STATISTICAL MODELS

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Initial Flaws Distribution and Size Effect of Fatigue Fracture

Distribution de défauts initiaux et l'effet de la longueur sur la rupture par fatigue

Fehlstellenverteilung und Maßstabseffekt beim Ermüdungsbruch

Vladimir V. BOLOTIN
Prof. Dr.
Russian Acad. of Sciences
Moscow, Russia

Vladimir V. Bolotin, born 1926, graduated from Moscow Institute of Railways Engineers in bridges and tunnels in 1948, received degree of Doctor of Sciences (Engineering) in 1952. Professor of Moscow Institute of Energetics, Member of Russian Academy of Sciences and Russian Academy of Engineering.

SUMMARY
A survey of random factors influencing the reliability and lifetime of structural components is presented with an emphasis on the initial flaws distribution and specific difficulties in the extrapolation of laboratory data to in-service conditions of full-scale structures. A far-reaching generalization is suggested of the probabilistic models of fatigue taking account of sets of initial and newly-created flaws as well as loads and actions randomly varying in time. Special attention is paid to the prediction of the fatigue life of ropes and cables consisting of a large number of wires. The influence of the wires' interaction on the size effect in ropes and cables is discussed.

RÉSUMÉ
Une approche des facteurs aléatoires qui interviennent sur la fiabilité et la durée de vie des composants structurels est ici présentée en mettant l'emphase sur la distribution des défauts initiaux et sur la difficulté particulière d'extrapoler les résultats obtenus en laboratoire aux structures réelles en conditions de service. On propose une généralisation des modèles probabilistiques de fatigue en tenant compte des défauts initiaux et des défauts d'apparition ultérieure. On prête une attention spéciale à la prédiction de la vie de fatigue des filins et des câbles formés d'un grand nombre de fils. A ce sujet, une discussion est engagée sur l'influence de l'interaction des fils sur l'effet d'échelle des différents câbles.

ZUSAMMENFASSUNG
In einer Übersicht der zufälligen Einflüsse auf die Zuverlässigkeit und Lebensdauer von Traggliedern werden die Verteilung anfänglicher Fehlstellen und die besonderen Schwierigkeiten bei der Extrapolation von Labordaten auf die Betriebsbedingungen in wirklichen Strukturen hervorgehoben. Eine weitreichende Verallgemeinerung der probabilistischen Ermüdungsmodelle berücksichtigt Gruppen bestehender und frischer Fehlstellen sowie stockastische Einwirkungen. Besondere Aufmerksamkeit gilt der Vorhersage der Ermüdungslebensdauer von Seilen und Kabeln mit zahlreichen Drähten. Dabei wird der Einfluß der Drahtinteraktion auf den Maßstabseffekt diskutiert.

1. INTRODUCTION

Fatigue failure is a phenomenon not easy to describe and predict quantitatively in a precise and reliable way even in the framework of the modern fracture mechanics. This phenomenon consists of several stages: dispersed microdamage accumulation, initiation of nuclei of macroscopic cracks, development of short cracks and the further formation of macroscopic cracks propagating up to the final failure. In wires, ropes and cables composed of thin fibers, fatigue failure is a result of fracture of a certain number of neighbouring fibers. Interaction between the fractured and whole fibers during the damage process is a complicated phenomenon from the viewpoint of structural mechanics, too. Additional complications arise due to the large statistical scatter of laboratory and field data, and that makes the application of probabilistic models necessary for the prediction of reliability indexes and lifetime for full-scale engineering systems.

Experimental analysis helps only partially since only comparatively short specimens could be tested meanwhile wires and cable used in large engineering systems are many times longer. Probabilistic models based on simple and transparent concepts often fail to predict more or less precisely reliability indexes for much longer structural components. The same situation takes place in prediction of the service lifetime using results of short time testing. The problem of reliable extrapolation of laboratory tests data on much longer structural components and much longer times is in fact the central point of this Workshop.

Random factors influencing on fatigue failure can be divided in the three groups: randomness of material properties; random fiaws and imperfections of structural components; random loads, actions and enviromental conditions. In turn, each group consists of several subgroups. For example, it is expedient to distinguish the randomness inherent to the microstructure of materials, and the batch-to-batch scattering of properties of commercial materials born from instabilities and imperfections of the manufacturing process. Flaws and imperfections of different origin and various shape, size and position enter in the second group of random factors. At last, loads and actions form a broad variety with random and/or uncertainly defined parameters.

The discrepancy between the predicted fatigue life of structural components and that observed in field condition is born from different sources. Among them is the difference in behaviour of short laboratory specimens and long structural components originated, in particular, from the differnce in load transfer and the relative input of the anchorage into the resulting reliability. Enviromental conditions are difficult to reproduce in laboratory tests, moreover, their long-time effects on mechanical properties. Non--stationary random loading influences on the fatigue life, and often in a non-trivial way (as example, effects of overloadings may be mentioned). Contrary to that, most laboratory tests are performed in stationary, regular cycle loading. Another obstacle to perform a reliable extrapolation of laboratory data is born from the use of oversimplified probabilistic models and nonadequate statistical techniques. The trust in universality of Weibull's model and the brave extrapolation of statistical data obtained from poor samples are, probably, the most evident examples of such oversimplification.

Later on, the following items concerning the problem of strength, lifetime and reliability prediction of long structural components are discussed: (a) probabilstic models of structural reliability in the presence of sets of randomly distributed flaws; (b) probabilistic models of the fatigue crack growth with the special attention to various sources of scattering of results; (c) interaction between single wires in ropes and cables and its account in probabilistic models; (d) some aspects of the extrapolation of short specimens and/or short-time fatigue tests.

2. RELIABILITY AND LIFETIME IN THE PRESENCE OF RANDOM SETS OF FLAWS

Consider a structural component or a specimen (later - a body) under, generally, nonstationary cycling loading. The body contains a number of cracks, cuts, pores, and flaws that can develop in macroscopic cracks which growth results into the final failure of the body. The flaws differ in origin (initial, new-born, detected and admitted during inspection, non-detected), and in size, shape and position. Later on, all crack-like flaws of macroscopic size, say, of the order of 1 mm and more are called cracks. Unite the cracks with similar features in sets. To identify cracks in their position, divide the body into domains M_1, ..., M_I that are, generally, may overlap. Dimension of this domains may be different. For brevity, use the same notations for domains and their measures, e.g. for the length of one-dimentional domains, for the surface of two-dimentional ones, etc. Choose a standard measure M_{0i} for each domain, say, the unit of the corresponding measure. For simplicity, let cracks of each set are described with a single size parameter a_{ij}, i = 1, ...,J where j is the number of j-th set, and J is the total number of sets. Assume that the failure of the body occurs when at least one of the cracks attains the corresponding critical size a_{ij}^{*}. Then the reliability (survival) function is

$$R(t) = P\left\{ \begin{matrix} \max a_{ij}(x,t) < a_{ij}^{*}(x,t);\; x \in M_i \\ i=1,\ ...,\ I;\quad j=1,\ ...,\ J \end{matrix} \right\} \qquad (1)$$

Here P{ } is probability of the event in braces, x is reference vector.

If the density of macroscopic cracks is sufficiently low, it is possible to neglect their interaction. Then Poisson model is valid for each set of cracks, for each domain, and for a body as a whole. Eq.(1) resuits into

$$R(t) = \exp\left[-\sum_{i=1}^{I} \sum_{j=1}^{J} \int_{M_i} \mu_{ij}(a_{ij}^{*};t)\, \frac{dM_i}{M_i^0} \right] \qquad (2)$$

where $\mu_{ij}(a_{ij};t)$ is the expected number of cracks from j-th set in i-th domain which size a_{ij} attains the critical magnitude a^*_{ij} at the time t.

Eq.(2) presents a far-going generalization of probabilistic models of reliability against fatigue failure based on Weibull's or double exponential (Gumbel's) distributions [10,11]. In particular, Eq.(2) takes into account nonhomogeneities that may be both continuous, accounted with integrals, and discrete, accounted with the division of the body into domains. One can to present $\mu_{ij}(a_{ij};t)$ in the form

$$\mu_{ij}(a_{ij};t) = \mu_{ij}(t)[1- F_{ij}(a_{ij};t)] \qquad (3)$$

where $\mu_{ij}(t)$ is the expected total number of cracks from the considered sets, and $F_{ij}(a_{ij};t)$ is the probability distribution function of crack sizes up to the time t. Determination of $\mu_{ij}(t)$ and $F_{ij}(a_{ij};t)$ is the subject of the probabilistic fracture mechanics. Many aspects of the reliability assessment can be taken into account with this model since it includes the cracks initiation and growth, inspection procedures, decision making, replacement ant repair, etc. We do not go here into details that can be found in book [5] and paper [6].

3. RANDOMNESS OF MATERIAL PROPERTIES AND FATIGUE CRACK GROWTH

To estimate functions $\mu_{ij}(t)$ and $F_{ij}(a_{ij};t)$ entering into Eqs.(2) and (3), solution are to be found of certain stochastic equations describing the evolution of cracks and crack-like flaws. A number of studies have been performed during the last decades dedicated either to the randomization of the known (deterministic) equations of fatigue crack growth or to the use of (also already known) mathematical models describing such irreversible stochastic processes that may be interpreted in terms of fatigue damage. A very important question of the real origin of randomness, as a rule, remains out of the area of these studies. Meanwhile, it is necessary to make difference between the inherent randomness of the material's microstructure which we call within-a-specimen scatter, and batch-to-batch or even specimen-to-specimen scatter. In addition, there is such an important factor as randomness of initial conditions - that of the size, position and shape of flaws at the beginning of the considered time segment. This factor takes an intermediate position between the two kinds of randomnesses. In our opinion, since the initial flaw distribution varies significally between specimens (and, moreover, between components of real structures), this type of randomness is to be attributed to specimen-to-specimen randomness. In fact, the crack tip blunting due to corrosion or overloading in the previous life can effect essentially on the duration of the initiation stage and on the early crack growth rate.

The influence of initial conditions on the fatigue life has been, generally, underestimated and almost not investigated. Consider, for example, the popular experimental data by Virkler et al. [12] (see Fig.1,a where sample functions of fatigue crack growth are shown schematically). One cannot miss a striking point: the lines corresponding to specimens cut from the same sheet with the same initial crack size and tested under strictly controlled conditions intersect rarely. It means that the scatter of crack growth is born not only from the point-to-point randomness of mechanical properties, but also, and not in a lesser degree, from the specimen-to-specimen scatter. It is strange that, to the author's knowledge, it has not been emphasized by those who made the comparison of theoretical models with experimental data. For example, in book [1] where a Markov type model was proposed for damage accumulation processes, both Virkler's and the corresppondina simulated sample functions are presented. The latters are shown schematically in Fig.1,b. There is an evident difference in the behaviour of sample functions: opposite to the experimental curves, the simulated ones intersect violently. It means that Fig.1,a and b represent quite different random processes although the single-point cumulative distribution functions $F(N|a)$ and $F(a|N)$ fit the experimental data satisfactorily.

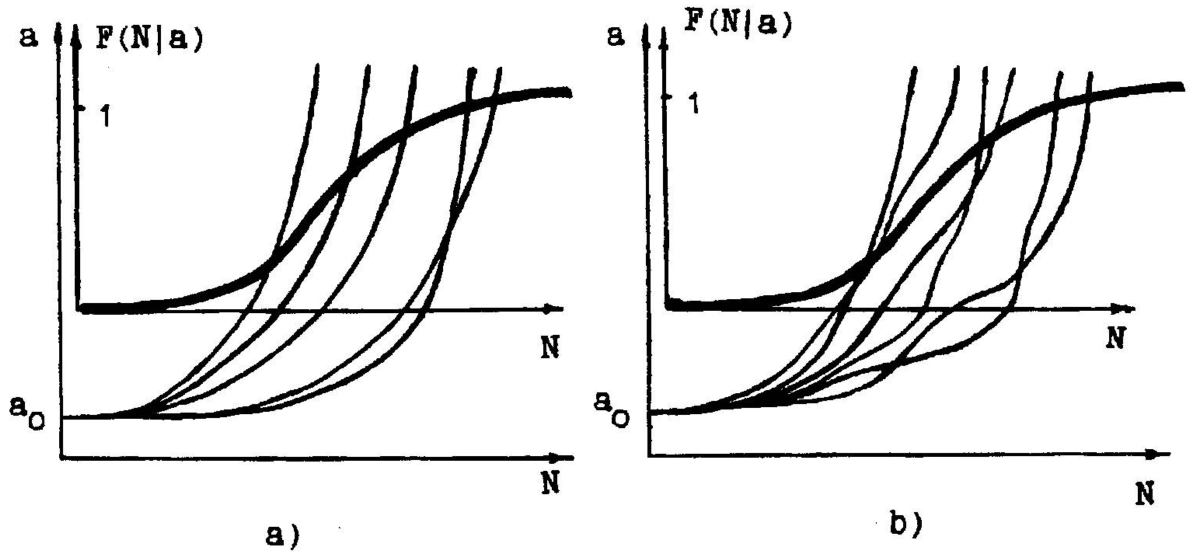

Fig. 1 Schematic comparison of experimental (a) and simulated (b) sample functions of fatigue crack growth

A special analysis is required to understand this phenomenon more profoundly. A large volume of numerical simulation has been performed recently by author and his associates with the use of equation [4,8]

$$\frac{da}{dN} = \frac{\lambda[(\Delta K - \Delta K_{th})/K_f]^m}{[(1 - K^2_{max}/K^2_{Ic})^{1/\alpha} - \omega_f(a.N)]} \qquad (4)$$

Here ΔK is stress intensity factor range; K_f, ΔK_{th} and K_{Ic} are material properties parameters, i.e. fatigue toughness, threshold fatigue toughness and fracture toughness, respectively; λ is scale parameter of the order of material's local nonhomogeneities, m and α are positive power exponents; $\omega_f(a,N)$ is the measure of microdamage accumulated in the far field, before the material's particles approach to the crack front. Eq.(4) takes into account the microdamage accumulation both in the far field and in the processing zone. In addition, the energy balance in the system cracked body - loading is included into Eq.(4). Thus, the equation is valid as at the low stresses, near the threshold fatigue toughness, as well as on the terminal stage when a crack advances in an accelerated way.

