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Initial Flaws Distribution and Size Effect of Fatigue Fracture

Distribution de défauts initiaux et l'effet de la longueur sur la rupture par fatigue

Fehlstellenverteilung und Maßstabseffekt beim Ermüdungsbruch

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SUMMARY
A survey of random factors influencing the reliability and lifetime of structural components is presented with an emphasis on the initial flaws distribution and specific difficulties in the extrapolation of laboratory data to in-service conditions of full-scale structures. A far-reaching generalization is suggested of the probabilistic models of fatigue taking account of sets of initial and newly-created flaws as well as loads and actions randomly varying in time. Special attention is paid to the prediction of the fatigue life of ropes and cables consisting of a large number of wires. The influence of the wires' interaction on the size effect in ropes and cables is discussed.

RÉSUMÉ
Une approche des facteurs aléatoires qui interviennent sur la fiabilité et la durée de vie des composants structurels est ici présentée en mettant l'emphase sur la distribution des défauts initiaux et sur la difficulté particulière d'extrapoler les résultats obtenus en laboratoire aux structures réelles en conditions de service. On propose une généralisation des modèles probabilistiques de fatigue en tenant compte des défauts initiaux et des défauts d'apparition ultérieure. On prête une attention spéciale à la prédiction de la vie de fatigue des filins et des câbles formés d'un grand nombre de fils. A ce sujet, une discussion est engagée sur l'influence de l'interaction des fils sur l'effet d'échelle des différents câbles.

ZUSAMMENFASSUNG
In einer Übersicht der zufälligen Einflüsse auf die Zuverlässigkeit und Lebensdauer von Traggliedern werden die Verteilung anfänglicher Fehlstellen und die besonderen Schwierigkeiten bei der Extrapolation von Labordaten auf die Betriebsbedingungen in wirklichen Strukturen hervorgehoben. Eine weitreichende Verallgemeinerung der probabilistischen Ermüdungsmodelle berücksichtigt Gruppen bestehender und frischer Fehlstellen sowie stockastische Einwirkungen. Besondere Aufmerksamkeit gilt der Vorhersage der Ermüdungslebensdauer von Seilen und Kabeln mit zahlreichen Drähten. Dabei wird der Einfluß der Drahtinteraktion auf den Maßstabseffekt diskutiert.

1. INTRODUCTION

Fatigue failure is a phenomenon not easy to describe and predict quantitatively in a precise and reliable way even in the framework of the modern fracture mechanics. This phenomenon consists of several stages: dispersed microdamage accumulation, initiation of nuclei of macroscopic cracks, development of short cracks and the further formation of macroscopic cracks propagating up to the final failure. In wires, ropes and cables composed of thin fibers, fatigue failure is a result of fracture of a certain number of neighbouring fibers. Interaction between the fractured and whole fibers during the damage process is a complicated phenomenon from the viewpoint of structural mechanics, too. Additional complications arise due to the large statistical scatter of laboratory and field data, and that makes the application of probabilistic models necessary for the prediction of reliability indexes and lifetime for full-scale engineering systems.

Experimental analysis helps only partially since only comparatively short specimens could be tested meanwhile wires and cable used in large engineering systems are many times longer. Probabilistic models based on simple and transparent concepts often fail to predict more or less precisely reliability indexes for much longer structural components. The same situation takes place in prediction of the service lifetime using results of short time testing. The problem of reliable extrapolation of laboratory tests data on much longer structural components and much longer times is in fact the central point of this Workshop.

Random factors influencing on fatigue failure can be divided in the three groups: randomness of material properties; random flaws and imperfections of structural components; random loads, actions and enviromental conditions. In turn, each group consists of several subgroups. For example, it is expedient to distinguish the randomness inherent to the microstructure of materials, and the batch-to-batch scattering of properties of commercial materials born from instabilities and imperfections of the manufacturing process. Flaws and imperfections of different origin and various shape, size and position enter in the second group of random factors. At last, loads and actions form a broad variety with random and/or uncertainly defined parameters.

The discrepancy between the predicted fatigue life of structural components and that observed in field condition is born from different sources. Among them is the difference in behaviour of short laboratory specimens and long structural components originated, in particular, from the differnce in load transfer and the relative input of the anchorage into the resulting reliability. Enviromental conditions are difficult to reproduce in laboratory tests, moreover, their long-time effects on mechanical properties. Non-stationary random loading influences on the fatigue life, and often in a non-trivial way (as example, effects of overloadings may be mentioned). Contrary to that, most laboratory tests are performed in stationary, regular cycle loading. Another obstacle to perform a reliable extrapolation of laboratory data is born from the use of oversimplified probabilistic models and nonadequate statistical techniques. The trust in universality of Weibull's model and the brave extrapolation of statistical data obtained from poor samples are, probably, the most evident examples of such oversimplification.

