitely solved.
Design engineers also need to extrapolate from small laboratory specimens to the actual length of structures such as cable-stayed or suspended bridges. In order for this extrapolation to be made with reasonable reliability, extra knowledge is required. Thus, we can conclude that further work, research and discussion is needed in order to achieve an adequate design of reliable and economic prestressed structures, bridges and similar structures.
As primary aims for the statistical analysis of fatigue data, the ASTM proposes:

- to estimate certain fatigue properties of a material or a component (together with measures of the reliability) from a given set of data.
- to give objective procedures for comparing two or more sets of fatigue data
- to provide information on the most efficient use of a limited number of test specimens and on the number of test specimens required to give a specific degree of confidence in the test results.

However, these objectives, which are mainly stated by statistical experts seem to correspond to the task of the testing engineer, but do not exactly coincide with those of practitioners. An engineer is generally faced with the problem of obtaining *design values*.
Though several aspects of fatigue could be considered for discussion, we shall concentrate primarily on the problem of the determination of design values and their main associated aspects (testing strategies and so on). We also assume that we start from laboratory tests and that we wish to extrapolate these results to actual structures. Thus, the problem of *extrapolation* will be one of our main concerns.
In this paper, more than stating particular problems and trying to give solutions to them, our intention is to present an overview of the existing open questions in order to motivate discussion during the workshop and promote a minimal consensus about some existing controversial problems.
In sections 2. to 6. we shall analyze single elements, that is wires, and we shall postpone the study of composed elements, such as cables or strands, till section 9.. However, it is worthwhile mentioning that the latter can always be considered as single elements if the analysis is based on adequate testing data.

Model	Reference	$S-N$ curves
1	[17]	$\ln(N) = A + B\Delta\sigma$
2	[17]	$\ln(N) = A + B\ln(\Delta\sigma)$
3	[15]	$\ln\left(\frac{N}{N_0}\right) = A\ln\left(\frac{\Delta\sigma}{\Delta\sigma_0}\right) + B\left\{\ln\left(\frac{\Delta\sigma}{\Delta\sigma_0}\right) + \frac{1}{\alpha}\ln\left[1+\left(\frac{\Delta\sigma}{\Delta\sigma_0}\right)^{-2\alpha}\right]\right\}$
4	[2]	$(N-N_0)(\Delta\sigma - \Delta\sigma_0) = Aexp\left[-C(\Delta\sigma - \Delta\sigma_0)\right]$
5	[5]	$\ln\left(\frac{N}{N_0}\right)\ln\left(\frac{\Delta\sigma}{\Delta\sigma_0}\right) = A$

Table 1: Different models for the $S-N$ curves.

In the following, we shall analyze different aspects of fatigue including the problem of fatigue modelling.
It shoulkd be noted that in the present terminology N refers to the logarithm of the number of cycles to failure, while $\Delta\sigma$ can represent either the stress range or its logarithm.

2. MECHANICAL ASPECTS OF FATIGUE

As mechanical aspects of fatigue we understand those directly influencing the mechanical fatigue resistance of the piece. Two main factors have been considered in the past as being determinant in design:

- stress level and
- size effect.

2.1 Stress level

It is well known from testing that the higher the stress level, $\Delta\sigma$, the lower the fatigue lifetime, N. Figure 1 shows a typical lifetime-stress level curve for a given piece. Different elements, due to their random selection, show different behaviour (see Figure 2). Knowledge of the precise shape and properties of these curves might be important for design values. Several models have been proposed in the past for these curves. Some of them are shown in Table 1.

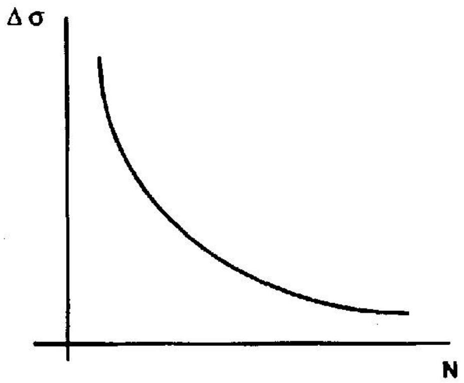

Figure 1: Lifetime-stress level failure curve for a given piece

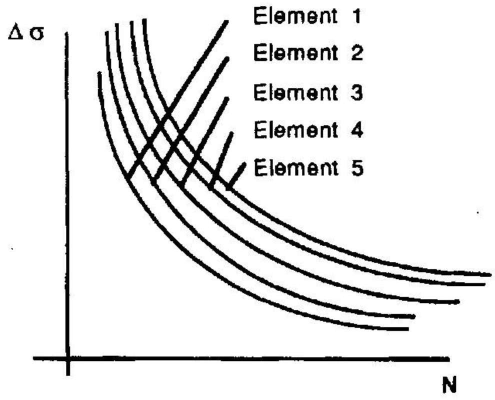

Figure 2: Lifetime-stress level failure curves for different elements or percentile curves.

Models 1 and 2 lead to families of parallel straight lines when using $ln(N)$ and either $\Delta\sigma$ or $ln(\Delta\sigma)$, respectively. Model 5 and some special cases of models 3 and 4 lead to families of hyperbolas on a logarithmic scale (see figure 3). Other variants of models 3 and 4 lead to more complex families.

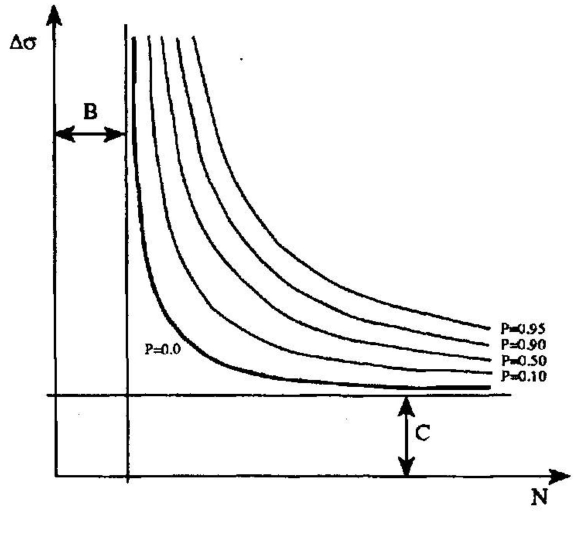

Figure 3: Whöler curves (percentiles)

The shape of the $S-N$ curves depends markedly on the material and testing conditions. Thus, models for a given material and testing condition cannot be used for other materials and/or conditions.

The lifetime-stress level curves can be non-intersecting (monotonic) as in figure 2 or they can intersect as shown in figure 4. Those defending non-monotonicity explain that different types of flaws or cracks can have different associated failure rates. Others argue that, because failure is governed by the stress intensity factor, no intersection occurs. Nevertheless, even in the first case an equivalent non-intersecting family (the percentile family) exists that leads to the same statistical distribution of N for given $\Delta\sigma$ or $\Delta\sigma$ given N. Thus, we can always think of a (fictitious) set of non-intersecting curves associated with elements of increasing strength. This is an interesting physical interpretation of percentile curves.

Perhaps the most commonly accepted and relevant property of these curves is the existence

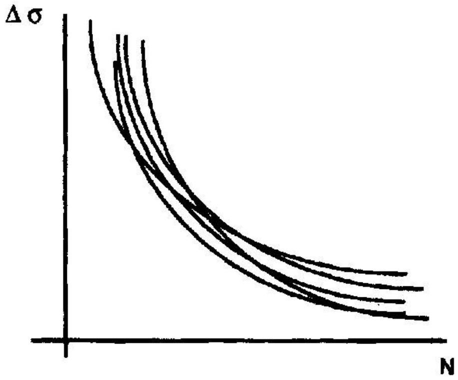

Figure 4: Intersecting lifetime-stress level failure curves

of an *endurance limit* (a stress level below which fatigue failure does not occur). A discussion on this existence remains to be clarified. Some important questions that arise are: does an endurance limit exist?, what are the practical implications if we make an erroneous assumption about this existence?

2.2 Size effect

Size effect, that is, the influence of length on the fatigue life of longitudinal elements, is another factor to be considered. Due to the fact that one element of length ks can be considered as divided into k small pieces of length s (see Figure 5), and that any of these pieces is subjected to the same fluctuating stresses as the whole piece, then the *weakest link principle* states that the strength of the element is that of the weakest piece, i.e.

$$N = min(N_1, N_2, \dots, N_k) \tag{1}$$

This extreme property of N will be the key for its statistical behaviour.

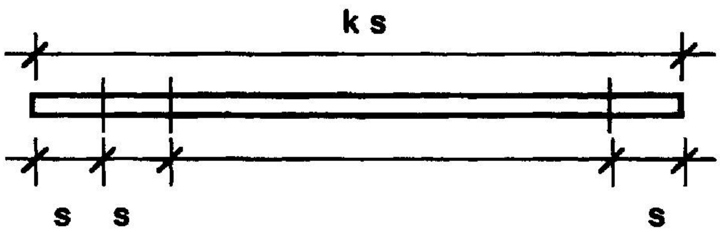

Figure 5: Longitudinal element and constituting pieces

An important aspect to be considered is the source of flaws or cracks and their spatial (longitudinal) occurrence. Among other questions, the following need to be answered: does the existence of a flaw or the occurrence of a crack in a given piece influence the occurrences in other pieces or do they behave independently? This mechanical behaviour, which is related to the fabrication process, plays an important role in the fatigue strength.

3. STATISTICAL ASPECTS OF FATIGUE

Fatigue life shows a markedly random character which implies that statistical analysis cannot be obviated. In real structures many factors influence fatigue life, such as stress levels, tem-

peratures, corrosion, quality of the elements, etc. In this study, we shall consider only stress level and the quality of the elements. In addition the random variation of stress levels is not considered. On the contrary, we assume a constant deterministic stress level. In other words, what is studied is what happens to an element under constant stress fatigue, which is previous to the problem of random stresses.
In the following paragraphs we shall illustrate some statistical aspects that can definitely help in the modelling of fatigue problems.

3.1 Stress level

Once the element (wire) is given, that is, selected at random from a given population (the factory production), the flaws or cracks are given. That is, we can assume that its lifetime is deterministic, equal to the strength of its weakest flaw and follows a law similar to that shown in figure 1, as is indicated by Fracture Mechanics Theory. This implies that every element has a unique and fixed associated failure curve $S-N$.
Consequently, we can state that the random character of lifetime comes from the fact that in longitudinal elements chosen at random from a given population (factory), some elements have larger cracks or weaker flaws than others (see figure 2).
The statistical influence of the stress level has been analyzed by means of the well known Whöler field, that is, the isopercentile lines associated with stress level and lifetime. Several models have been accepted for a log linear scale of lifetimes: some are linear (a family of straight lines) and some, non-linear.
In the Whöler field, we can think of two different random variables:

1. The lifetime N for a given stress range, $\Delta\sigma$.
2. The stress range $\Delta\sigma$ leading to a given lifetime, N.

Figure 6 shows one example of the probability density functions of these two random variables.

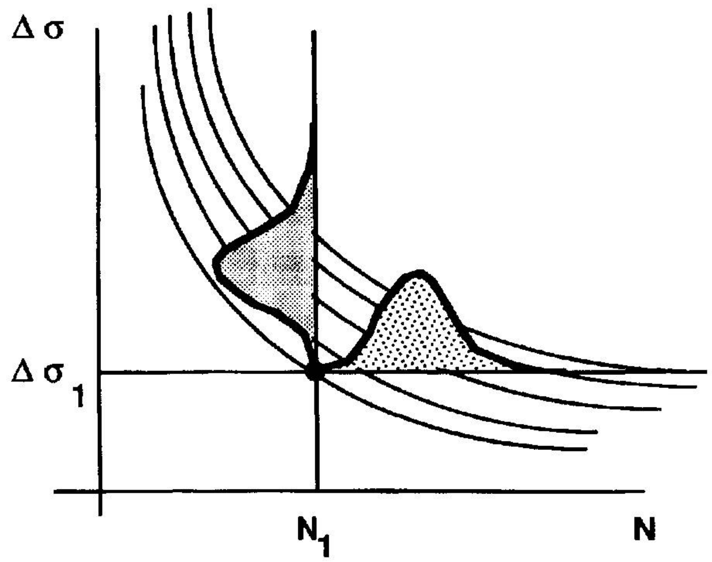

Figure 6: Two random variables: N for given $\Delta\sigma$ and $\Delta\sigma$ for given N

They lead to what has been called the *compatibility condition* (see reference [5]) that states that their survival functions (the survival function (s.f.), $S(N)$, is a function that gives the probability of surviving a period of duration N) must coincide, i.e.

$$S_1(N, \Delta\sigma, x) = S_2(\Delta\sigma, N, x) \quad (2)$$

where $S_1(N, \Delta\sigma, x)$ is the s.f. of N given $\Delta\sigma$ and the length x, and $S_2(\Delta\sigma, N, x)$ is the s.f. of $\Delta\sigma$ given N and the length x. This condition will be assumed in the following by using a single function $S(N, \Delta\sigma, x)$.

3.2 Size effect

From a statistical point of view, the analysis of the size effect can be made equivalent to the analysis of the influence of length on the reliabiliy function. Several models have been given in the past to solve this problem (see reference [6]). Unfortunately, most of them are based on the assumption of independence of the lifetime of non-overlapping pieces.
If independence holds, and based on the weakest link principle, we state that the s.f. of the lifetime of one element of length x is given by

$$S(N, \Delta\sigma, x) = S(N, \Delta\sigma, x_0)^{x/x_0} \quad (3)$$

This means that the graph of the survival function moves to the left, becomes steeper with increasing length x (see figure 7) and degenerates to a step function for infinite length.

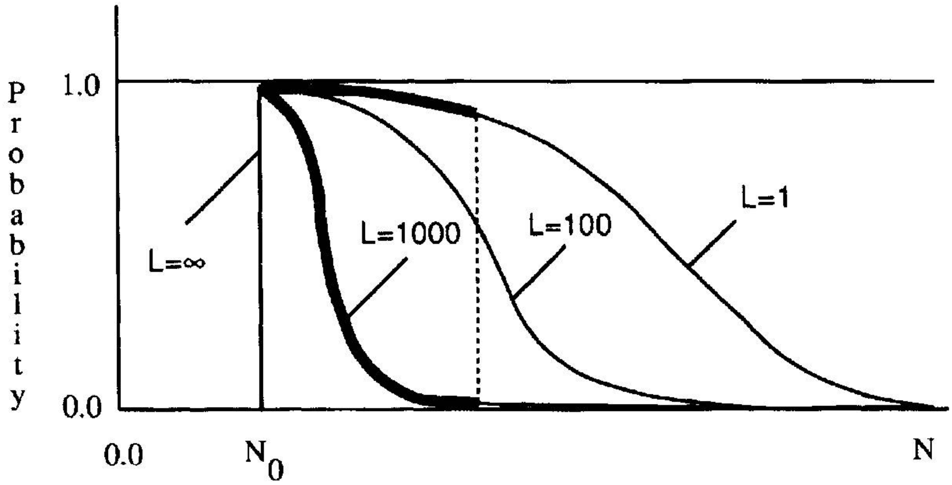

Figure 7: Survival function for different lengths 1, 100, 1000 and ∞.

Figure 8 shows the Whöler field for two different lengths. Note that the 1 probability of survival curve (or zero-percentile) coincides.
Experience shows that for small lengths the independence assumption can be inadequate and, when used for extrapolation purposes, can lead to either unsafe or very costly designs. This important problem has not yet been solved. Nevertheless we can state that the problem of dependence is crucial and the most critical factor to be considered in the analysis of size effect.
In addition to the above problems there are other statistical concepts that need some clarification because of their influence on design. One important fact to be taken into consideration is that only the tail behaviour influences the survival function for design purposes. Thus, mean values are not adequate for fatigue analysis. Initially, and only with the purpose of illustrating this property, we shall assume that the hypothesis of independence of the lifetimes of neighbouring pieces holds. According to expression (3), the value of the s.f. for a given N and length

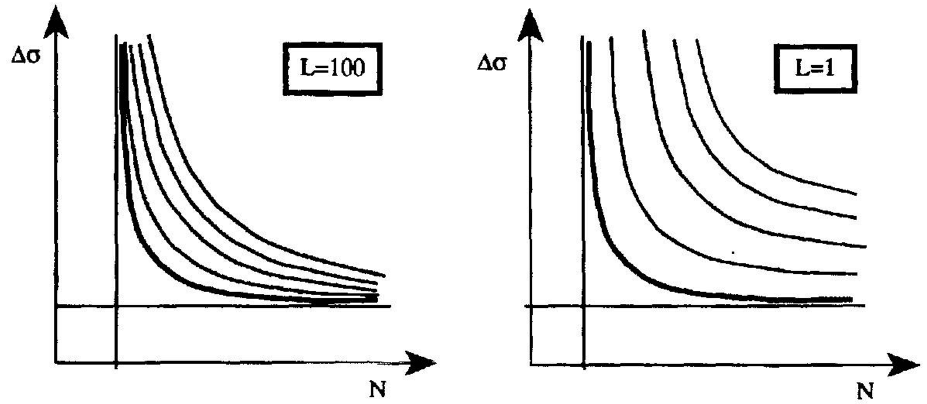

Figure 8: Whöler fields for two different lengths.

x coincides with the value of the s.f. for the same N and length x_0 but raised to a power of (x/x_0). In other words, expression (3) shows that the value of $S(N, \Delta\sigma, x)$ for any interval of N can be calculated from the value of of $S(N, \Delta\sigma, x_0)$ in the same interval. Thus, the main part of the s.f. for a length x comes from the tail of the s.f. for length x_0 (see the thick lines in figure 7). If two survival functions should hypothetically share the same left tail for $L = 1$ (see Figure 9), then, the associated survival functions for $L = 100$, would coincide in a wide range of the probability scale and would differ only by small differences in the rest. Some designers are unaware of this fact, which determines the modelling and testing aspects of fatigue as will be shown. If in addition to this, we take into account the fact that design values are associated with very small percentiles (in the left tail), then we arrive at the conclusion that we need to know much smaller percentiles of the cdf associated with length x_0. This illustrates the statistical difficulties of the problem.

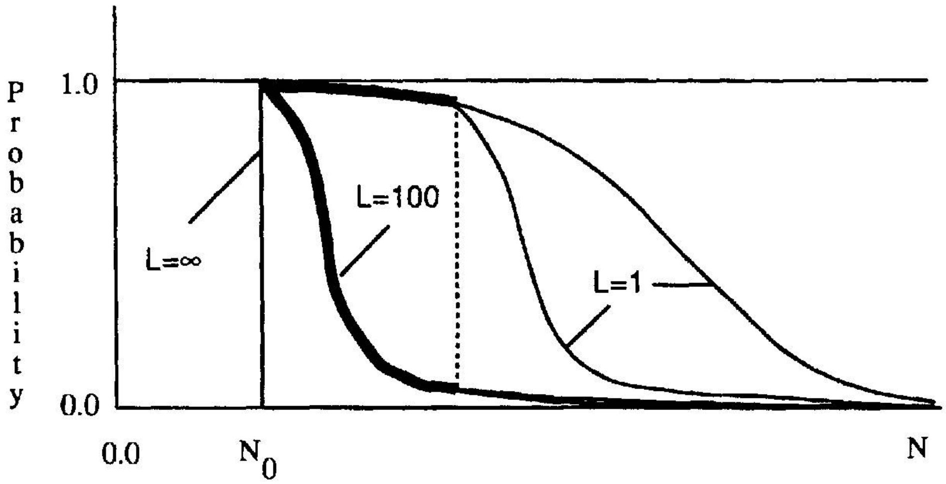

Figure 9: Two survival functions for L=1 leading to practically the same design curve for L=100.

Figure 10 shows a typical sample of lifetimes in Weibull paper obtained from lab specimens of length $L = 10$ cm. and the part of the cdf that could be determined for a length of 10 m. in the case of independence. Note the sample zone (the zone covered by the sample data), that corresponds to the part of the cumulative distribution function that can be calculated for length $100L$ (thick line in the figure), the extrapolation zone, that is, the zone where the cdf must be extrapolated based on the assumption of a Weibull model (linearity) and the design zone, which is the zone where the design values are calculated.

