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## Length Effect on Fatigue of Wires and Prestressing Steels

### Influence de la longueur sur la fatigue des fils et des aciers de précontrainte

### Einfluß der Länge auf die Ermüdung von Drähten und Spannstählen

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#### SUMMARY

A statistical approach, based on the independence assumption, is proposed here for the consideration of the length effect on the fatigue resistance. As a limiting condition for its applicability the test length must experimentally demonstrate to be longer than a certain threshold length. The validity of the model is currently being tested in two experimental programs on wires and prestressing steels. In spite of the promising results obtained, more research is needed for its general application in design.

#### RÉSUMÉ

On propose ici un modèle statistique, fondé sur l'hypothèse d'indépendance, pour l'étude de l'effet de la longueur sur la résistance à fatigue. La condition d'applicabilité de cette approche implique de vérifier expérimentalement que la longueur d'essai dépasse celle d'une valeur de seuil. La validité du modèle est actuellement à l'étude dans deux programmes expérimentaux sur des fils et des aciers de précontrainte. En dépit des résultats prometteurs obtenus jusqu'ici, il est nécessaire de continuer la recherche si l'on envisage une application ultérieure dans la pratique du dimensionnement.

#### ZUSAMMENFASSUNG

Auf der Grundlage der Unabhängigkeitsvoraussetzung wird ein statistisches Modell zur Berücksichtigung des Längeneinflusses auf die Ermüdungsfestigkeit vorgeschlagen. Als Voraussetzung für dessen Anwendbarkeit muß experimentell nachgewiesen werden, daß die Testlänge einen bestimmten Schwellenwert übersteigt. Die Gültigkeit des Modells wird derzeit in zwei experimentellen Programmen an Drähten und Spannstählen überprüft. Trotz der vielversprechenden, bisher erhaltenen Ergebnisse ist weitere Forschung bis zum Einsatz in der Bemessungspraxis nötig.



## 1. INTRODUCTION

It is generally accepted that the fatigue life of longitudinal elements is conditioned by the existence of flaws along its surface, derived from the manufacturing process, handling and storage. If the random distribution of the flaws shows no correlation along the element, the fatigue resistance of neighbouring pieces can be assumed to be independent and, as a consequence, the fatigue analysis can be based on the hypothesis of independency.

On the contrary, if such a correlation exists, the dependency effect must not be ignored and a suitable model which takes into account dependency should be used.

## 2. THE PROBLEM OF EXTRAPOLATION

In order to assess of their fatigue properties, many mechanical and structural elements cannot be tested on a real scale because of the high costs involved or even due to physical impossibility. This is the case of very long elements such as crane ropes, tendons in cable stay bridges or similar structures. Consequently, prediction of the fatigue resistance for long elements must follow on from extrapolation of test results usually obtained for short or very short specimens. This means that the fatigue life of one element of length  $s$  is the minimum fatigue life of  $s/r$  independent elements of length  $r$  in which the former is hypothetically divided.

Therefore, the survival function for the element of length  $s$  can be derived from the survival function of the element of length  $r$  ( $r > s$ ) by means of the expression:

$$G_s(x) = \text{Prob}(X_s > x) = |\text{Prob}(X_r > x)|^{s/r} = |G_r(x)|^{s/r} \quad (1)$$

$X_s$  and  $X_r$  being the fatigue lives of the two elements.

However, several experimental studies [1, 5, 11] evidence the non-validity of the independence model for extrapolation of fatigue life gained from short specimens in order to obtain the fatigue life of larger elements. To the contrary, extrapolation based on relatively large elements seems to lead to good results [5]. This fact has been theoretically justified since unless strong dependence exists between the strengths of neighbouring pieces, the asymptotic behaviour is that of independence [2,10].

According to [6], the transition between the dependence and independence assumption is governed by a certain threshold value of the length, say  $s_0$ , beyond which extrapolation based on Eq. 1 is valid. This will be further discussed in the following section.

## 3. SUGGESTED MODEL AND ITS JUSTIFICATION

Without theoretical justification, Bogdanoff and Kozin [1] suggest the expression:

$$G_s(x) = |G_r(x)|^{k(s,r)} \quad (2)$$

for general conversion of the survival function of one specimen of length  $r$  into that of another larger one,  $s$ , still within the dependence domain. The function  $k(s,r)$  is an unknown function to be determined experimentally (see Fig. 1). If independence holds, then  $k(s,r) = s/r$  and Eq. (2) becomes Eq. (1).