Later on, the following items concerning the problem of strength, lifetime and reliability prediction of long structural components are discussed: (a) probabilstic models of structural reliability in the presence of sets of randomly distributed flaws; (b) probabilistic models of the fatigue crack growth with the special attention to various sources of scattering of results; (c) interaction between single wires in ropes and cables and its account in probabilistic models; (d) some aspects of the extrapolation of short specimens and/or short-time fatigue tests.

2. RELIABILITY AND LIFETIME IN THE PRESENCE OF RANDOM SETS OF FLAWS

Consider a structural component or a specimen (later - a body) under, generally, nonstationary cycling loading. The body contains a number of cracks, cuts, pores, and flaws that can develop in macroscopic cracks which growth results into the final failure of the body. The flaws differ in origin (initial, new-born, detected and admitted during inspection, non-detected), and in size, shape and position. Later on, all crack-like flaws of macroscopic size, say, of the order of 1 mm and more are called cracks. Unite the cracks with similar features in sets. To identify cracks in their position, divide the body into domains $M_1, \ldots, M_I$ that are, generally, may overlap. Dimension of this domains may be different. For brevity, use the same notations for domains and their measures, e.g. for the length of one-dimentional domains, for the surface of two-dimentional ones, etc. Choose a standard measure M_{0i} for each domain, say, the unit of the corresponding measure. For simplicity, let cracks of each set are described with a single size parameter a_{ij}, i = 1, ...,J where j is the number of j-th set, and J is the total number of sets. Assume that the failure of the body occurs when at least one of the cracks attains the corresponding critical size a^*_{ij}. Then the reliability (survival) function is

$$R(t) = P\left\{ \begin{matrix} \max a_{ij}(x,t) < a^*_{ij}(x,t);\ x \in M_i \\ i=1,\ \ldots,\ I;\quad j=1,\ \ldots,\ J \end{matrix} \right\} \qquad (1)$$

Here P{ } is probability of the event in braces, x is reference vector.

If the density of macroscopic cracks is sufficiently low, it is possible to neglect their interaction. Then Poisson model is valid for each set of cracks, for each domain, and for a body as a whole. Eq.(1) resuits into

$$R(t) = \exp\left[-\sum_{i=1}^{I} \sum_{j=1}^{J} \int_{M_i} \mu_{ij}(a^*_{ij};t)\ \frac{dM_i}{M_i^0} \right] \qquad (2)$$

where $\mu_{ij}(a_{ij};t)$ is the expected number of cracks from j-th set in i-th domain which size a_{ij} attains the critical magnitude a^*_{ij} at the time t.

Eq.(2) presents a far-going generalization of probabilistic models of reliability against fatigue failure based on Weibull's or double exponential (Gumbel's) distributions [10,11]. In particular, Eq.(2) takes into account nonhomogeneities that may be both continuous, accounted with integrals, and discrete, accounted with the division of the body into domains. One can to present $\mu_{ij}(a_{ij};t)$ in the form

$$\mu_{ij}(a_{ij};t) = \mu_{ij}(t)[1- F_{ij}(a_{ij};t)] \qquad (3)$$

where $\mu_{ij}(t)$ is the expected total number of cracks from the considered sets, and $F_{ij}(a_{ij};t)$ is the probability distribution function of crack sizes up to the time t. Determination of $\mu_{ij}(t)$ and $F_{ij}(a_{ij};t)$ is the subject of the probabilistic fracture mechanics. Many aspects of the reliability assessment can be taken into account with this model since it includes the cracks initiation and growth, inspection procedures, decision making, replacement ant repair, etc. We do not go here into details that can be found in book [5] and paper [6].

3. RANDOMNESS OF MATERIAL PROPERTIES AND FATIGUE CRACK GROWTH

To estimate functions $\mu_{ij}(t)$ and $F_{ij}(a_{ij};t)$ entering into Eqs.(2) and (3), solution are to be found of certain stochastic equations describing the evolution of cracks and crack-like flaws. A number of studies have been performed during the last decades dedicated either to the randomization of the known (deterministic) equations of fatigue crack growth or to the use of (also already known) mathematical models describing such irreversible stochastic processes that may be interpreted in terms of fatigue damage. A very important question of the real origin of randomness, as a rule, remains out of the area of these studies. Meanwhile, it is necessary to make difference between the inherent randomness of the material's microstructure which we call within-a-specimen scatter, and batch-to-batch or even specimen-to-specimen scatter. In addition, there is such an important factor as randomness of initial conditions - that of the size, position and shape of flaws at the beginning of the considered time segment. This factor takes an intermediate position between the two kinds of randomnesses. In our opinion, since the initial flaw distribution varies significally between specimens (and, moreover, between components of real structures), this type of randomness is to be attributed to specimen-to-specimen randomness. In fact, the crack tip blunting due to corrosion or overloading in the previous life can effect essentially on the duration of the initiation stage and on the early crack growth rate.