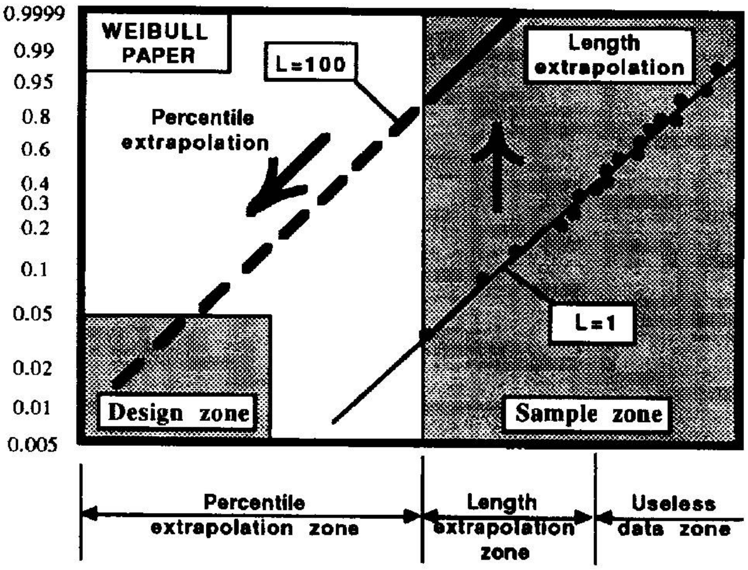

Figure 10: Extrapolation based on laboratory data

The hypothesis of independence, on which the previous approach is based, can be relaxed and replaced by asymptotical independence. By asymptotic independence we mean that extrapolation can be made using expresion (3) if x_0 is large enough. Being more precise we mean that the following relation holds

$$\lim_{x_0 \to \infty} \frac{S(N, \Delta\sigma, kx_0)}{S(N, \Delta\sigma, x_0)^k} = 1 \qquad (4)$$

Experience demonstrates that for short lengths independence does not hold. However, for large lengths physical and theoretical reasons justify this assumption. Thus, it is extremely important to determine a threshold value of length above which we can use this assumption. On the other hand the asymptotic independence assumption is the only possibility to reasonably extrapolate far from laboratory data.
It is well known that:

- If independence or assymptotic independence hold, only three limit distributions (Gumbel, Weibull and Frechet for minima) are possible
- due to the non-negative character of lifetime, the Frechet distribution can be excluded
- any Gumbel distribution can be approximated as closely as desired by Weibull distributions.

Thus, the Weibull distribution with survival function

$$S(N) = \exp\left[-\left(\frac{N - N_0}{\delta}\right)^\alpha\right] \qquad (5)$$

would be the right choice.

4. MODELLING ASPECTS OF FATIGUE

In this section we shall discuss some of the properties which are required in a mathematical or statistical model of fatigue failure.

4.1 Consistency

A primary condition is the *consistency* of the model, that is, its validity under changes in length. In order to illustrate this property we show two examples.

First, let us assume (see reference [1]) that the lifetime of lab specimens follows a normal distribution and that the independence assumption (expression (3)) holds. Then, if we select a lab specimen length $x_0 = 10$ *cm*, according to the model, the s.f. for length x becomes

$$S(N,x) = \left[\Phi\left(\frac{N-\mu_{10}}{\sigma_{10}}\right)\right]^{x/10} \tag{6}$$

where μ_{10} and σ_{10} are the mean and the standard deviation associated with $x_0 = 10cm.$, and $\Phi(N)$ is the s.f. of the standard $N(0,1)$ distribution.

Thus, the survival function (6) is not normal for $x \neq x_0$.

However, if we select a lab specimen of length $x_0 = 100$ cm. we obtain

$$S(N,x) = \left[\Phi\left(\frac{N-\mu_{100}}{\sigma_{100}}\right)\right]^{x/100} \tag{7}$$

It is clear that expressions (6) and (7) are not coincident for the same value of x. This is due to the fact that the normal family is not stable (closed) under minimum operations (see equation (1)). Thus, according to this model only one length can have a normal distribution. Consequently, this model is not consistent. A simple modification of the model (including the length as one more parameter) leads to consistency. This is called the extended normal model which includes the mean, the standard deviation and the length as parameters. This new model only states that lab specimens follow an extended normal model depending on three parameters. We should only obtain a normal model for lab specimens by coincidence.

Secondly, Bogdanoff and Kozin, [3], based on some experimental results, state that the s.f. of the lifetime is given by

$$S(N,x) = S(N,x_0)^{H(x,x_0)} \tag{8}$$

where $H(x,x_0)$ is an arbitrary positive function such that $H(x,x_0) > 1,\ \forall x > x_0$. This model is consistent if and only if we have (see reference [6]):

$$\begin{aligned} S(N,x_1) &= S(N,x_0)^{H(x_1,x_0)} \\ S(N,x_2) &= S(N,x_1)^{H(x_2,x_1)} \\ S(N,x_2) &= S(N,x_0)^{H(x_2,x_0)} \end{aligned} \tag{9}$$

which implies

$$H(x_2,x_0) = H(x_2,x_1)H(x_1,x_0) \Rightarrow H(x_2,x_0) = \frac{q(x_2)}{q(x_0)} \tag{10}$$

with $q(x)$ an arbitrary increasing function. Thus, our degrees of freedom are reduced to a function of one single variable and not to one of two variables as equation (8) seems to indicate. One interesting tool for obtaining consistent models is the use of functional equation techniques (see Castillo and Ruiz-Cobo, [8]). Note that equation (8) is a functional equation which is implicit in the function S, and whose solution leads to the general forms of the S and H functions satisfying (8).

Length (mm)	Shape parameter
140	2.99
1960	4.99
8540	10.2

Table 2: Experimental results reported by Castillo et al. for fatigue lifetime of wires.

4.2 Compatibility

The *compatibility* condition was already mentioned in section 3.. Castillo et al., [5], derived a Weibull model based on this property which leads to a functional equation. This model shows that for a given length, the s.f. becomes

$$S(N, \Delta\sigma) = \exp\left\{-\left[\frac{(N-N_0)(\Delta\sigma-\Delta\sigma_0)-\lambda}{\delta}\right]^{\alpha}\right\} \qquad (11)$$

Castillo et al., [7], also gave a Gumbel model based on the same property. In this case they found

$$S(N, \Delta\sigma) = \exp\{-\exp[-AN - B\Delta\sigma + C]\} \qquad (12)$$

Both equations (11) and (12) are derived from the original functional equations, and represent compatible solutions for the fatigue analysis.

4.3 Asymptotic behaviour

One outstanding property for a candidate model of fatigue is asymptotic stability. This means that models belong to that family not only for finite length but for lengths going to infinity. According to section 3., models (11) and (12) are the only ones having this property, that is, the limit models for $n \to \infty$ are also Weibull and Gumbel, respectively.
However, these models only take into account the influence of the stress level on lifetime. Thus, the natural extension of these models when dealing with the influence of length are the same Weibull and Gumbel models, but with parameters depending of length, with the only exception of the location parameter that, due to physical considerations, remains constant, that is, independent of length.

5. EXPERIMENTAL ASPECTS OF FATIGUE

Some internal features of the lifetime models can be tested by experimentation. The results reported by Fernández-Canteli et al.,[9], Castillo et al., [5], Phoenix et al., [14], etc. confirm the dependence of the scale and shape parameters on length. All of them coincide in showing an increase of the scale parameter with increasing lengths. On the contrary, they show either an increase or a decrease of the shape parameter with increasing lengths (see Tables 2 and 3), depending on the material tested.
Experimental results, reported by Spindel and Haibach, [15], Nishijima, [13], Haibach et al., [11], Bastenaire, [2], Castillo et al., [5], etc. confirm that the families in table 1 could be adequate. In general, the available data suggest the existence of an endurance limit for the $S-N$ curves.

Length (mm)	Shape parameter
5	6.5
10	5.4
50	4.3
500	4.0

Table 3: Experimental results reported by Phoenix et al. for creep rupture of IM-6 wires.)

6. DESIGN ASPECTS OF FATIGUE

In this section we shall try to answer the question of what we really need for design purposes? The following four alternatives can be taken into consideration

- the whole Whöler field
- the endurance limit
- one or some percentile curves
- confidence intervals

During the past decades a lot of effort has been devoted to the study of the whole Whöler field. It is clear that this knowledge is sufficient for design purposes, but is such an amount of information reasonable? or can we solve the design problem with less information?

In fact what we are looking for is the design value of the stress level, that is, a value leading to a very small probability of fatigue failure for a given very large number of cycles (normally 2×10^6 or 10^7) (see the design zone in figure 10). In addition, this is for large lengths (think of suspended or cable-stayed bridges).

Using, just for the sake of reasoning, the Castillo et al. model (11), the $\Delta\sigma$ associated with a given probability of failure p and a given lifetime N, is:

$$\Delta\sigma = \Delta\sigma_0 + \frac{\lambda + \delta\left\{\frac{x_0}{x}\left[-\ln\left(1-p\right)\right]^{\frac{1}{\alpha}}\right\}}{N - N_0} \tag{13}$$

Normally, design values are associated with very small values of p and very large values of N. In addition, $x >> x_0$. Thus, we are interested in the limiting cases:

$$\lim_{p\to 0}\Delta\sigma = \lim_{x\to\infty}\Delta\sigma = \Delta\sigma_0 + \frac{\lambda}{N - N_0} \tag{14}$$

which is the zero-percentile curve. This, for $N \to \infty$ becomes

$$\lim_{N\to\infty, p\to 0}\Delta\sigma = \lim_{N\to\infty, x\to\infty}\Delta\sigma = \Delta\sigma_0 \tag{15}$$

that is, the endurance limit.

Consequently, the design value is expected to be close to the zero-percentile curve or to the endurance limit if the design number of cycles is large enough (see Figure 11).

This discovery has important consequences in the design of testing strategies as we shall see.

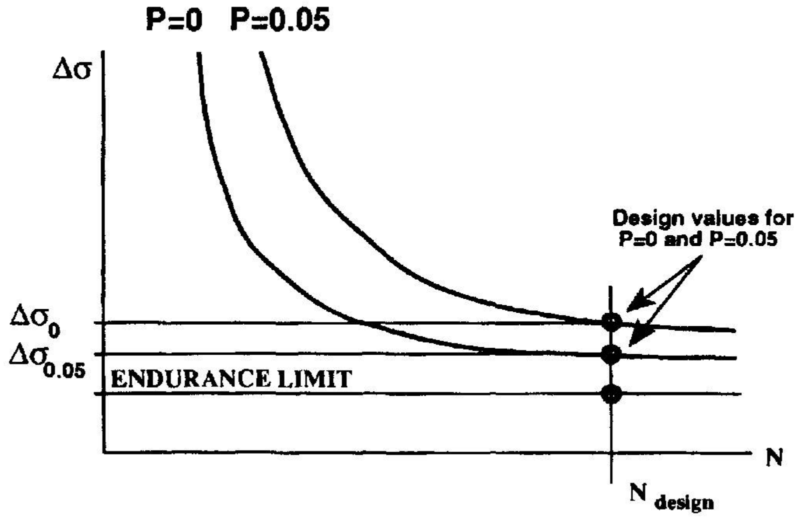

Figure 11: Illustration of the closeness of endurance limit and design values

For the general case of loading, the engineer needs a cumulative damage hypothesis in order to determine the design value (equivalent stress range). All the above also justifies the zero-percentile curve as the basis for this analysis.
Finally, confidence intervals are needed in order to give an idea of the precision and the reliability of the estimations.

7. THE PROBLEM OF EXTRAPOLATION

As we mentioned at the beginning of the paper, the engineer is faced with the problem of extrapolating fatigue strengths from lab results, normally obtained with very small specimens, to the large sizes of actual structures. In this step the independence or the weakened asymptotical independence assumptions are crucial and probably the only two objective alternatives we can make. Though the independence assumption has been proved not to be valid for small lengths, the asymptotical independence is sustained by the extreme value theory (see Galambos, [10], or Castillo, [4], that guaranties this behaviour even for cases of not too strong dependence of neighbouring pieces. Common sense and a bit of intuition allows also to suppose that for large lengths independence is a very reasonable assumption. Even though this is one of the topics for discussion, we shall indicate how the problem of extrapolation could be solved if the previous assumption holds.
An interesting model that takes care of dependence is the model in equation (8) together with the derived consistency condition (10). This model with the extra condition of a Weibull survival function becomes

$$S(N,\Delta\sigma,x) = \exp\left\{-\frac{q(x)}{q(x_0)}\left[\frac{(N-N_0)(\Delta\sigma-\Delta\sigma_0)-\lambda}{\delta}\right]^{\alpha}\right\} \qquad (16)$$

Castillo et al., [6], have shown that this is the most general model satisfying equation (8) and coincides with their non-stationary Poisson model in which flaws occur due to a nonstationary Poisson process with intensity $\lambda(x)$, changing with the position x. They have also shown that the Marshall Olkin model is of this type.

For asymptotic independence we must have

$$\lim_{x_0 \to \infty} \frac{q(kx_0)}{kq(x_0)} = 1 \tag{17}$$

Since the present maximal lengths for actual wires in stayed cable bridges is larger than 250 m. and the usual length used in lab tests is in the range of 200 mm., then the value of k becomes close to 1250. This involves an extrapolation very far beyond lab results, which implies a high risk, specially if the independence assumption has not been validated.
Figure 12 shows an example, reported by Castillo et al., [6], of a $q(x)$ function which satisfies the above condition.

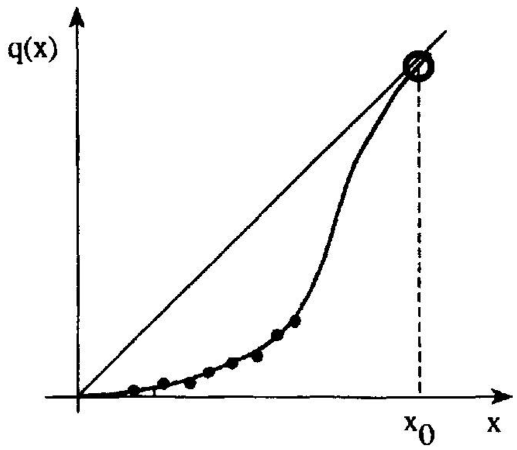

Figure 12: Example of asymptotic independence reported by Castillo et al., [6].

Following all the above we arrive at the model

$$S(N, \Delta\sigma, x) = \exp\left\{ - \left[\frac{(N - N_0)(\Delta\sigma - \Delta\sigma_0) - \lambda}{\delta(x)} \right]^{\alpha(x)} \right\} \tag{18}$$

that is, a Weibull model for the random variable $(N - N_0)(\Delta\sigma - \Delta\sigma_0)$ with constant location parameter and scale and shape parameters depending on length.
We shall assume that condition (4) holds and more precisely that we have

$$\lim_{x_0 \to \infty} \frac{\alpha(kx_0)}{\alpha(x_0)} = 1 \quad ; \quad \lim_{x_0 \to \infty} \frac{\left(\frac{1}{k}\right)^{\frac{1}{\alpha(x_0)}} \delta(x_0)}{\delta(kx_0)} = 1 \tag{19}$$

Note that equation (19) implies that the model (18) satisfies (4). This means that for large enough x_0 we have

$$S(N, \Delta\sigma, x) \cong \exp\left\{ - \left(\frac{x}{x_0}\right) \left[\frac{(N - N_0)(\Delta\sigma - \Delta\sigma_0) - \lambda}{\delta(x_0)} \right]^{\alpha(x_0)} \right\} \tag{20}$$

Consequently we must derive a threshold value x_0 above which equation (20) becomes a reasonable approximation.
Models (16) and (18), with restrictions (17) and (19) are equivalent for long lengths.

8. TESTING ASPECTS

Before starting this section, we remind the reader that the testing strategy should be oriented to the aim motivating the testing. It is not the same planning tests for determining the whole Whöler field as planning tests for determining the zero-percentile curve including the endurance limit.
In this section, some of the classical testing questions based on the discussion above and with the primary purpose of finding design values are discussed. In particular we address the following questions:

- What should be recommended: testing short or long elements?
- Are run-outs useful?

According to the above, testing should be addressed to the obtention of the zero-percentile curve above the N_{design} (number of cycles used in design) or the endurance limit.
As we have mentioned before (see figure 3), the zero-percentile curve is the only one that is independent of length. This facilitates things to a great degree and avoids the problem of dependence.
Let $N_0 = N_0(\Delta\sigma)$ be the zero-percentile curve. Then, we have

$$N_0(\Delta\sigma) \leq min(N_1, N_2, \dots, N_p) \tag{21}$$

where, $N_1, N_2, \dots, N_p$ are the fatigue lives of p specimens tested at the stress range $\Delta\sigma$.
Expression (21) gives only an upper bound for $N_0(\Delta\sigma)$. However, we can get a better estimate from the left tail of the distribution using the asymptotic Weibull model above.
Note that only left tail data should be used. This notably influences the testing strategies because tests can be stopped after the number of cycles associated with given percentiles. The problem of designing an optimal testing strategy is outside the aims of this paper.
In order to estimate the zero-percentile curve we shall divide the $S - N$ plane into two zones: the testing zone and the extrapolation zone, which are bounded by the limit number of cycles N_{limit} used in testing.
If the design number of cycles is smaller than N_{limit} then, extrapolation is not necessary and information beyond that number of cycles is irrelevant. In this case, testing, including at least three conveniently chosen levels of $\Delta\sigma$, allows the obtention of the zero-percentile curves.
On the contrary, if $N_{design} > N_{limit}$ then, the portion of the zero-percentile curve must be obtained by extrapolation (see Figure 13).
In any case, testing of the longest possible elements (10 m or more) is strongly recommended due to the fact that:

- Smaller values of k (real/lab length ratio) are obtained, which implies a weaker extrapolation.
- Closer sample values to the zero-percentile curve are obtained and hence, a better estimate follows.
- Shorter lifetimes and lower costs are involved, compensating the larger cost of ad hoc testing machines
- More information is gained because testing one long element, say of length ks, is equivalent to testing k elements of length s.

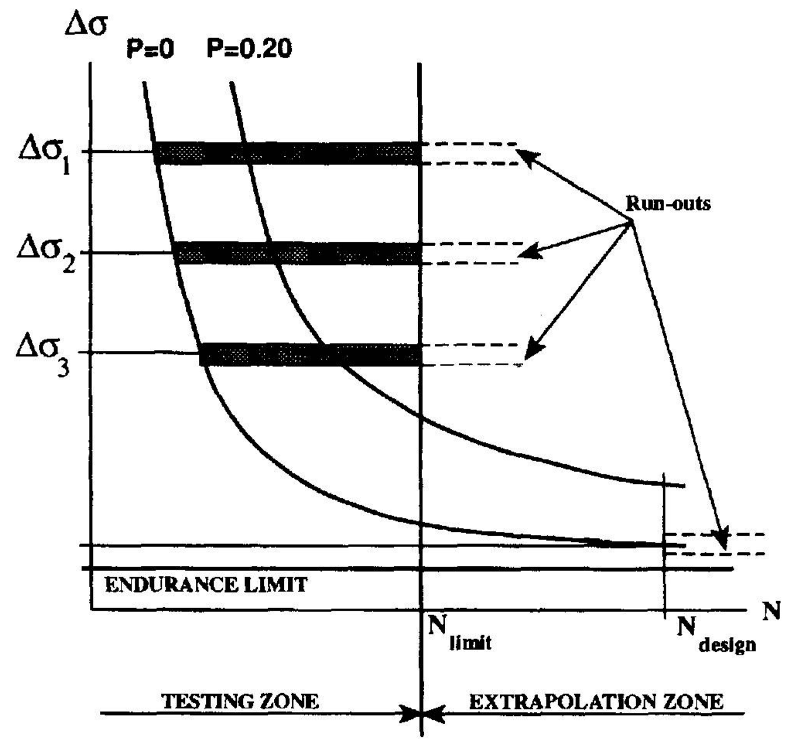

Figure 13: Illustration of some problems associated with the estimation of the zero-percentile curve.

- The threshold value associated with the asymptotic independence assumption is more likely to be overcome.

9. COMPOSED ELEMENTS

When dealing with composed elements we usually work with two variables: the number of wires, m, and the length, usually denoted by n, i.e. the number of times the element contains an elementary piece of given length, s_0 (see figure 14).

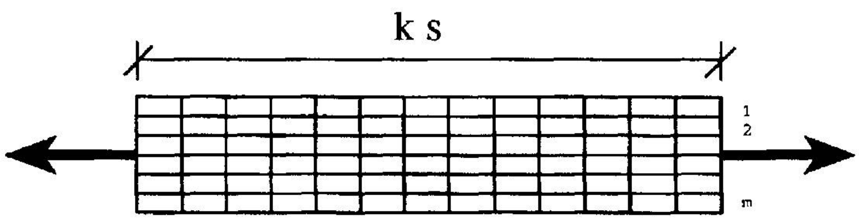

Figure 14: Composed element of m wires and length ks

Due to the presence of these two parameters, several limit distributions can be analyzed:

1. Constant n and very large m. This case is not useful for the fatigue problem.
2. Constant m and very large n. This case can be useful for fatigue analysis.
3. Very large n and m with m/n=k. This case has sense for fatigue analysis.
4. Very large n and m with m/n=0. This case has sense for fatigue analysis.

