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## Fatigue of Reinforcing Steel in Consideration of the Length Influence

Essais de fatigue sur les aciers d'armature passive, tenant compte de l'influence de la longueur

Ermüdungsdaten von Armierungsstahl unter Berücksichtigung des Längeneinflusses

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### **SUMMARY**

A continually occurring problem of engineers when designing components of structures is the availability of proper design values. In the literature, test results are documented for many different materials, but in rare cases only can values concerning the fracture or survival probability be found. In this context a statistical model is presented which enables the engineer to analyse test results and extract the design values with required fracture probability from a Wöhler-field. This model is applied to test data for reinforcing steel of different length and the results of the analysis are discussed.

### **RÉSUMÉ**

La disponibilité des valeurs caractéristiques des matériaux pose un problème perpétuel à l'ingénieur pour le dimensionnement des éléments de construction. On trouve bien dans la littérature des résultats d'essai pour les matériaux les plus divers, toutefois les valeurs de probabilité de rupture ou de survie ne sont données que dans de rares cas. Dans ce contexte, le modèle statistique présenté permet à l'ingénieur d'analyser les résultats d'essai et de tirer les valeurs de résistance nécessaires avec la probabilité de rupture demandée du diagramme de Wöhler. Ce modèle a été appliqué aux résultats d'essais effectués sur des aciers d'armature de longueurs diverses et les résultats des calculs sont discutés.

### **ZUSAMMENFASSUNG**

Eine immer wiederkehrende Problematik des Ingenieurs bei der Auslegung von Strukturbauteilen stellt die Verfügbarkeit der benötigten Materialkennwerte dar. In der Literatur sind wohl für die unterschiedlichsten Materialien Versuchsergebnisse dokumentiert, jedoch in den seltensten Fällen finden sich Angaben über die Bruch- oder Überlebenswahrscheinlichkeiten. In diesem Zusammenhang wird ein statistisches Modell vorgestellt, das dem Ingenieur erlaubt, Versuchsergebnisse zu analysieren und die benötigten Festigkeitswerte mit der geforderten Bruchwahrscheinlichkeit dem Wöhlerfeld zu entnehmen. Dieses Modell wird auf Versuchsergebnisse von Armierungsstahl unterschiedlicher Länge angewendet und die Berechnungsergebnisse werden diskutiert.



## 1. INTRODUCTION

Since mechanical structures were used more and more to move people quickly between different locations the problem of fatigue (metallic materials under repeated loading tend to fail after a certain number of cycles) became obvious. Especially when human beings lost their lives during accidents caused by fatigue, discussions started again about structural safety or probability of failure.

It is not new that for materials exist no accurate correlation between load level and cycles to failure. Whenever material is tested under dynamic loads, results (cycles to failure) always show an amount of scatter.

Depending on the number of tests a failure probability can then be related to each test result by means of statistic calculation.

If these results shall be used for a fatigue analysis it is desirable to choose a value which corresponds to a low probability of failure or a high probability of survival respectively.

Therefore engineers need more detailed information on the material with respect to failure probability.

## 2. CURRENT DESIGN SITUATION

Usually when engineers are developing structural components finite element analyses are involved in a very early stage to get an idea of the stress distribution in order to be able to optimise the design. Not only for static but as well for dynamic (fatigue or fracture) analyses material data is very important. A lot of data was collected in literature but for an actual design task in most cases the needed data can't be found because metal alloy doesn't match exactly, the heat treatment is different, or testing was performed for an other kind of loading etc. If material data can't be found in literature a company has to perform tests by itself or give an order to an external laboratory. These fatigue tests are expensive and so it's important to get a maximum of information out of a few tests.

Nowadays material data derived from laboratory are applied often for strength analysis without taking into consideration shape or size effects (Fig. 1). In other words, an extrapolation of material data from laboratory conditions to reality is rarely carried out.

The size (length) of the specimen used in the laboratory is normally not comparable with a real structure. This effect may have considerable influence on fatigue life predictions. When testing e.g. longer specimen the probability of the occurrence of larger cracks increases and therefore the fatigue lives decrease [2].