Some numerical results are presented in Figs.2-4. They were obtained in assumption that the fatigue toughness is a random function along the path of the crack given in the form $K_f(x) = I_0 + I_1u(x)$. Here I_0 is the minimal toughness, I_1 is a measure of toughness fluctuations, and u(x) is a stationary ergodic function of the coordinate x measured along the crack trajectory. A normalized Rayleigh function with a broad-band power spectral density has been used for modelling the point-to-point randomness. As to the other parameters of Eq.(4), the magnitudes of ΔK_{th} and K_{Ic} have been assumed connected deterministically with K_f, and for λ, m and α deterministic magnitudes have been taken. Since no information is available on the properties of the material at the tips of initiating cracks, the stationary distribution has been taken for $K_f(a_0)$ where a_0 is the initial crack size. Computations were made for central opening mode cracks in specimens of the given fixed width.

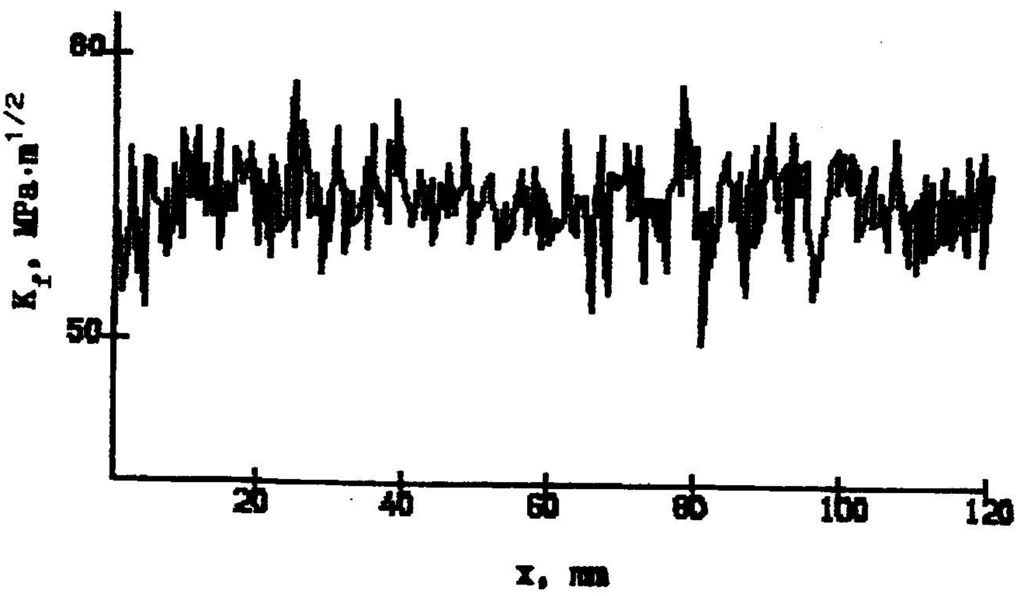

Fig. 2 Sample function of fatigue toughness along the crack path

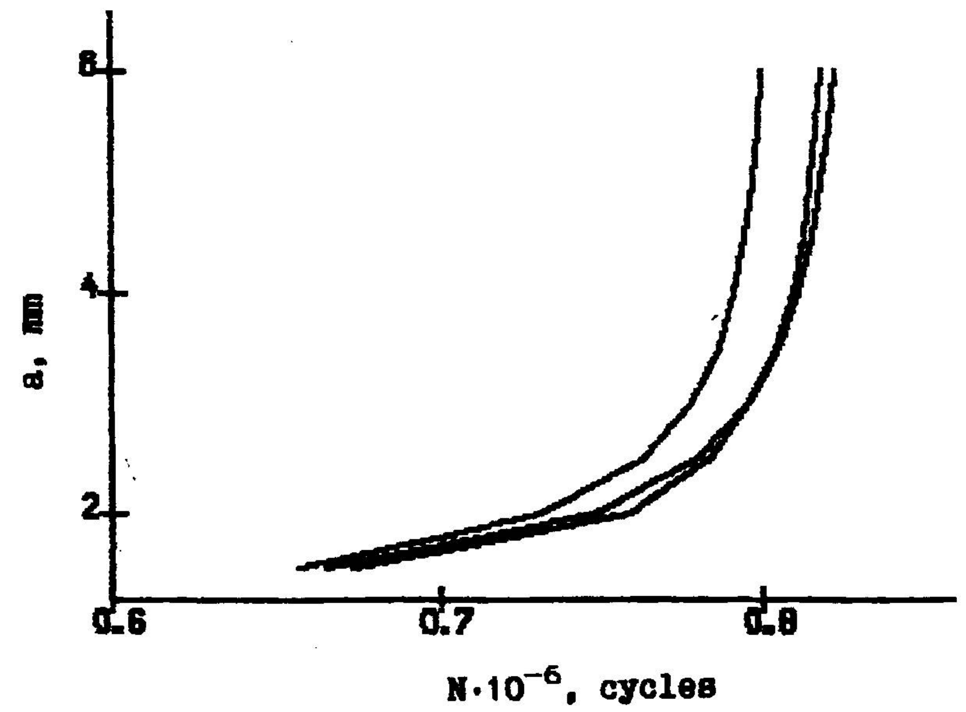

Fig. 3 Scatter of the crack size versus cycle number due to the influence of the point-to-point variability of mechanical properties and of the initial conditions

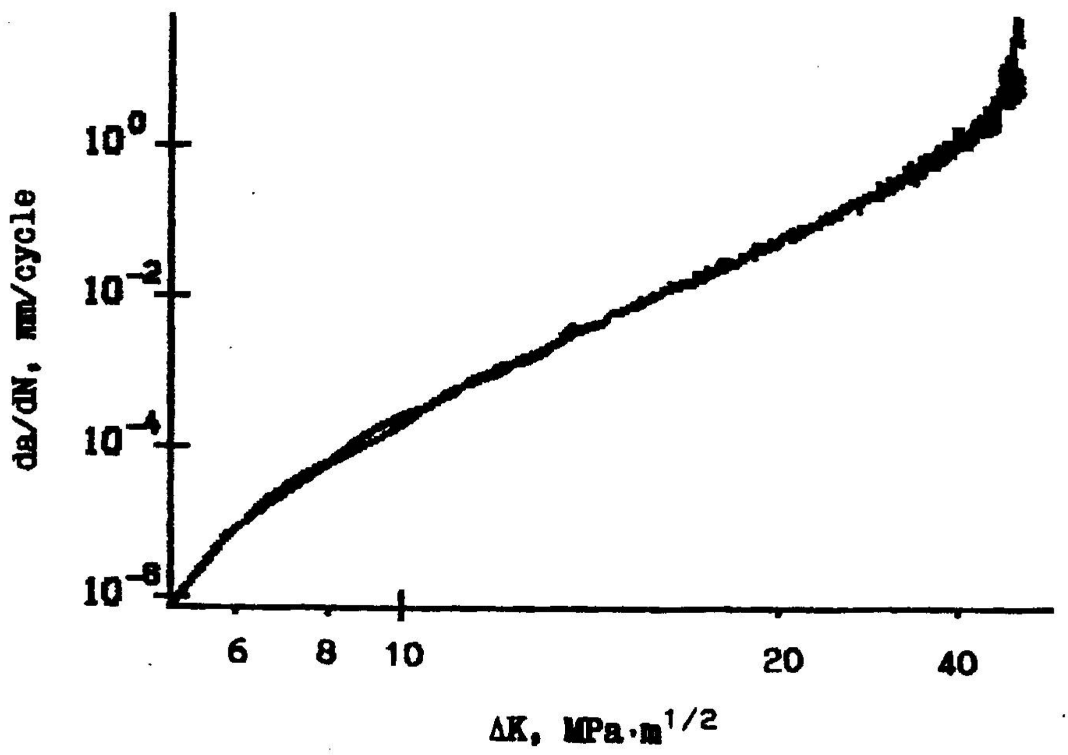

Fig. 4 Scatter of fatigue crack growth rate as function of the range of stress intensity factor

Fig.2 shows the variability of $K_I(x)$ along the crack trajectory. At the assumed nonhomogeneity scale of the order of 1 mm, sample functions $K_I(x)$ appear to be well-mixed. Nevertheless, the sample functions a(N) display a significant scatter (see Fig.3). This result appears unexpected. More minute analysis shows that the curves go apart at the earlier stages of crack growth. Hence, random initial conditions are mostly responsible for the large lifetime scatter at the later stages. Note that the "worst" and the "best" of 15 samples functions are plotted in Fig.3. At last, Fig.4 presents the relationship between the crack growth rate da/dN and the range ΔK of the stress intensity factor. Scatter of these sample functions is hidden due to the log-log scale, and becomes very significant on the terminal stage.

To close this brief discussion, we have to stress once again that not only sizes, shapes and positions of initial flaws but also material properties near the flaws can effect essentially on the crack growth rate and the fatigue life.

4. STRUCTURAL MODELS OF FLOWS INTERACTION IN ROPES AND CABLES

Wires and cables are longitudinal items, and for a given cross section a single scale parameter, the length L enters into the size effect analysis. Therefore, it may be awaited that the size strength effect is easier to describe and predict for wires and cables than that for structural components of more arbitrary shape. But in fact, ropes and cables consist, as a rule, of a large number of thin fibres, strands, etc. interacting in a rather complex way. In some aspects, fatigue and fracture of such composite structures are more difficult to model analytically and numerically than of monolithic bodies, even of complicated shape.

When single wires begin to fail, the load redistribution takes place,and that influence on the fatigue life of neighbouring wires. Moreover, the strength size effect is inherent not only to strands and cables, but, in a larger degree, to single wires. The tensile strength of single wire specimens of a comparatively short length is higher than that of wires working jointly in a strand of the same length. But on larger length, say, of the same order as that in an actual structure, one can observe that the strength of a strand is much higher than that of the summed strength of the jointed long wires. This is an effect of interaction of wires, when, due to the friction between wires, a kind of redundancy occurs increasing the load carrying capacity of long structural components. The same conclusion concerns the fatigue life and the reliability of wires and cables against fatigue failure. A rather close situation takes place in fiber composite materials where high performance fibers are connected in a monolith with a polymer or metal matrix that redestributes stresses around the ruptured fibers. Statistical models were suggested in [3,7] to predict the strength and fatigue life of unidirectional fiber composites under tension along fibers. Analogous models were used [5] to assess reliability of the core of the nuclear reactor composed of a large number of fuel elements. Later on a preliminary discussion is presented how to extend these models upon ropes and cables.

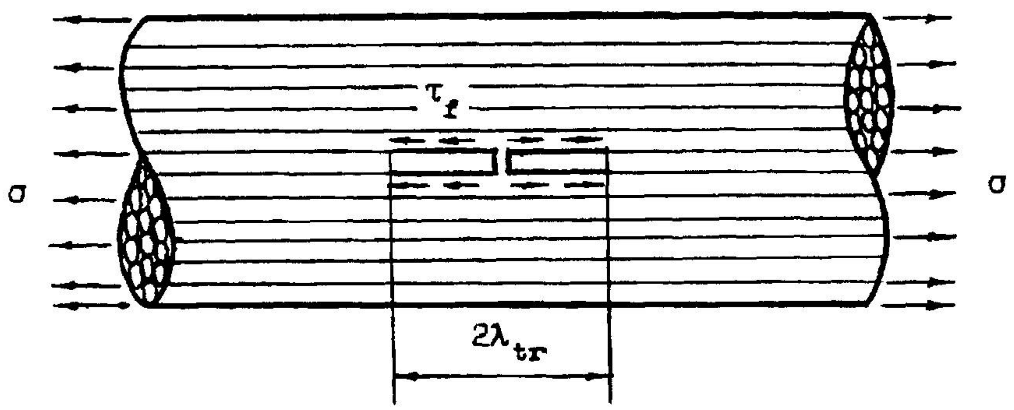

Fig. 5 Interaction of single fibre rupture in a parallel bundle; λ_{tr} is transfer length of load re-distribution

For simplicity consider a parallel strand composed of n wires where $n \gg 1$ (Fig.5). Denote with σ the nominal normal stress uniformly distributed upon the summed cross section of wires and with τ_f the ultimate friction stress between wires. When a wire is fractured, its load is to be redistributed among the neighbours, and the half-length of the redistribution domain may be estimated with the order of magnitude as [3]

$$\lambda_{tr} \approx r(\sigma/2\tau_f) \tag{5}$$

Here r is radius of wires. The stress in neighbours becomes $\varkappa\sigma$ where $\varkappa$ is stress concentration factor.

The transfer length λ_{tr} plays one of the main parts in the interaction between wires or fibers in threads, ropes and cables. Let, for example, the short-time strength σ of single wires satisfy the three-parameter Weibull distribution

$$F(\sigma) = 1 - \exp\left[-\frac{\lambda}{\lambda_0}\left(\frac{\sigma - \sigma_0}{\sigma_c}\right)^{\alpha}\right] \tag{6}$$

Here λ is the half-length of the wire segment, λ_0 is scale parameter, e.g. $\lambda_0 = r$, and σ_0, σ_c, α are material parameters of wires. At $\sigma \leqslant \sigma_0$ we put instead of Eq.(6) $F(\sigma) \equiv 0$.

It is very probable that when parallel wires interact in a strand, their individual input into the strand strength is characterzed with Eq.(6) where λ is evaluated from Eq.(5). In fact, if a single wire ruptures, the "naked" length of the neighbouring wires is of the order of λ_{tr}. The same situation takes place in fatigue damage. One of the simplest models is related to Weibull distribution of the fatigue life, i.e. analogous of Eq.(6):

$$F(N|\sigma) = 1 - \exp\left[-\frac{\lambda}{\lambda_0}\left(\frac{s - s_0}{s_c}\right)^{\alpha}\left(\frac{N - N_0}{N_c}\right)^{\beta}\right] \qquad (7)$$

Compared with the notation of Eq. (6), s is characteristic magnitude of cyclic loading, say, the stress range in wires. Material constants s_c, s_0, N_c, N_0, α and β are related to fatigue lifetime distribution of single wires with the length λ. At $s < s_0$ or $N < N_0$ one have to put $F(N|s) \equiv 0$.

