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# Modelling the Fatigue Strength and Lifetime of Wires and Cables

## Modélisation de la résistance à la fatigue et durée de vie des fils et câbles

## Modellierung der Ermüdungsfestigkeit und Lebensdauer von Drähten und Kabeln

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**SUMMARY**
This paper considers issues in modelling the fatigue strength and lifetime of wires and cables including size effects and extreme lower tail probabilities important to reliability in life safety applications. Examples are drawn from cables made of advanced composites in order to make two basic points. First, while a good fiber or wire model is an essential starting point, estimating certain extreme parameters and quantiles from test data may not be important to the reliability performance of a cable with series-parallel, load-sharing structure. Second, series-parallel load-sharing models afford the opportunity to pursue better designs for cable structures.

**RÉSUMÉ**
Cette communication traite les problèmes de modélisation de la résistance à la fatigue et de la durée de vie des fils et des câbles. Elle tient compte de l'effet d'échelle et de la probabilité de rupture extrêmement faible, facteurs importants quant à la fiabilité des réalisations impliquant un risque humain. Elle présente deux remarques fondamentales à partir d'exemples de câbles en matériaux composites hautement performants. Premièrement, bien qu'un modèle de fibre ou de fil d'excellente qualité soit essentiel au départ, une estimation, faite à partir de données expérimentales pour certains paramètres extrêmes et résultats statistiques, n'est absolument pas importante pour la fiabilité d'un câble à structure sérielle-parallèle avec répartition de charge. Deuxièmement, les modèles sériels-parallèles avec répartition de charge offrent la possibilité de mieux concevoir les structures de câbles.

**ZUSAMMENFASSUNG**
Dieser Beitrag behandelt Aspekte der Ermüdungsfestigkeit und Lebensdauer von Drähten und Kabeln. Miteinbezogen sind Größeneffekte und Schadensereignisse mit extrem kleiner Wahrscheinlichkeit, die im Zusammenhang mit Sicherheitsaspekten in der Anwendung wichtig sind. Beispiele werden aufgezeigt für Kabel aus Hochleistungsverbundstoffen, um zwei grundlegene Punkte anzusprechen: Obwohl, erstens, ein gutes Fasern- oder Drahtmodell ein wichtiger Ausgangspunkt ist, muß eine Abschätzung von gewissen extremen Parametern und Größen aus Testdaten nicht unbedingt wichtig sein für die Zuverlässigkeit eines Kabels mit seriell-paralleler, lastverteilender Struktur. Zweitens, seriell-parallele, lastverteilende Modelle bieten die Möglichkeit, besseren Konstruktionen für Kabelstrukturen nachzugehen.

# 1. INTRODUCTION

## 1.1 Preface

Our interest in steel cables and strands for cable stayed bridges and other suspended structures is a natural outcome of experiences with practical problems of cable and socket performance. For several years, one of us (SLP) has been a technical consultant to the Arecibo radar-radio telescope observatory, funded by the U.S. National Science Foundation. The feed systems for the telescope are supported by a large steel suspended structure, having twelve 7.6 cm diameter main cables and fifteen 8.3 cm diameter cables constructed of bridge strand of typical helical construction. Originally, this structure was designed to be a limited life, structure (about ten years) so that safety factors in many of the cables are *less than two*! Wire breakage in these cables has been experienced over its approximately 25 years of operation, and has been studied from both a mechanics and metallurgy perspective including dissection of a removed cable. This has led to unique corrosion protection efforts which have largely been effective in suppressing wire breakage, and many decades of useful life are expected. Some of our results have been published [1].

Most of the our experience, however, has been in advanced composites (mostly graphite, glass and Kevlar 49 aramid fibers in epoxy matrices) and ropes and cables (mostly Kevlar 29 aramid fibers in various untwisted and twisted constructions). Our interest has been in statistical modelling of the strength and lifetime in creep-rupture and fatigue of these fibrous structures and we have also done considerable experimental work on individual fibers, strands and bundles and unidirectional composites in order to validate various theories. We believe this experience brings a different perspective to the issues being addressed by this workshop as these issues are not unique to steel wires and strands. We would like to share a few observations through examples. Our comments are motivated in large measure by helpful material in the introductory lectures elsewhere in the workshop proceedings.