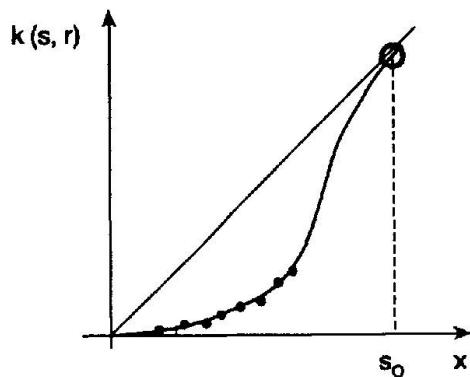


Fig. 1 Function  $k(s, r)$

The validity of Eq. (2), at least within a certain range, has been confirmed by analyzing experimental fatigue results in viscose yarn of differing length [11].

Assuming the number of flaws in a longitudinal element to be a nonstationary Poisson's process of intensity  $\lambda(s)$ , Castillo et al.[6] developed a statistical model which takes into account the influence of the manufacturing process on the lifetime of the piece from which the survival function is found to be:

$$G_T(x) = |\exp(-F(x))|^{m(r)} \quad (3)$$

where  $m(r)$  measures the frequency of appearance of flaws and  $F(x)$  represents the lifetime of a single flaw, respectively. The uniqueness of a c.d.f.,  $F(x)$ , as representative of all possible flaws can be reasonably accepted according to the Fracture Mechanics approach.

Consequently, the survival function for a different,  $s$ , is governed by

$$G_S(x) = |G_T(x)|^{m(s)/m(r)} \quad (4)$$

By means of the functional equation theory [8] it can be shown that  $m(s)/m(r) = k(s, r)$  and Eq. (4) is identical to Eq.(2).

The model includes the case of independence, for which  $m(s) = x$ , and also the case of asymptotic behaviour, in which  $F_S(x)$  goes over into that of independence [12] and the following equation holds:

$$m(n\Delta s) / m(\Delta s) = \infty \quad (5)$$

where  $\Delta s$  is the length of a piece; so that

$$G_{n\Delta s}(x) = |G_{\Delta s}(x)|^n \quad (6)$$

The model comprises still the B-model of Bogdanoff-Kozin, the statistical inconsistencies of which have been pointed out in [3]. Thus for large  $n$



and

$$G_{n\Delta s}(x) = |G_{\Delta s}(x)|^k \quad (8)$$

this means that after some given length the size effect vanishes (the so called saturation effect). However, neither statistical justification for saturation exists, nor experimental evidence for it has been glimpsed in the two test programs reported in the following section.

Because of the asymptotical confluence of  $k(s,r)$  into  $k=s/r$ , the determination of the threshold value  $s_0$ , mentioned in the above section, has only academic significance in research. The extrapolation of the results using equation (2) are hence possible as soon as it can be experimentally proved that  $k$  corresponds to the independence domain.

Now a general fatigue model such as the Weibull regression model suggested in [5,7], can be used in order to describe the Wöhler-field as a whole, thus allowing the extrapolation for designing both length and number of cycles to failure (estimation of the endurance limit).

#### 4. EXPERIMENTAL PROGRAM

Two experimental programs have been launched in order to validate the possible use of the independence model for extrapolation by testing long enough elements:

##### 4.1 Prestressing steels

This program has focused on the study of fatigue properties of prestressing steel (wires and stands) with relation to length and started out as a collaboration between the ETH-Zürich, the EMPA Dübendorf, and the Spanish Universities of Cantabria and Oviedo.

The aims of the study are:

- To corroborate the usefulness of the independence model for extrapolation of fatigue test results for both prestressing wire and 7-wire prestressing strands, verifying at the same time that the chosen minimal length has surpassed the hypothetical threshold length.
- To ascertain that the study of dependence-independence is unrelated to the stress range since the fatigue behaviour is assumed to be conditioned solely by the initial flaw distribution in the element. This would lead to the possibility of recommending a high stress range for testing in order to reduce the test duration.
- To study the influence of the test frequency in order to confirm or reject the possibility of extrapolating fatigue results obtained from tests at high frequency with very short specimens.