Let a bundle (strand, cable, etc.) composed of n parallel wires fails when at least one of its segment with the length λ_{tr} contains at least n_* ruptured wires. Then the probability distribution for the fatigue life of a bundle of the length L may be estimated as [7]

$$F(N|\sigma) = 1 - \exp\left[-\frac{L}{\lambda_0}\left(\frac{\lambda_{tr}}{\lambda_0}\right)^{n_*}\left(\frac{s - s_0}{s_c}\right)^{\alpha n_*}\left(\frac{N - N_0}{N_c}\right)^{\beta n_*}\right] \qquad (8)$$

Some conclusions can be made from Eq.(8) concerning the size effect due to the wires interaction. For example, the shape parameter of the fatigue life distribution is βn_* for bundles instead of β for single wires. The shape parameter of ultimate stress is $(\alpha - 1)n_*$ instead of α, respectively. To describe more intimate mechanisms of interaction, the load redistribution and stress concentration during the sequential ruptures of single wires are to be taken into consideration. The simplest way to account for the load redistribution is to replace s in Eq.(8) with $(\varkappa s)/(1 - n_*/n)$. Such an approach is similar to the well-known "bundle-of-fibres" model by Daniels. There is no place and time to go into further details, and we send the reader to the survey paper [7] where other references can be found.

5. THE PROBLEM OF EXTRAPOLATION OF SHORT-LENGTH AND SHORT-TIME TEST RESULTS UPON FULL-SCALE STRUCTURES

It has beeh shown above, and maybe, not for the first time, that the extrapolation problem is complicated with a number of factors that are not yet have been studied sufficiently and even not understood completely. Among them are the presence of cracks and crack-like flaws of different origin, shape, position and size; the complex mechanisms of macrocrack initiation and growth due to the random scatter of material properties, initial conditions, loads and enviromental actions; and at last, the interaction between flaws and ruptures of single wires and fibers composing full-scale ropes and cables. In the present dicussion, let limit ourselves with the role of diversity of flaws properties.

Return to Eq.(2) which is of a rather general nature. To make the discussion more concrete, consider the two special cases of Eq.(2), the Weibull-type equation

$$R(N|s) = \exp\left[-\frac{L}{L_0} f(N,s)\right] \tag{9}$$

and the Gumbel-type (double-exponential) equation

$$R(N|s) = \exp\left[-\frac{L}{L_0}\left[1 - \exp\{f(N,s)\}\right]\right] \tag{10}$$

where the notation is used

$$\sum_{i=1}^{I}\sum_{j=1}^{J}\left(\frac{s - s_{0ij}}{s_{cij}}\right)^{\alpha_{ij}}\left(\frac{N - N_{0ij}}{N_{cij}}\right)^{\beta_{ij}} H(s - s_{0ij})H(N - N_{0ij}) \equiv f(N,s) \tag{11}$$

with Heaviside function $H(\cdot)$. Eqs.(9)-(11) correspond to a stationary loading with a single characteristic stress parameter s. All the flaws are related to the total cable length L with the reference length L_0 assumed equal to the all sets of flaws. The meaning of other parameters in Eq.(10) is understandable from the comparison with Eqs.(2), (3), (7) and (8). Account of different thresholds s_{0ij} and N_{0ij} is a reasonable assumption since for each type of wires and each kind of flaws a certain minimal stress level and a certain minimal cycle number are required to produce macroscopic damage. For those who want to argue this point, it is enough to remind the low-cycle and high-cycle fatigue mechanisms with their own areas on s,N plane [9]. In addition, various damage mechanisms are expected in exterior and interior wires of ropes and wires, in wires of spiral cables with various angles, etc.

Let R_* is the specified reliability index. The admissible pairs of s and N satisfy to equation

$$f(N,s) = -\frac{L_0}{L}\ln R_* \tag{12}$$

in the case of Eq.(9), and to equation

$$f(N,s) = \ln\left(1 - \frac{L_0}{L}\ln R_*\right) \tag{13}$$

if Eq. (10) is preferable. Thus, the size ratio L/L_0 and the specified reliability index R_* enter into equations with respect to the admissble stress level (at a given cycle number) or to the admissible cycle number (at a given stress level).

The difficulties of extrapolation of short specimens tests on much longer, full-scale structures are obvious even from a such elementary consideration. Let L_0 is the length of a specimen, and L is the full-scale length. To ensure a confident estimation of the left-hand side of Eqs.(12) and (13), the non-failure probability of specimens should be of the order

$$R_0 = R_*^{L_0/L} \tag{14}$$

For example, if $R_* = 0.999$, and $L = 10^2 L_0$, Eq.(14) yields $R_0 \approx 0.9$. It means that tests are to be performed at stress levels much higher than that in the actual structure. On the other hand, at the higher stress level quite different mechanisms of damage begin to act, and that makes the extrapolation procedure rather questionable.

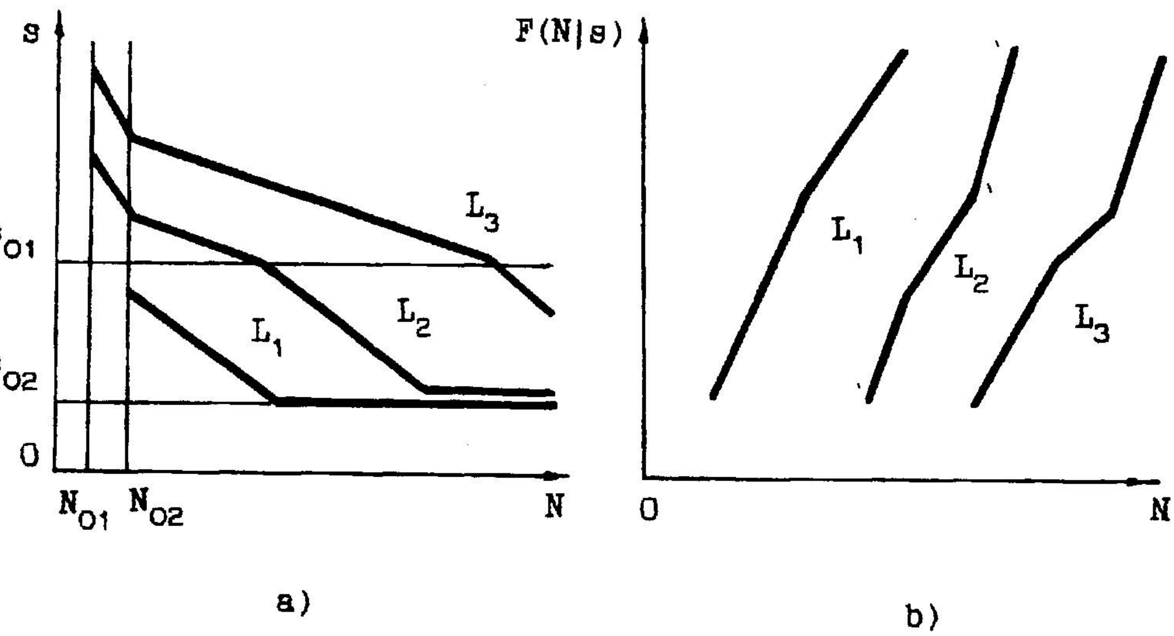

Fig. 6 Fatigue fractile curves (a), and probability distribution functions of fatigue life (b) at various lengths L > L > L

The situation is illustrated in Fig.6,a where fatigue curves are shown schematically corresponding to a given R_* and various lengths $L_1 > L_2 > L_3$. When a curve goes from one region $s > s_0$, $N > N_0$ to another , its character changes. No similitude is awaited of fatigue curves at the same R_* and at various L. This discrepancy appears in probability distribution functions, too, as it is shown in Fig.4,b.

Conclusion

It has been shown that the problem of extrapolation of laboratory fatigue tests of specimens upon the large-scale engineering structures meets significant difficulties. They are born, in particular, and probably in the first line, from the existence of several mechanisms of fatigue damage which relative inputs into the lifetime vary when the size of a structural component varies. Therefore, the coarse extrapolation upon much larger sizes and/or much larger lifetimes is awaited to be unsatisfactory. To overcome these difficulties, the following ways may be used:

- experimental study of the size effect on fatigue in conditions most close to the field ones including the pilot tests (very long specimens, very long durations, modelling of environmental actions, etc.);
- monitoring of loads, actions, stress-strain fields, flaws distribution and damage in existing and new-build structures;

- development of advanced probabilistic and structural models of fracture and fatigue of wires, threads, ropes and cables with account of all random factors effecting on their lifetime and structural reliability.

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Classical and Bayesian Analyses of Fatigue Strength Data

Analyses classique et bayesienne appliquées aux données de résistance à la fatigue

Klassische und Bayes'sche Analysen von Ermüdungsfestigkeitsdaten

Barry C. ARNOLD
Professor
University of California
Riverside, CA, USA

Enrique CASTILLO
Professor
University of Cantabria
Santander, Spain

José María SARABIA
Assistant Professor
University of Cantabria
Santander, Spain

Barry C. Arnold graduated from Stanford University in 1965 with a Ph. D. in Statistics.

Enrique Castillo graduated from the Polytechnical University of Madrid in 1969. Awarded a Ph. D. in Civil Engineering from Northwestern University in 1972 and in Mathematics from Complutensis University Madrid in 1974.

José María Sarabia graduated from Valladolid University in 1986 with a Ph. D. in Mathematics from the University of Cantabria.

SUMMARY
Functional equations and basic engineering principles suggest that fatigue length of longitudinal elements may be modelled using a Weibull distribution with a virtual length function entering as a scale parameter. When applied to yarn data, a quadratic virtual length function is supported by both classical and Bayesian analyses. Related simplified models and models exhibiting asympotic independence are also investigated.

RÉSUMÉ
Les équations fonctionelles et les principes technologiques fondamentaux permettent d'envisager une modélisation de l'effet de la longueur sur la fatigue des élements longitudinaux à l'aide d'une distribution de Weibull, en introduisant une fonction de longueur virtuelle en tant que paramètre d'échelle. Dans le cas des données de fibres textiles une fonction virtuelle quadratique peut être admise pour la longueur tant par l'analyse classique que par l'analyse bayesienne. Des modèles simplifiés et des modèles qui mettent en évidence une indépendance asymptotique sont également étudiés.

ZUSAMMENFASSUNG
Funktionalgleichungen und grundlegende Ingenieurprinzipien deuten darauf hin, daß der Einfluß der Länge auf die Ermüdung von Längselementen unter Anwendung einer Weibull-Verteilung mit einer virtuellen Längenfunktion als Maßstabparameter modelliert werden kann. Auf Ergebnisse an Garnen angewandt, wird eine quadratische virtuelle Längenfunktion sowohl durch traditionelle als auch durch Bayes'sche Analyse unterstützt. Ähnliche vereinfachte Modelle und solche, die asymptotische Unabhängigkeit aufweisen, werden ebenfalls untersucht.

1. INTRODUCTION

Longer wires are weaker than short ones of the same diameter. That much has been "obvious" for centuries. How does this relationship between length and strength manifest itself? A standard experimental paradigm involves the use of samples of the material in question of equal diameters but varying lengths. The samples are subjected to repeated vibrational stress and failure times are observed. There will be considerable variability but, in general, the short samples will have longer survival times. Development of an appropriate stochastic model will be especially desirable. A important use of the model will be to predict the strength (as manifested by failure times) of elements of lengths different from those actually studied. Extrapolation will inevitably be involved. Typically only short samples can be studied but, in fact, much longer elements are of interest. We study one meter long samples of cable and try to predict how a bridge cable will behave. Such enthusiastic extrapolation is risky. It is however inevitable. Small errors in the fitted parameters have little effect on predictions for short samples but may result in enormous ranges of uncertainty when we extrapolate to longer samples. This cannot be avoided. All we can do is provide the engineer with the available information, sparse and uncertain though it may be. Basically our goal is to provide predictions and estimated reliabilities of predictions for long elements based on experiments using short elements. If the predictions are too crude, then it will be quite appropriate to call for new experiments, undoubtedly involving longer samples. To illustrate these ideas, we will reanalyze the Picciotto yarn data. Short lengths of yarn (less than or equal to a meter in length) were studied. Extrapolation to longer lengths is desirable. In particular, it is of interest to know whether an assumption of asymptotic independence is tenable; i.e. for long elements, if one is twice as long as another, is it twice as weak?

2. DEVELOPMENT OF THE MODEL

The survival function for an element of length x will be denoted by $\bar{F}(t,x)$. It represents the probability that an element of length x will survive at least t units of time (often measured in cycles). An accelerated failure model would be one in which the distribution of failure times depended on the length x only through a scale factor. That is

$$\bar{F}(t,x) = \bar{F}_0(h(x)t) \tag{2.1}$$

where F_0 is the distribution of failure times for an item of unit length and so, by convention, $h(1) = 1$. Castillo and Ruiz-Cobo [5] use an argument involving functional equations to arrive at (2.1) but many experimenters will be happy to accept such a model in which the scale of but not the shape of the survival distribution depends on element length.

Bogdanoff and Kozin [2] suggest a model of the form

$$\bar{F}(t,x) = [\bar{F}(t,y)]^{N(y,x)} \tag{2.2}$$

for some function $N(y,x)$. Castillo et al [4] show that this necessarily implies that the model is of the form

$$\bar{F}(t,x) = [\bar{F}_0(t)]^{q(x)} \tag{2.3}$$

for some base survival function $F_0(t)$ and arbitrary non-negative function $q(x)$. This is recognizable as a proportional hazards model in the sense of Cox [6].

Assuming that we find the accelerated risk paradigm, (2.1), and the proportional hazards paradigm, (2.3), to both be compelling we are forced to conclude that a Weibull model is appropriate with some function of length entering as a scale parameter. Specifically our model is of the form

$$\bar{F}(t,x) = [e^{-t^c}]^{q(x)} \tag{2.4}$$

where $q(x)$ is a non-negative function. Finally a parametrically parsimonious model might assume that $q(x)$ is a low degree polynomial.