The influence of initial conditions on the fatigue life has been, generally, underestimated and almost not investigated. Consider, for example, the popular experimental data by Virkler et al. [12] (see Fig.1,a where sample functions of fatigue crack growth are shown schematically). One cannot miss a striking point: the lines corresponding to specimens cut from the same sheet with the same initial crack size and tested under strictly controlled conditions intersect rarely. It means that the scatter of crack growth is born not only from the point-to-point randomness of mechanical properties, but also, and not in a lesser degree, from the specimen-to-specimen scatter. It is strange that, to the author's knowledge, it has not been emphasized by those who made the comparison of theoretical models with experimental data. For example, in book [1] where a Markov type model was proposed for damage accumulation processes, both Virkler's and the corresppondíng simulated sample functions are presented. The latters are shown schematically in Fig.1,b. There is an evident difference in the behaviour of sample functions: opposite to the experimental curves, the simulated ones intersect violently. It means that Fig.1,a and b represent quite different random processes although the single-point cumulative distribution functions $F(N|a)$ and $F(a|N)$ fit the experimental data satisfactorily.

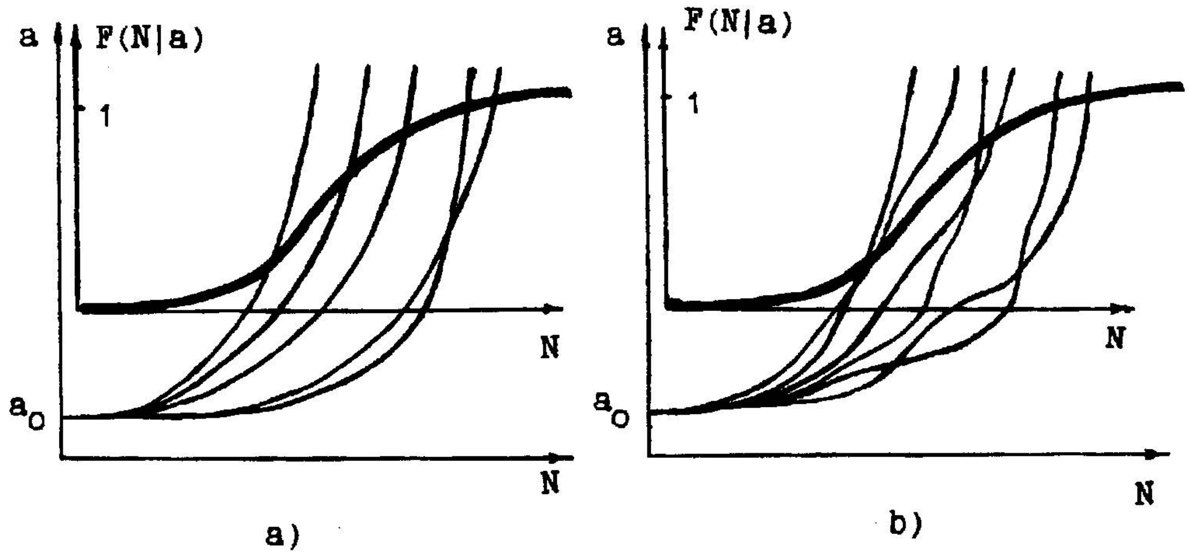

Fig. 1 Schematic comparison of experimental (a) and simulated (b) sample functions of fatigue crack growth

A special analysis is required to understand this phenomenon more profoundly. A large volume of numerical simulation has been performed recently by author and his associates with the use of equation [4,8]

$$\frac{da}{dN} = \frac{\lambda[(\Delta K - \Delta K_{th})/K_f]^m}{[(1 - K_{max}^2/K_{Ic}^2)^{1/\alpha} - \omega_f(a.N)]} \qquad (4)$$

Here ΔK is stress intensity factor range; K_f, ΔK_{th} and K_{Ic} are material properties parameters, i.e. fatigue toughness, threshold fatigue toughness and fracture toughness, respectively; λ is scale parameter of the order of material's local nonhomogeneities, m and α are positive power exponents; $\omega_f(a,N)$ is the measure of microdamage accumulated in the far field, before the material's particles approach to the crack front. Eq.(4) takes into account the microdamage accumulation both in the far field and in the processing zone. In addition, the energy balance in the system cracked body - loading is included into Eq.(4). Thus, the equation is valid as at the low stresses, near the threshold fatigue toughness, as well as on the terminal stage when a crack advances in an accelerated way.