## 1.2 Reliability Goals and Realities

Whether we are talking about steel or polymeric composite cables in applications involving life safety, **the key design problem is to establish wire and cable structures and parameters such that the probability of failure over a specified service lifetime, loading and environment is a very small number, say < $10^{-6}$.** This must be true not only for a *single* cable but for *all* cables of a structure viewed collectively as a system, whereby any one failure will produce collapse of the system. As has been pointed out by Castillo and Fernández-Canteli in their introductory lecture, this sort of requirement imposes prohibitive needs for experimentation if such reliability targets are to be verified by brute force experimentation. **It is not possible to verify such low probabilities of failure empirically since the *number* of required replications of a fatigue test is prohibitive (> $10^7$ in the above example), the specimen *sizes* must be huge (cables much longer than typical test facilities can handle) and the *times* for testing must be enormous (years).** Furthermore experience gathered on performance near the mean of a distribution may be very misleading with respect to performance in the extreme lower tail, which cannot be observed.

This brings us to the need pursue accurate models. Designers in the past have often been satisfied with mean values of fatigue strength and lifetime (and on occasion coefficients of variation) followed by application of large safety factors based on longstanding experience. The modern reliability approach, however, is to seek probabilistic assessment through careful modelling, with the goal of determining the full probability distribution of fatigue strength (the probability for each possible value of the cyclic stress range $\Delta\sigma$ that the fatigue lifetime will reach say 2 x $10^6$ load cycles) but especially in the extreme lower tail (say probabilities of $10^{-6}$ or less). This must be done not only for a single wire of laboratory length but for a full cable.

Much of the effort in the literature seems to be devoted to building a realistic probability model for the failure of a *single* wire including statistical estimation procedures for model parameters , and size or length effects about which there has been considerable controversy. Castillo and Fernández-Canteli in their introductory lecture have identified many key issues which from our experience are also relevant to the field of composite materials. But like composite materials, cables are redundant structures for which there is considerable load-sharing among wire members. This means that bundle models, chain-of-bundles models and other lattice models have great potential for describing how a cable is ultimately to perform, especially in the lower tail region of the probability distribution; single wire models cannot do this job alone, and in fact, one must to be careful about becoming preoccupied with wire model issues that may emerge as largely unimportant in a series-parallel, load-sharing structure. There have been a few attempts to build such bundle models, notably by Fernández-Canteli and coworkers [2-4], Stallings [5], and Tanaka and coworkers [6,7] with some success. But this is just a beginning, and we believe their is great potential based on what is known about these models in the context of polymer cables and composites. Unfortunately efforts so far have had little effect on the development of international standards for testing and design [8], but on the other hand, this shortcoming can be viewed as a great opportunity for the future.

We do not want to give the impression that the quest for better models and better cables is purely a mathematical exercise in statistics devoid of the realities of the current base of experience. In their introductory lectures Esslinger and Gabriel and Nürnberger have pointed out many issues related to manufacturing processes, environmentally driven corrosion and clamping and socketing, all of which may overwhelm idealized statistical predictions. Still, good models can help us identify what is theoretically possible but also what may actually be unimportant to our goal. Models can also help us identify strategies for *structural and materials design and engineering* in order to focus on innovative solutions to the key materials and mechanics problems that are identified.

In what follows we wish to discuss issues of wire and cable reliability performance many of which have been raised in the introductory lecture of Castillo and Fernández-Canteli. We will begin from the perspective of fibrous composites as a means of illustrating some key points. We will focus mainly on static strength as the concepts are simpler. Admittedly, steel wires have considerable ductility and small variability in ultimate strength as compared to fibers used in composites, however, the issues we raise have close analogies with respect to fatigue strength and lifetime of steel cables.

## 2. A PERSPECTIVE FROM COMPOSITE MATERIALS

### 2.1 Experience with Fibers

Our laboratory experience has largely been with advanced fibers such as Kevlar 49 by The duPont Company or IM-6 or AS-4 fibers by Hercules, Inc. These fibers vary from 5 to 12 $\mu$m in diameter depending on the specific material which means that a 1 cm length already has an aspect ratio (length/diameter ratio) of 1,000 to 2,000. In comparison to a steel wire with a diameter of 5 mm, this corresponds to a wire length of 500 cm to 1,000 cm! We routinely tension test fibers of lengths up to 20 cm or aspect ratios of 20,000 to 40,000, which corresponds to steel wire lengths of 10,000 to 20,000 cm. In a few cases [9] we have also determined strength statistics for fiber segments at an aspect ratio of only about 40, that is at lengths of 0.20 to 0.40 mm. So our experience spans fiber aspect ratios from 40 to 40,000 or *three orders of magnitude in length.* Certainly one would think that distributions motivated by extreme value statistics would naturally apply, but the experimental reality less simple.