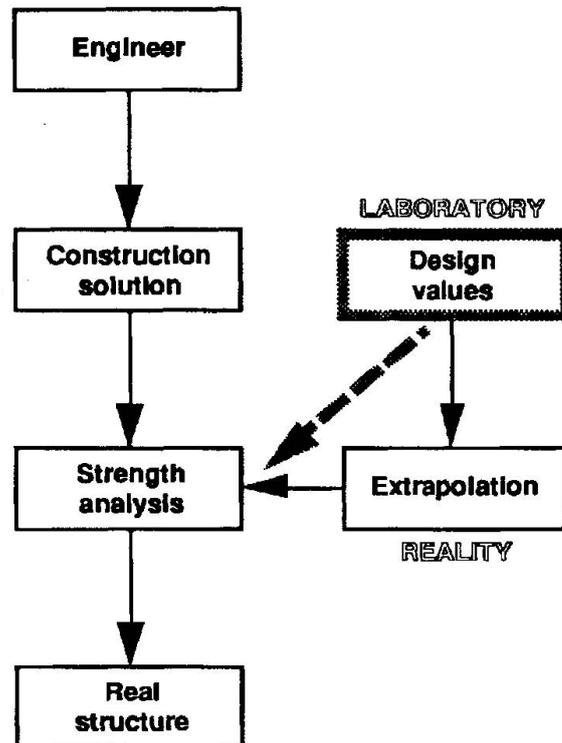


Fig. 1 Everlasting problem of engineers

### 3. OBTAINING MATERIAL PROPERTIES

#### 3.1. Performing Wöhler-testing

Material data describing fatigue life are determined normally by carrying out Wöhler-tests (Fig. 2). The size influence mentioned above is associated with the specimen geometry used for testing. This aspect will be discussed in the next chapter.

In order to obtain a proper shape of the Wöhler-curve the definition of the number of load levels on the one hand and the amount of the loads on the other hand are very important. It should be emphasised that the range of load levels should cover the stresses the engineer is interested in. Extrapolations of Wöhler-curves to values beyond the tested range should be avoided, especially in the range of high cycle fatigue, estimations of curve trends may lead to serious errors.

Beside the fracture probability related to a number of cycles to failure there is an other statistical value which defines the certainty for the fracture probability to be true. This statistical value is called the confidence level. Statements for material behaviour are usually expected to have a high confidence level (e.g. 95%). This can be obtained by performing numerous tests per load level. Of course many tests cost a lot of time and money. So again a compromise has to be made to get a satisfactory confidence level and to keep the costs low at the same time.

The relationship between testing time (test frequency) and test costs is quite obvious. What is not well known, is the aspect that the test frequency also may have an influence on fatigue life i.e. on the number of cycles to failure. What kind of effects become active when increasing the frequency has not yet been clarified but test results at EMPA for prestressing wire showed longer lives for tests with a higher frequency [3].



To minimise time and cost for testing, a final aspect has to be taken into consideration. When testing in the region of the endurance limit (if there exists one), it is favourable to define a limit number of cycles beyond that a test may be aborted. This value is called runout limit and specimen which have run through that limit are treated as "real" runouts. The difficulty of course is to define the value for that boundary because a specimen might fail a few cycles after that limit and doesn't therefore represent a runout. As discussed in chapter 6.2 the choice of this value may influence the evaluation of fatigue data. So it seems to be reasonable to set the limit beyond the region of cycles the engineer is interested in.

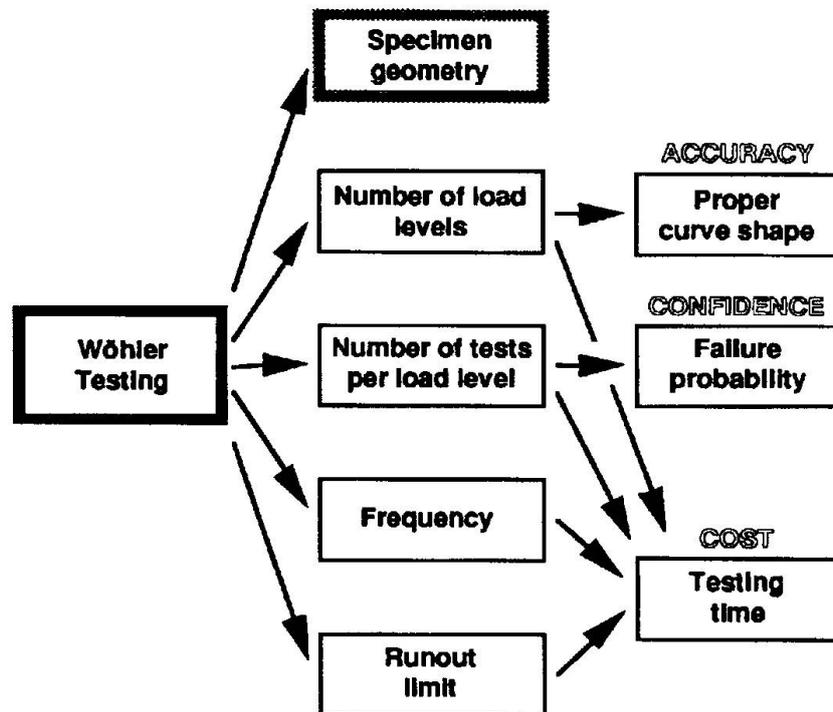


Fig. 2 Load controlled Wöhler-testing

### 3.2. Size effect

In order to perform Wöhler-testing a convenient specimen should be designed. In this context the geometry of the specimen need to fulfil some limitations which are shown in Fig. 3 below.

First of all the technical specifications of the testing machine in the laboratory (i.e. the proof length and the load capacity) usually force the designer of the specimen (depending on the problem) to deviate from reality (the specimen cannot be that long or wide as the real structure because it won't fit into the machine or the loads become too large).

As a second aspect the available facilities are often not able to manufacture the specimen's geometry.

Finally there are international standards which should be taken into consideration because testing results may then be comparable to data measured elsewhere.