Some numerical results are presented in Figs.2-4. They were obtained in assumption that the fatigue toughness is a random function along the path of the crack given in the form $K_f(x) = I_0 + I_1 u(x)$. Here I_0 is the minimal toughness, I_1 is a measure of toughness fluctuations, and u(x) is a stationary ergodic function of the coordinate x measured along the crack trajectory. A normalized Rayleigh function with a broad-band power spectral density has been used for modelling the point-to-point randomness. As to the other parameters of Eq.(4), the magnitudes of ΔK_{th} and K_{Ic} have been assumed connected deterministically with K_f, and for λ, m and α deterministic magnitudes have been taken. Since no information is available on the properties of the material at the tips of initiating cracks, the stationary distribution has been taken for $K_f(a_0)$ where a_0 is the initial crack size. Computations were made for central opening mode cracks in specimens of the given fixed width.

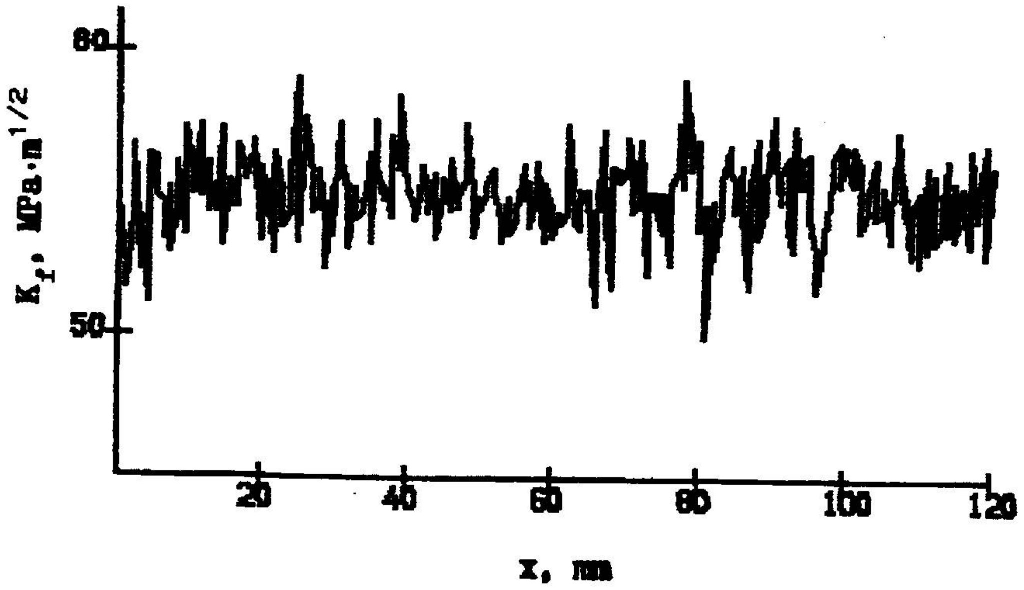

Fig. 2 Sample function of fatigue toughness along the crack path

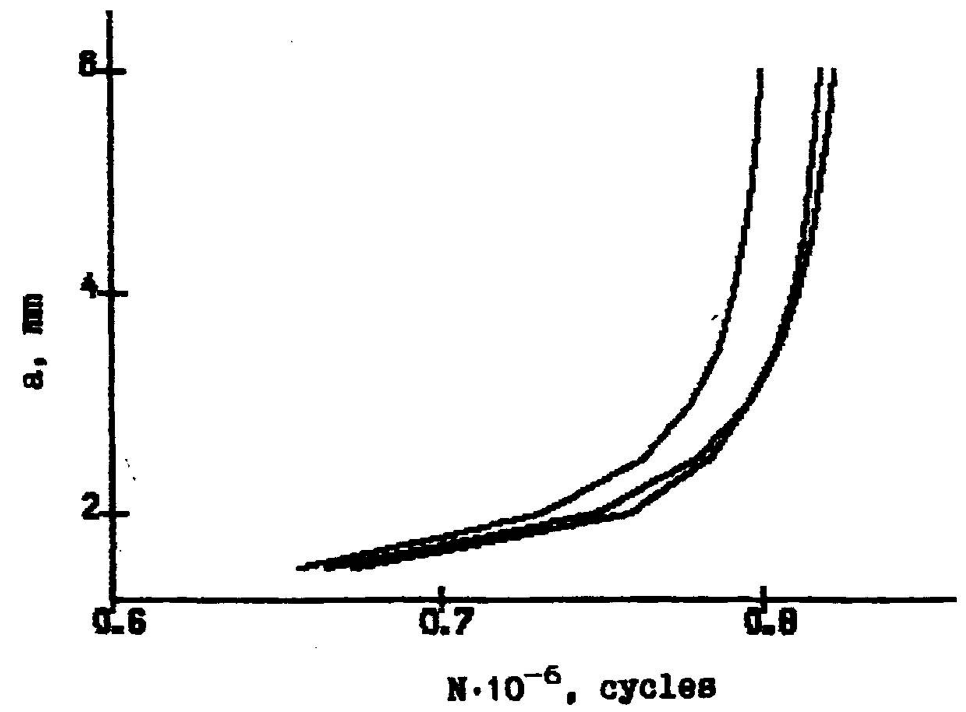

Fig. 3 Scatter of the crack size versus cycle number due to the influence of the point-to-point variability of mechanical properties and of the initial conditions