Fibers typically are produced as very lightly twisted yarn wrapped on spools. A yarn may have from 200 to more than 10,000 fibers in a cross section. Variability in fiber strength comes from flaws which are randomly distributed along the length either on the surface or within the fiber interior. As one might expect strength statistics may vary from spool to spool within the same lot. But more interesting, fibers differ in properties across a yarn. This is because the processing conditions from hole to hole in a spinnerette through which the fibers were originally extruded are not identical, and a fiber may have a common microstructure along its length, but slightly different from its lateral neighbor. Typically one finds that a Weibull weakest-link model works quite well but not over all length scales and not without the need for modification.

For a given spool and a given location along a spool (spanning say a few yards of yarn), the following version of the Weibull model for fiber strength often works quite well. Suppose we sample fibers from *across* a yarn and tension test them at arbitrary gage length $\mathcal{L}$ relative to some reference length $\mathcal{L}_0$. Experiments show that over a range of gage lengths $\mathcal{L}$, the distribution function for strength accurately follows the Weibull distribution

$$F(\sigma; \mathcal{L}) = 1 - \exp\{-(\mathcal{L}/\mathcal{L}_0)^{\alpha}(\sigma/\sigma_{\mathcal{L}_0})^{\rho}\}, \qquad \sigma \geq 0 \tag{1}$$

where $\sigma$ is fiber stress, $\mathcal{L}$ is the actual fiber gage length, $\mathcal{L}_0$ is a convenient reference length, $\sigma_{\mathcal{L}_0}$ is the Weibull scale parameter for strength measured at reference length $\mathcal{L}_0$, $\rho$ is the Weibull shape parameter for strength, and $\alpha$ is a parameter satisfying $0 < \alpha < 1$. Note that the strength versus length relationship is given by

$$\sigma_{\mathcal{L}} = \sigma_{\mathcal{L}_0}(\mathcal{L}_0/\mathcal{L})^{\alpha/\rho} \tag{2}$$

where the exponent is *not* the usual $1/\rho$.

When fiber segments are sampled from *along* a given fiber it often occurs that the model

$$\underline{F}(\sigma; \mathcal{L}) = 1 - \exp\{-(\mathcal{L}/\mathcal{L}_0)(\sigma/\underline{\sigma}_{\mathcal{L}_0})^{\rho^*}\}, \qquad \sigma \geq 0 \tag{3}$$

works well with $\rho^* = \rho/\alpha$, though $\underline{\sigma}_{\mathcal{L}_0}$ would vary from fiber to fiber. The strength versus length relationship is then given accurately by

$$\underline{\sigma}_{\mathcal{L}} = \underline{\sigma}_{\mathcal{L}_0}(\mathcal{L}_0/\mathcal{L})^{1/\rho^*} \tag{4}$$

as expected. (The theoretical underpinnings of the above 'empirical' model are discussed in Watson and Smith [10] both for fibers and composite strands.) Figure 1 shows