Cases (2) and (4) lead to Weibull or Gumbel models and model (3) leads to normal models. Thus, a discussion on the validity of the assumptions implied by models (2) to (4) is relevant for fatigue analysis.
The single element model is also applicable to the case of composed elements. We only need to test these elements and fit one of the above models. The results of the experiments will take into account the structure of the bundle of wires. If we have easy access to these kind of tests there is no sense in modelling a bundle based on the properties of single wires because many more sources of error are present in this approach, as for example, the effect of fretting.

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Fatigue Testing of Wires and Strands: Test Procedures and Experimental Studies

Essai à la fatigue de fils et de torons: Méthodes d'essai et études expérimentales

Ermüdungsprüfung von Spanndrähten und Spannlitzen: Versuchstechnik, und -ergebnisse

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SUMMARY
A survey is given, which is more related to material and tests than to theory, on the subject of size effects and the fatigue behaviour of prestressing steel. Possibilities of life determination are indicated and the presentation of test results is discussed. Some comments and recommendations are given for the experimental performance of these fatigue tests and possible test strategies are discussed. New test results are shown shortly and porposals for future investigations are given.

RÉSUMÉ
Le présent rapport traite des effects de la dimension des éprouvettes sur le comportement à la fatigue de l'acier de précontrainte. Il aborde les aspects pratiques, tels que "matériau" et "essai", par opposition à une approche théorique. Les durées de vie probables sont indiquées et les résultats d'essais sont présentés. Quelques commentaires et recommandations sont données sur la valeur expérimentale des essais de fatigue et sur des programmes éventuels d'essai. Des résultats récents sont présentés et des recherches futures sont proposées.

ZUSAMMENFASSUNG
Das Ermüdungsverhalten von Spannstahl wird in einem Überblick vor allem bezüglich der Auswirkung der Prüflänge dargestellt. Diese Darstellung richtet sich weniger auf die Theorie als vielmehr auf die Praxis der Versuchstechnik und auf das Werkstoffverhalten aus. Möglichkeiten für eine Berechnung der Lebensdauer werden kurz angesprochen und die Darstellung von Versuchsergebnissen diskutiert. Hinweise und Empfehlungen zur Versuchstechnik werden gegeben und Strategien zur Ausführung derartiger Versuche diskutiert. Neuere Ergebnisse von Ermüdungsversuchen an Drähten und Litzen unterschiedlicher Prüflänge werden mitgeteilt und Vorschläge für zukünftige Untersuchungen unterbreitet.

INTRODUCTION

In nature and in technology two basic terms are used to describe the geometry of a body: shape and size.

The shape is given by the ratio of the lengths observed, whereas the size is determined by their absolute values. It is astonishing that the Latin word "ratio" not only means "proportion", but also, in the figurative sense, "reason". Human reason relates intellectual values to each other.

Within the scope of this workshop the influence of size, more precisely length, on the fatigue strength of high strength prestressing wires and strands will be examined.

From practical experience we all know that the fatigue strength of ropes and tendons is primarily determined by the fatigue properties of the anchorages. The fatigue strength of the anchorage thus limits the loading permitted in service. The fatigue strength of the free length of a pretensioning element is usually considerably higher.

There are two reasons why it is necessary and worthwhile to investigate the effect of length on the fatigue strength of prestressing steels:

- the quality testing of prestressing steels
- the theoretical treatment and description of the length effect with the aid of statistical laws.

The procedure for quality testing prestressing steels is laid down in national and international standards. The dependence of fatigue strength on the test length is little known today and is hardly ever treated in the standards. Thus the specification of a required fractile with a certain confidence level only makes sense when a corresponding minimum test length is also stated. This fact is insufficiently anchored in the heads of the specialists who create the standards.

The practical engineer can only handle an effect such as the size effect on fatigue strength when the relevant laws are expressed in a formula and, better still, illustrated in easily interpreted diagrams.

Within the scope of this workshop, both topics will be dealt with and discussed.

HISTORICAL BACKGROUND

Timoshenko [1] has traced back the history of materials strength to the times of the ancient Egyptians, Greeks and Romans. Knowledge of the size effect goes back at least as far as Leonardo da Vinci (ca. 1500 AD), who made experimental studies on the dependence of the (static) strength of iron wires on their length and observed that long wires are weaker than short wires of the same diameter. Chaplin [2], around 1880, applied the weakest link theory to the effect of length on the tensile strength of metal bars of constant cross-sectional area.

Among the earliest authors to discuss the size effect on fatigue were Peterson [3] in 1930, Weibull [4] in 1939 and Aphanasiev [5] in 1948. The statistical theory of extreme values (weakest-link theory) plays an important role in studies of the size effect on material strength. Weibull [6] also used extreme value theory to give the first reasonably satisfactory explanation of the volume effect on material strength.

Freudenthal and Gumbel [7] employed the Weibull distribution to describe the life of fatigue loaded components. Freudenthal gives in [8] a starting point for taking into account the volume of highly loaded materials. Castillo et al. [9] showed theoretically that for fatigue results under constant amplitude loading, the Weibull distribution is the only distribution that meets the requirements of stability, compatibility and limit conditions.

A comprehensive literature review up to the year 1976 on the subject of "size effect" was carried out by Harter [10].

In more recent times it is above all the works of Heckel and co-workers [11-15] that deal theoretically and experimentally with the size effect on fatigue strength.

EFFECT OF LENGTH, EFFECT OF SIZE

The length effect is a part of the size effect and this may be divided up, according to Kloos [16], into the following basic effective mechanisms (Fig.1):

- geometric aspect
- technological aspect
- statistical aspect

A complete decoupling of these size effect mechanisms is experimentally impossible. By holding various parameters constant, however, the influence of the individual mechanisms may be observed separately under certain conditions.

GEOMETRICAL EFFECT

In this category belong all the influences that are based on the geometrical differences between the components. The geometrical aspect of the size effect may be explained with the aid of the different stress gradients in the components and with the help of the conceptions macro and micro supporting effect created by Neuber [17].

The geometrical aspect includes all effects that cause a change in the crack growth rate and in the final crack length during the fatigue life. Hence this effect shows up essentially only after the crack initiation stage.

TECHNOLOGICAL ASPECT

The technological influence is based on the effects of size, shape and distribution of crack nuclei in the volume of the material on the crack initiation and propagation. During mechanical and thermal treatment, namely, cross-section and volume dependent changes of this kind often arise in the material.

Effects caused by mechanical and thermal surface treatment processes with elements of differing size (e.g. differing extents of residual stress fields or hardening zones) must also be included in the technological aspect.

Technological effects may influence the crack initiation and propagation phase. With components of differing size they lead to differences in the life. With parts of equal size that were fabricated from different regions of a larger material volume, they lead to different amounts of scattering in the life.

STATISTICAL EFFECT

The statistical size effect enables one to explain the scatter in the number of cycles to rupture in fatigue tests on completely (ideally) identical samples.

If one considers, for instance, a long wire under repeated loading in tension, then with appropriate loading at a certain point, namely at the crack nucleus with the least resistance (weakest link), a crack is initiated that finally leads to rupture. If both fragments are further tested with the same loading, then further cycles are possible before failure, i.e. the life of the two fragments is greater. Continuation of this procedure yields for the fragments the same scatter in the number of cycles to fracture as one would have obtained by dividing the wire at the start into a number of short lengths and testing these separately.

The most sensitive crack nucleus observed on the long wire has an upper limit to its cracking susceptibility. The susceptibility to cracking can only be of such a magnitude that the wire does not already rupture during the fabrication process as a result of the loading inherent in the process (e.g. during drawing). One may therefore expect long wires to show a low number of cycles to rupture with low scatter. The shorter the wire, the higher the mean number of cycles to rupture and the higher the life scatter.

The effect of the statistical aspect with a randomly shaped component in a condition of inhomogeneous stress may be described, according to Böhm [12], with the aid of the so-called stress integral. Since the crack initiates preferentially at the surface, the stress integral is usually calculated for the surface. With the dimension of an area, it represents a measure for the size of the highly stressed surface. With increasing size of the highly stressed material surface, the probability of the existence of larger crack nuclei grows accordingly. These lead to shorter lifes.

If one tests wires and strands of the same geometrical dimensions that originate from the "identical" material and "identical" fabrication, then the importance of the geometrical and technological aspects is very much reduced and the effect of the statistical aspect may be examined almost without hindrance. Identical geometrical dimensions also mean that the number of cycles to rupture may be used as a clearly observable measure for comparison, and not the technical crack, which is difficult to find.

PHASES IN FATIGUE BEHAVIOUR

Under cyclic loading, changes of state occur in the material from the first cycle onwards. If flaws that may be designated as cracks do not exist at the beginning, then hardening and softening processes lead initially to microscopic fatigue cracks. Inhomogeneities such as inclusions, phase boundaries, etc. also lead to the formation of crack nuclei on account of the stress concentrations caused by them. Fatigue cracks arise mainly at or just below the surface, since at this location (a) the highest loading often exists (bending, torsion, notch effect), (b) the attack of any corrosive media takes place, (c) a certain surface roughness exists or (d) surface flaws or damage have arisen during fabrication or transport. In multi-component systems like strands, the fatigue behaviour is mostly determined by the response of the contacting surfaces to fretting [21].

With continued cyclic loading, microcracks may propagate on account of their notch effect. The so-called microcrack growth takes place within the order of magnitude of a few grain diameters. A precise distinction between crack formation and microcrack growth is impossible. During the phase of microcrack growth, the propagation rate is still very strongly influenced by the immediate environment. If the crack leaves the region dominated by the material microstructure, the further growth may be calculated by the methods and laws of linear elastic fracture mechanics, provided that the correction functions for the stress intensity are available. The corresponding crack size is termed the technical crack. From this size onwards, the material behaves quasi homogeneously. If the crack has attained a size that is critical for the momentary loading, then final fracture occurs as a result of unstable crack propagation.

The life of cyclically loaded components may thus be divided up into four sections:

- Crack-free phase
- Formation of one or more crack nuclei and microcrack growth

- Stable crack propagation
- Unstable crack propagation (final fracture)

The typical scatter in the life of cyclically loaded samples and components may be explained by the irregularities in the material microstructure or in the size of the crack nuclei originally present. Depending on the local loading, cracks that may possibly develop further to a microcrack occur at various times because of randomly distributed flaws with varying influences. The actual (stable) crack propagation from technical crack up to failure gives rise to little scatter with components of identical geometry and fabrication under constant test conditions.

FLAWS AND LIFETIME CALCULATION

From the efforts to be able to deal with the fracture mechanism in cracked components arose the discipline of fracture mechanics, which is also the basis for calculating the propagation behaviour of cracks. The flaws distributed in the material may be regarded in their effect as ficticious initial cracks. The possibility thus arises of assigning to each flaw, according to its sharpness, a certain crack length. The flaws are then represented by a distribution of their crack lengths. Moreover, it is not the basic distribution of all cracks that is characteristic for the life of a cyclically loaded component, but the distribution of the largest cracks, since these are responsible for the later fracture. According to Schweiger [13], the distribution of the largest initial cracks is usually expressed in the form of an exponential function:

$$F(a) = \exp\left[-\left(\frac{a}{a_v}\right)^{-c}\right] \qquad (1)$$

The parameters a_v and c are measures of the absolute size and scatter, respectively, of the maximum crack lengths (Fig. 2).

According to [18], fatigue failures in cold drawn wires were always initiated at surface flaws with depths between 30 and 100 µm. These cracks were probably produced during the drawing process [18]. If cracks are already present, the initiation period may be neglected and the life calculated by integration of the Paris Law [19]:

$$\frac{da}{dn} = C(R) \cdot (\Delta K)^{m(R)} \qquad (2)$$

where a is the crack length, da/dn is the crack growth rate per cycle, ΔK is the stress intensity range and C(R) and m(R) are experimental coefficients dependent on the stress ratio R. Both the coefficients C and m are found to be independent of crack depth, mean stress and frequency [20].

For crack growth rates below 10^{-9} m/cycle, a threshold value of the stress intensity range ΔK_{th} has been observed. The threshold stress intensity range is dependent upon the stress ratio R and the following empirical relationship was found [20]:

$$\Delta K_{th} = 5.54 - 3.43\,R\,[\mathrm{MPa \cdot m^{1/2}}] \qquad (3)$$

During the fatigue of strands, the formation of crack nuclei is primarily caused by fretting corrosion [21]. On a seven-wire strand with a centre wire and six outer wires, the line contact length is twelve times the strand length: six contact lines between the outer wires and six between the outer wires and the centre wire. Whether and to what extent local compression and displacements occur along the line contacts depends primarily on the diameter and relevant tolerances in the outer and centre wires. It is known that the fatigue strength of components that are in contact with each other is considerably reduced. Patzak [22] states that the fatigue strength of components subjected to fretting corrosion may be reduced to as low as 10 % of the value obtained without fretting. Hence the reduction of fatigue strength by fretting corrosion may exceed that caused by other factors, such as stress concentrations as a result of the notch effect in the anchorage.

According to DIN 50900 [23], fretting corrosion is understood to be the damage of materials which contact each other under a normal force and execute oscillatory movements of very small extent, i.e. ca. 0.1 to 300 µm. In the contact surfaces, mechanical and physicochemical processes work together, whereby the chemical processes are initiated by the mechanical. When at least one of the contact pairs is a metal, oxidation phenomena occur in the fretting regions. Because of the small fretting movements, the wear particles arising, which as oxides have a volume 2.2 times that of the metal, can leave the fretting region only slowly or not at all. In the friction loaded surface, local stress peaks arise that are superposed on the cyclic loading, which may give rise to premature cracking. Since the notch sensitivity of steel increases with increasing tensile strength, the loss of fatigue strength in high-strength steels is especially marked owing to the sharp notch effect of the cracks. In the fatigue loading of steel components subjected to fretting corrosion, it must be additionally taken into account that even with small stress ranges, fractures occur after two million load cycles. An infinite life range in the classical sense does not exist [24]. According to [24], the fatigue strength decreases with increasing compression and increasing relative displacement in the µm range. A general statement on the influence of these two parameters, however, appears to be difficult, since it is the combination of these values existing in a specific case that is decisive. With regard to the effect of the loading frequency, the majority of investigators [25] state that the values in the finite and infinite life ranges decrease with falling frequency. In tests [25] the fatigue strength at 2.9 Hz was 20 % lower than at 50Hz. In the low frequency tests, ca. 20 % higher fretting coefficients arose at low cycle numbers than at high frequency. The influence of frequency with fretting corrosion may thus be explained by the fact that at low frequencies the time dependent corrosion effect is of higher importance, and on the other hand, that the fretting indices are increased by ca. 20 % though oxide formation.

GRAPHIC REPRESENTATION

The results of fatigue tests with constant amplitude loading are summarized in the so-called S-N diagram (Wöhler diagram). The scales on both axes are normally logarithmic. On the abscissa, the life is plotted as a number of cycles. The ordinate represents the loading, in the case of prestressing steel, usually the range $\Delta\sigma$ of the stress. The failure criterium laid down is either a previously defined crack, usually between 0.1 and 1 mm in size, or the fracture of the sample, here the rupture of a single wire. In one-step Wöhler tests on steel, a pronounced limiting number of cycles appears, which depends on the loading and the environment. For steel under normal laboratory conditions, a value of two million cycles is reported.

The not insignificant scatter of life and strength values on fatigue loading is generally taken into account by statistical evaluation. Apart from S-N curves for a failure probability of 50 %, one often finds additional curves for 10 and 90 %. The scatter of test results on a load level is characterized with the aid of various distribution functions. Buxbaum [26] names as the five most commonly used functions:

- Normal distribution
- Log normal distribution
- Linear exponential distribution
- Weibull distribution
- Arc sin $\sqrt{P}$ transformation

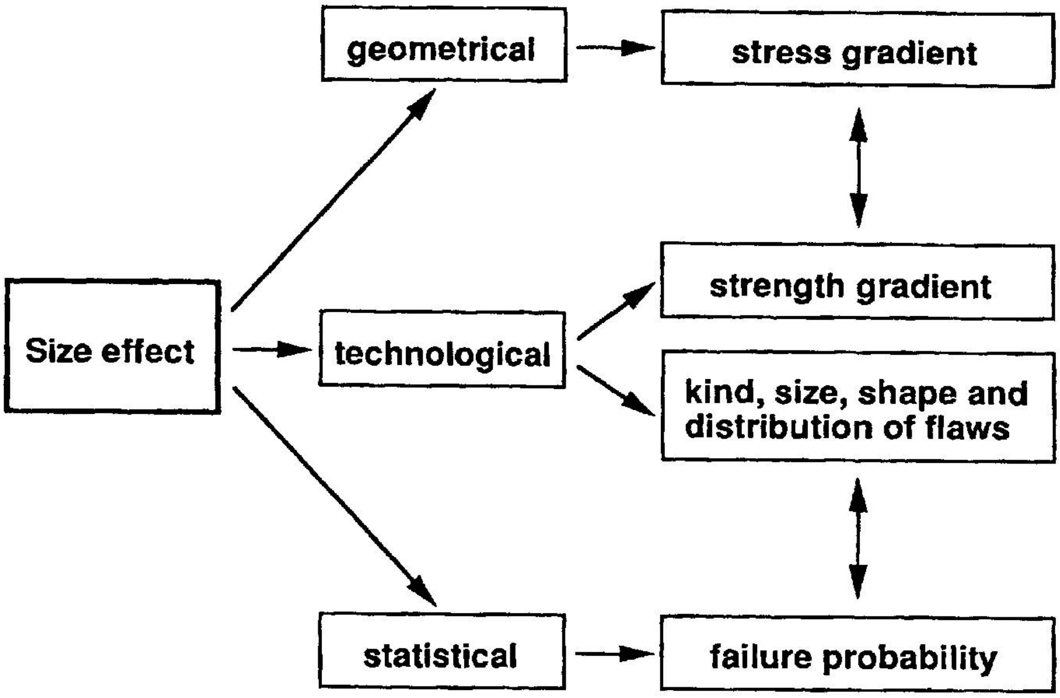

Fig. 1: The different aspects of the size effect, after Kloos [16]

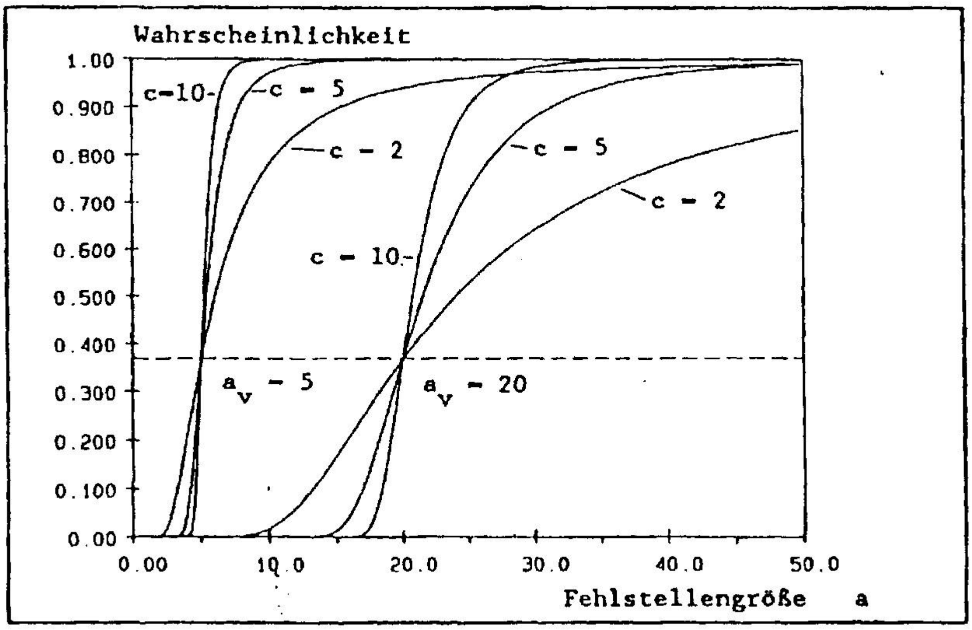

Fig. 2: Distribution function of the initial crack length for different parameter values, after Krä [15]

By suitable choice of the axes, a probability grid may be constructed for a distribution.