The tests were been carried out at the EMPA Dübendorf (Swiss Federal Laboratories for Material Testing and Research) .Results are given in Tables 1, 2 and 3. A detailed description of testing and devices is given in [9].

Specimen number	Number of cycles to failure (in thousand)	Specimen number	Number of cycles to failure (in thousand)
1	64	1	92
2	70	2	93
3	84	3	109
4	99	4	116
5	105	5	125
6	110	6	129
7	117	7	132
8	133	8	134
9	151	9	135
10	163	10	135
11	199	11	138
12	201	12	188
13	268		
$f = 62 \text{ Hz}$		$f = 3,5 \text{ Hz}$	

**Table 1** Fatigue test results for the study of the influence of the frequency in wires

Specimen number	Number of cycles to failure (in thousand)	Specimen number	Number of cycles to failure (in thousand)	Specimen number	Number of cycles to failure (in thousand)
1	205	1	206	1	143
2	225	2	206	2	179
3	256	3	237	3	200
4	266	4	257	4	209
5	268	5	260	5	210
6	279	6	272	6	216
7	283	7	276	7	218
8	293	8	307	8	222
9	300	9	318	9	223
10	521	10	331	10	225
11	1709	11	365	11	225
12	1762	12	393	12	236
13	2303	13	403	13	279
14	4230	14	413	14	303
		15	824	15	315
				16	325
$l=1,04 \text{ m}$		$l=2,03 \text{ m}$		$l=10,40 \text{ m}$	

**Table 2** Fatigue test results for the study of the influence of the length in preressing strands



Specimen number	Number of cycles to failure (in thousand)	Specimen number	Number of cycles to failure (in thousand)
1	49	1	153
2	53	2	162
3	56	3	165
4	59	4	190
5	60	5	229
6	60	6	235
7	60	7	237
8	60	8	280
9	60	9	297
10	61	10	804
11	62	11	866
12	63	12	1530
		13	1964
		14	2000 runout
		15	2000 runout
		16	2000 runout
$\Delta\sigma = 700 \text{ N/mm}^2$		$\Delta\sigma = 400 \text{ N/mm}^2$	

**Table 3** Fatigue test results for the study of the influence of the stress range in wires

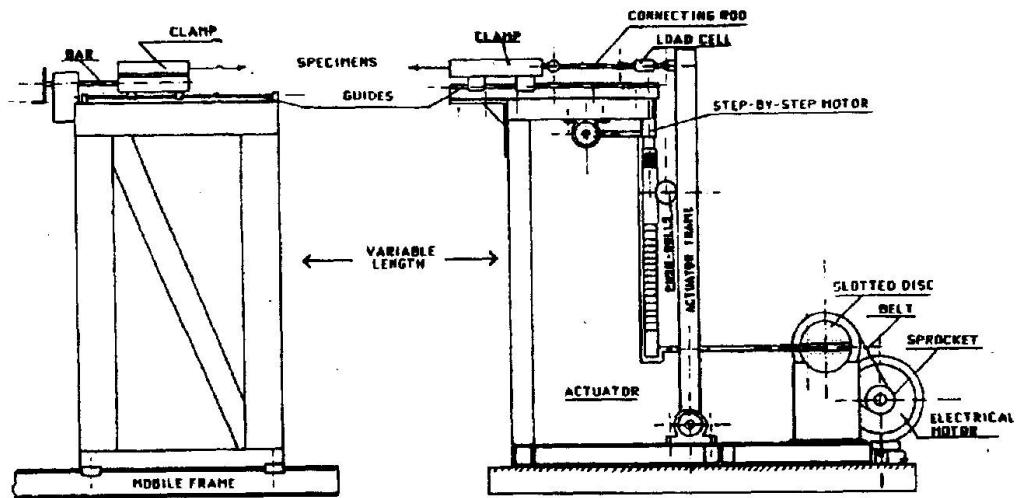
#### 4.2 Wires

The initial program on prestressing steels has been extended to another one devoted to the study of length effect on 0.5mm diameter hipo-eutectoid steel wire.