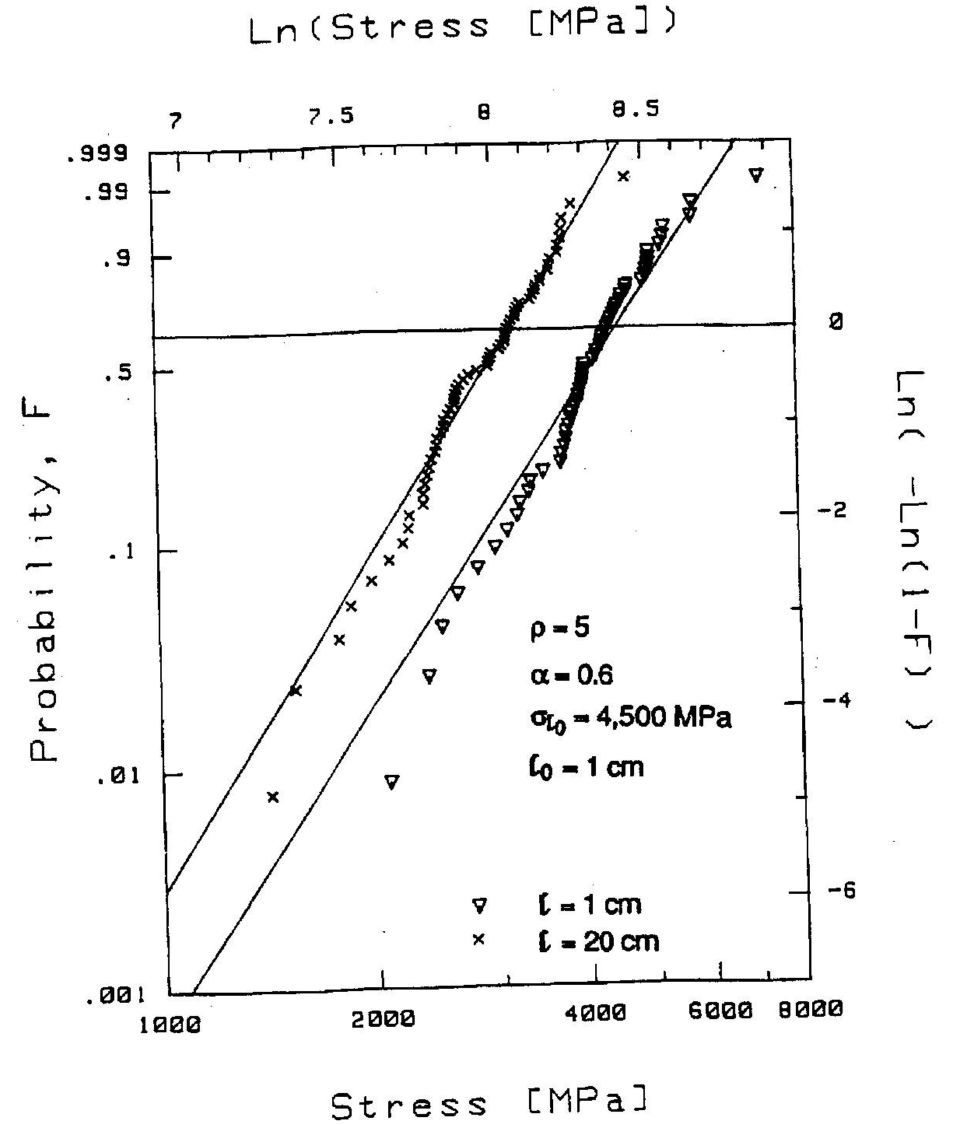


Fig. 1. Strength data for Hercules AS-4 graphite fiber on Weibull coordinates (Ref. [14])

experimental data for Hercules AS-4 graphite fibers tested at gage lengths $\ell$ of 1 and 20 cm. For this fiber we find $\rho = 5$ and $\alpha = 0.6$, $\rho^* = 8.3$ and $\sigma_{\ell_0} = 4{,}500$ MPa.

The question arises as to how well the model works with respect to extrapolation to much longer or shorter gage lengths of AS-4 fiber. Experience shows that reasonable extrapolations are possible down to $\ell = 0.25$ mm, which is approximately the effective load transfer length for fibers in a graphite/epoxy composite. At that length, the scale parameter for strength would be in the vicinity of 7,000 MPa, but further decreases in length may not produce the anticipated increases in strength. For IM-6 graphite fibers in this situation, it turns out that a rolloff in strength and rapid increase in Weibull shape parameter may already occur at such lengths [9]. The extrapolation works well for longer lengths of AS-4 graphite fiber also, up to perhaps 100 cm. At that length, the scale parameter would be about 2,600 MPa but the strength my drop of more rapidly at even longer lengths than eqn (4) predicts often accompanied by a sudden drop in Weibull shape parameter. So the model reasonably covers a strength range varying by a factor of almost three and a length range of over three orders of magnitude.

This model works well for the AS-4 graphite fibers at hand, but one finds great variety particularly in the value of $\alpha$. For Kevlar 49 we find $\rho \approx 8$ and $\alpha \approx 0.4$. For Hercules IM-6 fibers we have found $\alpha \approx 1.0$ over a limited range, but tending actually to drop to 0.8 for longer lengths. Moreover, values vary from spool to spool even in the same production lot. For spectra fibers one finds $\alpha \approx 0.1$ with only a very mild length effect at least up to several centimeters. For most fibers one finds 'drift' in the model in that the Weibull shape parameter will drift continually *downward* to smaller values as the length is increased starting with aspect ratios of about 40. This behavior is well known for glass fibers and we have seen it also in Hercules IM-6 graphite fibers and boron fibers.

Thus, our experience is that for the strength of single fibers one must anticipate the need for models of the form

$$F(\sigma; \ell) = 1 - \exp\{-g(\ell/\ell_0)\Lambda(\sigma/\sigma_{\ell_0})\}, \qquad \sigma \geq 0 \qquad (5)$$

where $g(\ell/\ell_0)$ and $\Lambda(\sigma/\sigma_{\ell_0})$ are a quite arbitrary functions not necessarily well represented by simple power forms leading to the usual Weibull distribution. Power approximations to $\Lambda(\sigma/\sigma_{\ell_0})$ for smaller and smaller values of stress may require smaller and smaller exponents leading to smaller Weibull shape parameters for longer and longer lengths. Even for very large aspect ratio,s $g(\ell/\ell_0) \approx \ell/\ell_0$ may not be the appropriate model for fibers in a bundle as the fibers have consistent differences in properties. We believe the same situation will occur for steel wires in a bundle as one must be concerned about the manufacturing homogeneity and source of the wires.