A failure probability may be assigned to the fatigue lives arranged in the order of increasing value according to

$$P = \frac{m}{(n+1)}, \qquad m = 1,2,...,n \text{ where} \qquad (4)$$

m - is the ordinal number and
n - is the total number of tested samples per load level

This estimate has proven itself in numerous works [13]. If one plots the pairs of values obtained in this way (failure probability and fatigue life) on the probability grid, then one obtains an approximately straight line. The statistical parameters of the distribution taken as a basis may be determined from the slope and position of this line. Although the Weibull distribution must be regarded on theoretical considerations [9] as the distribution valid for the fatigue tests of the given kind, it finds only slow acceptance in the practice of test evaluation. It may seem astonishing to the statistician that the engineer is more ready to think in terms of mean and standard deviation than in characteristic fatigue life and Weibull slope.

According to [9] the description of the S-N diagram may be given by the following relation

$$E\ (N, \Delta S, L) = 1-\exp\left[-\frac{L}{L_o}\left(\frac{(N-B)\ (\Delta S-C)}{D} + E\right)^A\right] \qquad (5)$$

N - number of cycles, ΔS - stress range, L - proof length

The five parameters have the following meaning:

A: Weibull slope parameter
B: asymptotic N-limit
C: endurance limit
D: scale fitting parameter obtained for an arbitrarily chosen reference length L_0
E: constant defining the S-N threshold curve below which a zero probability of fatigue failure exists

The percentile curves can be obtained by making E equal to a constant failure probability P. This model will be the base for serveral evaluations of tests and for a special computer program reported elsewhere during this workshop.

EXPERIMENTAL PERFORMANCE OF TESTS

Fatigue tests on wires and strands are mostly carried out on equipment with hydraulic or electromagnetic drive, less frequently with mechanical resonance drive. With the hydraulic equipment, a distinction must be made between volumetrically and servo-hydraulically controlled tests. Since smaller equipment of the same capacity generally costs less, runs faster and consumes less energy, these machines, and hence short test lengths, are normally considered to be advantageous.

The effects of the test and measuring equipment on the results of fatigue tests is generally underestimated. Taking this into account, it becomes possible to evaluate deviations of the experimental values from those predicted by models.

In the following we shall first deal more closely with the accuracy and the checking of test loads. With one step fatigue tests, one assumes that the maximum and minimum values of the cyclic loading remain constant, even over long test durations and millions of cycles. The limits to this "constancy" are given for a specific test apparatus by the short and long term accuracy of the cyclic loads. Fatigue tests on wires and strands represent here a rather special case, since with relatively high preloading, a high precision in the relatively small range is required. This is based on the fact that primarily the range and only secondarily the size of the "static" component of the load determines the fatigue life. If one assumes, for example, that on a test apparatus the maximum and minimum values of the required loading can be maintained with an accuracy of $\pm$ 2 %, whereby this figure must be regarded as absolutely realistic, then for a maximum stress $S_{max} = 0.7\ S_{ult}$ and a range $\Delta S = 0.2\ S_{ult}$, there results an accuracy of $\pm$ 12 % for the range, e.g. $\Delta S = 350 \pm 42$ N/mm^2. S_{ult} marks the ultimate strength of the material. This possible deviation should also not be forgotten when the results of fatigue tests show scatter. That the limits of the test loading, apart from this, also vary from cycle to cycle and hence strictly speaking a fatigue test is executed with a range that varies randomly from cycle to cycle must also be taken into account. Hydraulically driven test equipment in general, but particularly those which are volume controlled, react sensitively to changes in ambient temperature with changes of the test load. For this reason, unwanted load changes must be expected with a day/night cycle in the case of tests of longer duration.

In order to obtain an idea of the magnitude and accuracy of the real test loads, it is recommended to measure these in series with the fixed sample clamp by means of a load cell. The control of hydraulic test apparatus by means of pressure transducers in the hydraulic system can lead to considerable error, especially at high frequencies and with large moving masses.

The clamping of samples in the fatigue machine is always one of the critical points in a test. Samples that fail in the clamping region must be rejected and merely cause costs without a result. What is understood by the clamping region is not laid down in a way that is universally valid. In the testing of wires and strands it has become the custom to regard ruptures occurring within one or two diameters of the wire or strand from the clamping position as belonging to the clamping region and to disregard these in the evaluation. It should be noted that with short test specimens, the clamping region takes up a relatively large proportion of the free length. If one assumes, for example, an effective clamping region of two diameters length at each end, then with a wire of 7 mm diameter and 150 mm length, these regions amount to already 19 % of the total length. The probability thus increases, that "normal" flaws, which come to lie in the clamping region, are wrongly rejected.

The actual clamping of the specimens may be effected by form fit or material fit or friction fit. In practice a combination of these principles is usually employed. Well-designed clamping systems for fatigue tests are characterized by the avoidance of stress concentrations in the sample as far as possible and measures to prevent fretting corrosion. The latter may be avoided by careful choice of contact pairs (the materials of sample and clamp), above all at the interface between the clamp and the free length of specimen. It should be pointed out that there is a possibility of making the specimen itself insensitive to clamping fractures, e.g. by introducing local residual stresses in compression by rolling the wire in the clamping region. Experience with, and details of the possibilities given and applied will be covered in a special contribution to this workshop.

TEST STRATEGIES

In fatigue tests it is the aim to obtain as much information as possible about the S-N field with as few samples as possible, more precisely: with as small a total number of load cycles as possible. The engineer is primarily interested in curves for low failure probabilities of high confidence level, as well as the asymptotic limits for the fatigue strength and the minimum number of cycles to rupture. Depending on the assignment, knowledge of the entire S-N field or, separately, the finite or infinite life ranges are of predominant interest.

It is only natural that tests in the infinite life range are evaluated only in the direction of the loading and based on test strategies that are suitable for the treatment of dual (binomial) events, such as the staircase method [27] and the Probit method [28]. Since the statistical validity of dual (binomial) events is considerably lower than that of metric (variable) events, such as the life-time in the finite life range, one requires considerably more tests in the infinite life range in order to obtain the same statistical confidence as in the finite life range. Because of the large number of cycles needed in the infinite life range, a further increase in the expenditure of time and money occurs.

The limit number of cycles in the infinite life range should not be set too high, even when occasional fractures are to be expected after the limit number of cycles. Such rare fractures depress the mean value only negligibly, but they make the tests considerably longer and more expensive. The preference here should be given to low limit numbers of cycles combined with an increased number of tests, in order to increase the confidence in mean value and scatter.

Acceptance tests in the sense of proving a certain failure probability at a prescribed confidence level for a certain loading are often carried out in the sense of binomial results, since an agreement on the distribution function to be employed is unnecessary. One stipulates, for instance, that in the fatigue test, none out of seven samples may undergo fracture up to a prescribed limit number of cycles. This strategy is unsatisfying, since the statistical validity of these binomial events, as mentioned above, is considerably lower than with metric events. In addition, during the entire duration of the test, no information is gained on the chances of success, and after the first rupture, a new test series must be started. Here it would be far more economical of time and money to agree to a higher loading level with higher failure probability and to agree to acceptable number of cycles.

The finite life range is generally investigated by testing specimens on at least two load levels. With a fixed number of samples, one today prefers to test many samples at few levels, rather than few samples at many levels. Whether this method brings an improved "statistical efficiency" or merely a simpler test procedure is not yet clear. In the finite life range, it is best to start testing at the highest planned load level, since there the results are more quickly available and from this position one can more easily estimate the load levels required for the further tests.

Since the practicing engineer is interested in fatigue data with low failure probability, test specimens should always be as long as possible [9], provided this is allowed by the test equipment. If one reduces the results in each case to a certain test length, e.g. 1 metre, then the results from testing long specimens become not only considerably more informative, but also more favourable with regard to the expenditure of time and money.

TEST RESULTS

The Swiss Federal Institute for Testing and Research (EMPA) has over the years carried out numerous investigations on the length effect with prestressing wires and strands. Since the results will be discussed in a special contribution, they will be shown here only in summary.

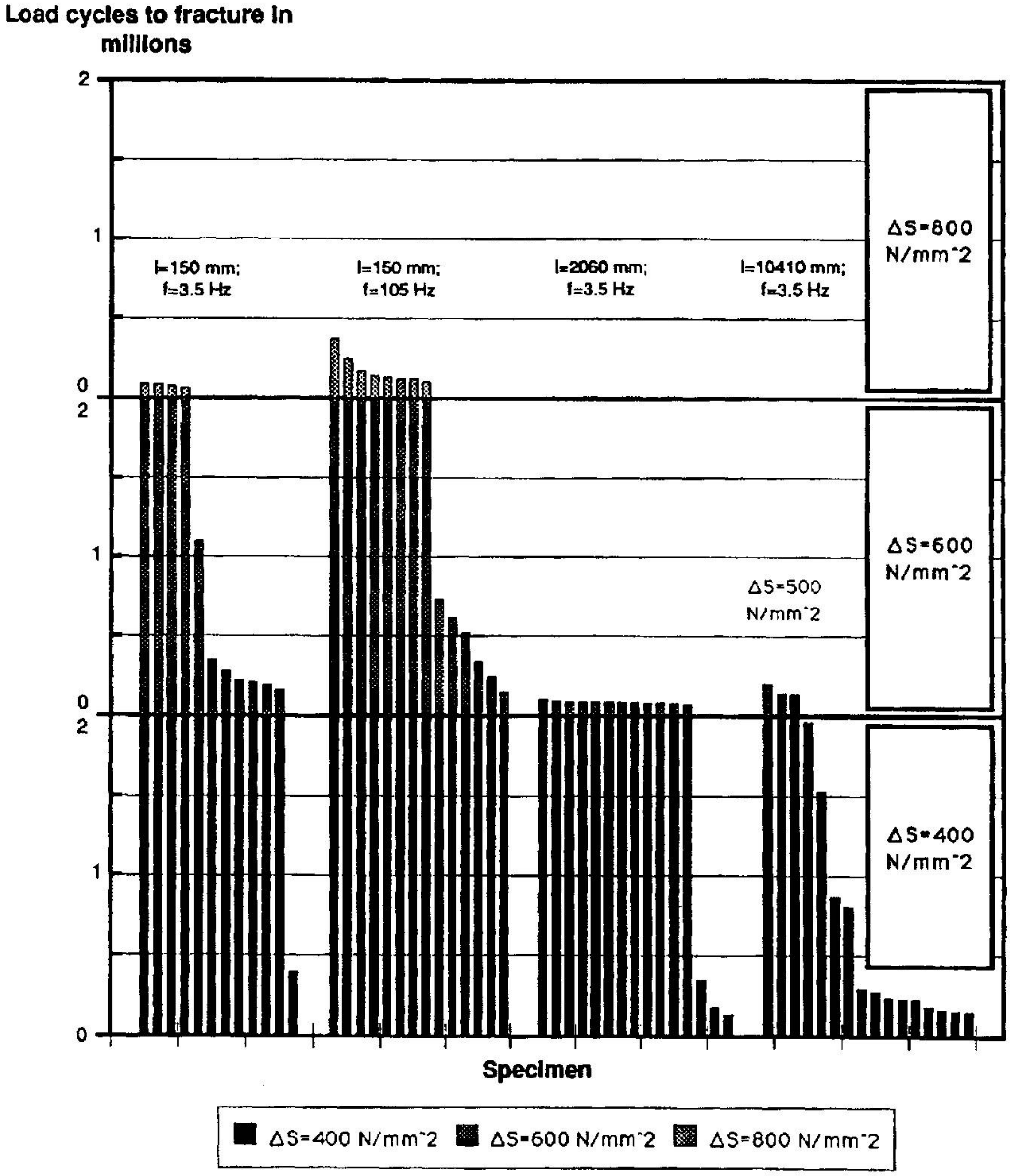

Fig. 3: **Summary of fatigue test results on prestressing wires for different proof length**

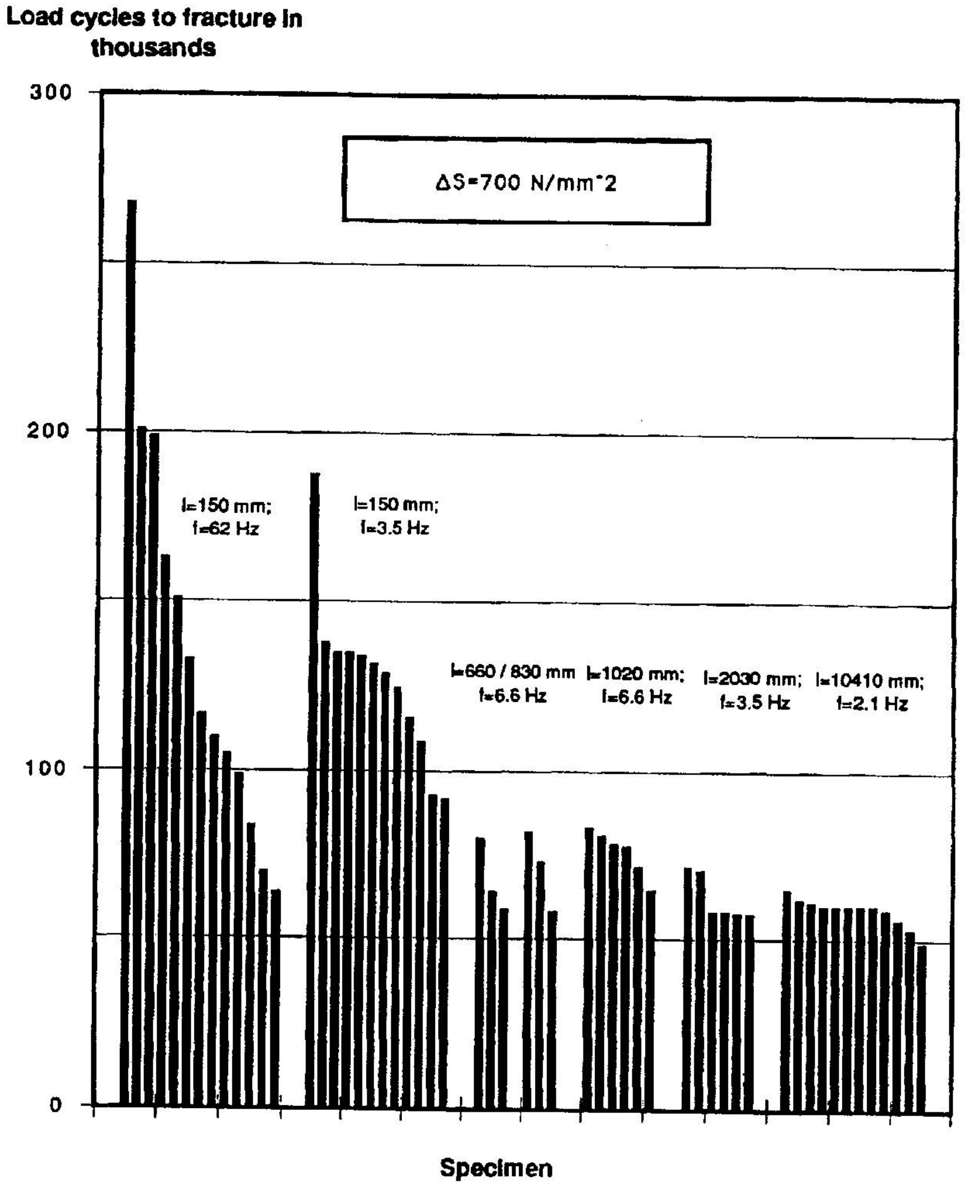

Fig. 4: Summary of fatigue test results on prestressing wires for different proof length

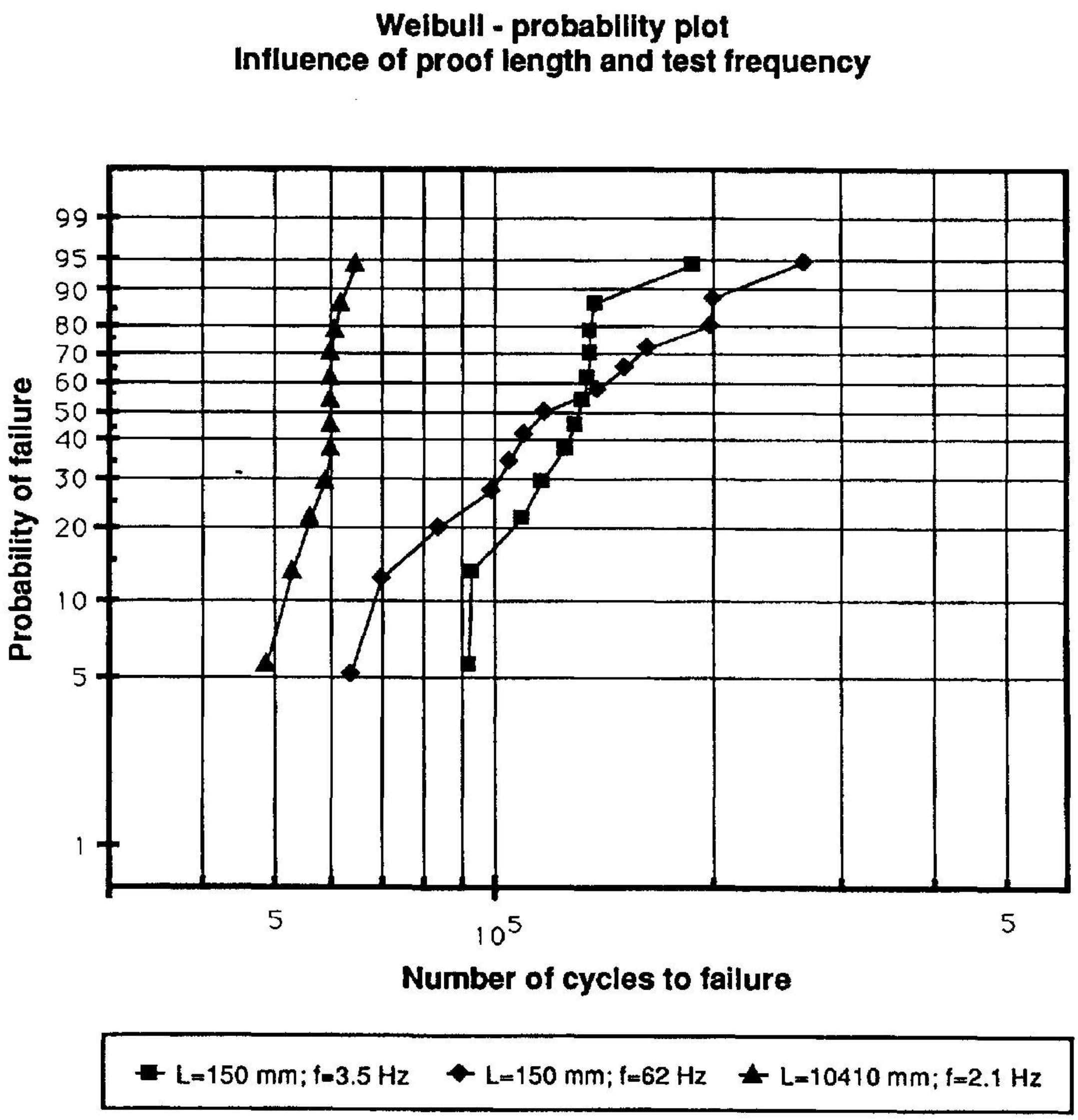

Fig. 5: Fatigue of prestressing wires

7 mm diam., stress range $\Delta\sigma = 700\ N/mm^2$

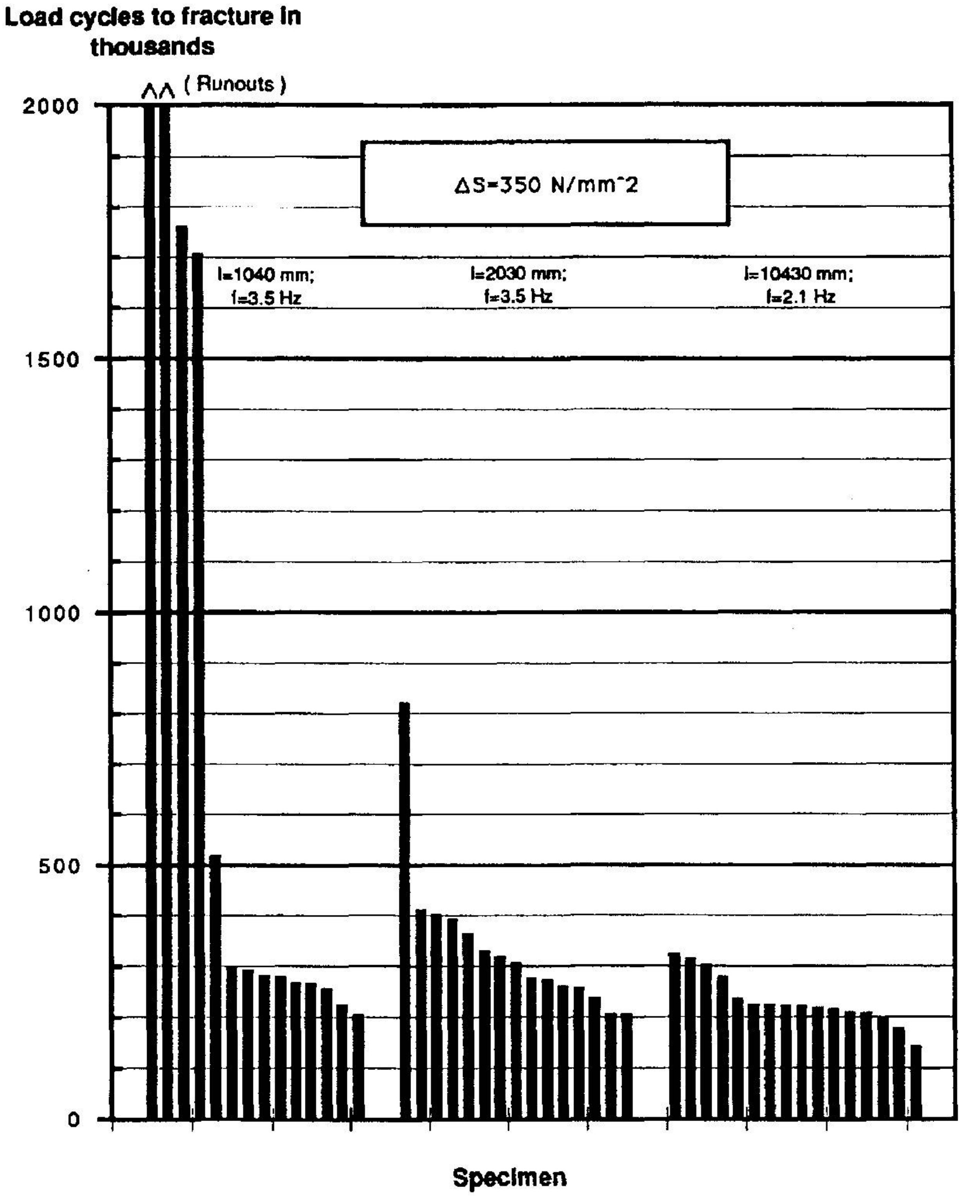

Fig. 6: **Summary of fatigue test results on strands for different proof length**