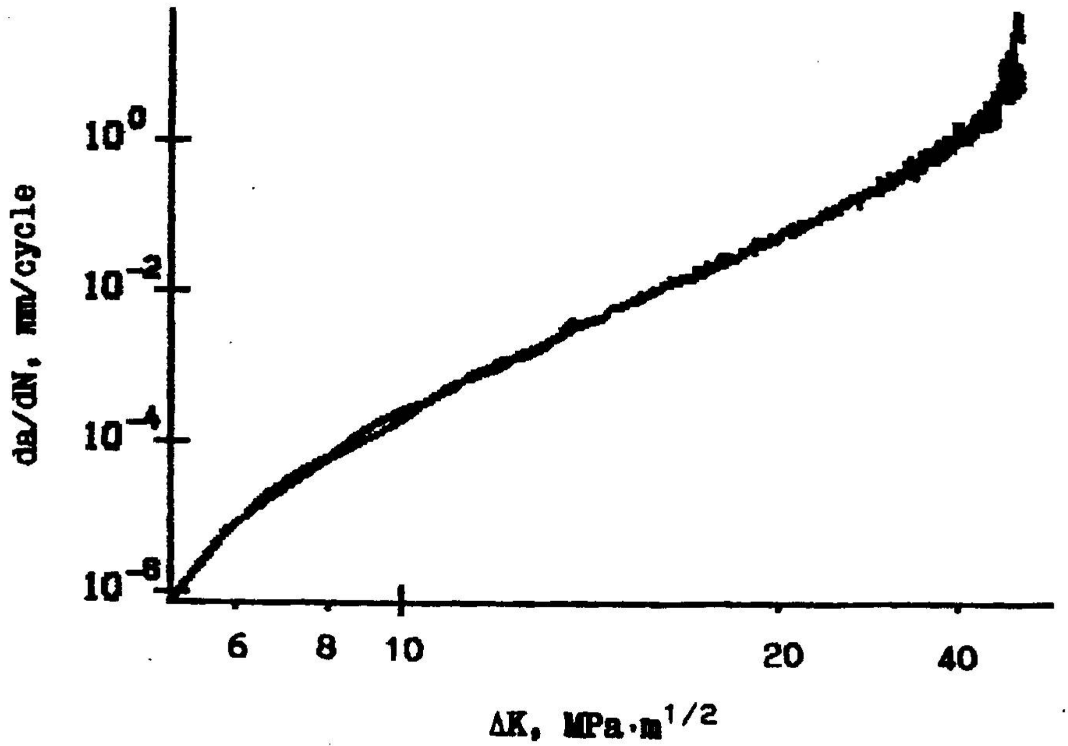

Fig. 4 Scatter of fatigue crack growth rate as function of the range of stress intensity factor

Fig.2 shows the variability of $K_I(x)$ along the crack trajectory. At the assumed nonhomogeneity scale of the order of 1 mm, sample functions $K_I(x)$ appear to be well-mixed. Nevertheless, the sample functions a(N) display a significant scatter (see Fig.3). This result appears unexpected. More minute analysis shows that the curves go apart at the earlier stages of crack growth. Hence, random initial conditions are mostly responsible for the large lifetime scatter at the later stages. Note that the "worst" and the "best" of 15 samples functions are plotted in Fig.3. At last, Fig.4 presents the relationship between the crack growth rate da/dN and the range ΔK of the stress intensity factor. Scatter of these sample functions is hidden due to the log-log scale, and becomes very significant on the terminal stage.

To close this brief discussion, we have to stress once again that not only sizes, shapes and positions of initial flaws but also material properties near the flaws can effect essentially on the crack growth rate and the fatigue life.