The goals of this experimental research are:

- To develop a testing device for analysing the influence of length in fatigue tests with the purpose of minimizing time and costs.
- To analyze and compare fatigue results for length/ diameter ratios comparable to the ones in real structures. This would apport sufficient data for testing the saturation assumption.
- To explore at the same time the trend of the  $k(x)$ -values for large lengths, probably much larger than the critical length, beyond which no dependence effect would be expected.
- To ascertain if the dependence study can be limited to a single level, the conclusions from it being extensive to the whole Wöhler-field

Economical and technical considerations lead to the choice of an electro- mechanical testing device. The authors intention being to test several specimens simultaneously with the possibility of varying the test length within a large range (say up to 20 m). Since such a non-conventional machine is not commercially available it was designed and built at the Dept. of Construction of the University Oviedo (see Fig. 2).



**Fig. 2** Testing device for wires

The way-controlled machine permits simultaneous testing of five specimens in horizontal position, moved by a step-by-step electrical motor, that allows a steady variation of the frequency (in the present program frequency was fixed at 2 Hz in order to avoid resonance effects for the thin wire). Depending on the Young modulus of the material, specimens of up to 20 m may be tested.

The relatively small forces required to stretch the small diameter wires permit the fixation of the actuator and the end rig frames directly to the floor of the laboratory.

The stress in the wires is measured individually for each wire by means of extensometric techniques; whilst the reading of the total applied load is made for the set of the five specimens as a whole. The generated load wave can be displayed during the test.

Some of the difficulties found using this kind of wire are:

- The stress-strain curve of this material exhibits a remarkable linearity, practically without any plastic deformation. As a consequence, the actual applied maximum stress level and the strength range, being constant for all the tests, imply, in fact, relative differences in the test conditions for the single wires and propitiate greater scatter for the fatigue results.
- The relative tolerances in the diameter of the tested wire may be not comparable to the ones in the prestressing steels, which incidentally are the main subject of the workshop.

The results for the fatigue life of 2,00 m. and 10,00 long wires are presented in Table 4.



Specimen number	Number of cycles to failure (in thousand)	Specimen number	Number of cycles to failure (in thousand)	Specimen number	Number of cycles to failure (in thousand)	Specimen number	Number of cycles to failure (in thousand)
1	55440	13	222362	1	20250	11	95155
2	80458	14	250155	2	39154	12	101164
3	127127	15	253134	3	61197	13	105107
4	129024	16	271344	4	67868	14	111030
5	129906	17	276612	5	70470	15	121638
6	138600	18	282822	6	76340	16	125086
7	173754	19	315849	7	79206	17	129046
8	190246	20	324955	8	84492	18	134255
9	196329	21	344669	9	94950	19	137057
10	208026	22	351282	10	95030	20	144348
11	221480	23	351282				
12	221495						
l=2.00m				l=10.00m			

**Table 4**

## 5. ANALYSIS OF RESULTS

### 5.1 Prestressing steels

The comparison of fatigue results for two markedly different test frequencies (see Fig. 3) demonstrate that neglecting parameters considered as secondary, such as the frequency, can lead to a groundless rejecting of the independence model, if fatigue results for different test conditions are used for comparison.

The analysis of Fig.4 enables us, with certain reservations, to accept the validity of the independence model for strands, if specimens of at least 2m long are used for testing and ulterior extrapolation. The poor prediction arising from the fatigue results for 1m long specimens are not conclusive, since, as reported in [9], surface failures in the outer wires of the strand caused by handling or storage damages are supposed to be determinant for the shorter lifetime measured. Due to the high rate of run-outs obtained in the wire tests, no sound results for length effect of prestressing wires has yet been achieved.

No definitive conclusions can be drawn concerning the question of whether the distributions resulting from different stress levels should be the same (see Fig.5), the study of independency could hence follow for a convenient high stress range.