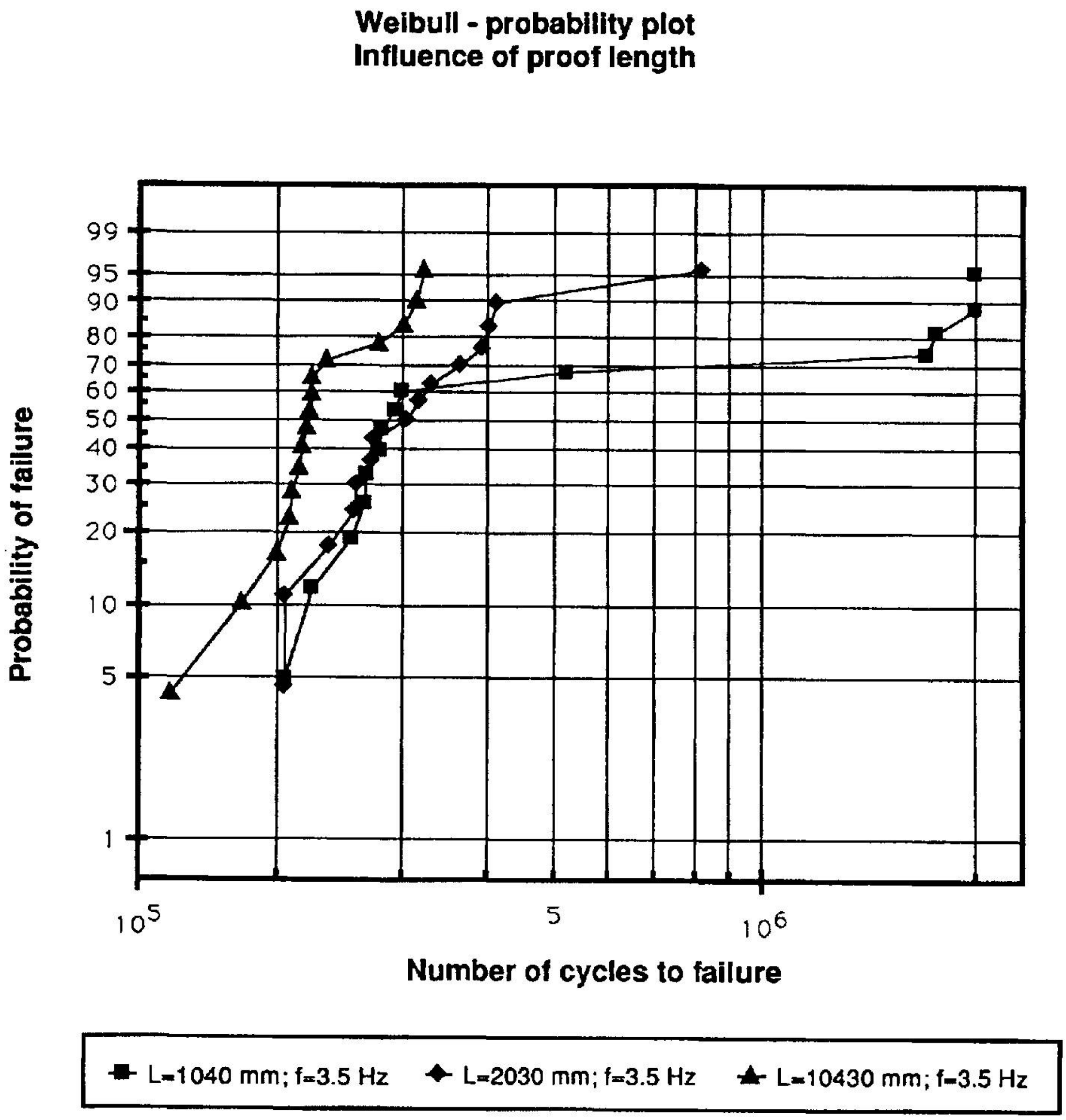

Fig. 7: **Fatigue of prestressing strands**
0.6 inch, stress range $\Delta\sigma = 350$ N/mm^2

WIRES

In Fig. 3 the results are shown for wires of between 150 and 10410 mm in length. For the 150 mm length, the test frequencies 3.5 and 105 Hz have also been investigated. In the tests, the maximum stress $S_{max} = 0.7\ S_{ult}$ was kept constant and with a limit number of cycles of two million, the stress range was raised from $\Delta S = 400\ N/mm^2$ over 600 to 800 N/mm^2. If one compares, at constant frequency, the number of cycles to rupture for the lengths 150 and 10410 mm, then one sees clearly the effect of the test length, i.e. shorter life with longer specimens. That such a length effect actually occurs also in the infinite life range with small stress ranges can ultimately be proved only by carrying out these time consuming tests to find the ratio of ruptures/total of tested specimens per level. With a stress range of $\Delta S = 400\ N/mm^2$, the results were for a test length of 150 mm 1 rupture in 12 samples, and for a test length of 10410 mm 13 ruptures in 16 samples.

Fig. 4 shows the results for the finite life range with a stress range of $\Delta S = 700\ n/mm^2$. The influence of the test length in the sense of larger means and bigger scatter at low test lengths and lower means and scatter with larger test lengths is clear. Plotting these results on the Weibull diagram (Fig. 5) confirms this observation. In connection with Fig. 5 there arise two questions: the model according to [9] predicts for differing test lengths the same Weibull slope, which is here not the case; it is at the present also unknown which phenomena cause a greater scatter with approximately the same mean at higher frequencies when the test length is kept constant and the frequency varied.

STRANDS

In Fig. 6 the results are plotted of fatigue tests on strands at a constant maximum load $S_{max} = 0.7\ S_{ult}$ and a stress range $\Delta S = 350\ N/mm^2$ for the test lengths 1040, 2030 and 10430 mm. The numbers of cycles to rupture shown here are for the rupture of the first wire of the seven wire strand. Although the influence of the test length is still discernable on comparing the numbers of cycles to rupture for 10430 mm as against 1040 or 2030 mm length, an increasing number of specimens show up here with decreasing test length that rupture only at higher numbers of cycles, Fig. 7. Apparently various damage mechanisms are present here. One could imagine that the importance of fretting corrosion as a crack initiating mechanism decreases with decreasing test length and hence at higher numbers of cycles to rupture, ruptures in the individual wires become effective that originate from "normal flaws".

SUGGESTED FURTHER WORK

- S-N diagram: ability to draw in curves with certain failure probability at a given confidence level. For the engineer, curves with low failure probability at high confidence level are important.
- S-N diagram: optimum conception and simple evaluation procedure for fatigue tests in the infinite life range with the goal of being able to state a load level with a low failure probability at high confidence level, with as few specimens as possible.
- Ability to determine, with non-destructive methods on wires, an equivalent initial crack length for flaws, which can lead via fracture mèchanics to an estimate of life.
- To gain more knowledge on the mechanism of fretting corrosion with strands in a greased or galvanized condition.

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Failure Mechanisms in Fatigue

Mécanismes d'endommagement en fatigue

Schädigungsmechanismen bei Ermüdung

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SUMMARY
The metallurgical and the structural engineering aspects of failure mechanisms and their effects are presented, which can lead to an early failure of tendons. The influence of the tendon-length was given only limited consideration. The deciding factors are disturbances ocurring at the clamps and the end anchorages as well as incomplete or damaged corrosion protection systems.

RÉSUMÉ
Les aspects métallurgiques et structuraux des mécanismes de ruine sont exposés, ainsi que leurs effets qui peuvent conduire à la rupture prématurée d'éléments tendus. L'influence de la longueur de l'élément tendu ne peut être prise en compte que de façon très limitée. Les éléments déterminants sont les désordres survenant au niveau de clavettes et des ancrages terminaux, ainsi que les systèmes de protection anti-corrosion incomplets ou endommagés.

ZUSAMMENFASUNG
Aus der Sicht der Metallurgen und des konstruktiven Bauingenieurs werden Schädigungsmechanismen und deren Auswirkungen geschildert, welche zu frühzeitigem Versagen von Zuggliedern führen können, wobei ein Einfluß der Zuggliedlänge nur sehr begrenzt berücksichtigt werden kann. Ausschlaggebend sind Störungen an Klemmpunkten und Endverankerungen sowie unvollständige oder zu Schaden gekommene Korrosions-schutzsysteme.

1. INTRODUCTION

High-strength materials, used today for bridging the ever increasing spans in the form of external pretensioning of concrete load bearing members or in the form of free tendons used for suspending or supporting bridge decks [1, 2], are thin fibres or metal wires which are combined into units of great load bearing capacity by means of sometimes complicated constructions. Since high tensile strengths can be produced economically only by small cross-sections (for example cold-drawing of steel, directed blending of plastics, quenching of glass), high-strength tendons with great loadbearing capacity have a large surface that has to be protected. n wires within one tendon have a n times larger surface than a cylindrical bar of the same total cross-section. Also, during bundling and stranding a lot of gaps and hollow spaces occur between the single elements (fig. 1) and the fibres and wires are less ductile and more sensitive to transversal stress due to their high-performance tensile strength. The surface quality and the impact of pollutants, which are intensified by the gaps and the unavoidable friction-motion are the main factors triggering fatigue fractures, which always start at the surface.

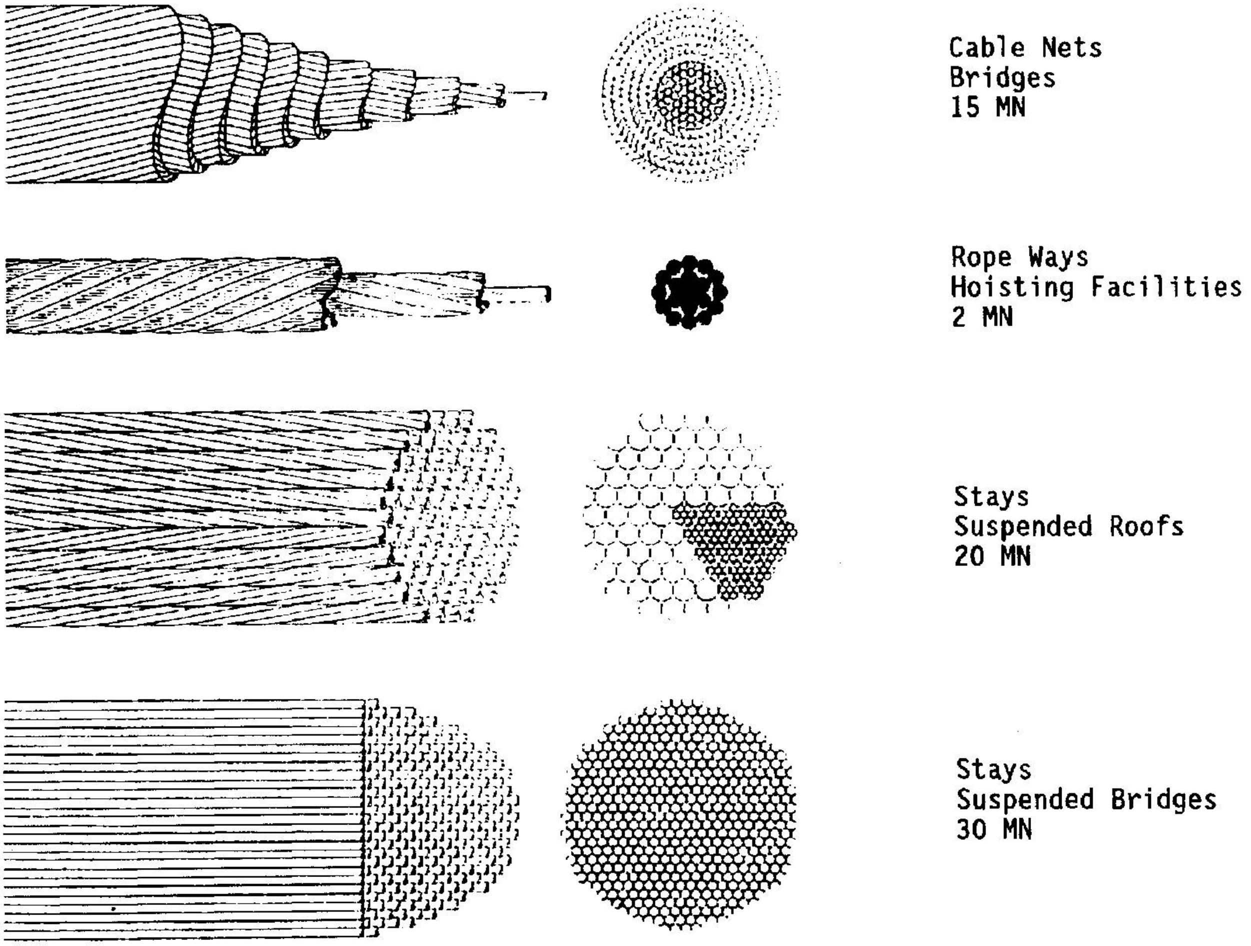


Fig.1 Cross-sections of various cable-constructions and their fields of application
- Locked coil rope
- Round strand rope
- Parallel strand cable
- Parallel wire cable

Building construction does not allow large free tendon lengths and the intensified impact of pollutants in the area of the many anchorages, clamps and deviations [3] places high demands on the protection of these details (fig. 2).

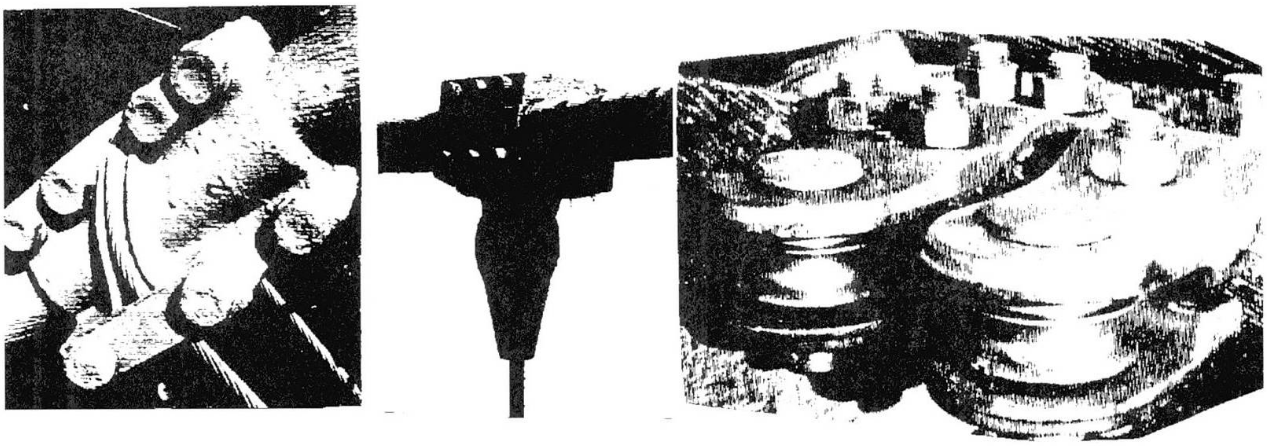


Fig.2 Clamped boundary-cables

Great cable lengths occur in case of suspension- and stay-cable-bridges as well as high, guyed masts. The selected load-bearing system also effects the security and the life expectancy, as does the form of the cross-section in the case of the large cables themselves. It is also relevant to security whether or not the selected construction of the cable should be redundant, sufficiently safe for inspection, and whether or not the parts should be replaceable, the wires and strands are bundled or should, as ropes, form a closed wire-system [4, 5].

The system can contain very long, free cables, which seem to be economical due to a small number of fittings (only 2 anchorages each), but they are subject to greater stresses due to wind- and traffic-induced deformations and vibrations. Therefore in the case of large bridge construction all variations ranging from the classical suspension-bridge-system to the simple stay-cable-bridge can be selected, according to the tasking and the marginal conditions [1] (fig. 3). This leads to a walk on the tightrope between the large, but easier deformable lengths and the increase in the number of fittings.

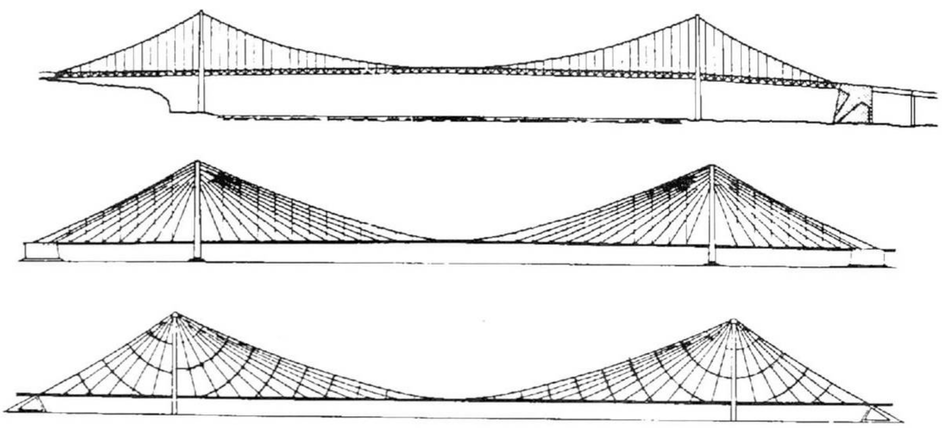


Fig.3 Loadbearing systems of large suspended bridges [1]

2. FATIGUE BEHAVIOUR OF WELL-PROTECTED SYSTEMS

2.1 Influence of material

According to mechanical and chemical influences, metal structural members as well as high-strength wires for ropes and bundles submitted to fatigue loading may undergo irreversible structural modifications. Based on the alternating loading, slip lines and slip bands may occur. Afterwards micro- and macro-cracks produced may extend and continue until fracture.

The properties of steel have an essential influence upon the fatigue behaviour [6-8]. For patented and cold-drawn wires out of unalloyed carbon steel, the fatigue strength increases by an increasing tensile strength until a cross-section reduction of about 80-86 % during drawing (fig. 4). For high-strength steel wires the surface quality has to be considered. Differences of surface roughness due to fabrication and surface imperfections (e.g. drawing marks) have a detrimental influence. Notch sensitivity rises by increasing strength. This is demonstrated in fig. 5 for the example of fatigue strength of smooth and ribbed prestressing steel wires of different strength.