4. STRUCTURAL MODELS OF FLOWS INTERACTION IN ROPES AND CABLES

Wires and cables are longitudinal items, and for a given cross section a single scale parameter, the length L enters into the size effect analysis. Therefore, it may be awaited that the size strength effect is easier to describe and predict for wires and cables than that for structural components of more arbitrary shape. But in fact, ropes and cables consist, as a rule, of a large number of thin fibres, strands, etc. interacting in a rather complex way. In some aspects, fatigue and fracture of such composite structures are more difficult to model analytically and numerically than of monolithic bodies, even of complicated shape.

When single wires begin to fail, the load redistribution takes place,and that influence on the fatigue life of neighbouring wires. Moreover, the strength size effect is inherent not only to strands and cables, but, in a larger degree, to single wires. The tensile strength of single wire specimens of a comparatively short length is higher than that of wires working jointly in a strand of the same length. But on larger length, say, of the same order as that in an actual structure, one can observe that the strength of a strand is much higher than that of the summed strength of the jointed long wires. This is an effect of interaction of wires, when, due to the friction between wires, a kind of redundancy occurs increasing the load carrying capacity of long structural components. The same conclusion concerns the fatigue life and the reliability of wires and cables against fatigue failure. A rather close situation takes place in fiber composite materials where high performance fibers are connected in a monolith with a polymer or metal matrix that redestributes stresses around the ruptured fibers. Statistical models were suggested in [3,7] to predict the strength and fatigue life of unidirectional fiber composites under tension along fibers. Analogous models were used [5] to assess reliability of the core of the nuclear reactor composed of a large number of fuel elements. Later on a preliminary discussion is presented how to extend these models upon ropes and cables.

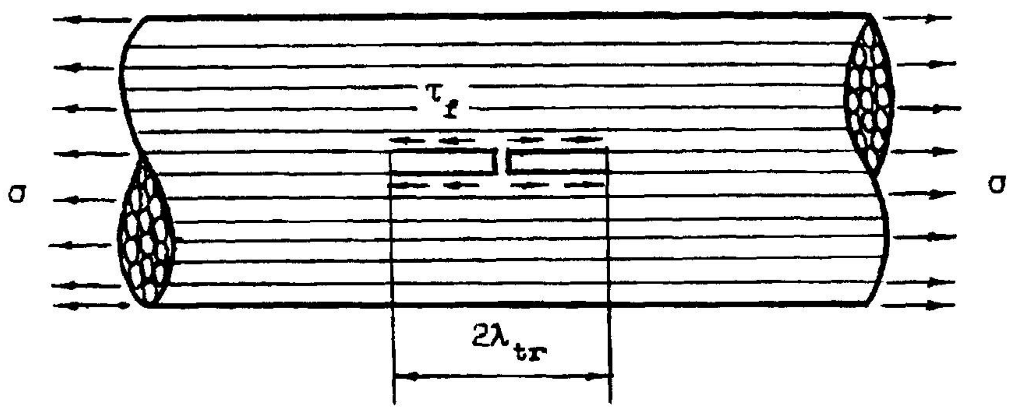

Fig. 5 Interaction of single fibre rupture in a parallel bundle; λ_{tr} is transfer length of load re-distribution

For simplicity consider a parallel strand composed of n wires where $n \gg 1$ (Fig.5). Denote with σ the nominal normal stress uniformly distributed upon the summed cross section of wires and with τ_f the ultimate friction stress between wires. When a wire is fractured, its load is to be redistributed among the neighbours, and the half-length of the redistribution domain may be estimated with the order of magnitude as [3]

$$\lambda_{tr} \approx r(\sigma/2\tau_f) \tag{5}$$

Here r is radius of wires. The stress in neighbours becomes $\varkappa\sigma$ where $\varkappa$ is stress concentration factor.

The transfer length λ_{tr} plays one of the main parts in the interaction between wires or fibers in threads, ropes and cables. Let, for example, the short-time strength σ of single wires satisfy the three-parameter Weibull distribution

$$F(\sigma) = 1 - \exp\left[- \frac{\lambda}{\lambda_0} \left(\frac{\sigma - \sigma_0}{\sigma_c} \right)^{\alpha} \right] \tag{6}$$

Here λ is the half-length of the wire segment, λ_0 is scale parameter, e.g. $\lambda_0 = r$, and σ_0, σ_c, α are material parameters of wires. At $\sigma \leqslant \sigma_0$ we put instead of Eq.(6) $F(\sigma) \equiv 0$.