All these limitations (beside the cost for manufacturing complicated surfaces) may lead to specimen geometries which differ more or less from the real component of a structure. A detailed investigation on the size effects between specimen and real structures can be found in ref. [4].

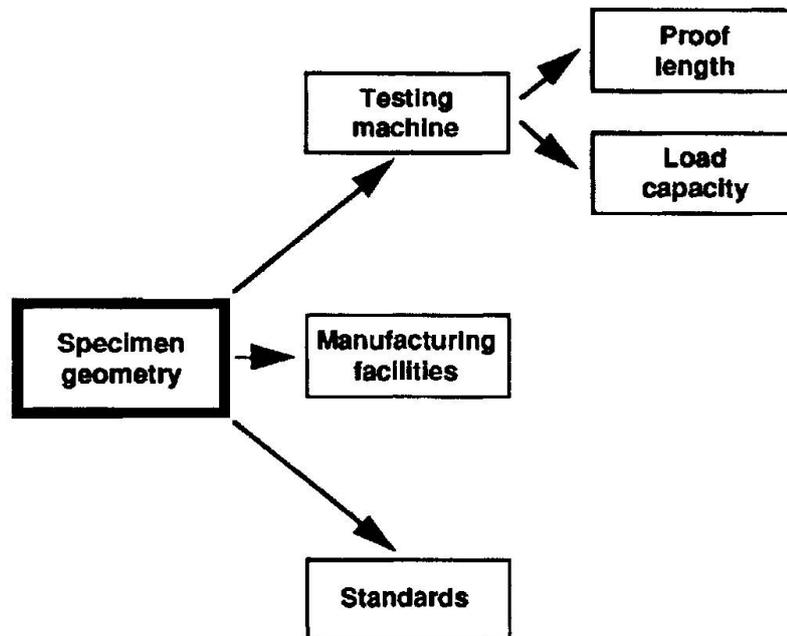


Fig. 3 Different aspects influencing the specimen's geometry

#### 4. USAGE OF WÖHLER-DATA

Wöhler data coming out of laboratory represent in most cases a summary of fatigue lives (number of cycles to failure) derived from testing specimen. In order to get an idea of the data, an engineer needs to create a graphical representation as show in Fig. 4.

In practice there will scarcely be such a large number of data available. However, the required design values have to be extracted out of a chart like this to carry out fatigue analysis and this characterises the problem many engineers have to deal with.

Looking at the example in Fig. 4 some material dependent characteristics may be pointed out for this Wöhler-field:

- For this material, an endurance limit seems to exist.
- A non constant character of scatter can be recognised for different load levels.
- The median curve (50% of failure probability) is non linear.



Still, two important informations are missed in Fig. 4:

- How do the curves for different failure probability  $P_B$  ( $P_B=5\%$ ,  $10\%$ ,  $50\%$ ,  $90\%$ ,  $95\%$ ) look like ?
- What is the influence of the length ?