3. INDEPENDENCE, ASYMPTOTIC INDEPENDENCE AND VIRTUAL LENGTH

The simplest model for fatigue strength corresponds to the choice $q(x) = mx$ in (2.3). In this model an element of length x behaves as if, when it divided into k subelements of length x/k, the subelements act independently and the full element survives if and only if all k subelements survive. This is the independence model. It undoubtedly only applies to long elements and then only approximately. Following Arnold, Castillo and Sarabia [1] we will say that the model exhibits asymptotic independence if

$$\lim_{x\to\infty}[q(\lambda x) - \lambda q(x)] = 0 \quad \forall\, \lambda > 1 \tag{3.1}$$

and exhibits strong asymptotic independence if

$$\lim_{x\to\infty} q(x)/mx = 1 \text{ for some } m > 0. \tag{3.2}$$

Both cases correspond to situations in which, for large x, $q(x)$ behaves like mx.

The function $q(x)/q(1)$ will, under independence, correspond to the length of the element (namely x). We will call this function the virtual length function. An element of the material of length x acts as if (under an independence assumption) it were of length $q(x)/q(1)$. A key restriction on $q(x)$ is that it be non-negative over the range of observed values of x. More importantly it should remain non-negative over the range of x values to which we wish to extrapolate.

4. THE PICCIOTTO DATA; LIKELIHOOD ANALYSIS

Yarn samples of length's 0.3(0.1)1.0 meters were studied by Picciotto [7]. A total of 797 observations were made; 99 or 100 for each value of x. The full data set is reported in [1] and [4] as well as in Picciotto [7]. The previous discussion suggests a model of the form

$$P(T > t|X = x) = \exp[-(\alpha + \beta x + \gamma x^2)t^\delta] \tag{4.1}$$

where a convenient normalization has been invoked so that T = (# of cycles to failure)/1000 and X corresponds to length measured in meters. The virtual length function is

$$q(x) = \alpha + \beta x + \gamma x^2. \tag{4.2}$$

Our observed levels of x cover the range 0.3 - 1.0. Our interest is in extrapolation to large values of x. Consequently it is reasonable to restrict α, β, γ in (4.2) so that $q(x)$ remains positive over the range $(0.3, \infty)$. A more stringent requirement that $q(x)$ remain positive over the half line $(0, \infty)$ would require $\alpha = 0$ and $\beta = 0$ so that $q(x)$ would assume the particularly simple form γx^2. This quadratic virtual life model is appealing in its simplicity but we must be wary, since this simplicity may be bought at a high price of reduced explanatory power in the region of interest for extrapolation. If, for some reason we were very interested in small, rather than large, values of x then of course a restriction that $q(x)$ remain positive for small values of x would be essential. For our present purposes, it is judged to not be essential. The model (4.1) may be fitted to the Picciotto data by standard maximum likelihood routines. Using a BMDP package the following estimated values of the four parameters were obtained.

parameter	estimate	asymptotic standard error	estimate/s.e.
α	-6.21	18.98	-0.33
β	-6.84	86.03	-0.08
γ	320.23	101.29	3.16
δ	1.97	0.05	37.99

The corresponding value of the log-likelihood is 1413.27. Note the negative values of α and β. If these are kept in the virtual length function then, for very small values of x (less than $0.3 - 1$, the observed range), the virtual length will be negative. Note also that the estimated value of δ namely 1.97 is remarkably close to 2, the value corresponding to a Rayleigh distribution.

Physical considerations do suggest that we set $\alpha = 0$ (so that $g(0) = 0$). The maximum likelihood estimates subject to this restriction are found to be

parameter	estimate	asymptotic standard error	estimate/s.e.
α	0		
β	-34.56	17.22	-2.01
γ	347.84	59.32	5.86
δ	1.97	0.05	38.26

The corresponding log-likelihood is 1413.21, only a tiny reduction from the value obtained for the four parameter model. Setting $\beta = 0$ (to guarantee $g(x) > 0$ on $(0, \infty)$) is a bit more costly. We find

parameter	estimate	asymptotic standard error	estimate/s.e.
α	0		
β	0		
γ	247.52	23.66	10.47
δ	1.91	0.04	45.40

with a corresponding log-likelihood of 1410.70. This is a barely significant reduction and the gain in parsimony and in the ability to extrapolate to small values of x may be judged to be worth the price. Over the observed range (0.3 – 1.0), the two fitted virtual length functions $-34.56x + 347.84x^2$ and $247.82x^2$ are very similar.

If we set $\alpha = 0$ and $\delta = 2$ and fit the resulting Rayleigh model, our maximum likelihood estimates of β and γ are given by

parameter	estimate	asymptotic standard error	estimate/s.e.
α	0		
β	-41.98	13.03	-3.22
γ	380.60	27.70	13.74
δ	2		

with a log-likelihood of 1413.04. This is remarkably close to the value 1413.27 obtained for the full four parameter model. Based on the available data, such a parsimonious Rayleigh model would be recommended. Further simplification by setting $\beta = 0$, though desirable in that it forces $g(x)$ to be positive over $(0, \infty)$, will be bought at a significant reduction in the log-likelihood (down to 1408.23). This extremely parsimonious model, namely

$$P(T > t|X = x) = \exp[-302.13(xt)^2],$$

merits reporting and would gain in stature if some physical interpretation could be found for the belief that $(xT)^2$ should be exponentially distributed. Further comment on this model will be found in Section 7.

5. BAYESIAN ANALYSIS

In practice it is to be expected that, not only would the engineer be able to argue in favor of a particular parametric model such as (4.1), but also he would have some insights into plausible values for the parameters in the model. If we denote the elicited prior for $(\alpha, \beta, \gamma, \delta)$ by $\varphi(\alpha, \beta, \gamma, \delta)$ (most likely of the form $\varphi_1(\alpha)\varphi_2(\beta)\varphi_3(\gamma)\varphi_4(\delta)$), then our posterior will be proportional to the following expression

$$h(\alpha, \beta, \gamma, \delta) = \varphi(\alpha, \beta, \gamma, \delta) \prod_{i=1}^{797} (\alpha + \beta x_i + \gamma x_i^2)\delta t_i^{\delta-1} \exp[-(\alpha + \beta x_i + \gamma x_i^2)t_i^\delta]. \tag{5.1}$$

Posterior means will serve as reasonable point estimates of the four parameters. Thus we would need to evaluate, for example

$$\tilde{\alpha} = \int_\Theta \alpha h(\alpha, \beta, \gamma, \delta) d\alpha d\beta d\gamma d\delta \Big/ \int_\Theta h(\alpha, \beta, \gamma, \delta) d\alpha d\beta d\gamma d\delta \tag{5.2}$$

where $\Theta = \{(\alpha, \beta, \gamma, \delta) : \delta > 0, \alpha + \beta x + \gamma x^2 > 0 \ \forall \ x \geq 0.3\}$. Such four dimensional numerical integrations may use up considerable amounts of computer time, which may be expensive. An alternative approximate approach involves iterative one dimensional averaging. Provided that the posterior is unimodal this will provide a reasonable approximation to the posterior means. In this approach admissible initial values $(\alpha_0, \beta_0, \gamma_0, \delta_0) \epsilon \Theta$ are chosen and are updated as follows

$$\alpha_1 = \int \alpha h(\alpha, \beta_0, \gamma_0, \delta_0) d\alpha \Big/ \int h(\alpha, \beta_0, \gamma_0, \delta_0) d\alpha$$

$$\beta_1 = \int \beta h(\alpha_1, \beta, \gamma_0, \delta_0) d\beta \Big/ \int h(\alpha_1, \beta, \gamma_0, \delta_0) d\beta$$

$$\gamma_1 = \int \gamma h(\alpha_1, \beta_1, \gamma, \delta_0) d\gamma \Big/ \int h(\alpha_1, \beta_1, \gamma, \delta_0) d\gamma$$

$$\delta_1 = \int \delta h(\alpha_1, \beta_1, \gamma_1, \delta) d\delta \Big/ \int h(\alpha_1, \beta_1, \gamma_1, \delta) d\delta \tag{5.3}$$

Then the process is repeated, now using $(\alpha_1, \beta_1, \gamma_1, \delta_1)$ as initial values. The integration in (5.3) is over the ranges of values for which the integrands are positive. The process is continued until stable values are obtained.

Implementation of a simplified version of the scheme (5.3) (in which we set $\alpha = 0$) with the Picciotto data, assuming improper uniform priors, yielded estimates of the form

$$\tilde{\beta} = -34.49$$

$$\tilde{\gamma} = 349.94$$

$$\tilde{\delta} = 1.973$$

As expected these are good approximations to the maximum likelihood estimates. More disparity would be encountered if a more precise prior were to be used.

6. WHAT SHOULD THE VIRTUAL LENGTH FUNCTION BE?

Data driven analysis suggested that a quadratic virtual length function is appropriate. For predictive purposes that may be adequate but, since many non-quadratic functions on the real line are well approximated by quadratic functions on the interval $(0.3, 1)$, extrapolation will be dangerous. Existence of a plausible physical explanation for quadratic virtual length would lessen the dangers of such projections. One possible explanation argues that failures occur at faults and that faults are distributed along the element according to a possibly non-homogenous Poisson process. An element with k faults will have a survival time function given by $[\bar{G}_0(t)]^k$. If the non-homogenous fault generating Poisson process has rate function $\lambda(x)$ then the expected number of faults in an element of length x will be $\int_0^x \lambda(y)dy$. This will be quadratic if λ is a linear function. Under such a scenario a Weibull model with a quadratic virtual length function might provide a good approximation to reality. The problem with this model is that our elements of length say $\frac{1}{4}$ were not made that length. They were cut from a longer piece. The fault generating process model is difficult to justify under such circumstances.

Quadratic virtual length has another negative feature. It manifestly fails to exhibit asymptotic independence. Yet asymptotic independence is surely a reasonable requirement for a model. A long element will fail if it fails in the first or second half of its length. These events should have equal probabilities and should be (roughly) independent. It doesn't take much arguing to convince yourself that at least asymptotic independence is appriopriate. Consequently the correct virtual length function will behave for small lengths in an approximately quadratic fashion and will be approximately linear for large x. In fact many researchers (see e.g. Castillo and Fernandez-Canteli [3]) argue that our main task is to determine a threshhold beyond which the linear virtual length model can be assumed to hold.

A three parameter virtual length function of the following form

$$g(x) = ax + b(e^{cx} - 1) \tag{6.1}$$

was considered in Arnold, Castillo and Sarabia [1]. They assumed a Rayleigh distribution so the survival model was of the form

$$P(T > t | X = x) = \exp\left[-[ax + b(e^{cx} - 1)]t^2\right]. \tag{6.2}$$

This virtual length function behaves roughly as a quadratic for $x < 1$ but, for large x, behaves like ax. For extrapolation purposes, particular interest will be focussed on the parameter a. If we apply this model to the Picciotto data, the likelihood surface exhibits a ridge. The likelihood can be made large by picking very small values of c together with appropriate large values of a and b. Three sets of parameter values which yield approximately equal values of the likelihood (and which are consequently essentialy equally plausible) are

a	b	c	log-likelihood
1011.5	1089.90	-1.00	1412.63
7859	79048	-0.10	1413.05
76290	7633183	-0.01	1413.07

The data based on lengths in the interval $(0.3, 1)$ support the plausibility of arbitrarily large values of a. Perhaps predictably, they are unable to assist us in determining the appropriate asymptotic slope.

7. THE SIMPLE MODEL

The simplest model with explanatory power corresponds to the choice $\alpha = 0, \beta = 0, \delta = 2$ in (4.1). For this model, as remarked in Section 4, $Y = (xT)^2$ will have an exponential distribution. The sample c.d.f. of the 797 Y_i's from the Picciotto data should look like $1 - e^{-\lambda y}$. Consequently a plot of $\log \bar{F}_n(y)$ (the log of the empirical survival function) vs y should be a line through the origin with negative slope ($= -\lambda$). The actual situation for the Picciotto data is as shown in Figure 1.

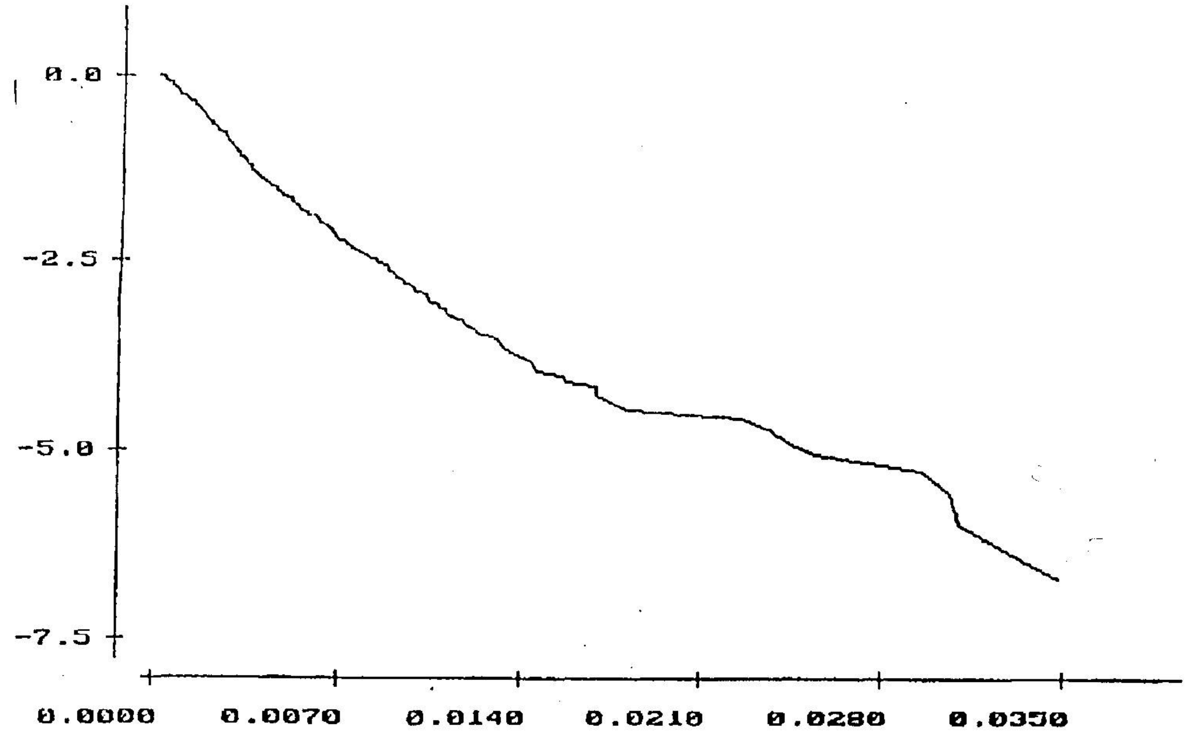

Figure 1: Plot of the logarithm of the empirical survival function ($\log \bar{F}_n(y)$) versus $y (= (xT)^2)$ for Picciotto data.