It is very probable that when parallel wires interact in a strand, their individual input into the strand strength is characterzed with Eq.(6) where λ is evaluated from Eq.(5). In fact, if a single wire ruptures, the "naked" length of the neighbouring wires is of the order of λ_{tr}. The same situation takes place in fatigue damage. One of the simplest models is related to Weibull distribution of the fatigue life, i.e. analogous of Eq.(6):

$$F(N|\sigma) = 1 - \exp\left[-\frac{\lambda}{\lambda_0}\left(\frac{s - s_0}{s_c}\right)^{\alpha}\left(\frac{N - N_0}{N_c}\right)^{\beta}\right] \qquad (7)$$

Compared with the notation of Eq. (6), s is characteristic magnitude of cyclic loading, say, the stress range in wires. Material constants s_c, s_0, N_c, N_0, α and β are related to fatigue lifetime distribution of single wires with the length λ. At $s < s_0$ or $N < N_0$ one have to put $F(N|s) \equiv 0$.

Let a bundle (strand, cable, etc.) composed of n parallel wires fails when at least one of its segment with the length λ_{tr} contains at least n_* ruptured wires. Then the probability distribution for the fatigue life of a bundle of the length L may be estimated as [7]

$$F(N|\sigma) = 1 - \exp\left[-\frac{L}{\lambda_0}\left(\frac{\lambda_{tr}}{\lambda_0}\right)^{n_*}\left(\frac{s - s_0}{s_c}\right)^{\alpha n_*}\left(\frac{N - N_0}{N_c}\right)^{\beta n_*}\right] \qquad (8)$$

Some conclusions can be made from Eq.(8) concerning the size effect due to the wires interaction. For example, the shape parameter of the fatigue life distribution is βn_* for bundles instead of β for single wires. The shape parameter of ultimate stress is $(\alpha - 1)n_*$ instead of α, respectively. To describe more intimate mechanisms of interaction, the load redistribution and stress concentration during the sequential ruptures of single wires are to be taken into consideration. The simplest way to account for the load redistribution is to replace s in Eq.(8) with $(\text{æ}\, s)/(1 - n_*/n)$. Such an approach is similar to the well-known "bundle-of-fibres" model by Daniels. There is no place and time to go into further details, and we send the reader to the survey paper [7] where other references can be found.

5. THE PROBLEM OF EXTRAPOLATION OF SHORT-LENGTH AND SHORT-TIME TEST RESULTS UPON FULL-SCALE STRUCTURES

It has beeh shown above, and maybe, not for the first time, that the extrapolation problem is complicated with a number of factors that are not yet have been studied sufficiently and even not understood completely. Among them are the presence of cracks and crack-like flaws of different origin, shape, position and size; the complex mechanisms of macrocrack initiation and growth due to the random scatter of material properties, initial conditions, loads and enviromental actions; and at last, the interaction between flaws and ruptures of single wires and fibers composing full-scale ropes and cables. In the present dicussion, let limit ourselves with the role of diversity of flaws properties.

Return to Eq.(2) which is of a rather general nature. To make the discussion more concrete, consider the two special cases of Eq.(2), the Weibull-type equation

$$R(N|s) = \exp\left[-\frac{L}{L_0} f(N,s)\right] \tag{9}$$

and the Gumbel-type (double-exponential) equation

$$R(N|s) = \exp\left[-\frac{L}{L_0}\left[1 - \exp\{ f(N,s) \}\right]\right] \tag{10}$$

where the notation is used

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\left(\frac{s - s_{0ij}}{s_{cij}}\right)^{\alpha_{ij}}\left(\frac{N - N_{0ij}}{N_{cij}}\right)^{\beta_{ij}} H(s - s_{0ij})H(N - N_{0ij}) \equiv f(N,s) \tag{11}$$

with Heaviside function $H(\cdot)$. Eqs.(9)-(11) correspond to a stationary loading with a single characteristic stress parameter s. All the flaws are related to the total cable length L with the reference length L_0 assumed equal to the all sets of flaws. The meaning of other parameters in Eq.(10) is understandable from the comparison with Eqs.(2), (3), (7) and (8). Account of different thresholds s_{0ij} and N_{0ij} is a reasonable assumption since for each type of wires and each kind of flaws a certain minimal stress level and a certain minimal cycle number are required to produce macroscopic damage. For those who want to argue this point, it is enough to remind the low-cycle and high-cycle fatigue mechanisms with their own areas on s,N plane [9]. In addition, various damage mechanisms are expected in exterior and interior wires of ropes and wires, in wires of spiral cables with various angles, etc.

Let R_* is the specified reliability index. The admissible pairs of s and N satisfy to equation

$$f(N,s) = -\frac{L_0}{L} \ln R_* \tag{12}$$

in the case of Eq.(9), and to equation

$$f(N,s) = \ln\left[1 - \frac{L_0}{L} \ln R_*\right] \tag{13}$$

if Eq. (10) is preferable. Thus, the size ratio L/L_0 and the specified reliability index R_* enter into equations with respect to the admissble stress level (at a given cycle number) or to the admissible cycle number (at a given stress level).