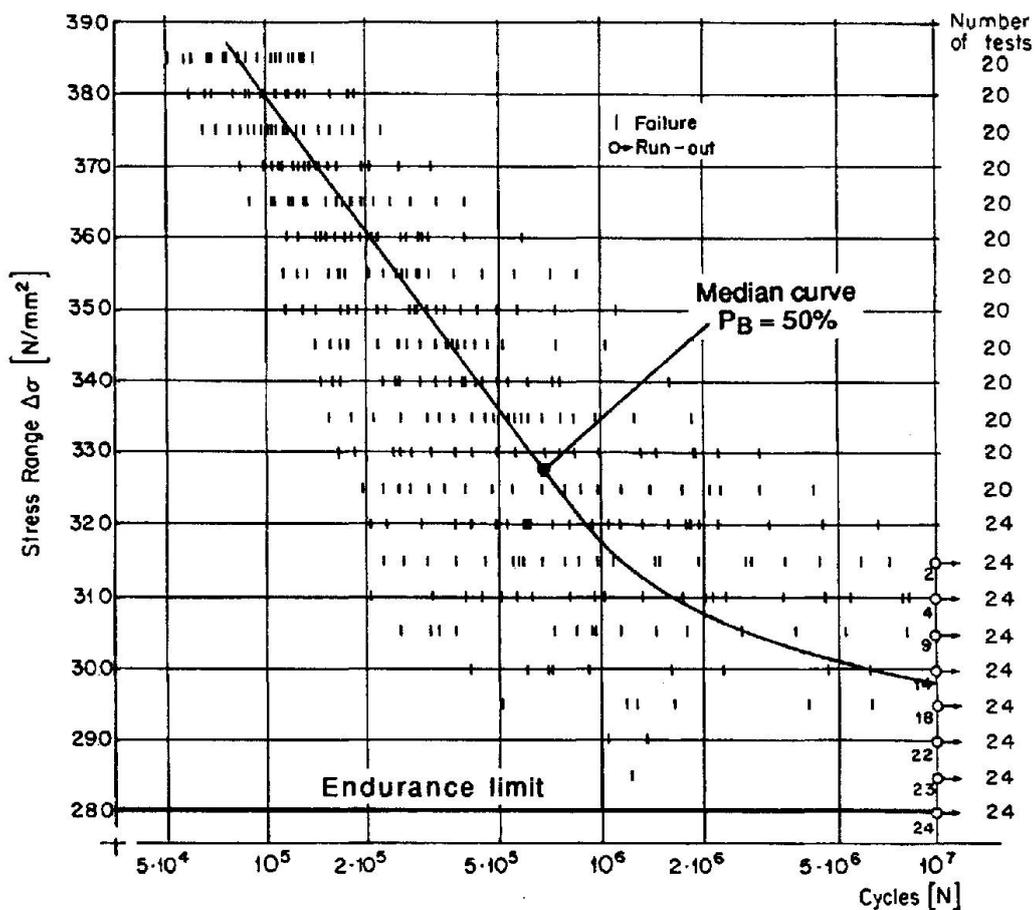


Fig. 4 Good experimental background

Just for illustration, some fatigue data of reinforcing steel measured at EMPA are presented in Fig. 5 to demonstrate the existence of a length influence:

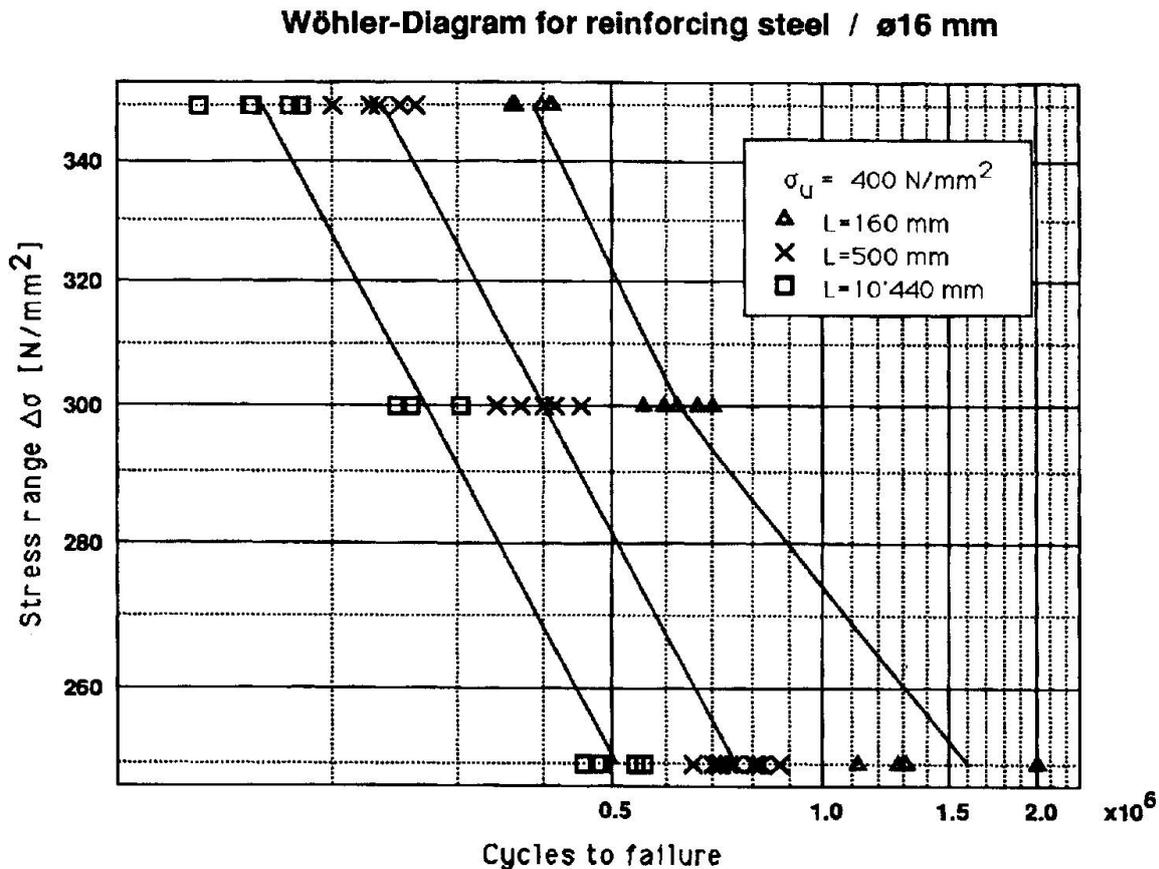


Fig. 5 Length influence for reinforcing steel

The material characteristics and the lack of information concerning failure probability mentioned above can be handled by a statistical model which is presented in the following chapters.

## 5. STATISTICAL MODEL FOR EVALUATING WÖHLER DATA

### 5.1. Development of the programme

A mathematical model for analysing data of wires, strands and cables, based on statistical requirements (compatibility, stability and limit conditions) was developed by E. Castillo and co-workers at the **University of Santander**, Spain. The methods, procedures and the theoretical background of the model was published in a IABSE Periodica already 1985 [1]. A computer programme of this model was first implemented for Macintosh but later rewritten for DOS-environments by F. M. Rodríguez at the Technical University of Gijón in 1989. This DOS-programme was sent to EMPA in 1990. For practical reasons this release was adapted and modified at EMPA. Thus, this new version will be discussed in the following chapters.