Elimination of the 8 largest Y_i's, regarded as potential outliers, improves the picture considerably. The empirical plot of the remaining 789 points, shown in Figure 2, might be judged to be acceptably linear. This may be advocated as a simplistic straw-man model for small values of the length x. We are still lacking a theoretical argument in favor of the quadratic virtual length which is implicit in such a model. We recognize that extrapolation to large lengths using such a model will be questionable since it will fly in the face of our belief of the plausibility of asymptotic independence.

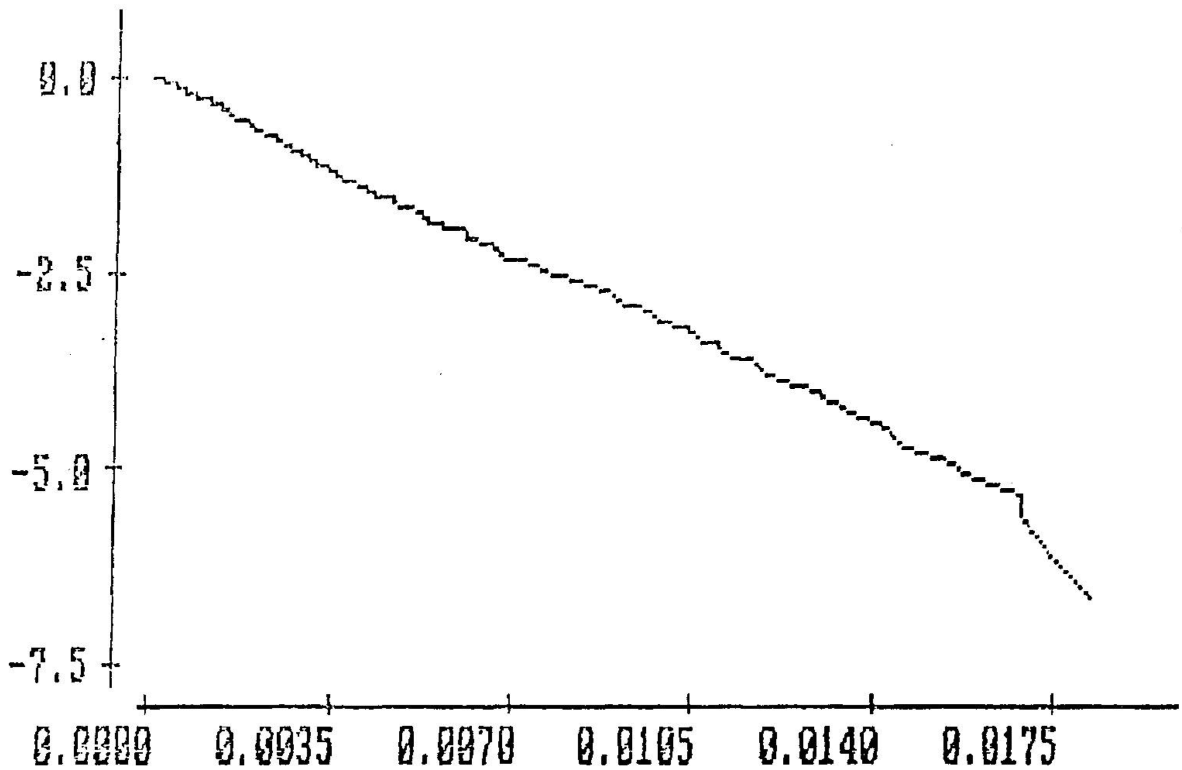

Figure 2: Plot of log $\bar{F}_n(y)$ versus y for the 789 Picciotto points remaining after deleting eight outliers.

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Modelling the Fatigue Strength and Lifetime of Wires and Cables

Modélisation de la résistance à la fatigue et durée de vie des fils et câbles

Modellierung der Ermüdungsfestigkeit und Lebensdauer von Drähten und Kabeln

S. Leigh PHOENIX
Professor
Cornell University
Ithaca, NY, USA

Leigh Phoenix, born in 1946, obtained his PhD degree from Cornell University in 1972, and after two years in industry returned to the Cornell faculty. Since then he has worked on probability aspects of the strength and fatigue of cables and composite materials.

Irene J. BEYERLEIN
Student
Clemson University
Clemson, SC, US

Irene Beyerlein, born in 1971, is a Clemson undergraduate in Mechanical Engineering.

SUMMARY

This paper considers issues in modelling the fatigue strength and lifetime of wires and cables including size effects and extreme lower tail probabilities important to reliability in life safety applications. Examples are drawn from cables made of advanced composites in order to make two basic points. First, while a good fiber or wire model is an essential starting point, estimating certain extreme parameters and quantiles from test data may not be important to the reliability performance of a cable with series-parallel, load-sharing structure. Second, series-parallel load-sharing models afford the opportunity to pursue better designs for cable structures.

RÉSUMÉ

Cette communication traite les problèmes de modélisation de la résistance à la fatigue et de la durée de vie des fils et des câbles. Elle tient compte de l'effet d'échelle et de la probabilité de rupture extrêmement faible, facteurs importants quant à la fiabilité des réalisations impliquant un risque humain. Elle présente deux remarques fondamentales à partir d'exemples de câbles en matériaux composites hautement performants. Premièrement, bien qu'un modèle de fibre ou de fil d'excellente qualité soit essentiel au départ, une estimation, faite à partir de données expérimentales pour certains paramètres extrêmes et résultats statistiques, n'est absolument pas importante pour la fiabilité d'un câble à structure sérielle-parallèle avec répartition de charge. Deuxièmement, les modèles sériels-parallèles avec répartition de charge offrent la possibilité de mieux concevoir les structures de câbles.

ZUSAMMENFASSUNG

Dieser Beitrag behandelt Aspekte der Ermüdungsfestigkeit und Lebensdauer von Drähten und Kabeln. Miteinbezogen sind Größeneffekte und Schadensereignisse mit extrem kleiner Wahrscheinlichkeit, die im Zusammenhang mit Sicherheitsaspekten in der Anwendung wichtig sind. Beispiele werden aufgezeigt für Kabel aus Hochleistungsverbundstoffen, um zwei grundlegene Punkte anzusprechen: Obwohl, erstens, ein gutes Fasern- oder Drahtmodell ein wichtiger Ausgangspunkt ist, muß eine Abschätzung von gewissen extremen Parametern und Größen aus Testdaten nicht unbedingt wichtig sein für die Zuverlässigkeit eines Kabels mit seriell-paralleler, lastverteilender Struktur. Zweitens, seriell-parallele, lastverteilende Modelle bieten die Möglichkeit, besseren Konstruktionen für Kabelstrukturen nachzugehen.

1. INTRODUCTION

1.1 Preface

Our interest in steel cables and strands for cable stayed bridges and other suspended structures is a natural outcome of experiences with practical problems of cable and socket performance. For several years, one of us (SLP) has been a technical consultant to the Arecibo radar-radio telescope observatory, funded by the U.S. National Science Foundation. The feed systems for the telescope are supported by a large steel suspended structure, having twelve 7.6 cm diameter main cables and fifteen 8.3 cm diameter cables constructed of bridge strand of typical helical construction. Originally, this structure was designed to be a limited life, structure (about ten years) so that safety factors in many of the cables are *less than two*! Wire breakage in these cables has been experienced over its approximately 25 years of operation, and has been studied from both a mechanics and metallurgy perspective including dissection of a removed cable. This has led to unique corrosion protection efforts which have largely been effective in suppressing wire breakage, and many decades of useful life are expected. Some of our results have been published [1].

Most of the our experience, however, has been in advanced composites (mostly graphite, glass and Kevlar 49 aramid fibers in epoxy matrices) and ropes and cables (mostly Kevlar 29 aramid fibers in various untwisted and twisted constructions). Our interest has been in statistical modelling of the strength and lifetime in creep-rupture and fatigue of these fibrous structures and we have also done considerable experimental work on individual fibers, strands and bundles and unidirectional composites in order to validate various theories. We believe this experience brings a different perspective to the issues being addressed by this workshop as these issues are not unique to steel wires and strands. We would like to share a few observations through examples. Our comments are motivated in large measure by helpful material in the introductory lectures elsewhere in the workshop proceedings.

1.2 Reliability Goals and Realities

Whether we are talking about steel or polymeric composite cables in applications involving life safety, **the key design problem is to establish wire and cable structures and parameters such that the probability of failure over a specified service lifetime, loading and environment is a very small number, say < 10^{-6}.** This must be true not only for a *single* cable but for *all* cables of a structure viewed collectively as a system, whereby any one failure will produce collapse of the system. As has been pointed out by Castillo and Fernández-Canteli in their introductory lecture, this sort of requirement imposes prohibitive needs for experimentation if such reliability targets are to be verified by brute force experimentation. **It is not possible to verify such low probabilities of failure empirically since the *number* of required replications of a fatigue test is prohibitive (> 10^7 in the above example), the specimen *sizes* must be huge (cables much longer than typical test facilities can handle) and the *times* for testing must be enormous (years).** Furthermore experience gathered on performance near the mean of a distribution may be very misleading with respect to performance in the extreme lower tail, which cannot be observed.

This brings us to the need pursue accurate models. Designers in the past have often been satisfied with mean values of fatigue strength and lifetime (and on occasion coefficients of variation) followed by application of large safety factors based on longstanding experience. The modern reliability approach, however, is to seek probabilistic assessment through careful modelling, with the goal of determining the full probability distribution of fatigue strength (the probability for each possible value of the cyclic stress range $\Delta\sigma$ that the fatigue lifetime will reach say 2×10^6 load cycles) but especially in the extreme lower tail (say probabilities of 10^{-6} or less). This must be done not only for a single wire of laboratory length but for a full cable.

Much of the effort in the literature seems to be devoted to building a realistic probability model for the failure of a *single* wire including statistical estimation procedures for model parameters , and size or length effects about which there has been considerable controversy. Castillo and Fernández-Canteli in their introductory lecture have identified many key issues which from our experience are also relevant to the field of composite materials. But like composite materials, cables are redundant structures for which there is considerable load-sharing among wire members. This means that bundle models, chain-of-bundles models and other lattice models have great potential for describing how a cable is ultimately to perform, especially in the lower tail region of the probability distribution; single wire models cannot do this job alone, and in fact, one must to be careful about becoming preoccupied with wire model issues that may emerge as largely unimportant in a series-parallel, load-sharing structure. There have been a few attempts to build such bundle models, notably by Fernández-Canteli and coworkers [2-4], Stallings [5], and Tanaka and coworkers [6,7] with some success. But this is just a beginning, and we believe their is great potential based on what is known about these models in the context of polymer cables and composites. Unfortunately efforts so far have had little effect on the development of international standards for testing and design [8], but on the other hand, this shortcoming can be viewed as a great opportunity for the future.

We do not want to give the impression that the quest for better models and better cables is purely a mathematical exercise in statistics devoid of the realities of the current base of experience. In their introductory lectures Esslinger and Gabriel and Nürnberger have pointed out many issues related to manufacturing processes, environmentally driven corrosion and clamping and socketing, all of which may overwhelm idealized statistical predictions. Still, good models can help us identify what is theoretically possible but also what may actually be unimportant to our goal. Models can also help us identify strategies for *structural and materials design and engineering* in order to focus on innovative solutions to the key materials and mechanics problems that are identified.

In what follows we wish to discuss issues of wire and cable reliability performance many of which have been raised in the introductory lecture of Castillo and Fernández-Canteli. We will begin from the perspective of fibrous composites as a means of illustrating some key points. We will focus mainly on static strength as the concepts are simpler. Admittedly, steel wires have considerable ductility and small variability in ultimate strength as compared to fibers used in composites, however, the issues we raise have close analogies with respect to fatigue strength and lifetime of steel cables.

2. A PERSPECTIVE FROM COMPOSITE MATERIALS

2.1 Experience with Fibers

Our laboratory experience has largely been with advanced fibers such as Kevlar 49 by The duPont Company or IM-6 or AS-4 fibers by Hercules, Inc. These fibers vary from 5 to 12 μm in diameter depending on the specific material which means that a 1 cm length already has an aspect ratio (length/diameter ratio) of 1,000 to 2,000. In comparison to a steel wire with a diameter of 5 mm, this corresponds to a wire length of 500 cm to 1,000 cm! We routinely tension test fibers of lengths up to 20 cm or aspect ratios of 20,000 to 40,000, which corresponds to steel wire lengths of 10,000 to 20,000 cm. In a few cases [9] we have also determined strength statistics for fiber segments at an aspect ratio of only about 40, that is at lengths of 0.20 to 0.40 mm. So our experience spans fiber aspect ratios from 40 to 40,000 or *three orders of magnitude in length.* Certainly one would think that distributions motivated by extreme value statistics would naturally apply, but the experimental reality less simple.

Fibers typically are produced as very lightly twisted yarn wrapped on spools. A yarn may have from 200 to more than 10,000 fibers in a cross section. Variability in fiber strength comes from flaws which are randomly distributed along the length either on the surface or within the fiber interior. As one might expect strength statistics may vary from spool to spool within the same lot. But more interesting, fibers differ in properties across a yarn. This is because the processing conditions from hole to hole in a spinnerette through which the fibers were originally extruded are not identical, and a fiber may have a common microstructure along its length, but slightly different from its lateral neighbor. Typically one finds that a Weibull weakest-link model works quite well but not over all length scales and not without the need for modification.