The difficulties of extrapolation of short specimens tests on much longer, full-scale structures are obvious even from a such elementary consideration. Let L_0 is the length of a specimen, and L is the full-scale length. To ensure a confident estimation of the left-hand side of Eqs.(12) and (13), the non-failure probability of specimens should be of the order

$$R_0 = R_*^{L_0/L} \tag{14}$$

For example, if $R_* = 0.999$, and $L = 10^2 L_0$, Eq.(14) yields $R_0 \approx 0.9$. It means that tests are to be performed at stress levels much higher than that in the actual structure. On the other hand, at the higher stress level quite different mechanisms of damage begin to act, and that makes the extrapolation procedure rather questionable.

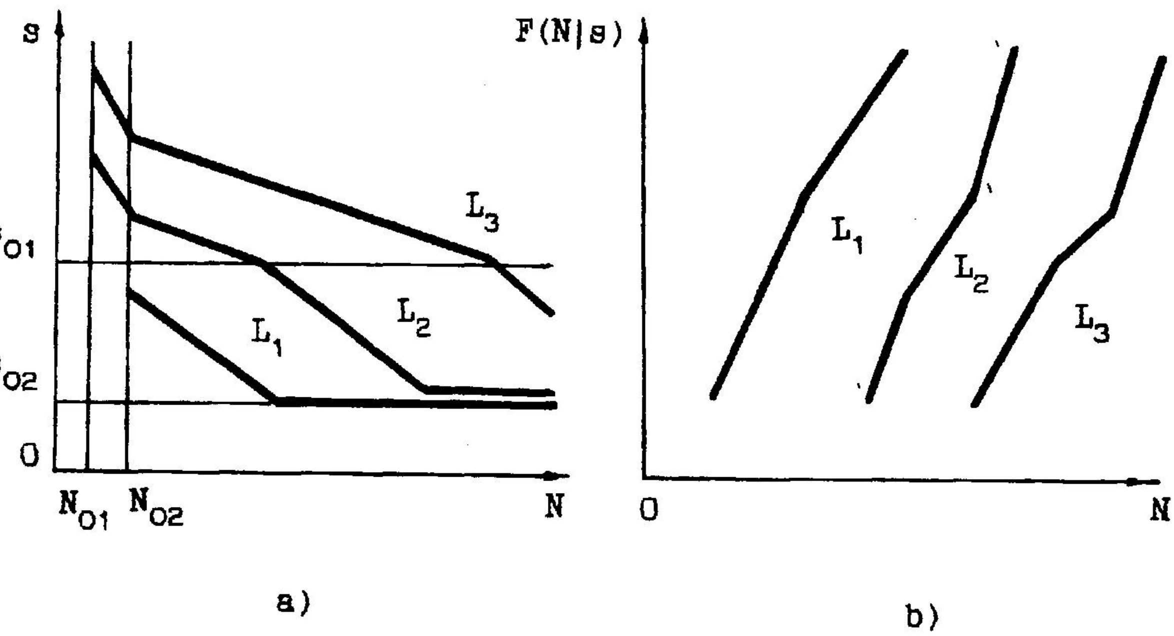

Fig. 6 Fatigue fractile curves (a), and probability distribution functions of fatigue life (b) at various lengths L > L > L

The situation is illustrated in Fig.6,a where fatigue curves are shown schematically corresponding to a given R_* and various lengths $L_1 > L_2 > L_3$. When a curve goes from one region $s > s_0$, $N > N_0$ to another , its character changes. No similitude is awaited of fatigue curves at the same R_* and at various L. This discrepancy appears in probability distribution functions, too, as it is shown in Fig.4,b.

Conclusion

It has been shown that the problem of extrapolation of laboratory fatigue tests of specimens upon the large-scale engineering structures meets significant difficulties. They are born, in particular, and probably in the first line, from the existence of several mechanisms of fatigue damage which relative inputs into the lifetime vary when the size of a structural component varies. Therefore, the coarse extrapolation upon much larger sizes and/or much larger lifetimes is awaited to be unsatisfactory. To overcome these difficulties, the following ways may be used:

- experimental study of the size effect on fatigue in conditions most close to the field ones including the pilot tests (very long specimens, very long durations, modelling of environmental actions, etc.);
- monitoring of loads, actions, stress-strain fields, flaws distribution and damage in existing and new-build structures;

- development of advanced probabilistic and structural models of fracture and fatigue of wires, threads, ropes and cables with account of all random factors effecting on their lifetime and structural reliability.

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