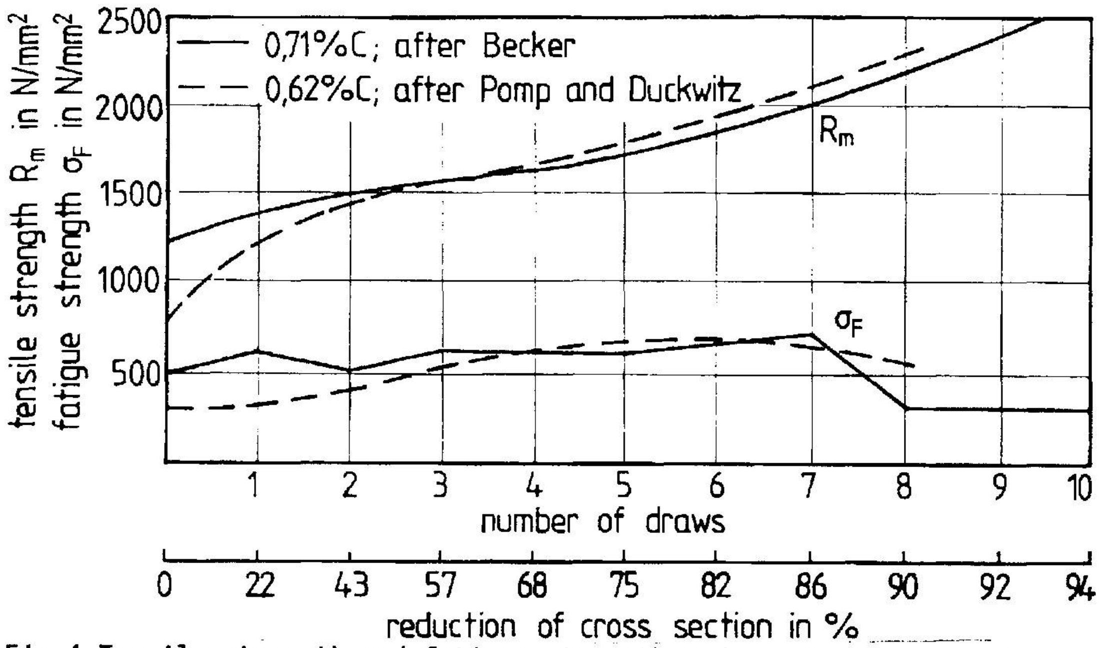


Fig.4 Tensile strength and fatigue strength under pulsating tensile stresses of cold drawn carbon steel wires [8]

For high-carbon steel wires with a degree of deformation > 65-75 % the fatigue strength is reduced by tempering [7]. Therefore, the non-heat-treated rope wires behave more favourably than the tempered prestressing steels. In [9] fatigue strength of 450 and 650 N/mm² were determined for non-tempered round wires with a strength of 1500 N/mm² and 1860 N/mm² respectively. Z-profile wires with a strength of 1470 N/mm² have a fatigue strength of 400 N/mm².

Strands and ropes have much lower fatigue strengths than single wires [7,9,11,12]. This is due to additional stress during roping, and friction between the wires (chapt. 3.4). Results of fatigue tests using unstranded galvanized wires, strands and ropes are shown in the Smith diagram in fig. 6 [11]. Worth noticing is the distance between the wire and the strand and also between the strand and the rope. The reduction of the fatigue strength of a wire increases with the wire strength during roping. Based on the results shown in [7], the following applies: If the strengths R_m are only 800-900 N/mm², the pulsating strength of the rope was only 0,25-0,35 R_m and for strength values of 1700-2000 N/mm², it is only 0,1-0,2 R_m.

Taking the dynamic behaviour into consideration, there is no advantage in using high-strength wires for the fabrication of ropes, as compared to low-strength wires.

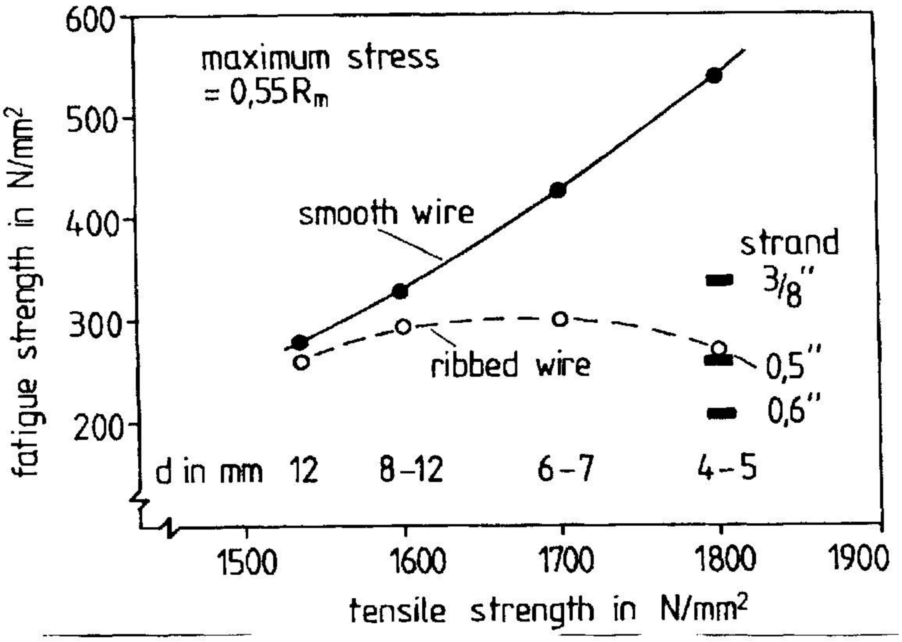


Fig.5 Fatigue strength of prestressing wires with smooth and ribbed surface and of strands (Nürnberger)

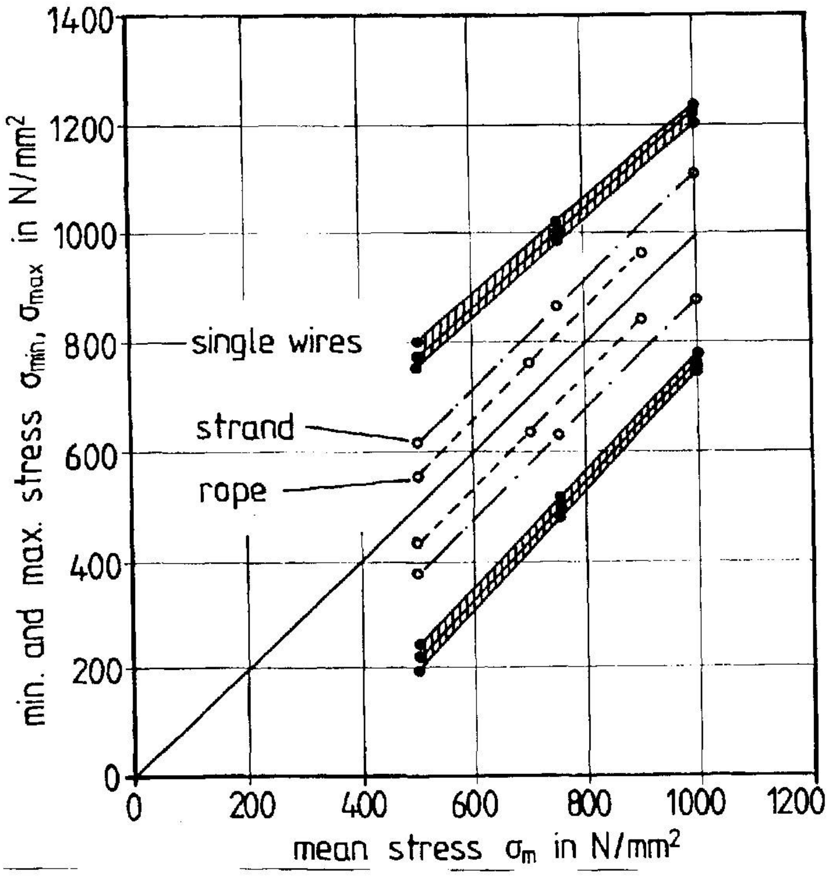


Fig.6 Fatigue resistance ability, Smith diagrams for ropes, strands (before roping) and wires (before stranding) (Becker) [11]

2.2 Influence of corrosion protection

Wires for ropes and bundles made of unalloyed steel could be protected against corrosion by metallic and/or organic coatings. The coating already improves the dynamic bearing capacity in the air.

The formation of a fatigue crack is influenced by physico-chemical interactions between the environment and the steel surface, activated by fatigue (chapt. 3.5). Not only liquids, but also gases and vapours may accelerate the deterioration process [13]. Dry air is already a surface-active medium and reduces the fatigue strength in comparison to the vaccuum. For high-strength steels the fatigue decreases by increasing the steam content in the air. Steel surfaces, activated by deformation, react with steam forming hydrogen which penetrates the steel and accelerates the formation of cracks as well as the crack propagation [14]. For the previously mentioned reasons, coatings impermeable to oxygen and steam (e.g. sufficiently thick reactive resins) improve the fatigue behaviour not only in corrosive environment (chapt. 3.5), but also in the air [13,15].

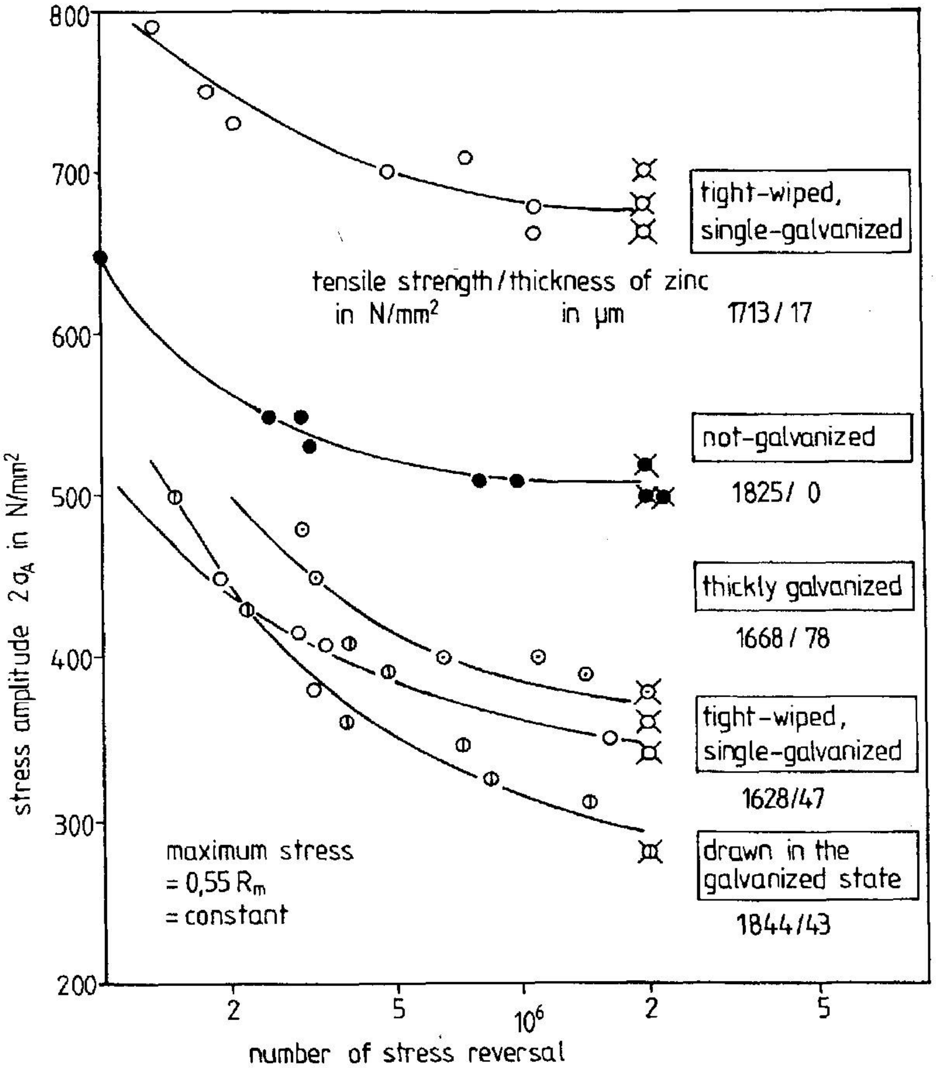


Fig.7 Results of fatigue tests of non-galvanized and galvanized cold-drawn and patented steel wires (tight-wiped of two fabrications) (Rehm, Nürnberger, Rieche) [17,18]

By galvanizing the wires, deterioration [7,16,17] as well as improvement [18] of the fatigue behaviour in the air may be established (fig. 7). An improvement of fatigue behaviour is due to:
- the sealing of the steel surface against the air,
- the thermal effect of the zinc bath which reduces the residual stresses.

An unfavourable effect is exerted by:
- the dipping of the steel surface, thus increasing the surface roughness as well as the amount of the absorbed hydrogen
- the notching effect of cracks in the iron-zinc-alloy layer (cracks result from handling and pulsation)

Electro-galvanizing may improve the fatigue strength (shielding effect), or the electrolytically released hydrogen may severely impair it [7,17].

3. INFLUENCE OF CHEMICAL ATTACK

3.1 Corrosion situation around tension members

Ropes and bundles (e.g. for the fastening of towers and bridges) are exposed to static/dynamic and corrosive influences. Corrosion promoting substances are water (high humidity, rain and condensation) and air (oxygen). Pollutants strengthen die attack. Extremely harmful sulphur dioxide is a result from burning fossile products. An industrial atmosphere contains up to 50 mg/m^3 of SO_2. After the oxidation and the reaction with water, sulphuric acid develops. The ph-content of atmospheric liquid substances, above all of dew and fog, may reach up to 3. Furthermore, the chlorides contained in de-icing salt-sprays are harmful. An oceanic athmosphere contains up to 0,1 mg/m^3. The corrosion of metals is generally enhanced by dust-deposits, which concentrate corrosive media and humidity.

3.2 Durability of organic and metallic coatings (galvanizing)

Organic coating

The coating has to separate the steel surface from the attacking media (passive corrosion protection). Plastics used as surface protection for metals show a certain permeability towards steam and oxygen. If the coatings are sufficiently thick, the resistance to diffusion is so high that corrosion underneath the coating may be neglected. As long as the protective system separates the steel wire from the surrounding area, no corrosion occurs.

According to [19], corrosive attacks are only possible if:
- the coating is not impermeable, e.g. pores exist,
- during construction the protective system is damaged, e.g. mechanically or by overexpansion,
- during operation mechanical effects (e.g. friction) or relative movements between the wires cause the coating to tear,
- under the influence of humidity and UV-light the coatings become brittle and tear under the mechanical loads or under the impact of temperature changes,
- permanent humidity causes the adhesive failure of mechanically sensitive coatings.

The following corrosive attacks may appear due to deteriorations of the coating (fig. 8) [20,21]:

Within the area of local defects oxygen-type corrosion may occur. If the coatings become (slightly) electrically conductive and are sufficiently permeable for oxygen (only in the case of thin coatings), corrosion will increase due to cell formation. The steel surface in the weak-point is the anode, and the coated steel surface is the cathode.

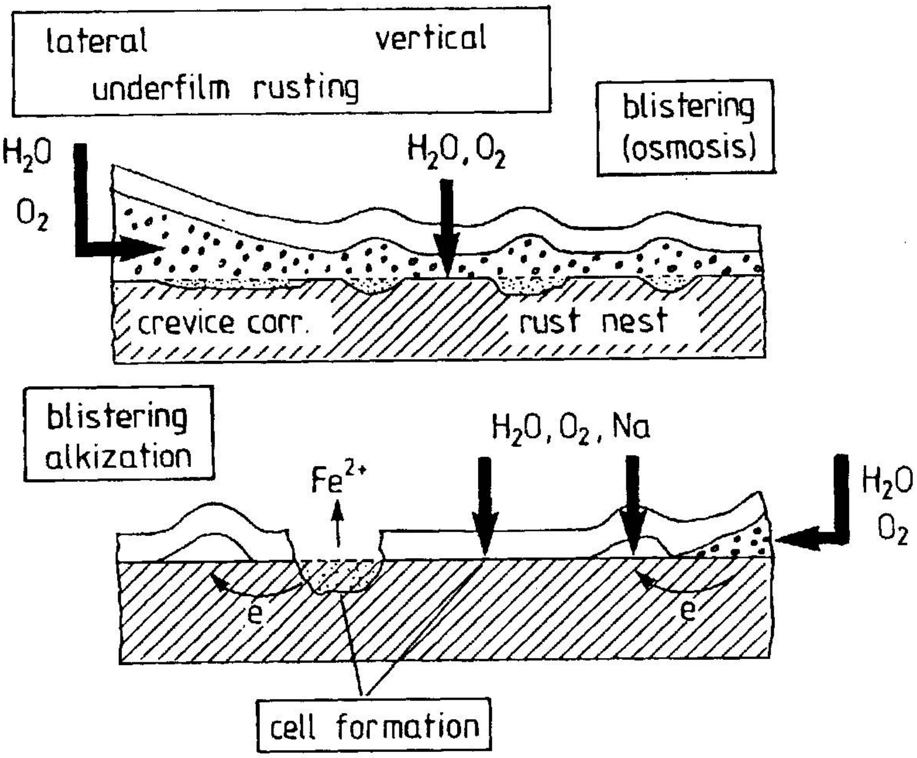


Fig.8 Possibilities of corrosion at coated surfaces

Near the defects in the coating or at the painting edges, an underfilm of rust occurs. Frequently it is supported by crevice-corrosion as a consequence of aeration cells. In the crevice, due to a hydrolysis and acidification of the liquid may occur, e.g.:

$$Fe\,Cl_2 + 2H_2O \rightarrow Fe\,(OH)_2 + 2\,H\,Cl$$

In the case of underfilm rusting from the boundary, lack of adhesion due to insuffi- cient pre-treatment of the basis (rust spots) has an especially unfavourable effect. If the attacking medium contains salt, the corrosion may be uniform or pit-shaped. Exclusively in the case of thin-coated steel in frequent contact with water, blistering occurs. These may burst open and start the corrosion. Blisters are the result of the cathodic reaction near corroding areas underneath the coating:

$$1/2\ O_2 + H_2O + 2\ Na^+ + 2e^- \rightarrow 2\ NaOH$$

Galvanizing

Zinc coatings on steel protect against corrosion because protective carbon layers develop under atmospheric corrosion conditions. Under weathering corrosions rates of 1-10 μm annually are possible, depending on the aggressiveness of the atmosphere [22]. If the surface is poorly ventilated (e.g. water accumulations at low points, in crevices, frequent condensation), the formation of passive film is hindered and the corrosion rate increases extensively [19,23].

For duplex systems a combination of galvanization and organic coatings is applied. Only alkaline-resistant coatings (zinc corrosion products have an alkaline reaction) adhere to zinc-coated surfaces, and in the case of older zinc-coated surfaces, the adhesiveness is improved. Problems may occur if the coating is permeable for steam, but prevents CO_2-diffusion. Under these conditions a voluminous corrosion product $Zn(OH)_2$ develops instead of the protective zinc carbonate. Consequently, zinc is removed quickly and the coating peels off. The adhesiveness of the coating is desicively reduced, if it has been applied to surfaces, polluted by the atmosphere. Zinc salts have hygroscopic qualities and represent centers of disturbance for the coatings.

3.3 Fatigue behaviour of corroded wires

Corroded wires have a lower dynamic bearing capacity than new ones. The notching effect, i.e. the type and frequency of corrosive attacks, have an impact [12,19,24]. In the case of pit-shaped attacks, the reduction of the fatigue strength is greater than in the case of mainly uniform attacks (fig. 9). In the case of a drawn wire, 0,2mm deep corrosion pits or a more uniform corrosion with a depth of 1mm cause a low fatigue strength similar to that of a new wire exposed to additional fretting corrosion.

3.4 Fretting corrosion and fatigue

Tests in the air showed that single wires have a much more favourable fatigue behaviour than strands and ropes manufactured out of them (fig. 6).