For a given spool and a given location along a spool (spanning say a few yards of yarn), the following version of the Weibull model for fiber strength often works quite well. Suppose we sample fibers from *across* a yarn and tension test them at arbitrary gage length ℓ relative to some reference length ℓ_0. Experiments show that over a range of gage lengths ℓ, the distribution function for strength accurately follows the Weibull distribution

$$F(\sigma;\ell) = 1 - \exp\{-(\ell/\ell_0)^{\alpha}(\sigma/\sigma_{\ell_0})^{\rho}\}, \qquad \sigma \geq 0 \qquad (1)$$

where σ is fiber stress, ℓ is the actual fiber gage length, ℓ_0 is a convenient reference length, σ_{ℓ_0} is the Weibull scale parameter for strength measured at reference length ℓ_0, ρ is the Weibull shape parameter for strength, and α is a parameter satisfying $0 < \alpha < 1$. Note that the strength versus length relationship is given by

$$\sigma_\ell = \sigma_{\ell_0}(\ell_0/\ell)^{\alpha/\rho} \qquad (2)$$

where the exponent is *not* the usual $1/\rho$.

When fiber segments are sampled from *along* a given fiber it often occurs that the model

$$\underline{F}(\sigma;\ell) = 1 - \exp\{-(\ell/\ell_0)(\sigma/\underline{\sigma}_{\ell_0})^{\rho^*}\}, \qquad \sigma \geq 0 \qquad (3)$$

works well with $\rho^* = \rho/\alpha$, though $\underline{\sigma}_{\ell_0}$ would vary from fiber to fiber. The strength versus length relationship is then given accurately by

$$\underline{\sigma}_\ell = \underline{\sigma}_{\ell_0}(\ell_0/\ell)^{1/\rho^*} \qquad (4)$$

as expected. (The theoretical underpinnings of the above 'empirical' model are discussed in Watson and Smith [10] both for fibers and composite strands.) Figure 1 shows

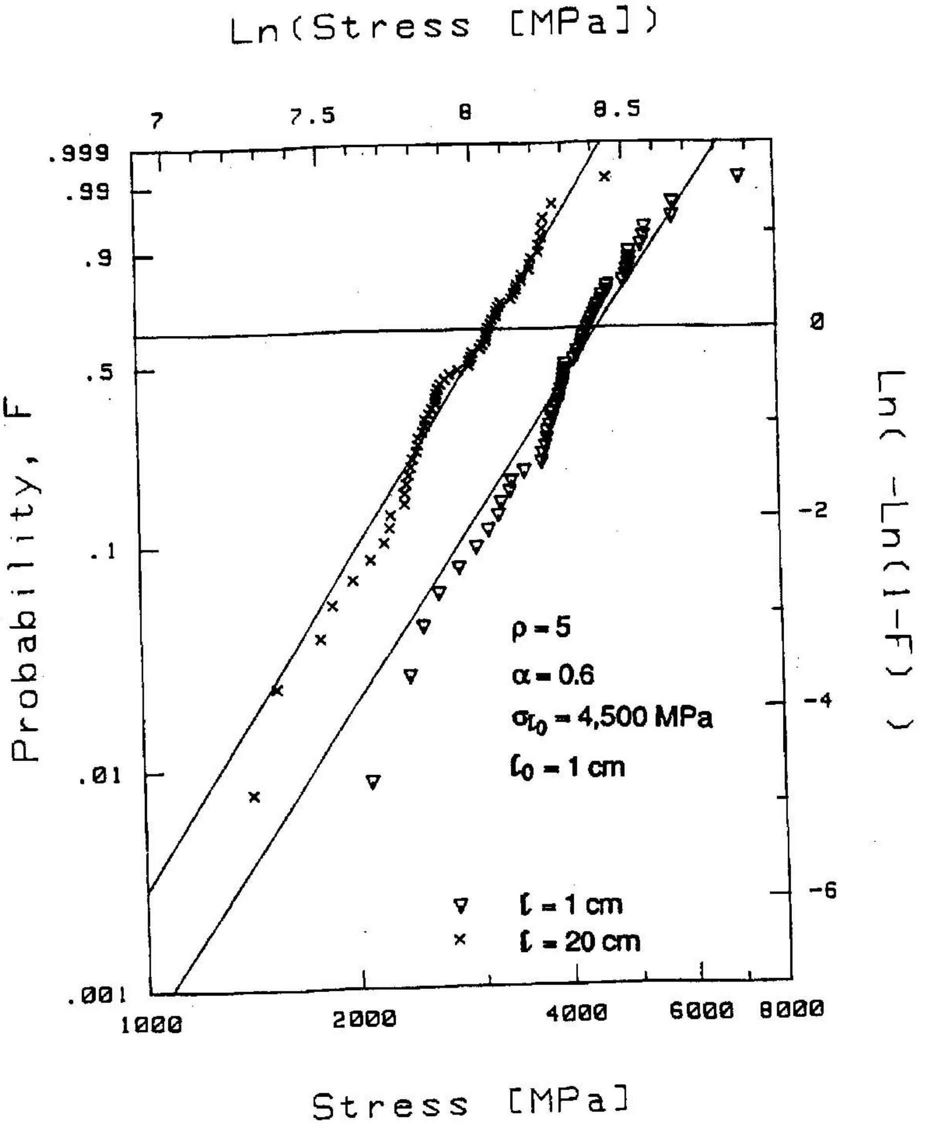

Fig. 1. Strength data for Hercules AS-4 graphite fiber on Weibull coordinates (Ref. [14])

experimental data for Hercules AS-4 graphite fibers tested at gage lengths ℓ of 1 and 20 cm. For this fiber we find $\rho = 5$ and $\alpha = 0.6$, $\rho^* = 8.3$ and $\sigma_{\ell_0} = 4{,}500$ MPa.

The question arises as to how well the model works with respect to extrapolation to much longer or shorter gage lengths of AS-4 fiber. Experience shows that reasonable extrapolations are possible down to $\ell = 0.25$ mm, which is approximately the effective load transfer length for fibers in a graphite/epoxy composite. At that length, the scale parameter for strength would be in the vicinity of 7,000 MPa, but further decreases in length may not produce the anticipated increases in strength. For IM-6 graphite fibers in this situation, it turns out that a rolloff in strength and rapid increase in Weibull shape parameter may already occur at such lengths [9]. The extrapolation works well for longer lengths of AS-4 graphite fiber also, up to perhaps 100 cm. At that length, the scale parameter would be about 2,600 MPa but the strength my drop of more rapidly at even longer lengths than eqn (4) predicts often accompanied by a sudden drop in Weibull shape parameter. So the model reasonably covers a strength range varying by a factor of almost three and a length range of over three orders of magnitude.

This model works well for the AS-4 graphite fibers at hand, but one finds great variety particularly in the value of α. For Kevlar 49 we find $\rho \approx 8$ and $\alpha \approx 0.4$. For Hercules IM-6 fibers we have found $\alpha \approx 1.0$ over a limited range, but tending actually to drop to 0.8 for longer lengths. Moreover, values vary from spool to spool even in the same production lot. For spectra fibers one finds $\alpha \approx 0.1$ with only a very mild length effect at least up to several centimeters. For most fibers one finds 'drift' in the model in that the Weibull shape parameter will drift continually *downward* to smaller values as the length is increased starting with aspect ratios of about 40. This behavior is well known for glass fibers and we have seen it also in Hercules IM-6 graphite fibers and boron fibers.

Thus, our experience is that for the strength of single fibers one must anticipate the need for models of the form

$$F(\sigma; \ell) = 1 - \exp\{-g(\ell/\ell_0)\Lambda(\sigma/\sigma_{\ell_0})\}, \qquad \sigma \geq 0 \qquad (5)$$

where $g(\ell/\ell_0)$ and $\Lambda(\sigma/\sigma_{\ell_0})$ are a quite arbitrary functions not necessarily well represented by simple power forms leading to the usual Weibull distribution. Power approximations to $\Lambda(\sigma/\sigma_{\ell_0})$ for smaller and smaller values of stress may require smaller and smaller exponents leading to smaller Weibull shape parameters for longer and longer lengths. Even for very large aspect ratio,s $g(\ell/\ell_0) \approx \ell/\ell_0$ may not be the appropriate model for fibers in a bundle as the fibers have consistent differences in properties. We believe the same situation will occur for steel wires in a bundle as one must be concerned about the manufacturing homogeneity and source of the wires.

The statistical theory of extremes suggests to us that there are only two limiting distributional forms, namely a Weibull form or the double exponential (Gumbel) form useful for tensile strength. But this can be a misleading concept as the above example shows. One must be prepared for well behaved possibilities that don't conform nicely to a Weibull or Gumbel model. In fact, we have a well developed lattice model for failure [11] with weakest-link properties but where a simple well behave distribution is far superior to a Weibull or Gumbel approximation *at all lengths and especially in the lower tail.* The simple Weibull model is not always a good approximation for a fiber in a given application since in a composite material where load sharing takes place, many length scales and stress ranges may be important simultaneously. Nevertheless in analytical models of systems involving load-sharing the Weibull model can give us considerable insight.

2.2 Experience with the Strength of Simple Composites

Armed only with such statistical models for fiber strength, what can be said immediately about the strength behavior of a fiber/epoxy composite? Short of bounding the strength from below, the answer is very little! To see this, we consider a simple graphite fiber/epoxy strand made from impregnating a graphite yarn with epoxy. A typical laboratory specimen might be 20 cm long and have 10,000 fibers in its cross section, yet it is still smaller in diameter than a shoelace! The total length of fiber in strand is now 200,000 cm or 2 kilometers. In fact, a key characteristic length in the composite is the effective load transfer length for a fiber in the epoxy matrix, being of the order of 40 fiber diameters or 0.25 mm. There are 8×10^6 such fiber elements in our composite strand.

If we apply the above Weibull model, eqn (1) and assume that the composite fails when the first fiber fails, we might predict by extrapolation that the strength of the composite parameter $4{,}500\times(200{,}000)^{-(0.6)/5}$ MPa = 1,040 MPa. Experiments show, however, that this prediction is false. In fact, the strength of such strands will follow approximately a Weibull distribution with a shape parameter of about 30 and a scale parameter of close to 4,500 MPa (fortuitously the value for 1 cm fibers). Furthermore if the strand length is increased by a factor of say 10 to 100, say, the strength will decrease very slowly approximately in proportion to $\ell^{-0.6/30}$, which is almost unmeasurable by experiment. What this means is that in a composite loaded say to 2,800 MPa there will be many fiber breaks -- of the order of one in every 50 cm of fiber in the composite; this is of the order of a total of 4,000 fiber breaks in the strand, yet at this load the composite has survived nicely! In fact the probability of failure is lower than 6.6×10^{-7}. **The presence of a huge number of fiber breaks is consistent with high reliability.**

There are two points: First, breaks may be monitored by acoustic emission in an attempt to predict impending composite failure, but experience has shown this to be largely an unproductive exercise. These breaks and for that matter the strengths of the weakest fibers tell us little about the strength performance of the composite, and are not a reason for its removal from service. Second, for prediction in the lower tail of the composite strength distribution, there is no need to characterize the strength of fibers beyond a length of 50 cm. In fact our models show that the fibers could actually be discontinuous with a mean length of about 10 cm, and the strength distribution for the composite would be negligibly altered. This is the power of fiber load-sharing through the matrix.

Over the past few years we and others have worked on the development of chain-of-bundles probability models to explain the above behavior. The basic idea is that the above composite strand can be partitioned into a chain of short bundles with each bundle having length equal to the effective load transfer length for a broken fiber next to an intact fiber in the epoxy matrix, which transfers the load through shear. This length is of the order 0.25 mm in the above example. Fibers within these bundles then share load according to a load-sharing rule which assigns the loads of failed fibers mostly onto the nearest surviving neighbors. This produces what amounts to a local redundancy and the composite will fail once a critical cluster of a few broken fibers develops which then becomes unstable. References for such models are Harlow and Phoenix [11] and Smith et al. [12] particularly the references therein. Phoenix and Tierney [13] consider such models in the setting of time dependent failure and fatigue adaptable to steel cables.

An interpretation of the above Weibull-like result is that the composite strand fails once a critical cluster of about six fibers develops, and the Weibull shape parameter for the composite, ρ_c turns out to be $6\times5 = 30$. The effective Weibull scale parameter for the composite, σ_c, is determined from the load-sharing in a fairly complex way [14]. We can write the approximate Weibull model for the composite as

$$F_c(\sigma;\ell) \approx 1 - \exp\{-(\sigma/\sigma_c)^{\rho_c}\}, \qquad \sigma \geq 0. \tag{6}$$

Yet if we increase the length of the composite (or for that matter its width) the model predicts that the shape parameter will actually *increase* very slowly aproximately in proportion to the log of the volume (since as the strength drops the critical cluster size grows). This increase occurs despite the fact that the fiber shape parameter decreases! Furthermore the Weibull distribution actually *overestimates* the probability of failure in the lower tail consistent with an increasing Weibull exponent applicable to that region. This is not just a prediction from the model. **All experiences with composite structures, orders of magnitude larger than the laboratory strand under discussion reveal that these general features are valid.**

2.3 An Example of a 4-fiber Composite 'Cable'

The number of fibers in the cross-section of a typical commercial yarn (a thousand or more) or the number of characteristic fiber elements in a small composite strand (at least 10^6) are orders of magnitude larger than the number of wires or wire elements in a typical steel cable. Thus one may attempt to argue that the above models and ideas have limited relevance, especially the benefits of load-sharing among elements. This is not true. In fact most of the benefits of localized load-sharing are realized with the interaction of very few fibers, and the effective degree of interaction actually grows as the log of the total volume, which is very slowly. For more global load sharing, the benefits grow even faster. The following experimental example [14] makes the point.

We have fabricated miniature composite 'cables' consisting of four AS-4 graphite fibers in parallel in a square cross-section and held together by an epoxy. The fibers were closely packed and the epoxy not only filled in the voids but also formed a fairly thin layer around the fibers. The epoxy volume fraction was about 30%. Specimens were fabricated for tension testing at two gage lengths, 1 cm and 20 cm. Note that these specimens had a diameter of about 16 μm (much less than a human hair so they would be useful as cables only to insects!) so that their aspect ratios were about 600 and 12,000, respectively. From a chain-of-bundles model point of view, the effective load transfer length, δ, among fibers in a cross section is about 0.15 mm, so the number of bundles, m, in the chain model is about 66 for the 1 cm specimens and about 1,333 for the 20 cm specimens; the total number of fiber elements 4m were 264 and 5,280 respectively. Figure 2 is a schematic of the composite including load-sharing configurations.