The statistical theory of extremes suggests to us that there are only two limiting distributional forms, namely a Weibull form or the double exponential (Gumbel) form useful for tensile strength. But this can be a misleading concept as the above example shows. One must be prepared for well behaved possibilities that don't conform nicely to a Weibull or Gumbel model. In fact, we have a well developed lattice model for failure [11] with weakest-link properties but where a simple well behave distribution is far superior to a Weibull or Gumbel approximation *at all lengths and especially in the lower tail.* The simple Weibull model is not always a good approximation for a fiber in a given application since in a composite material where load sharing takes place, many length scales and stress ranges may be important simultaneously. Nevertheless in analytical models of systems involving load-sharing the Weibull model can give us considerable insight.

## 2.2 Experience with the Strength of Simple Composites

Armed only with such statistical models for fiber strength, what can be said immediately about the strength behavior of a fiber/epoxy composite? Short of bounding the strength from below, the answer is very little! To see this, we consider a simple graphite fiber/epoxy strand made from impregnating a graphite yarn with epoxy. A typical laboratory specimen might be 20 cm long and have 10,000 fibers in its cross section, yet it is still smaller in diameter than a shoelace! The total length of fiber in strand is now 200,000 cm or 2 kilometers. In fact, a key characteristic length in the composite is the effective load transfer length for a fiber in the epoxy matrix, being of the order of 40 fiber diameters or 0.25 mm. There are $8 \times 10^6$ such fiber elements in our composite strand.

If we apply the above Weibull model, eqn (1) and assume that the composite fails when the first fiber fails, we might predict by extrapolation that the strength of the composite parameter $4{,}500\times(200{,}000)^{-(0.6)/5}$ MPa = 1,040 MPa. Experiments show, however, that this prediction is false. In fact, the strength of such strands will follow approximately a Weibull distribution with a shape parameter of about 30 and a scale parameter of close to 4,500 MPa (fortuitously the value for 1 cm fibers). Furthermore if the strand length is increased by a factor of say 10 to 100, say, the strength will decrease very slowly approximately in proportion to $\ell^{-0.6/30}$, which is almost unmeasurable by experiment. What this means is that in a composite loaded say to 2,800 MPa there will be many fiber breaks -- of the order of one in every 50 cm of fiber in the composite; this is of the order of a total of 4,000 fiber breaks in the strand, yet at this load the composite has survived nicely! In fact the probability of failure is lower than $6.6 \times 10^{-7}$. **The presence of a huge number of fiber breaks is consistent with high reliability.**

There are two points: First, breaks may be monitored by acoustic emission in an attempt to predict impending composite failure, but experience has shown this to be largely an unproductive exercise. These breaks and for that matter the strengths of the weakest fibers tell us little about the strength performance of the composite, and are not a reason for its removal from service. Second, for prediction in the lower tail of the composite strength distribution, there is no need to characterize the strength of fibers beyond a length of 50 cm. In fact our models show that the fibers could actually be discontinuous with a mean length of about 10 cm, and the strength distribution for the composite would be negligibly altered. This is the power of fiber load-sharing through the matrix.

Over the past few years we and others have worked on the development of chain-of-bundles probability models to explain the above behavior. The basic idea is that the above composite strand can be partitioned into a chain of short bundles with each bundle having length equal to the effective load transfer length for a broken fiber next to an intact fiber in the epoxy matrix, which transfers the load through shear. This length is of the order 0.25 mm in the above example. Fibers within these bundles then share load according to a load-sharing rule which assigns the loads of failed fibers mostly onto the nearest surviving neighbors. This produces what amounts to a local redundancy and the composite will fail once a critical cluster of a few broken fibers develops which then becomes unstable. References for such models are Harlow and Phoenix [11] and Smith et al. [12] particularly the references therein. Phoenix and Tierney [13] consider such models in the setting of time dependent failure and fatigue adaptable to steel cables.