## 5.2. Programme modifications

The statistical programme "ZURICH" received from the University of Gijon was written in Turbo Pascal 5.0 and for dialogues, error messages and help texts the Spanish language was used.

In order to understand all the modules and auxiliary files of the programme a translation into the German language was carried out. Additionally an English version was written for more general use. During translation the programme's name was changed to "**FANOW**" (Fatigue **AN**alysis **O**f **W**ires).

Applying the programme to practical fatigue data some of the input routines have been improved by adding default statements or keeping values already entered before. It became also obvious that some input procedures are not needed if the programme is used for more than one data set evaluation. Therefore the general programme's execution was simplified in a way that users may skip input routines for values which shouldn't be altered. In the meantime Turbo Pascal 6.0 was released, so the programme has been adapted to this newest Pascal version in spring 1992.

When trying to output diagrams on a printer the corresponding procedures showed not to work correctly for the used EPSON LQ-500 printer. Modifications within the initialisation and formatting commands solved these problems. In addition to that output of diagrams are also foreseen for pen plotters, but the procedures used to perform this task have not yet been tested.

## 5.3. Statistical model

This chapter is intended to give just a general idea of the programme's evaluation methods. For detailed theoretical information the reader is referred to ref. [1,5].

Initial for the programme to work correctly, is the availability of numbers of cycles to failure for at least **three** different load levels. Otherwise the programme will crash.

One of the fundamental theoretical findings is the fact that data in the Wöhler-field follows the Weibull distribution. In a first step the measured data are used to calculate the parameters (constants) of the Weibull-distribution to define its shape, one of the most important value. Beside this, also an endurance limit as well as an asymptotic limit of numbers of cycles to failure results from this calculation. With the known Weibull-distribution the model allows to determine a set of different failure probability curves (hyperbolas, see below), which may give the engineer the required and important design values as shown in Fig. 6.

$$(N - B) (\Delta\sigma - C) = D \left[ \left\{ -\frac{L_0}{L} \log(1 - P) \right\}^{\frac{1}{A}} - E \right]$$

For  $P=0$  and  $L=0 \Rightarrow$  Hyperbolas

- A: Weibull shape or slope parameter
- B: Asymptotic N limit
- C: Endurance limit
- D: Scale fitting parameter obtained for chosen reference length  $L_0$
- E: Constant defining S-N threshold curve (zero probability of failure)

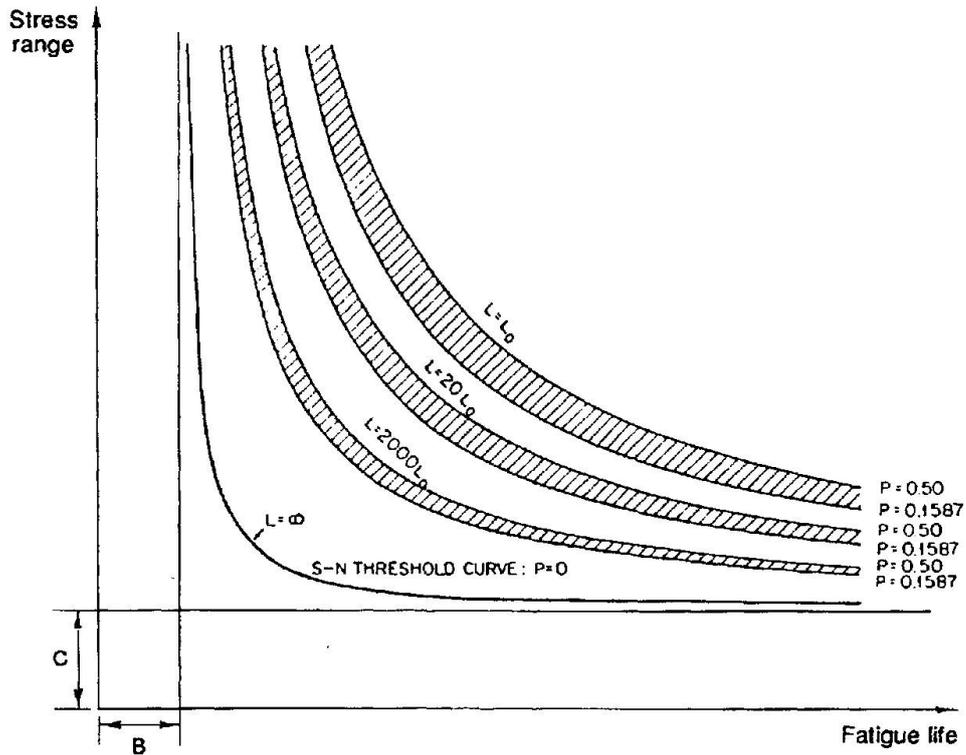


Fig. 6 Statistical model

As already mentioned in chapter 3.1 testing cost may be saved by defining a low runout limit. Information of runouts are incomplete and contain an uncertainty concerning the aspect if the specimen would fail or not. The programme FANOW assumes that the specimen will fail some time later and tries to predict the number of cycles this failure will happen.