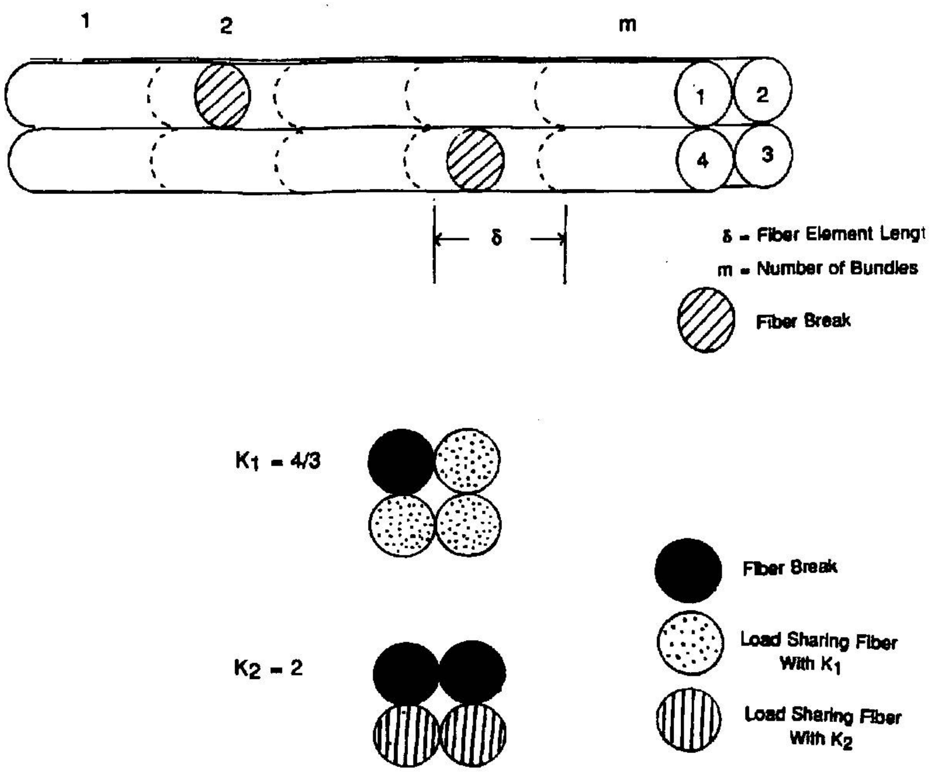

Fig. 2. Schematic of 4-fiber composite cable and load-sharing model (Ref. [14])

According to theory [14], we expect to see the emergence approximately of Weibull distributions of the form

$$F^{[k]}(\sigma) \approx 1 - \exp\{-4m^{\alpha}(\sigma/\sigma_{\delta,k})^{k\rho}\}, \qquad \sigma \geq 0, \qquad (7)$$

for the strength of the composite where k = 1,2,3 and 4, ρ is the Weibull shape parameter for the fiber, and α is a parameter discussed earlier in connection with the fiber. Also

$$\sigma_{\delta,k} = \sigma_{\delta}(d_k)^{-1/(k\rho)}, \qquad (8)$$

where

$$d_1 = 1, \quad d_2 \approx 3(4/3)^{\rho}, \quad d_3 \approx 6(4/3)^{\rho}(2)^{\rho}, \quad d_4 \approx 6(4/3)^{\rho}(2)^{\rho}(4)^{\rho} \qquad (9)$$

captures (approximately) the effect of load-sharing factors and configurations, and

$$\sigma_{\delta} = \sigma_{\ell_0}(\ell_0/\delta)^{\alpha/\rho} \qquad (10)$$

is the characteristic Weibull strength (scale parameter) for a fiber element of length δ. For lower and lower stress ranges, these Weibull distributions will apply in succession, where, $k = 1$ applies roughly for $3\sigma_{\delta}/4 < \sigma$, $k = 2$ applies roughly for $\sigma_{\delta}/2 < \sigma < 3\sigma_{\delta}/4$, $k = 3$ applies roughly for $\sigma_{\delta}/4 < \sigma < \sigma_{\delta}/2$ and $k = 4$ applies roughly for $0 < \sigma < \sigma_{\delta}/4$. Note that $k = 1$ corresponds to the strength of the weakest flaw in all four fibers, that is, the first fiber break or a 'weakest flaw' view of failure of the composite.

Figure 3 demonstrates that these features are largely observed in the experimental data for these composites [14]. The various lines are the Weibull distributions of eqn (7) for k = 1, 2, 3 and 4 and m = 66 and 1,333 for the two cases. For the calculations we have taken $\alpha = 0.6$ and $\rho = 5$ as in Figure 1, and $\delta = 0.15$ mm for which, we have $\sigma_{\delta} \approx 7{,}500$ MPa. The interpretation of the plots is that k is the critical cluster size (number of adjacent fiber breaks required for collapse) for that stress range. Clearly the composites do not fail with the first fiber failure. For the 1 cm composites, perhaps only the strongest specimen failed when one fiber failed, many of the remainder required two adjacent breaks, and the weakest few required three breaks to cause collapse. In fact in the extreme lower tail, the appropriate Weibull distribution will have k = 4 with Weibull shape parameter $4\rho = 20$.

For the 20 cm composites the fit is not quite as good, but it can be greatly improved by choosing $\delta = 0.25$ instead of $\delta = 0.15$. The appropriate Weibull distribution modelling the extreme lower tail has shape parameter $\rho_c = 20$ and scale parameter $\sigma_c = 2{,}850$ MPa calculated from eqns (7) to (10) in view of eqn (6). The stress level producing a 10^{-6} probability of failure is about 1,450 MPa. In this case failure requires four adjacent breaks prior to collapse. Incidentally Figure 1 shows that we have sufficient data on fibers to be confident of the fiber strength at that stress level. We do *not* need to know basic fiber statistics at extremely low probabilities of failure or extremely long lengths. Although we can never make and test enough specimens to prove the point (although filament wound pressure vessels with 10^9 times as much material behave as predicted) we are confident of these reliability predictions for this microscopic cable.

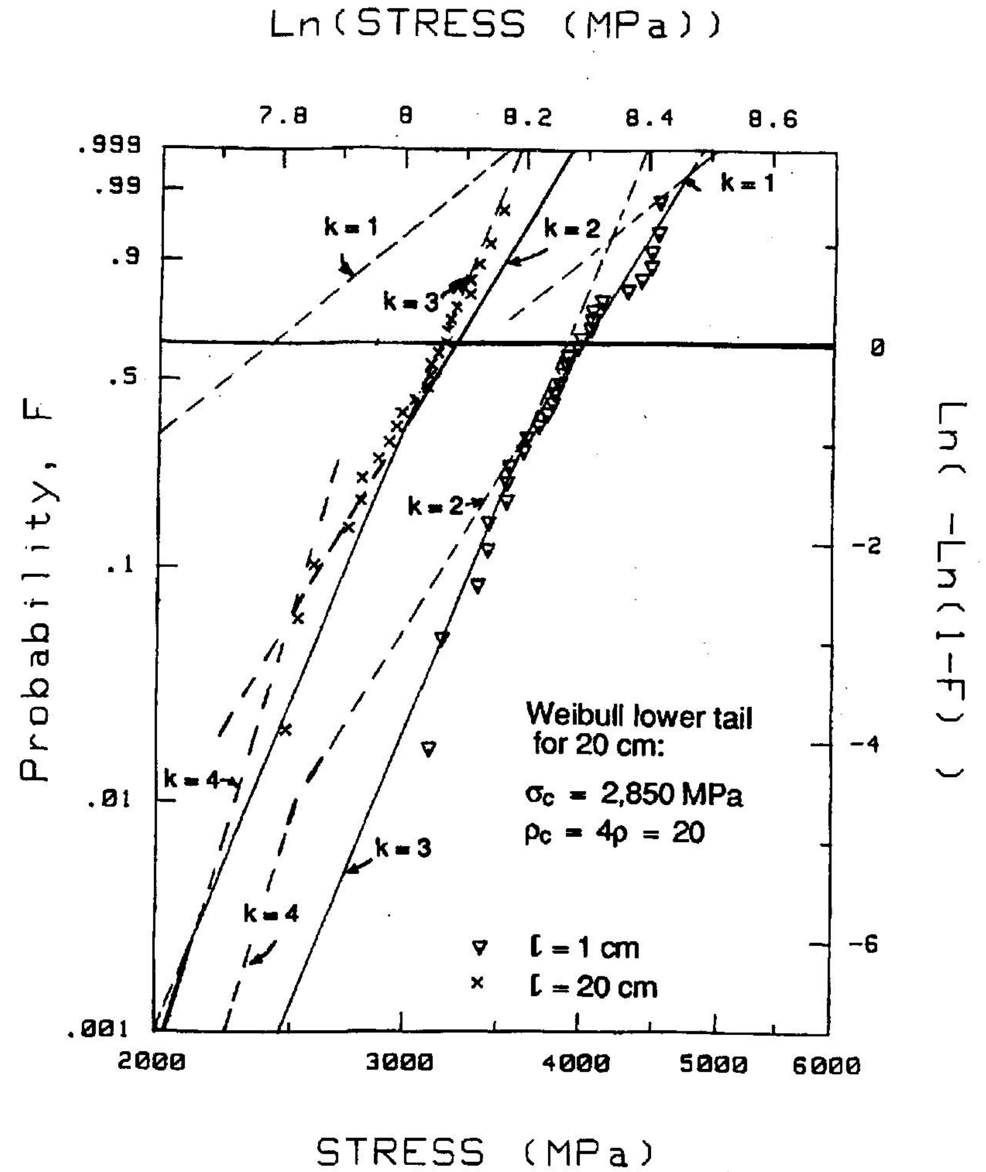

Fig. 3. Strength distributions for a composite 'cable' of four AS-4 graphite fibers in an epoxy matrix plotted on Weibull coordinates for two gage lengths (Ref [14])

3. THE PROMISE OF BUNDLE AND CHAIN-OF-BUNDLE MODELS FOR CABLES FATIGUE SETTINGS

Apart from the attempts described earlier [2-7] there are various bundle models and chain-of-bundles models ready to be adapted to steel wire bundles and cables for purposes of reliability prediction in fatigue. Phoenix [16] describes a bundle model of great flexibility, though it has seen little application thus far. One example of importance in the case of glass fibers is due to Kelly and McCartney [16]. This model should be adaptable to the wire fatigue model described in the introductory lecture of Castillo and Fernández-Cantelli. Other versions are also discussed by Phoenix [17] and [18]. Single fiber models are used to interpret experimental data in Wu et al. [19]. Smith and Phoenix [20] and Pitt and Phoenix [21] give various static and time dependent cases of the model applicable to cable systems.

As pointed out by Castillo and Fernández-Canteli in their introductory lecture, these models often lead to various limiting or approximating distributions for fatigue strength and lifetime, including Weibull, Gaussian and Gumbel distributions. In some important cases, however, other distributions arise [11] with far more power to model the extreme lower tails of interest in high reliability requirements. This power may be diluted by attempting to generate a classical extreme value form, especially when such is unnecessary.

Such models can help us identify not only what is theoretically possible but also what may actually be unimportant to our goal of reliable and efficient cables. Models can also help us identify strategies for *structural and materials design and engineering* in order to focus on innovative solutions to the key problems that are identified. In the process of failure in a typical laboratory fatigue test such models may put into perspective the value of data on wire failures along the way. It is not clear that current practises, particularly as they pertain to acceptance/reject standards, bear much connection to the performance of the extreme lower tails of distributions. This perhaps was the most important point raised in the analysis of Stallings [5]. It is clearly a point in our 4-fiber cable example.

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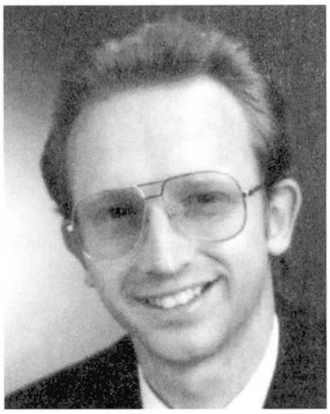
Fatigue of Reinforcing Steel in Consideration of the Length Influence

Essais de fatigue sur les aciers d'armature passive, tenant compte de l'influence de la longueur

Ermüdungsdaten von Armierungsstahl unter Berücksichtigung des Längeneinflußes

Roland KOLLER
Mech. Eng.
EMPA
Dübendorf, Switzerland



Roland Koller, born 1957, graduated in Mech. Eng. at the Fed. Inst. of Technology Zurich, Switzerland. For six years he was involved in fatigue and design analysis of lightweight structures. Now, he is responsible at the EMPA for problems concerning service loading.

SUMMARY
A continually occurring problem of engineers when designing components of structures is the availability of proper design values. In the literature, test results are documented for many different materials, but in rare cases only can values concerning the fracture or survival probability be found. In this context a statistical model is presented which enables the engineer to analyse test results and extract the design values with required fracture probability from a Wöhler-field. This model is applied to test data for reinforcing steel of different length and the results of the analysis are discussed.

RÉSUMÉ
La disponibilité des valeurs caractéristiques des matériaux pose un problème perpétuel à l'ingénieur pour le dimensionnement des éléments de construction. On trouve bien dans la littérature des résultats d'essai pour les matériaux les plus divers, toutefois les valeurs de probabiljté de rupture ou de survie ne sont données que dans de rares cas. Dans ce contexte, le modèle statistique présenté permet à l'ingénieur d'analyser les résultats d'essai et de tirer les valeurs de résistance nécessaires avec la probabilité de rupture demandée du diagramme de Wöhler. Ce modèle a été appliqué aux résultats d'essais effectués sur des aciers d´armature de longueurs diverses et les résultats des calculs sont discutés.