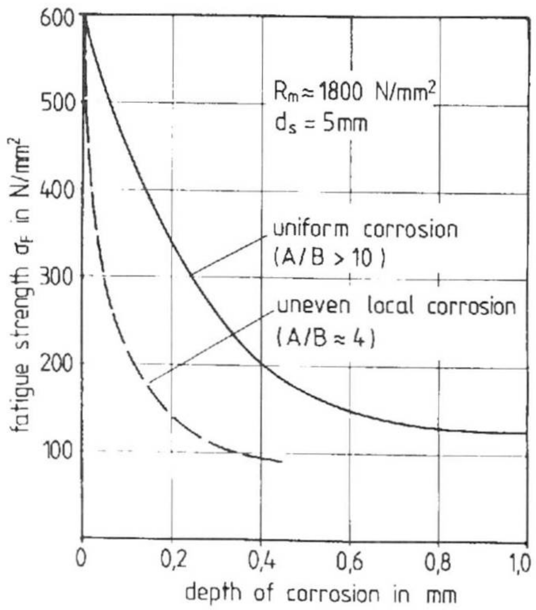
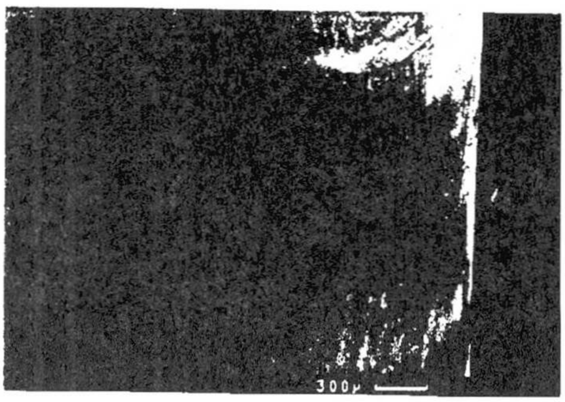


Fig.9 Fatigue strength under pulsating tensile stresses of corroded cold-drawn carbon steel wires
(A=diameter of corrosion, B=depth of corrosion) (Neubert, Nürnberger) [24]



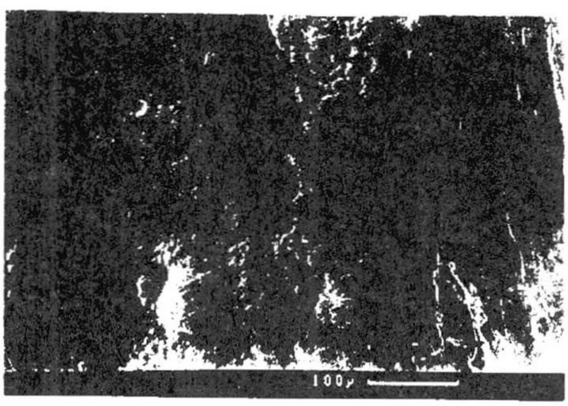
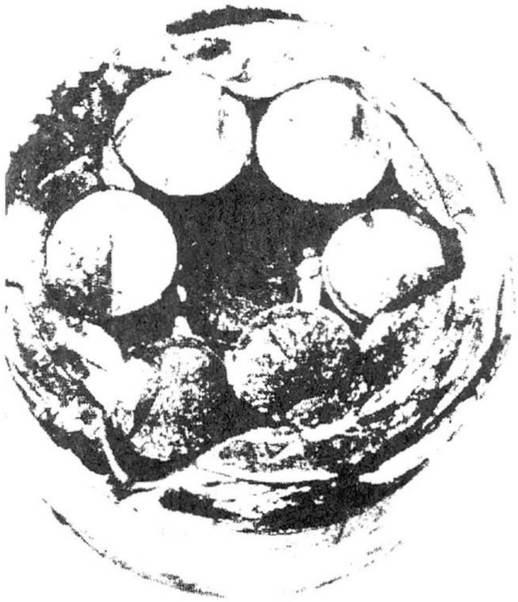


Fig.10 Fatigue cracks in the zone of fretting



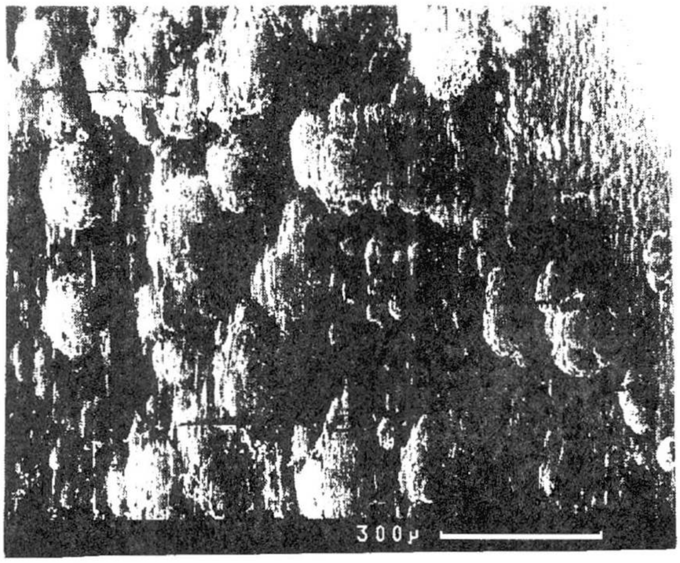


Fig.12 Pre-cracks at cold-drawn, unalloyed steel wire due to corrosion fatigue

Fig.11 Fatigue damage near fitting

In the rope, relative movements may occur between the wires. The result is a damage similar to the wear with a roughening of the wire surface. The surrounding air, i.e. the impact of an oxydizing medium, these friction surfaces are oxydized. Under fatigue loads, this fretting corrosion [25] causes additional tension stresses which constantly change their direction. Furthermore, the combination of mechanical and corrosive activity destroys the metal structure at the friction points, causing cracks which, in form of sharp notches, facilitate the development of permanent fractures [19,26] (fig. 10). Additional corrosive influences do not substantially increase the fretting corrosion compared to air.

Above all, due to the fretting corrosion 2 x 10^6 stress reversals were reached for the cable (for the maximum stress 42 % of the tensile strength), depending on the type of construction with the following stress amplitude:

open spiral cables: 180-200 N/mm^2
closed spiral calbes: 150-180 N/mm^2
parallel wire bundles: 200 N/mm^2

For single wires, a fatigue strength > 400 N/mm^2 is possible.

In addition, there are a number of structural influences such as clamps and anchorages which in the case of unaffected cables, can further reduce the above-mentioned fatigue cycles due to fretting corrosion. Cables anchored by cast zinc alloys, the entries of the wires into the casting are highly endangered due to the fretting corrosion and the development of fatigue cracks [27], since here the relative shifts and the compression between the wires reach their maximum. Fig. 11 shows the most frequent development of a permanent fracture in the cast of an experimental anchoring system. The fatigue strength of this type of wire are at 120-140 N/mm^2 in the sockets. Besides the reduction of the stress amplitude, there are structural possibilities to reduce the impact of the fretting corrosion [27,28]. The unfavourable mechanical stress may be decreased by reducing the relative shifts and the local compressions as well as the friction coefficient.

The simplest method would be to apply a durable lubrication layer between the contact surfaces. Soft-metal-coatings such as zinc and aluminium also have a favourable effect, contrary to hot-dip galvanizing, because the brittle iron-zinc alloy layer increases the wear. Also omitting the corrosive medium prevents fretting corrosion. Appropriate measures are the exclusion of the atmosphere (air) by durable coatings and the sealing of the load supporting details (e.g. with plastic) [29].

3.5 Corrosion fatigue

Besides the fretting corrosion occuring in the contact areas of the wires due to the air, corrosion fatigue cracking may develop in the case of single wires, cables or bundles under dynamic loads [7,19,26]. This means the favourite development and the spreading of cracks, caused by simultaneous corrosion [30]; Consequently this reduces the tolerable fatigue strength. No specific corrosive medium is necessary, however, the corrosion fatigue cracking depends on the type and the concentration of the corrosive medium. Fig. 13 shows the behaviour of cold-drawn wires in different media: Even water produces a reduction of the fatigue strength, though negligible from an engineering point of view. Acid water (acid rain, dew etc.) and solutions containing chloride (sea water, de-icing salt spray) produce a lower fatigue resistance than fretting corrosion in the air (100 N/mm^2 compared to 120-140 N/mm^2).

In the case of unalloyed steels, cracks preferably occur in corrosion pits., resulting in a pitted surface with numerous cracks (fig. 12). Therefore, corrosion fatigue in cables is promoted by structural conditions (e.g. formation of cracks with aeration cells) and by conditions caused by the surrounding media (e.g. chlorides), since local corrosion attacks are favoured.

Also a major factor is the frequency or duration: With decreasing frequency or increasing duration, the influence of the corrosion on the development and the spreading of cracks and thus the difference to the behaviour in air is more prominent (fig. 13).

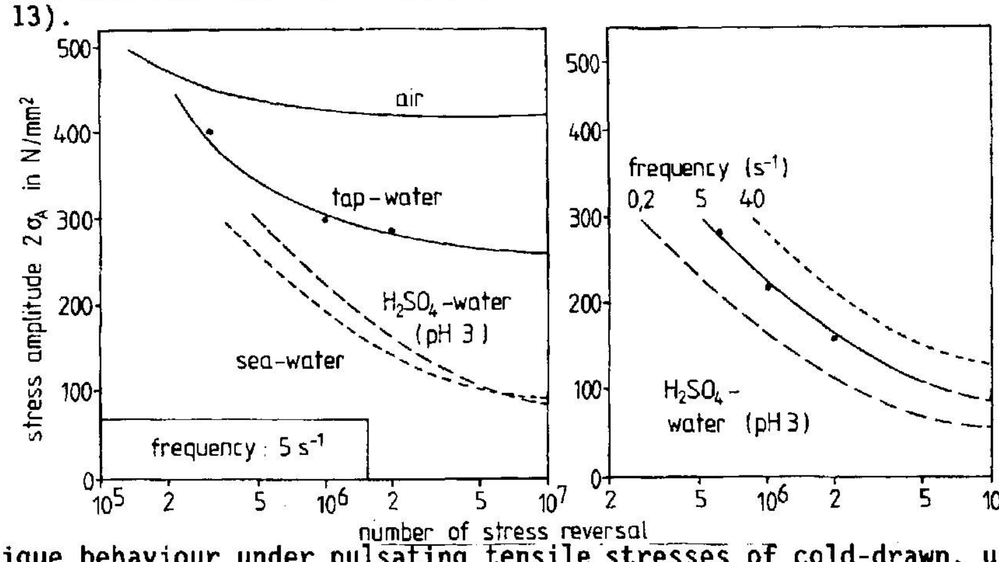


Fig.13 Fatigue behaviour under pulsating tensile stresses of cold-drawn, unalloyed steel wires ($R_m \approx 1750$ N/mm^2) in air and corrosion-promoting solutions (Nürnberger)

The corrosion fatigue behaviour of high-strength cable wires is improved by galvanization [31]. Fig. 14 illustrates, that in chloride containing solutions, as well as in sulphuric acid, the number of cycles to fracture is higher in the case of galvanized wires. The improvements due to galvanizations increase with decreasing frequency.

By manufacturing high-strength strands and cables out of austenitic, stainless steels, the general corrosion behaviour as well as the stress corrosion fatigue behaviour may be improved [32]. An additional protection by organic coatings is not necessary. Fig. 15 shows the results of fatigue tests under pulsating tensile stresses using strands out of high-strength, austenitic steel wires in chloride solution; results of tests with unalloyed steels are quoted for comparison. This shows that strands out of unalloyed steel behave much more unfavourable in chloride solution than in air. Here the influence of the frequency is much greater than in the case of unalloyed steels. At low frequencies the differences between the two materials are not prominent. The behaviour of high-alloyed steels depends upon its susceptibility towards pitting corrosion, since cracks start in corrosion areas. Stainless steels are susceptible towards fretting corrosion, which has an unfavourable effect on the fatigue behaviour with and without additional corrosion.

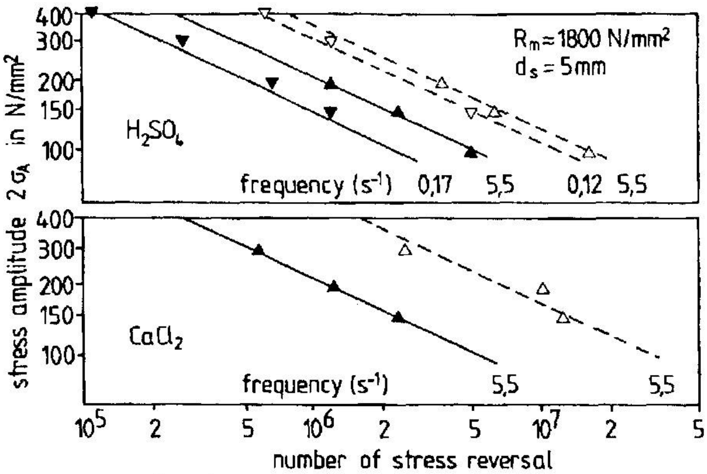


Fig.14 Fatigue tests under pulsating tensile stresses with galvanized and hot-galvanized drawn wires in diluted sulphuric acid (pH3) and 3% $CaCl_2$-solution (Wiume, Nürnberger) [31]

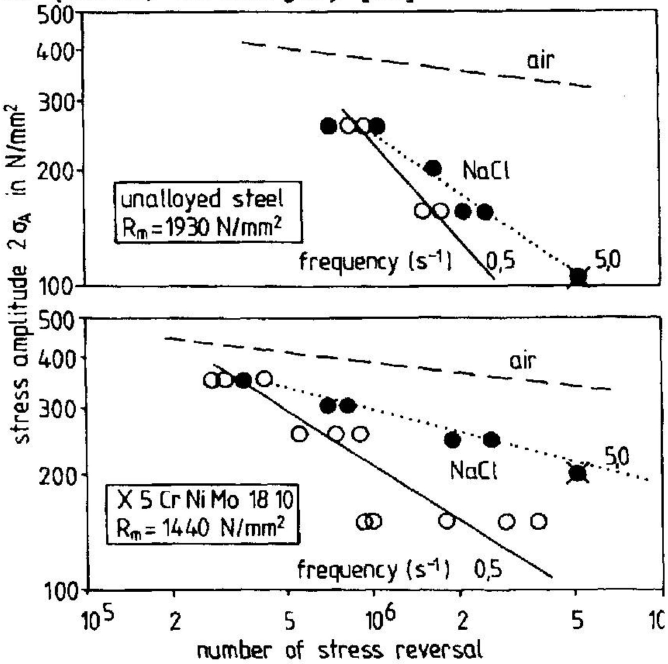


Fig.15 Corrosion fatigue behaviour of strands of high-strength wires of unalloyed and stainless steel in 3,5% NaCl-solution (Nürnberger, Wiume, Beul) [32]

4. WIRE MATERIAL AND WIRE PRODUCTION

Nowadays unalloyed carbon steel with 0.85 % C is a sufficiently examined and tested material available for large tendons. Its characteristics, relevant to its use, were examined mainly dependent on the ratio of Fe to C and the subsequent treatment (fig. 13). The alloys Mn and Si in essence improve the homogeneity of the material during pouring, hot-rolling, cold-drawing and the subsequent temperature treatment. Only recently the result were released [33], which prove a positive influence of a higher Si-percentage in the steel without this interfering with the galvanization process. This way the strength-reducing influence of heat-treatment on the cold-deformed steel-structure can even be reduced.

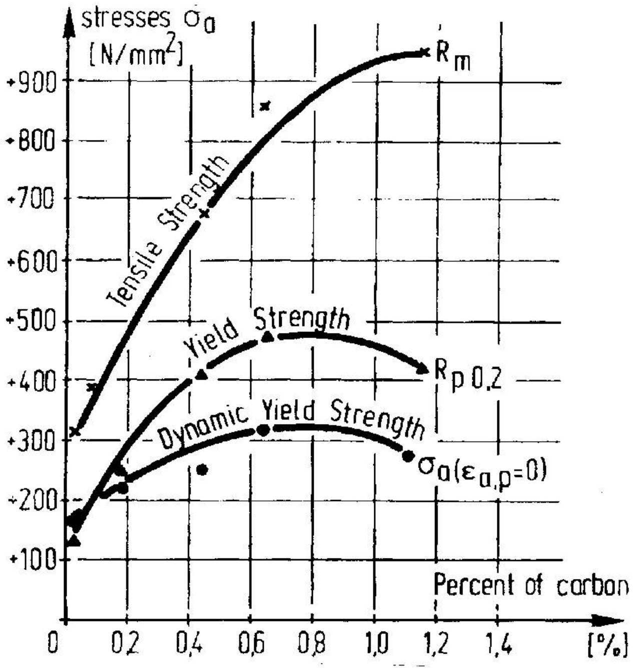


Fig.16 Different strengths depending on the carbon content of the iron (Fe) [34,35]

Whether or not the dynamic creep-limit [34], which can be regarded as the ultimate upper limit of a strictly elastic inflation stress [35] (fig. 16), is influenced by the higher Si-percentages could not yet be proven [36]. However, it also must be put in relation to the fatigue strength of even a long tendon. Therefore, literature lists only few test-series due to the great expenditure required for fatigue tests [37, 38, 39], and the evaluation of the results shown in fig. 17 are linked to certain production-charges and cannot claim to be generally valid, but they coincide very well with the figures in fig. 6.

5. TYPES OF CABLES

In a large tendon, wires can be placed parallel, semi-parallel and as wire-spirals of a rope (fig. 1). Elements produced in this way can be combined into larger dimensions by running parallel, being intertwined or stranded for a second time; this creates rope-bundles, strand-bundles or cable lay ropes. Depending on the wire-geometry, the size of the changes in the rope-force and an angle-deviation at the entry of the fittings, relativ movements will occur in a tendon (fig. 18), which nowadays can already be calculated [40] [41]. Spirally twisted wires move against each other, when looking at the neighboring wires of the same layer. The wires of neighboring layers for example in a spiral rope, rotate counter-clockwise around the point of contact. Here a relative movement occurs when the wires are pressed into each other due to the radial forces of the spiral geometry and an area of contact exists or shaping wires are pressed on top of each other on the surface (fully locked and half-locked coil rope).

	Prestressing Wire Ø 7mm (patented, cold drawn, stranded, heat treated)		Rope Wire Ø 7mm (patented, cold drawn, tested after stranding)		Prestressing Strand Ø 0,6' (patented, cold drawn, stranded, heat treated)	
	$\bar{x}$	s	$\bar{x}$	s	$\bar{x}$	s
Fatigue Strength (bare wire) $2\sigma_A$ [N/mm²] Specimen length ~ 500mm	420	40	360	45	250	30
Exponent of Inclination in the Wöhler-Diagram K	8,5	–	–	–	7,5	–
Tensile Strength (bare wire) R_m [N/mm²] Specimen length ~100mm	1770	36	1570	52	1670	42
Yield Strength $R_{p0,2}$ [N/mm²]	1560	39	1270	50	1470	52
Uniform Deformation A_{gl} [%]	36	6	20	4	40	7

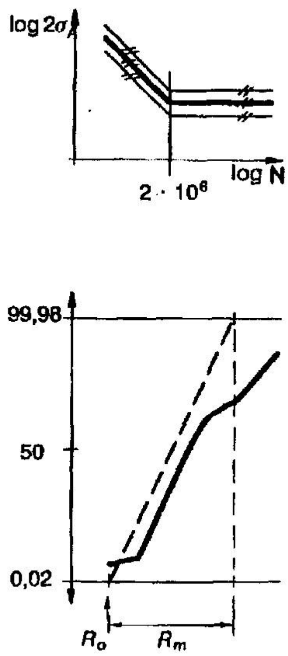
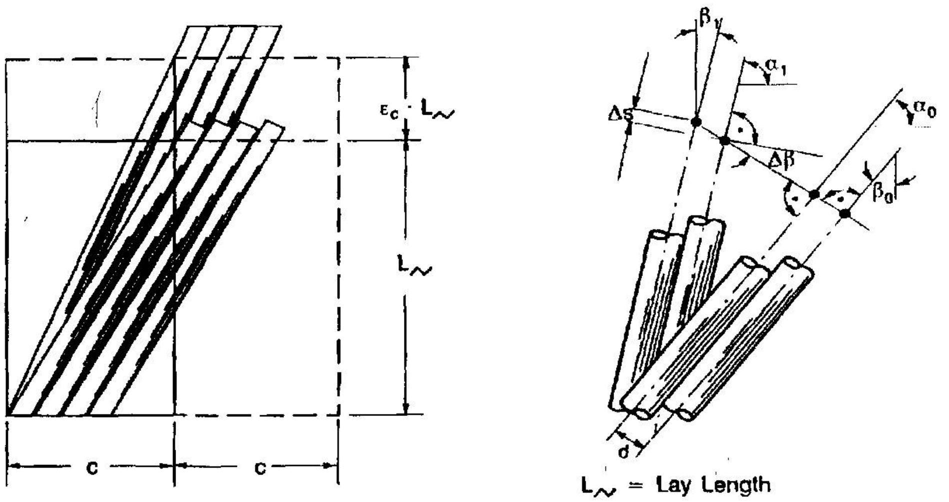


Fig.17 Statistic values of short specimen rupture tests (≈ 100mm) and fatigue tests (≈ 500mm) determined out of a special charge of these wires and strands



$$\text{ctg}\ \alpha_0 = \frac{\pi \cdot D_w}{L_\sim}$$

$$\text{ctg}\ \alpha_1 = \frac{\pi \cdot D_w(1 - \varepsilon_R)}{(1 + \varepsilon_c)\ L_\sim}$$

$L_\sim$ = Lay Length
D_w = Diameter of the Layer
ε_c = Cable Elongation
ε_R = Decreasing of Diameter

$$\Delta s = d \cdot \text{ctg}\Delta\alpha = d\ \frac{\text{ctg}\alpha_0\ \text{ctg}\alpha_1 + 1}{\text{ctg}\alpha_0 - \text{ctg}\alpha_1}$$

Fig.18 Relative movement Δs and rotation Δβ of wires in stranded cables [43]

The extent of the pressure, which determines the effects of the friction-corrosion just as the size of the friction-distance does [25], can also be established [42] [43]. It primarily depends on the size of the tendon force.