The user just has to specify within the data set the number of cycles to failure which represents this runout limit. Using the Weibull-distribution, determined before, the programme extrapolates these runouts by calculating an estimation of a number of cycles to failure for each runout and associates these numbers to the corresponding runouts (Fig. 7).

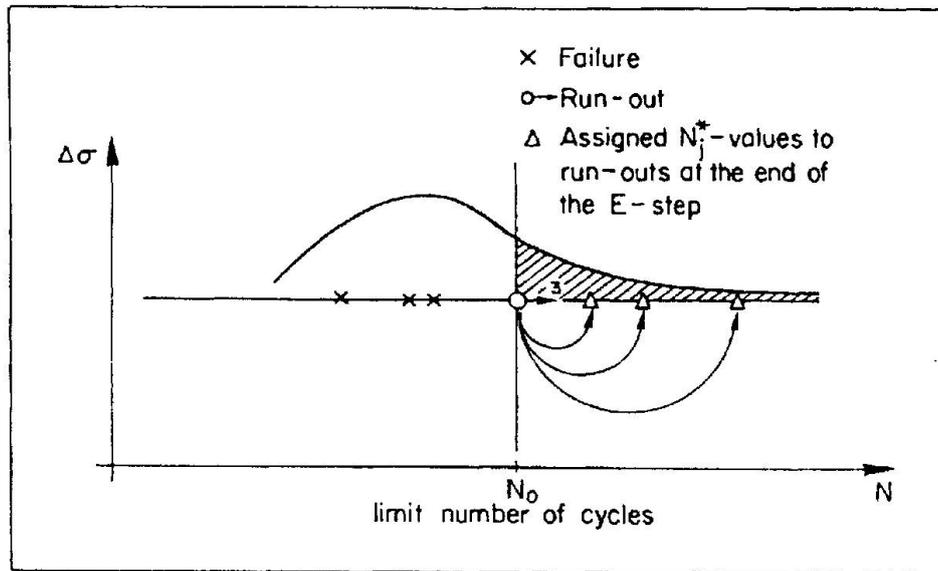


Fig. 7 Considering runouts

## 6. PRACTICAL EXAMPLE

### 6.1. Length influence on fatigue data of reinforcing steel

Bars of reinforcing steel ( $\varnothing=16\text{mm}$ ) have been tested at EMPA for three different length ( $L=160\text{mm}$ ,  $L=500\text{mm}$ ,  $L=10440\text{mm}$ ) at up to five stress levels ( $\Delta\sigma=200\text{N/mm}^2$ ,  $\Delta\sigma=220\text{N/mm}^2$ ,  $\Delta\sigma=250\text{N/mm}^2$ ,  $\Delta\sigma=300\text{N/mm}^2$ ,  $\Delta\sigma=350\text{N/mm}^2$ ). An upper stress limit was kept constant at  $\sigma_u=400\text{N/mm}^2$ . Three test frequencies ( $f=2.5\text{Hz}$ ,  $f=3.5\text{Hz}$ ,  $f=10\text{Hz}$ ) have been used for testing. The influence of the different frequencies was considered as negligible (the differences are small).

In order to evaluate the measured fatigue lives, three different data sets (one for each length) have been created for the programme "FANOW". The results of the analyses are plotted in Fig. 8 to 10.

The measured numbers of cycles to failure (test data) are represented in the Wöhler-field by unfilled squares, whereas the filled squares mark the extrapolated runouts. Accordingly, the vertical dotted line shows the chosen runout limit of  $2 \cdot 10^6$  cycles [6].

As main result of the programme's evaluation a field of failure probability curves (hyperbolas for  $P_B=1\%$ ,  $P_B=5\%$ ,  $P_B=50\%$ ,  $P_B=95\%$  and  $P_B=99\%$ ) can be found on these plots.