An interpretation of the above Weibull-like result is that the composite strand fails once a critical cluster of about six fibers develops, and the Weibull shape parameter for the composite, $\rho_c$ turns out to be $6\times5 = 30$. The effective Weibull scale parameter for the composite, $\sigma_c$, is determined from the load-sharing in a fairly complex way [14]. We can write the approximate Weibull model for the composite as

$$F_c(\sigma; \ell) \approx 1 - \exp\{-(\sigma/\sigma_c)^{\rho_c}\}, \qquad \sigma \geq 0. \tag{6}$$

Yet if we increase the length of the composite (or for that matter its width) the model predicts that the shape parameter will actually *increase* very slowly aproximately in proportion to the log of the volume (since as the strength drops the critical cluster size grows). This increase occurs despite the fact that the fiber shape parameter decreases! Furthermore the Weibull distribution actually *overestimates* the probability of failure in the lower tail consistent with an increasing Weibull exponent applicable to that region. This is not just a prediction from the model. **All experiences with composite structures, orders of magnitude larger than the laboratory strand under discussion reveal that these general features are valid.**

## 2.3 An Example of a 4-fiber Composite 'Cable'

The number of fibers in the cross-section of a typical commercial yarn (a thousand or more) or the number of characteristic fiber elements in a small composite strand (at least $10^6$) are orders of magnitude larger than the number of wires or wire elements in a typical steel cable. Thus one may attempt to argue that the above models and ideas have limited relevance, especially the benefits of load-sharing among elements. This is not true. In fact most of the benefits of localized load-sharing are realized with the interaction of very few fibers, and the effective degree of interaction actually grows as the log of the total volume, which is very slowly. For more global load sharing, the benefits grow even faster. The following experimental example [14] makes the point.

We have fabricated miniature composite 'cables' consisting of four AS-4 graphite fibers in parallel in a square cross-section and held together by an epoxy. The fibers were closely packed and the epoxy not only filled in the voids but also formed a fairly thin layer around the fibers. The epoxy volume fraction was about 30%. Specimens were fabricated for tension testing at two gage lengths, 1 cm and 20 cm. Note that these specimens had a diameter of about 16 μm (much less than a human hair so they would be useful as cables only to insects!) so that their aspect ratios were about 600 and 12,000, respectively. From a chain-of-bundles model point of view, the effective load transfer length, $\delta$, among fibers in a cross section is about 0.15 mm, so the number of bundles, m, in the chain model is about 66 for the 1 cm specimens and about 1,333 for the 20 cm specimens; the total number of fiber elements 4m were 264 and 5,280 respectively. Figure 2 is a schematic of the composite including load-sharing configurations.

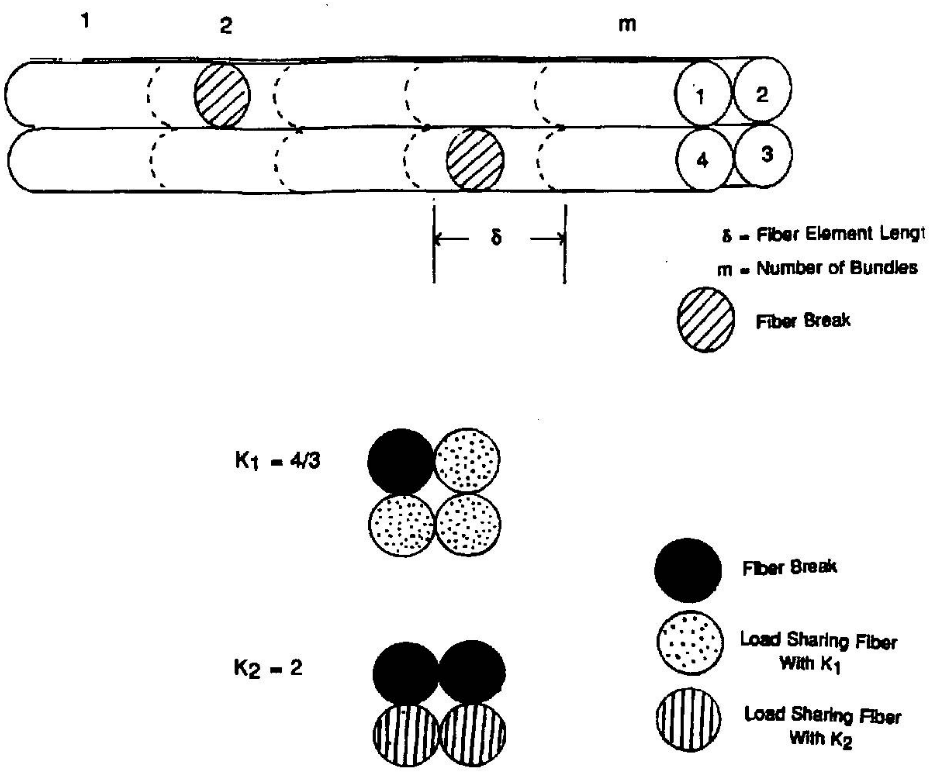


Fig. 2. Schematic of 4-fiber composite cable and load-sharing model (Ref. [14])