In tests and with the help of cross-checks, the friction coefficient was determined to be between 0.12 [44] and 0.15 [45], and a rope will act the same way as a total cross-section as long as this static friction is not overcome. In the case of movement, energy is lost for overcoming the friction.

Of course the friction coefficient also depends on the friction partners and the filling compound of the rope. If up to now only purely galavanized or galfan-galavanized wires are used (stainless steel wires are still rare), the friction coefficient will be extremely dependent on the quality of the lubricant, which has to resist high pressures and may not be rolled out. For example pure oils only lubricate a little because they are rolled out, oils with a high pigment-content (for example with zinc dust) increase the friction, which, in laboratory rope tests, can already be noticed from the temperature of the test element. Lubricants with aluminum pigments and poly-waxes showed favourable lubricating effects.

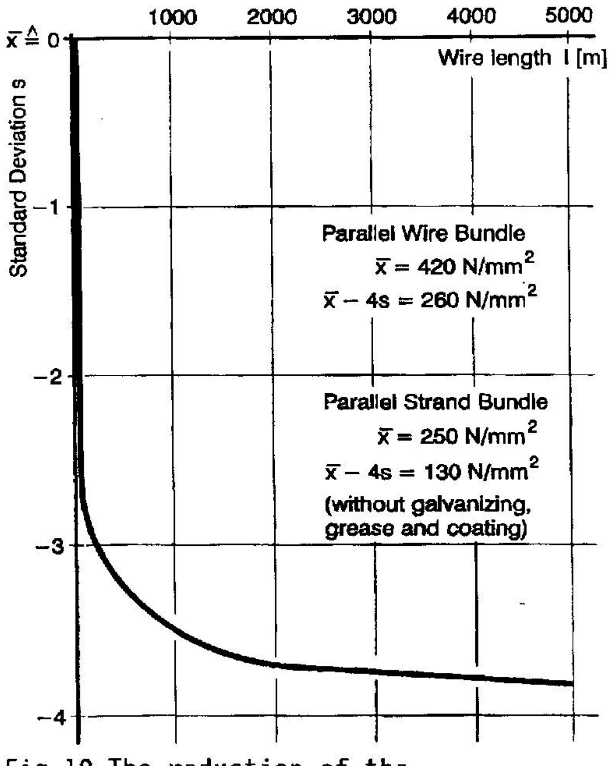


Fig.19 The reduction of the fatigue strength of a wire bundle out of over 100 elements, depending on the length

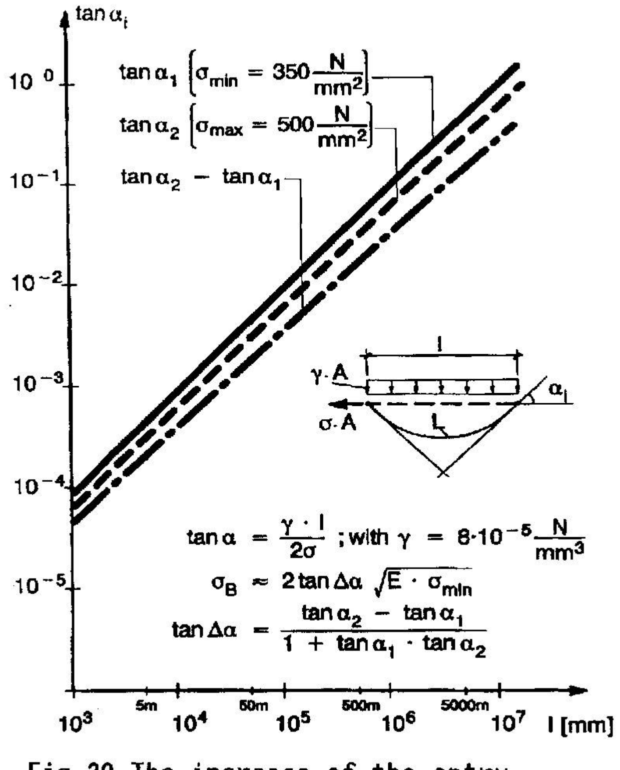


Fig.20 The increase of the entry angle at the end of a cable, sagging under dead load, depending on the length if the cable-tensio increases from 350 N/mm² to 500 N/mm²

The length-effect is extremely affected by the bonding between the single wires. In case of ropes it is a result of the friction between the wires [46], in case of wire- or strand-bundles it may result out of the induced concrete injection. This bonding only allows the broken wire to be effective as tension increase of the remaining cross-section over a short distance. It only has to be taken into consideration whether or not the broken ends damage the neighboring wires ahead of time.

Fig. 19 shows the decrease in strength for a bundle [35]. This clearly shows that the fatigue of an indefinitely long bundle or even a rope moves towards a limit, which has to be considered to be the mean of coincidental variables of the fatigue strength of the entire tendon without the scattering [39] and which has to correspond to a scale which might co-relate to the creep-limit [35]. It is necessary to determine the amount of stress, up to which no tearing occurs at dents or notches [47], consequently up to which the material stabilizes itself and is deformed only elastically. This should lead to the determination of the fatigue stress which can be endured by an indefinitely long bundle.

6. THE STRUCTURAL TENSION ELEMENT

The freely spanned tendons do not experience substantial relativ movements between the wires or serious bending stresses along their entire free length, neither due to a change in the sag, nor due to stimulated vibrations. A crucial factor are the changes in the entry-angles at the end anchorages, which lead to bending stresses in the tendon in the same way as the large deformations without expansion of a boundary cable with boundary clamps, or of a transversally pressed suspension cable equipped with hanger clamps found in suspension bridges can cause such additional stresses[48]. As an example in the case of a lateral spanned cable fig. 20 shows the change in the angle being dependent on the length, due to an increase in the load from 350 to 500 N/mm^2 longitudinal stress of the tendon.

Due to the weight limitation during assembly, the possibility of sufficient corrosion protection and the demand for replaceability of the tension elements, very large tendon units are introduced, which are more and more dessected, in order to make them accessible and to be able to inspect them. The access to the elements up to the point of anchorage is a far greater gain than the advantages resulting out of the redundancy of the dessected cable (fig. 21). The effect single flaws occurring during the production of the corrosion protection have on the security of the entire cable are kept down [49].

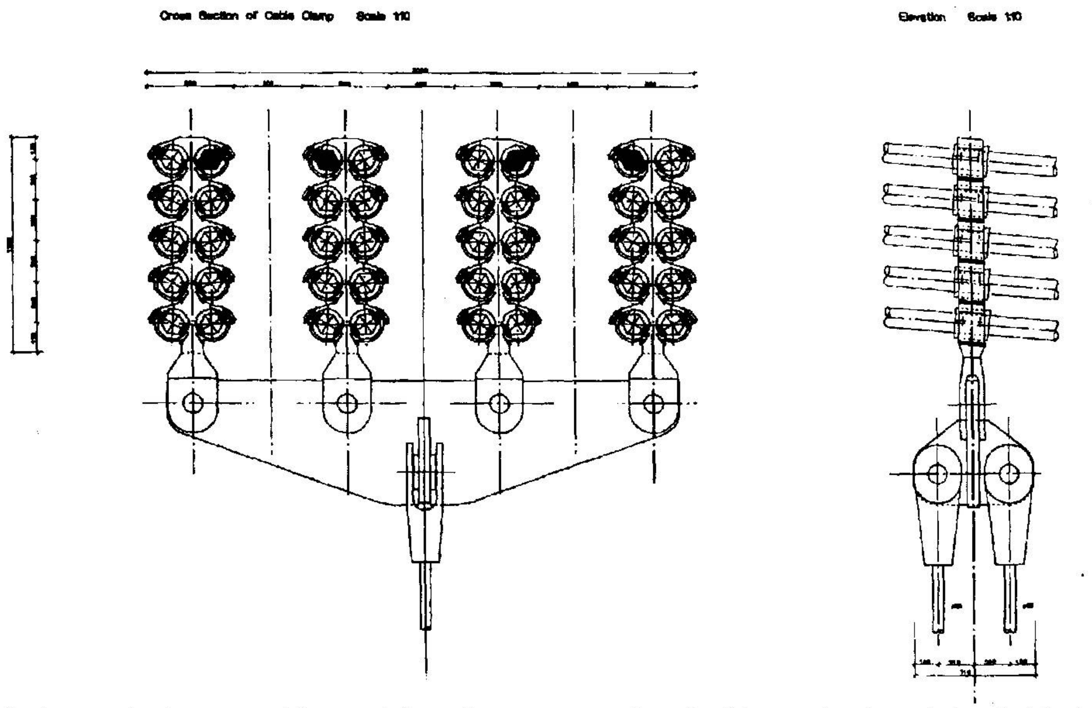


Fig.21 A dessected supporting cable of a suspension bridge, designed by Schlaich Bergermann and Partners

7. STRUCTURAL DETAILING

7.1 Clamping

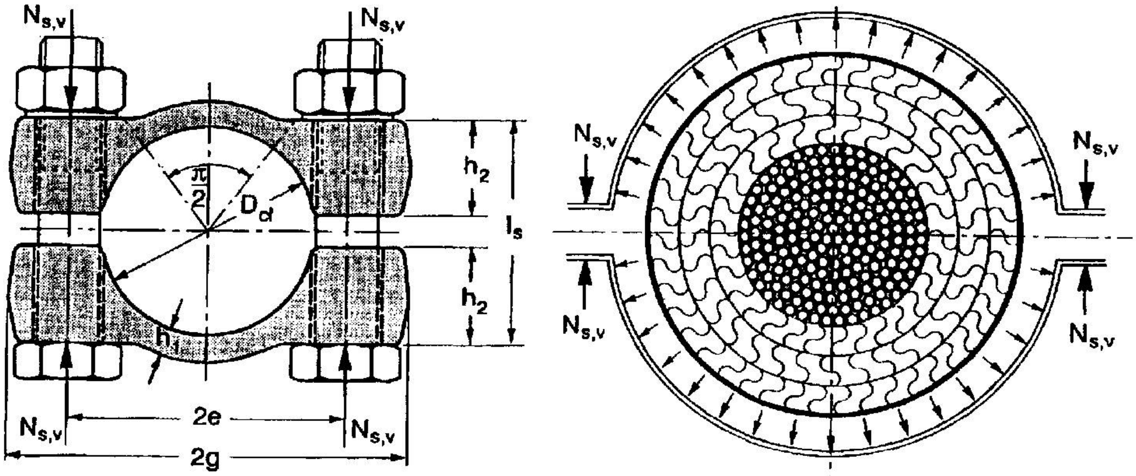
Loads are induced into a continuous cable via clamps by friction, which is activated between the cable and the clamps, which are usually pressed-on by high-strength screws [50]. The tension element becomes more resistant in the clamp-area, if the clamp absorbs part of the tensile stress [51]. The leap in resistance is sudden, and it depends on the cross-section-ratio between the clamp and the tendon, on the adhesive pressure and on the friction coefficient, whether the movement between the clamp and the tendon comes to a halt. In this case it has to be considered that the tendon - especially the ropes -, withdraw from the clamp-pressure with increasing axial force, and that a friction movement is hardly hindered [52] (fig. 22). A serious change in the medium tension of the dynamic inflation stress would have a very unfavourable effect.

In order for the energy resulting due to friction not to cause friction corrosion, the area of the relative movement has to be carefully protected against entering air and dampness [26]. Friction corrosion can still lead to the failure of a wire, even after very high number of load changes.

7.2 Anchoring

Three anchoring systems are mainly used for deducing loads from high-strength wires of tendons with large loadbearing capacity (fig. 23):
- the wedge anchorage in the case of tensioning strands,
- the compressed, small heads of thick wires and strands, which are supported by a perforated sheet
- the metal-bonding of wire- and strand-bundles as well as cables.

The wedges, the compressed, small button heads and the bonding-cone have to be formed in such a way that the load is deduced over the shortest possible length, without noticably reducing the resistance against static stress [25]. The fact remains, that the anchorage is the weak-point of the entire tendon under inflating load, if additional measures are not taken to reduce the local stresses.



$$\Delta N_{s,v} = \frac{\pi \cdot D_{cl} \cdot \Delta F_c}{E_{c,p} \cdot A_c} \cdot \mu_c \frac{1}{\frac{\pi \cdot D_{cl}}{2 \cdot b_{cl} \cdot h_1 \cdot E_{cl}} + \frac{2 l_s}{E_s A_s}}$$

μ_c = 0,3 for parallel wire
μ_c = 1,5 for not prestretched spiral ropes
μ_c = 0,8 for prestretched spiral ropes
$.._c$ = related to cables
$.._{cl}$ = related to clamps
$.._s$ = related to screws

Fig.22 The release of prestretched screws $\Delta N_{s,v}$ of a cable clamp, depending on the increase of the cable force ΔF_c

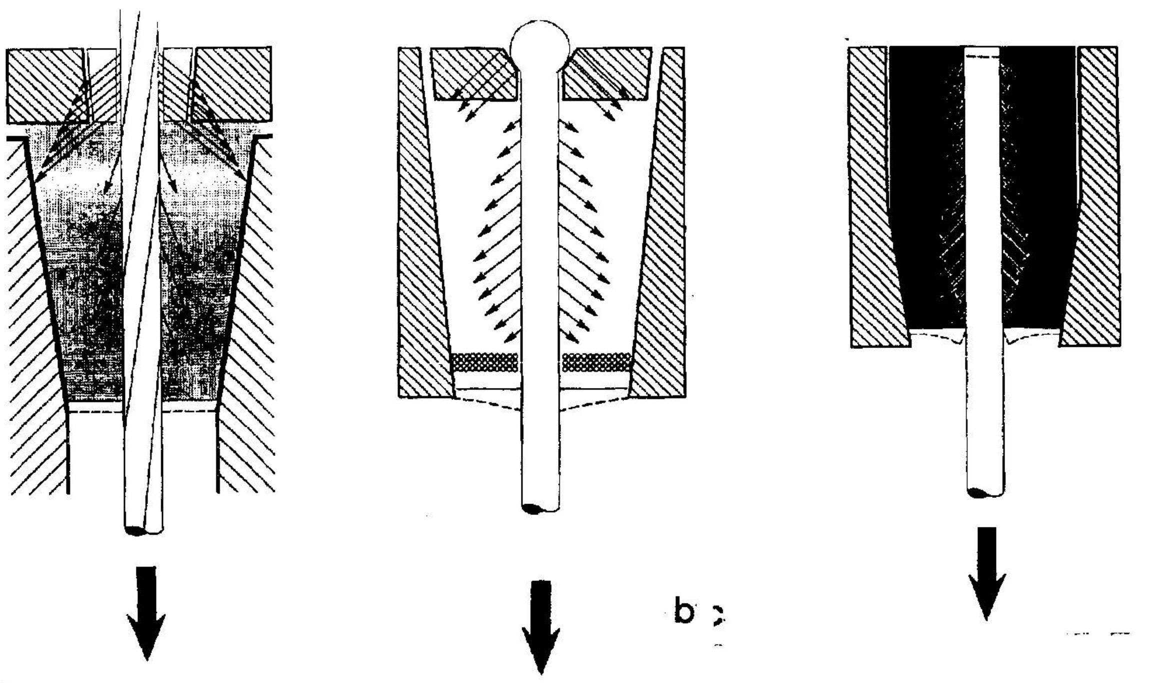


Fig.23 Anchoring mechanisms of
a) a wedge anchorage of strands with a cement grouted trumpet to lower the life load amplitudes in the anchoring zone
b) a steel bullet-resin compound for wire and strand bundles being additionally fixed in a perforated sheet by button heads
c) a metal cast cone for wire and strand bundles as well as ropes being fixed by friction caused by transverse, radial pressure

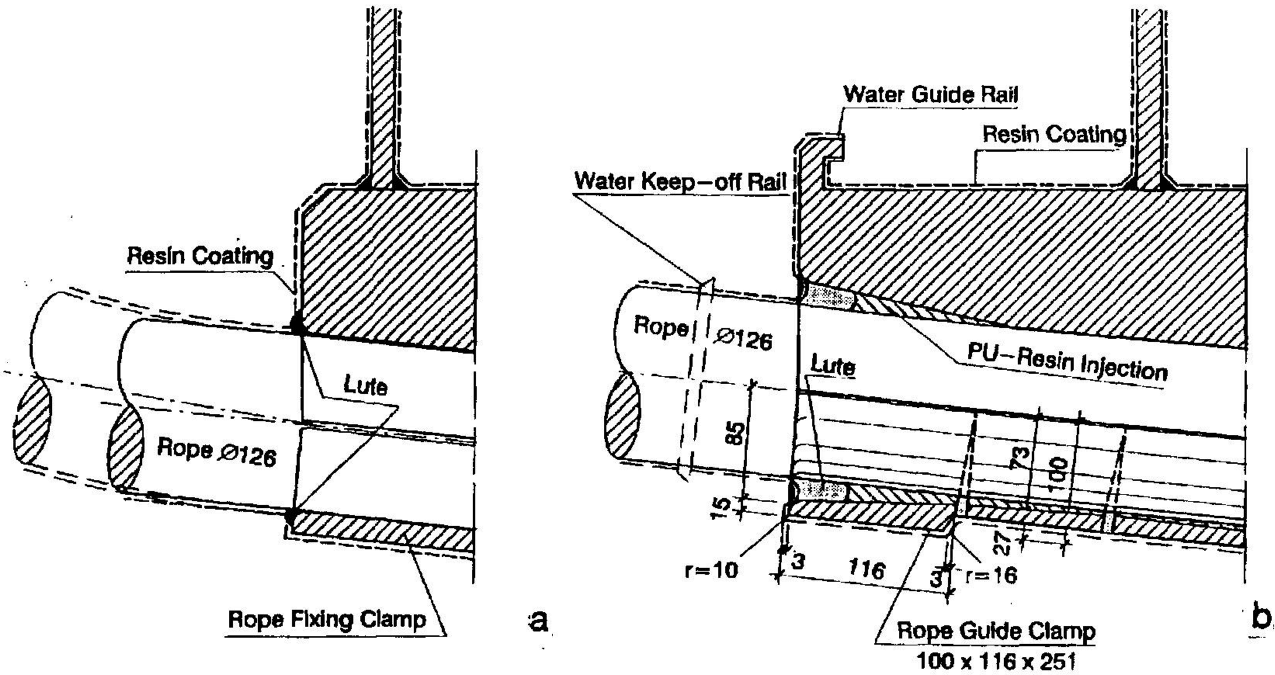


Fig.24 Constructive and coating measures to obtain good corrosion protection for ropes in the fitting areas
a) without a trumpet zone
b) with trumpet zone to avoid great bending stresses

One of the most common measures is injecting two-component plastics into the anchorage area. This is to prevent air and dampness from entering. Since the quality of such an injection cannot be checked, there is also the possibility to only have the static anchorage absorb a basic load (usually the dead load) and to intercept load fluctuations by using interim transmission constructions. Fig. 23 a) shows how the load change can be absorbed in the case of a trumpet injected with cement [53], and b) how the interim ball-plastic-mixture usually transferes the entire static load on to the socket and how the Epoxy-bonding receives the additional stresses [54], and how nowadays it is attempted in the case of the metal-bondings to extremely compress the entry into the cone in a transversal direction. However, a large gap between the tendon and the socket lets the area of entry vibrate with the expansion of the wires at the exit-area of the cone, respectively allows it to approach a final state in the course of several load changes, which eliminates further relative movements between wire and cone [55]. Laboratory-tests are proving that the anchorages are not the weak-point of a high-strength tendon. It is not known what they can really sustain and what stresses they are subjected to.

8. CORROSION PROTECTION

Close attention has to be paid to corrosion protection (fig. 24), since in the case of a damage assessment, the influence of corrosion always comes into play, i.e.: the corrosion protection was faulty or showed isolated damages which were not discovered in time [56] [57]. The controversy today is, if the protection is so complete that it lasts for a long time, but is hard to inspect, or if it is kept to a minimum and that the regular inspections are part of the protection.

A combination of individually protected single strands [58] or ropes out of galfan-coated wires [59], filled only with poly-waxes, are now regarded to be a practical alternative to the coating-systems of cables which are very expensive and have to be renewed.

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