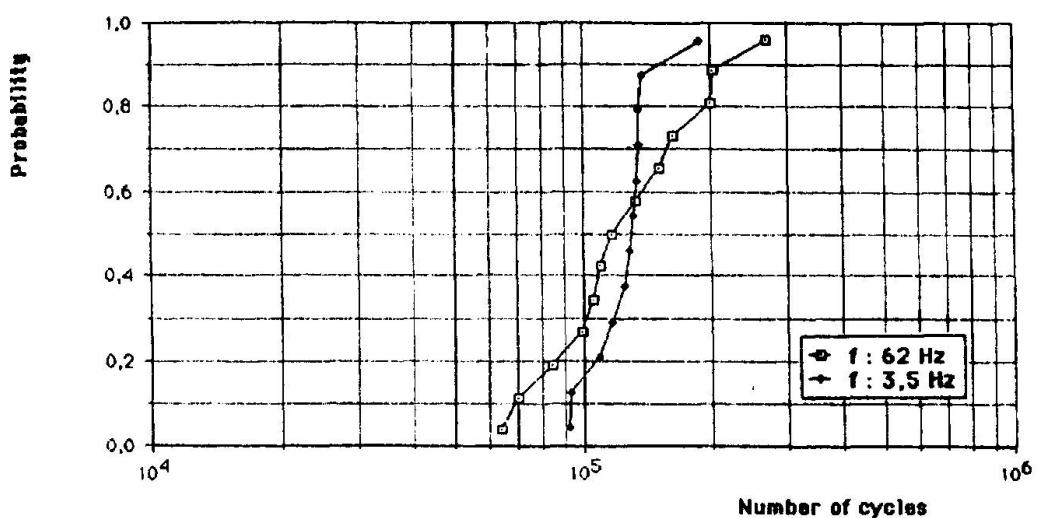


Fig. 3. Fatigue test results on prestressing wire for differing frequencies ( $l=0,150\text{m}$ ,  $\Delta\sigma=700\text{ N/mm}^2$ )

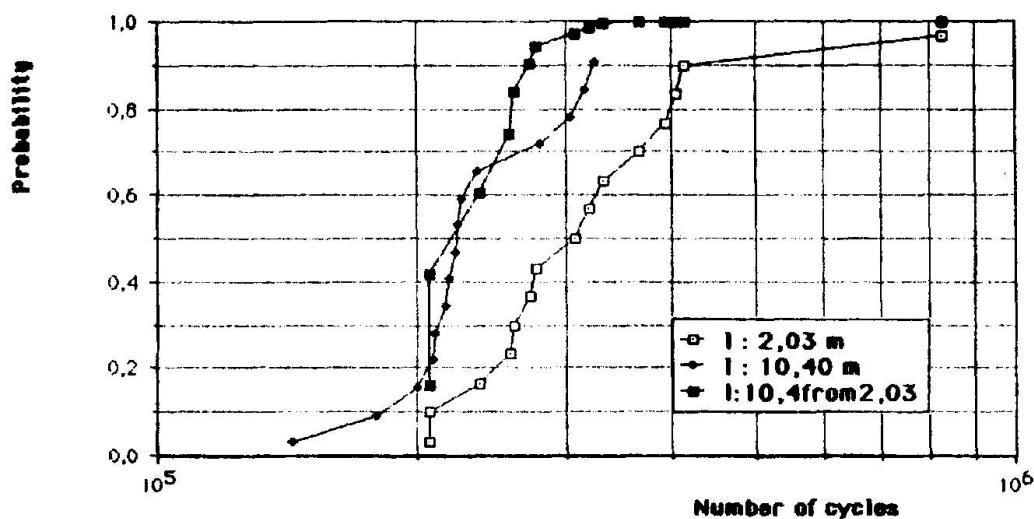


Fig. 4. Fatigue test results on prestressing strand for differing length ( $f=3.5\text{ Hz}$ ,  $\Delta\sigma=700\text{ N/mm}^2$ )