According to theory [14], we expect to see the emergence approximately of Weibull distributions of the form

$$F^{[k]}(\sigma) \approx 1 - \exp\{-4m^{\alpha}(\sigma/\sigma_{\delta,k})^{k\rho}\}, \qquad \sigma \geq 0, \tag{7}$$

for the strength of the composite where k = 1,2,3 and 4, $\rho$ is the Weibull shape parameter for the fiber, and $\alpha$ is a parameter discussed earlier in connection with the fiber. Also

$$\sigma_{\delta,k} = \sigma_{\delta}(d_k)^{-1/(k\rho)}, \tag{8}$$

where

$$d_1 = 1, \quad d_2 \approx 3(4/3)^{\rho}, \quad d_3 \approx 6(4/3)^{\rho}(2)^{\rho}, \quad d_4 \approx 6(4/3)^{\rho}(2)^{\rho}(4)^{\rho} \tag{9}$$

captures (approximately) the effect of load-sharing factors and configurations, and

$$\sigma_{\delta} = \sigma_{\ell_0}(\ell_0/\delta)^{\alpha/\rho} \tag{10}$$

is the characteristic Weibull strength (scale parameter) for a fiber element of length $\delta$. For lower and lower stress ranges, these Weibull distributions will apply in succession, where, $k = 1$ applies roughly for $3\sigma_\delta/4 < \sigma$, $k = 2$ applies roughly for $\sigma_\delta/2 < \sigma < 3\sigma_\delta/4$, $k = 3$ applies roughly for $\sigma_\delta/4 < \sigma < \sigma_\delta/2$ and $k = 4$ applies roughly for $0 < \sigma < \sigma_\delta/4$. Note that $k = 1$ corresponds to the strength of the weakest flaw in all four fibers, that is, the first fiber break or a 'weakest flaw' view of failure of the composite.

Figure 3 demonstrates that these features are largely observed in the experimental data for these composites [14]. The various lines are the Weibull distributions of eqn (7) for k = 1, 2, 3 and 4 and m = 66 and 1,333 for the two cases. For the calculations we have taken $\alpha = 0.6$ and $\rho = 5$ as in Figure 1, and $\delta = 0.15$ mm for which, we have $\sigma_\delta \approx 7{,}500$ MPa. The interpretation of the plots is that k is the critical cluster size (number of adjacent fiber breaks required for collapse) for that stress range. Clearly the composites do not fail with the first fiber failure. For the 1 cm composites, perhaps only the strongest specimen failed when one fiber failed, many of the remainder required two adjacent breaks, and the weakest few required three breaks to cause collapse. In fact in the extreme lower tail, the appropriate Weibull distribution will have $k = 4$ with Weibull shape parameter $4\rho = 20$.

For the 20 cm composites the fit is not quite as good, but it can be greatly improved by choosing $\delta = 0.25$ instead of $\delta = 0.15$. The appropriate Weibull distribution modelling the extreme lower tail has shape parameter $\rho_c = 20$ and scale parameter $\sigma_c = 2{,}850$ MPa calculated from eqns (7) to (10) in view of eqn (6). The stress level producing a $10^{-6}$ probability of failure is about 1,450 MPa. In this case failure requires four adjacent breaks prior to collapse. Incidentally Figure 1 shows that we have sufficient data on fibers to be confident of the fiber strength at that stress level. We do *not* need to know basic fiber statistics at extremely low probabilities of failure or extremely long lengths. Although we can never make and test enough specimens to prove the point (although filament wound pressure vessels with $10^9$ times as much material behave as predicted) we are confident of these reliability predictions for this microscopic cable.