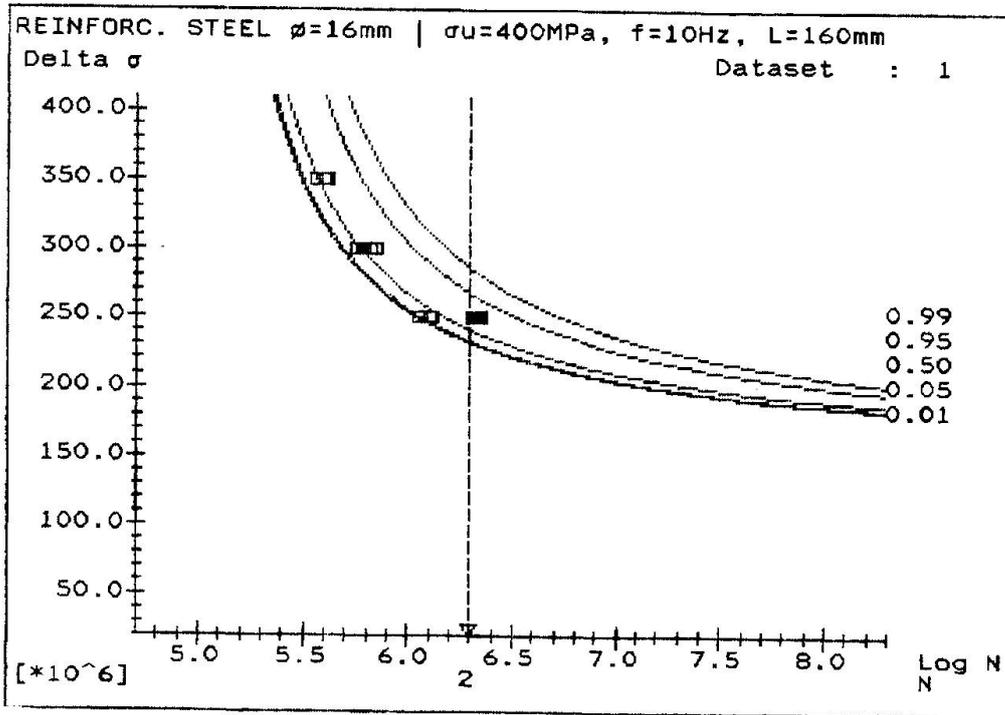


Fig. 8 Evaluation of reinforcing steel, L = 160 mm

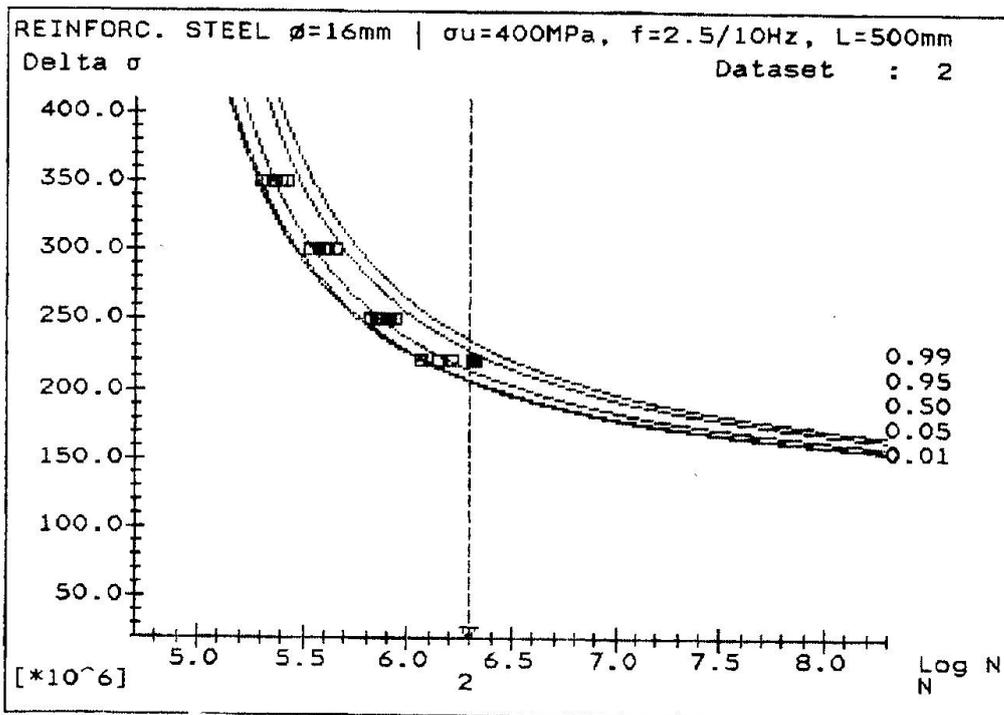


Fig. 9 Evaluation of reinforcing steel, L = 500 mm

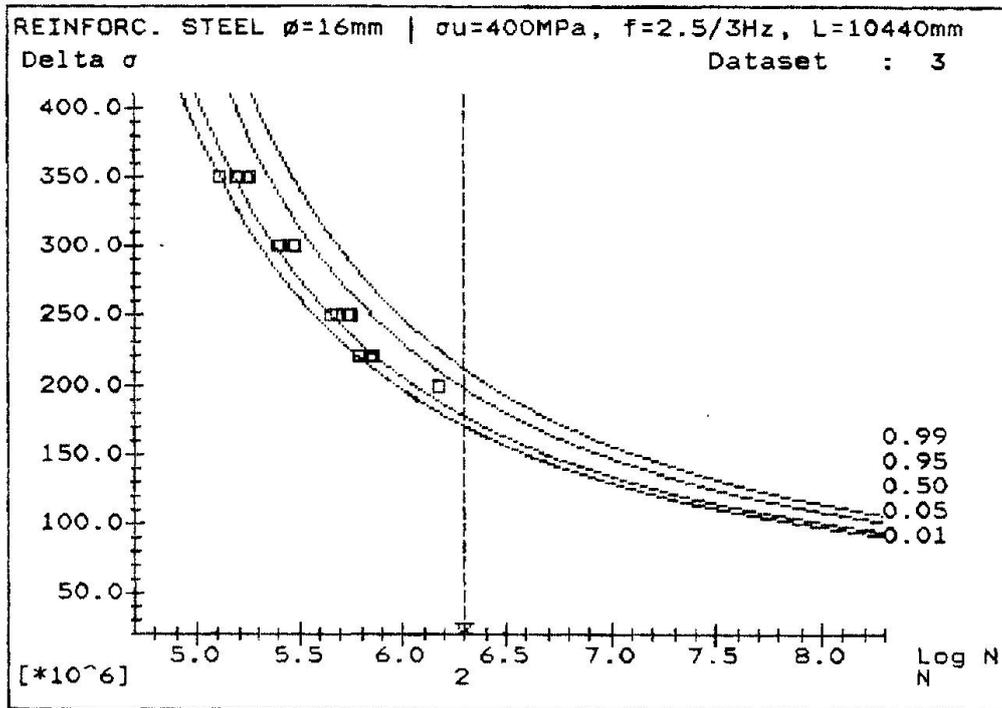


Fig. 10 Evaluation of reinforcing steel,  $L = 10'440 \text{ mm}$

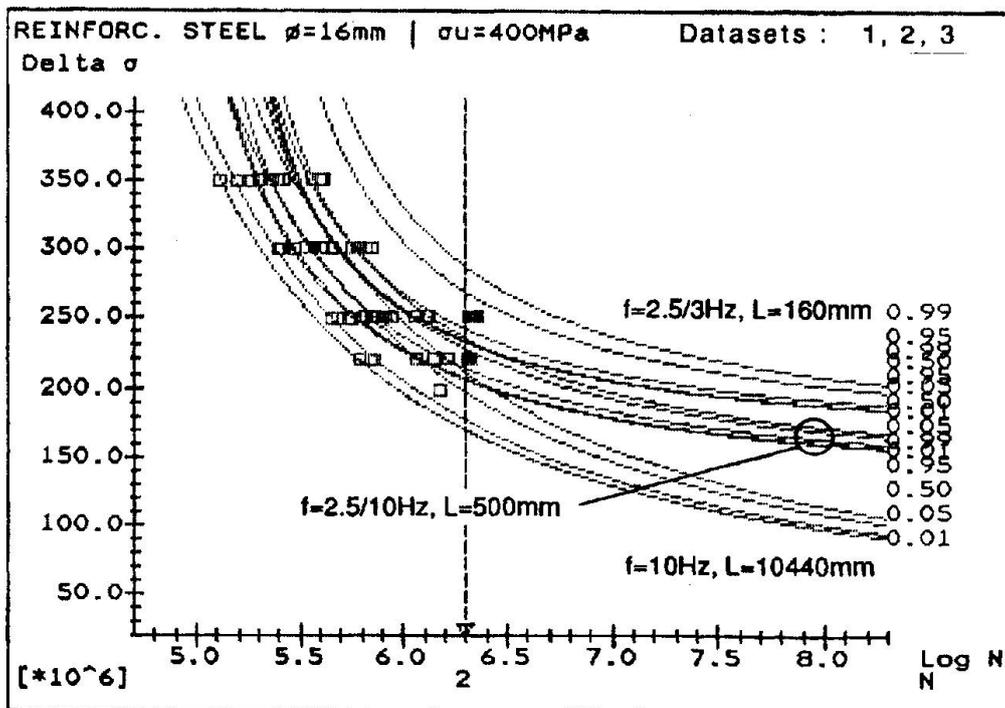


Fig. 11 Length influence for reinforcing steel ( $L=160\text{mm}$ ,  $500\text{mm}$ ,  $10'440\text{mm}$ )

## 7. CONCLUSIONS

Based on the experience made during evaluation of fatigue data for reinforcing steel this statistical model turned out to be a valuable tool for processing and presentation of Wöhler-test-data. However, it should be kept in mind that the model is still in development. As pointed out in the former chapter the user should avoid trying to extrapolate data beyond the range where Wöhler-testing was carried out due to the sensibility of the model to the number of load levels as well as to the choice of the runout limit.

During development of the statistical model an assumption (simplification) was made to derive a solution from a functional equation. This assumption consisted of setting the Weibull-slope-parameter constant for the whole range of loading. One of the future tasks is to discuss in detail the influence of this simplification.

An other open question is the circumstance, that there seems to be no or little influence of the number of tests per load level on the estimated probability curves. In order to demonstrate the difference of just a few test results per load level compared with many tests on the same level a confidence interval for each failure probability curve should be included into the model. The more test results the engineer has for evaluation the narrower the confidence interval for one failure probability curve will be. This problem is worth to pay attention to in future work.

The programme "FANOW" as well is still in development. By the moment evaluations can be carried out just for the actual test length. Future improvements of the code are planned with the object of being able to alter the length and predicting the new Wöhler-field based on the measured data set.

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