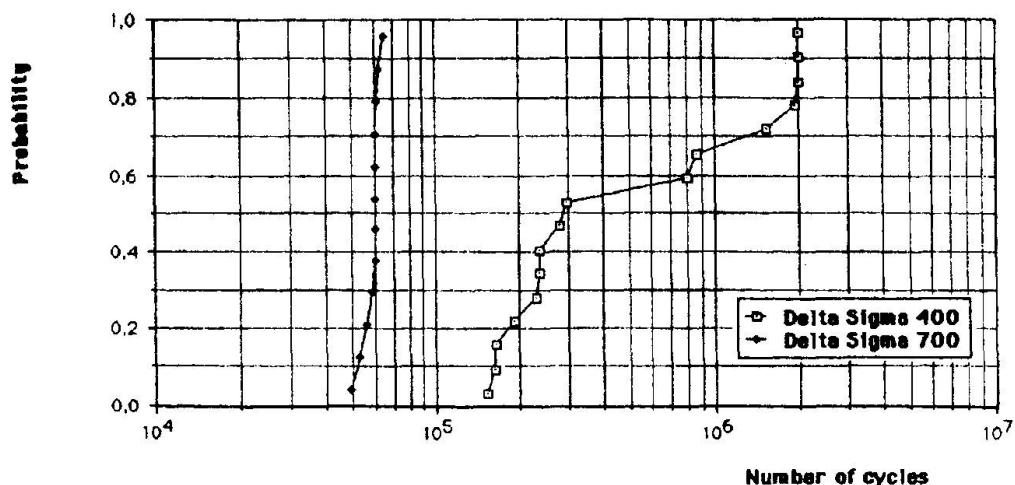


Fig. 5. Fatigue test results on prestressing wire for differing stress range ( $l=0,15\text{m}$ ,  $f=3.5\text{ Hz}$ )



### 5.2 Wires

The research program is still being followed, so that a comparison can only be made for extrapolation purposes. The fatigue data obtained for 10m long specimens and the regression line derived from the data for 2 m. long specimens show a fair agreement. (Fig.6).

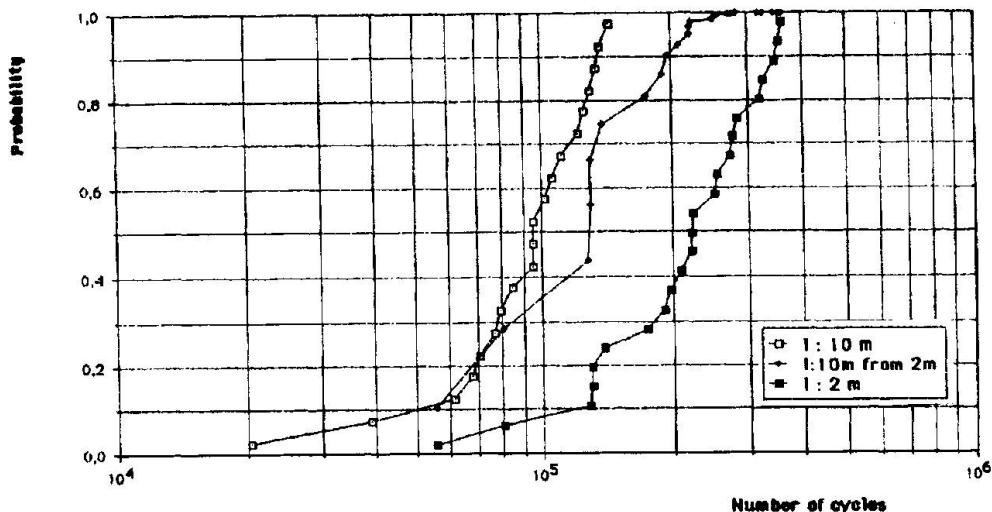


Fig. 6. Fatigue test results on, hypoeutectoid wire for different length ( $f=2\text{Hz}$ ,  $\Delta\sigma=747 \text{ N/mm}^2$ )

## 6. CONCLUSIONS

As general remarks for both experimental programs it can be stated that:

- Since the left tail of the distributions, corresponding to the low probabilities of failure (design region), is determinant for ascertaining or rejecting the initial assumptions, further testing is needed in order to come to reliable and definitive conclusions related to the presented model. Up till then, the proposed model seems to be founded on sound assumptions without physical or statistical inconsistencies, and reasonably well supported experimentally; and therefore acceptable for practical design purposes.
- The graphical representation of the results obtained shows that the different sets of data seem to follow Gumbel rather than Weibull distributions. This should be clarified in the course of the ongoing research. Nevertheless, since, as a regression model with the Gumbel instead of Weibull assumption proposed in [5] has been derived by Castillo, (see [2]), and as stated in [4], any Gumbel distribution can be approximated as closely as desired by Weibull distributions.

## 7. ACKNOWLEDGEMENTS

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