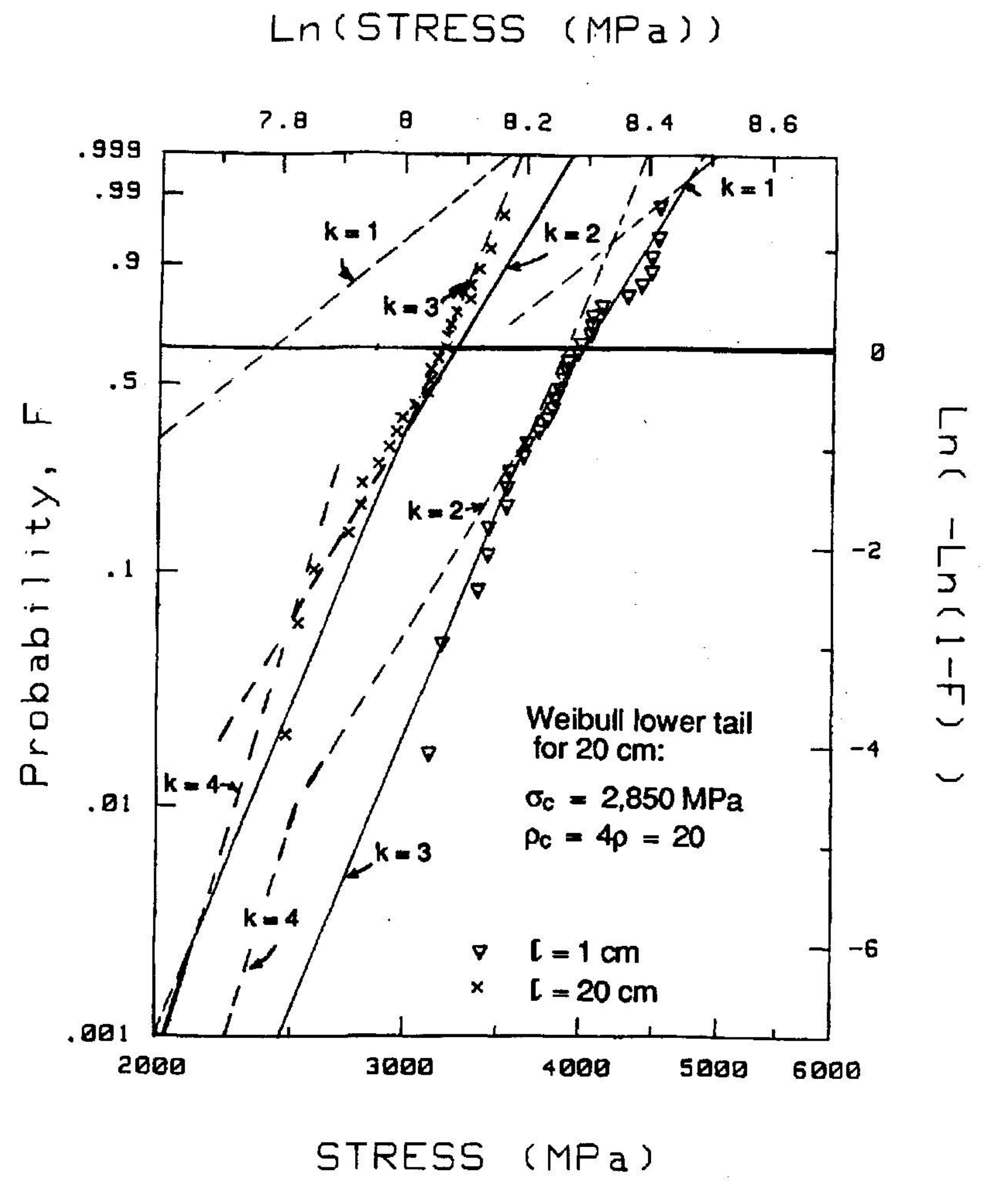


Fig. 3. Strength distributions for a composite 'cable' of four AS-4 graphite fibers in an epoxy matrix plotted on Weibull coordinates for two gage lengths (Ref [14])

## 3. THE PROMISE OF BUNDLE AND CHAIN-OF-BUNDLE MODELS FOR CABLES FATIGUE SETTINGS

Apart from the attempts described earlier [2-7] there are various bundle models and chain-of-bundles models ready to be adapted to steel wire bundles and cables for purposes of reliability prediction in fatigue. Phoenix [16] describes a bundle model of great flexibility, though it has seen little application thus far. One example of importance in the case of glass fibers is due to Kelly and McCartney [16]. This model should be adaptable to the wire fatigue model described in the introductory lecture of Castillo and Fernández-Cantelli. Other versions are also discussed by Phoenix [17] and [18]. Single fiber models are used to interpret experimental data in Wu et al. [19]. Smith and Phoenix [20] and Pitt and Phoenix [21] give various static and time dependent cases of the model applicable to cable systems.

As pointed out by Castillo and Fernández-Canteli in their introductory lecture, these models often lead to various limiting or approximating distributions for fatigue strength and lifetime, including Weibull, Gaussian and Gumbel distributions. In some important cases, however, other distributions arise [11] with far more power to model the extreme lower tails of interest in high reliability requirements. This power may be diluted by attempting to generate a classical extreme value form, especially when such is unnecessary.

Such models can help us identify not only what is theoretically possible but also what may actually be unimportant to our goal of reliable and efficient cables. Models can also help us identify strategies for *structural and materials design and engineering* in order to focus on innovative solutions to the key problems that are identified. In the process of failure in a typical laboratory fatigue test such models may put into perspective the value of data on wire failures along the way. It is not clear that current practises, particularly as they pertain to acceptance/reject standards, bear much connection to the performance of the extreme lower tails of distributions. This perhaps was the most important point raised in the analysis of Stallings [5]. It is clearly a point in our 4-fiber cable example.

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