ZUSAMMEMFASSUNG
Eine immer wiederkehrende Problematik des Ingenieurs bei der Auslegung von Strukturbauteilen stellt die Verfügbarkeit der benötigten Materialkennwerte dar. In der Literatur sind wohl für die unterschiedlichsten Materialien Versuchsresultate dokumentiert, jedoch in den seltensten Fällen finden sich Angaben über die Bruch- oder Überlebenswahrscheinlichkeiten. In diesem Zusammenhang wird ein statistisches Modell vorgestellt, das dem Ingenieur erlaubt, Versuchsergebnisse zu analysieren und die benötigten Festigkeitswerte mit der geforderten Bruchwahrscheinlichkeit dem Wöhlerfeld zu entnehmen. Dieses Modell wird auf Versuchsresultante von Armierungsstahl unterschiedlicher Länge angewendet und die Berechnungsergebnisse werden diskutiert.

1. INTRODUCTION

Since mechanical structures were used more and more to move people quickly between different locations the problem of fatigue (metallic materials under repeated loading tend to fail after a certain number of cycles) became obvious. Especially when human beings lost their lives during accidents caused by fatigue, discussions started again about structural safety or probability of failure.
It is not new that for materials exist no accurate correlation between load level and cycles to failure. Whenever material is tested under dynamic loads, results (cycles to failure) always show an amount of scatter.
Depending on the number of tests a failure probability can then be related to each test result by means of statistic calculation.
If these results shall be used for a fatigue analysis it is desirable to choose a value which corresponds to a low probability of failure or a high probability of survival respectively.
Therefore engineers need more detailed information on the material with respect to failure probability.

2. CURRENT DESIGN SITUATION

Usually when engineers are developing structural components finite element analyses are involved in a very early stage to get an idea of the stress distribution in order to be able to optimise the design. Not only for static but as well for dynamic (fatigue or fracture) analyses material data is very important. A lot of data was collected in literature but for an actual design task in most cases the needed data can't be found because metal alloy doesn't match exactly, the heat treatment is different, or testing was performed for an other kind of loading etc. If material data can't be found in literature a company has to perform tests by itself or give an order to an external laboratory. These fatigue tests are expensive and so it's important to get a maximum of information out of a few tests.
Nowadays material data derived from laboratory are applied often for strength analysis without taking into consideration shape or size effects (Fig. 1). In other words, an extrapolation of material data from laboratory conditions to reality is rarely carried out.
The size (length) of the specimen used in the laboratory is normally not comparable with a real structure. This effect may have considerable influence on fatigue life predictions. When testing e.g. longer specimen the probability of the occurrence of larger cracks increases and therefore the fatigue lives decrease [2].

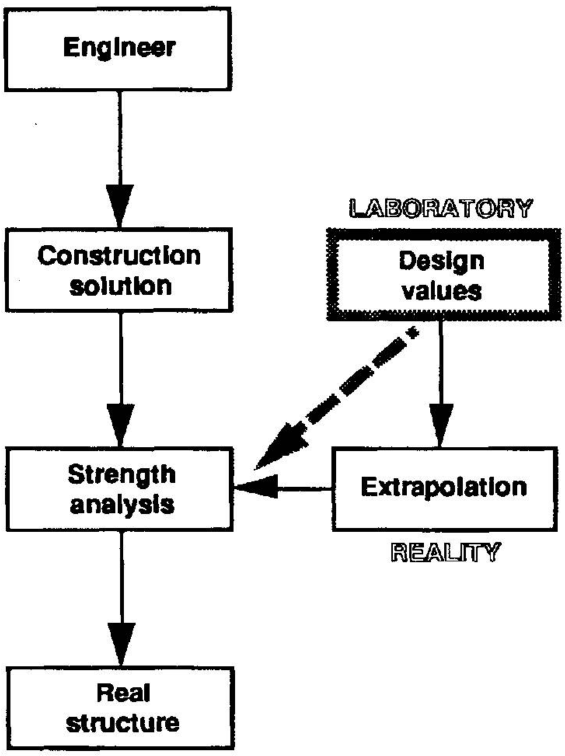


Fig. 1 Everlasting problem of engineers

3. OBTAINING MATERIAL PROPERTIES

3.1. Performing Wöhler-testing

Material data describing fatigue life are determined normally by carrying out Wöhler-tests (Fig. 2). The size influence mentioned above is associated with the specimen geometry used for testing. This aspect will be discussed in the next chapter.
In order to obtain a proper shape of the Wöhler-curve the definition of the number of load levels on the one hand and the amount of the loads on the other hand are very important. It should be emphasised that the range of load levels should cover the stresses the engineer is interested in. Extrapolations of Wöhler-curves to values beyond the tested range should be avoided, especially in the range of high cycle fatigue, estimations of curve trends may lead to serious errors.
Beside the fracture probability related to a number of cycles to failure there is an other statistical value which defines the certainty for the fracture probability to be true. This statistical value is called the confidence level. Statements for material behaviour are usually expected to have a high confidence level (e.g. 95%). This can be obtained by performing numerous tests per load level. Of course many tests cost a lot of time and money. So again a compromise has to be made to get a satisfactory confidence level and to keep the costs low at the same time.
The relationship between testing time (test frequency) and test costs is quite obvious. What is not well known, is the aspect that the test frequency also may have an influence on fatigue life i.e. on the number of cycles to failure. What kind of effects become active when increasing the frequency has not yet been clarified but test results at EMPA for prestressing wire showed longer lives for tests with a higher frequency [3].

To minimise time and cost for testing, a final aspect has to be taken into consideration. When testing in the region of the endurance limit (if there exists one), it is favourable to define a limit number of cycles beyond that a test may be aborted. This value is called runout limit and specimen which have run through that limit are treated as "real" runouts. The difficulty of course is to define the value for that boundary because a specimen might fail a few cycles after that limit and doesn't therefore represent a runout. As discussed in chapter 6.2 the choice of this value may influence the evaluation of fatigue data.
So it seems to be reasonable to set the limit beyond the region of cycles the engineer is interested in.

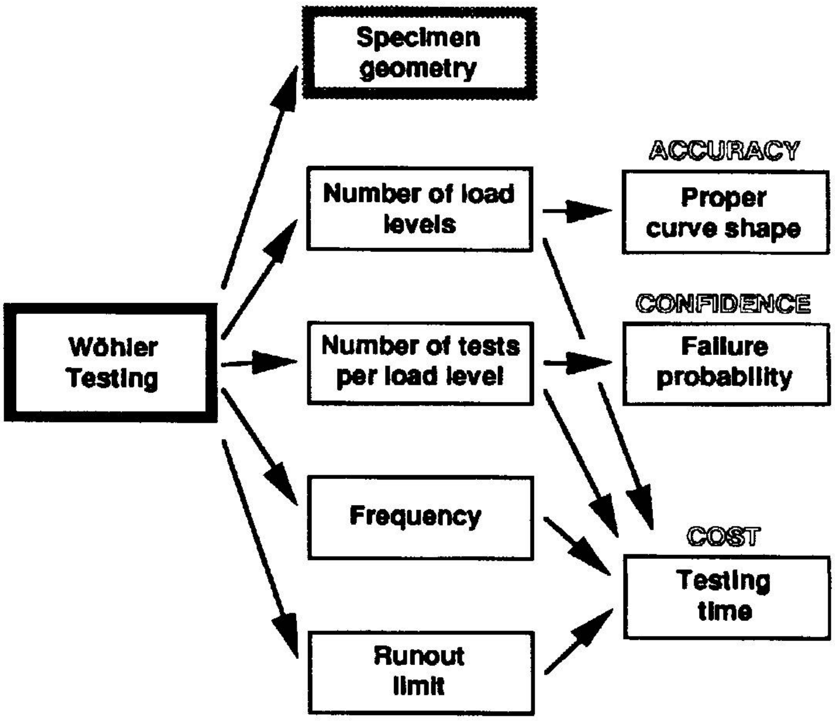


Fig. 2 Load controlled Wöhler-testing

3.2. Size effect

In order to perform Wöhler-testing a convenient specimen should be designed. In this context the geometry of the specimen need to fulfil some limitations which are shown in Fig. 3 below.
First of all the technical specifications of the testing machine in the laboratory (i.e. the proof length and the load capacity) usually force the designer of the specimen (depending on the problem) to deviate from reality (the specimen cannot be that long or wide as the real structure because it won't fit into the machine or the loads become too large).
As a second aspect the available facilities are often not able to manufacture the specimen's geometry.
Finally there are international standards which should be taken into consideration because testing results may then be comparable to data measured elsewhere.

All these limitations (beside the cost for manufacturing complicated surfaces) may lead to specimen geometries which differ more or less from the real component of a structure. A detailed investigation on the size effects between specimen and real structures can be found in ref. [4].

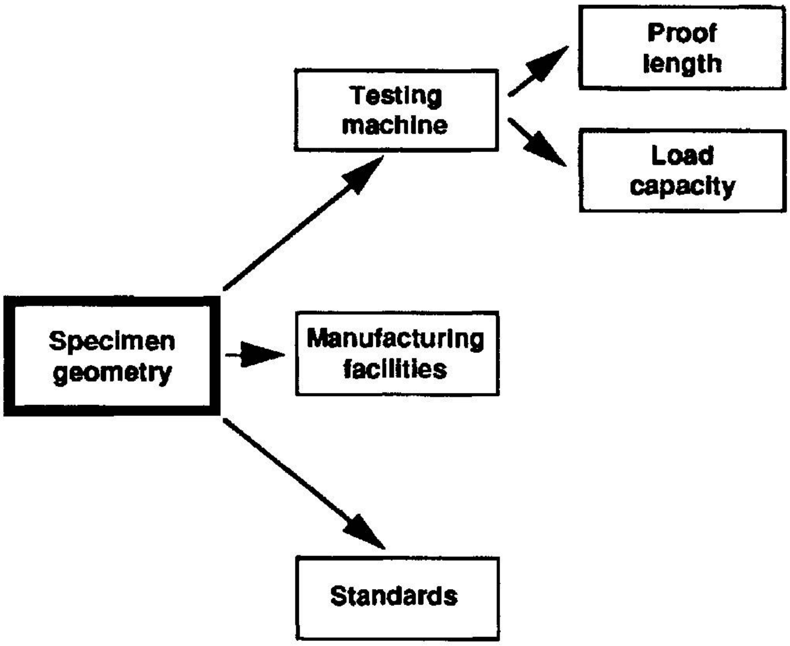


Fig. 3 Different aspects influencing the specimen's geometry

4. USAGE OF WÖHLER-DATA

Wöhler data coming out of laboratory represent in most cases a summary of fatigue lives (number of cycles to failure) derived from testing specimen. In order to get an idea of the data, an engineer needs to create a graphical representation as show in Fig. 4.

In practice there will scarcely be such a large number of data available. However, the required design values have to be extracted out of a chart like this to carry out fatigue analysis and this characterises the problem many engineers have to deal with.

Looking at the example in Fig. 4 some material dependent characteristics may be pointed out for this Wöhler-field:

- For this material, an endurance limit seems to exist.
- A non constant character of scatter can be recognised for different load levels.
- The median curve (50% of failure probability) is non linear.

Still, two important informations are missed in Fig. 4:

- How do the curves for different failure probability P_B (P_B=5%, 10%, 50%, 90%, 95%) look like ?
- What is the influence of the length ?

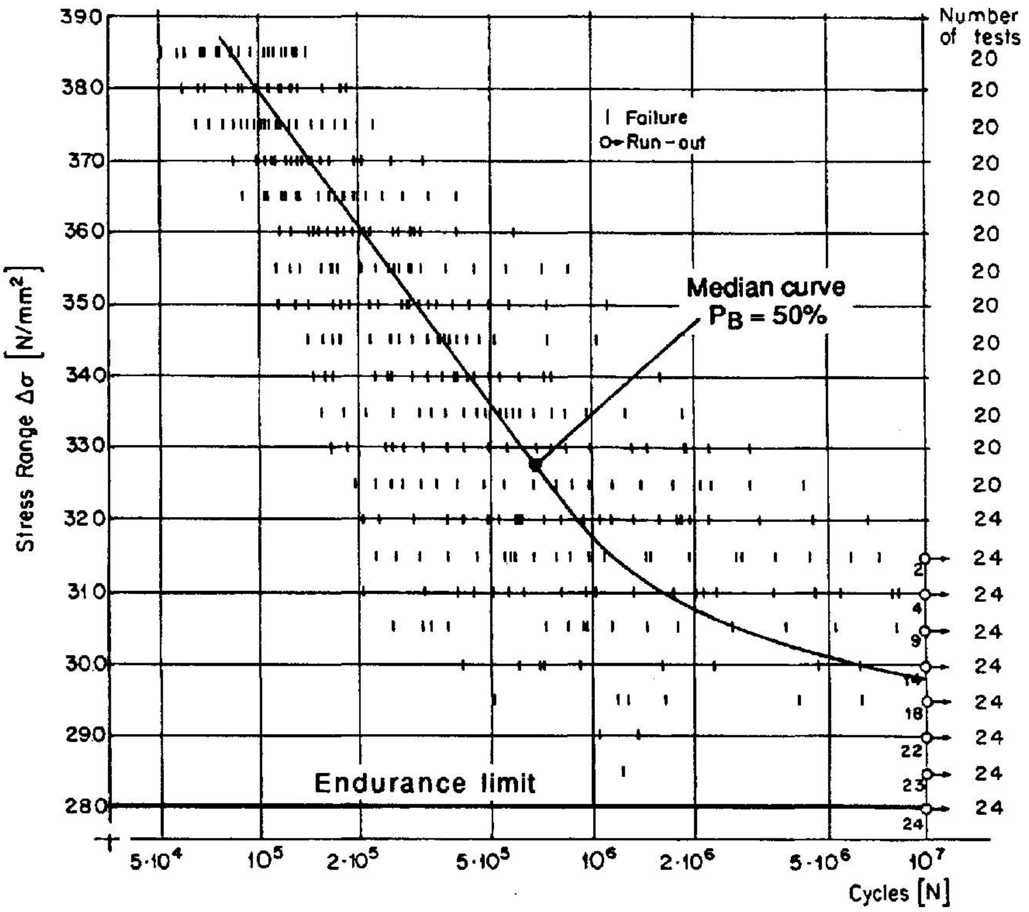


Fig. 4 Good experimental background

Just for illustration, some fatigue data of reinforcing steel measured at EMPA are presented in Fig. 5 to demonstrate the existence of a length influence:

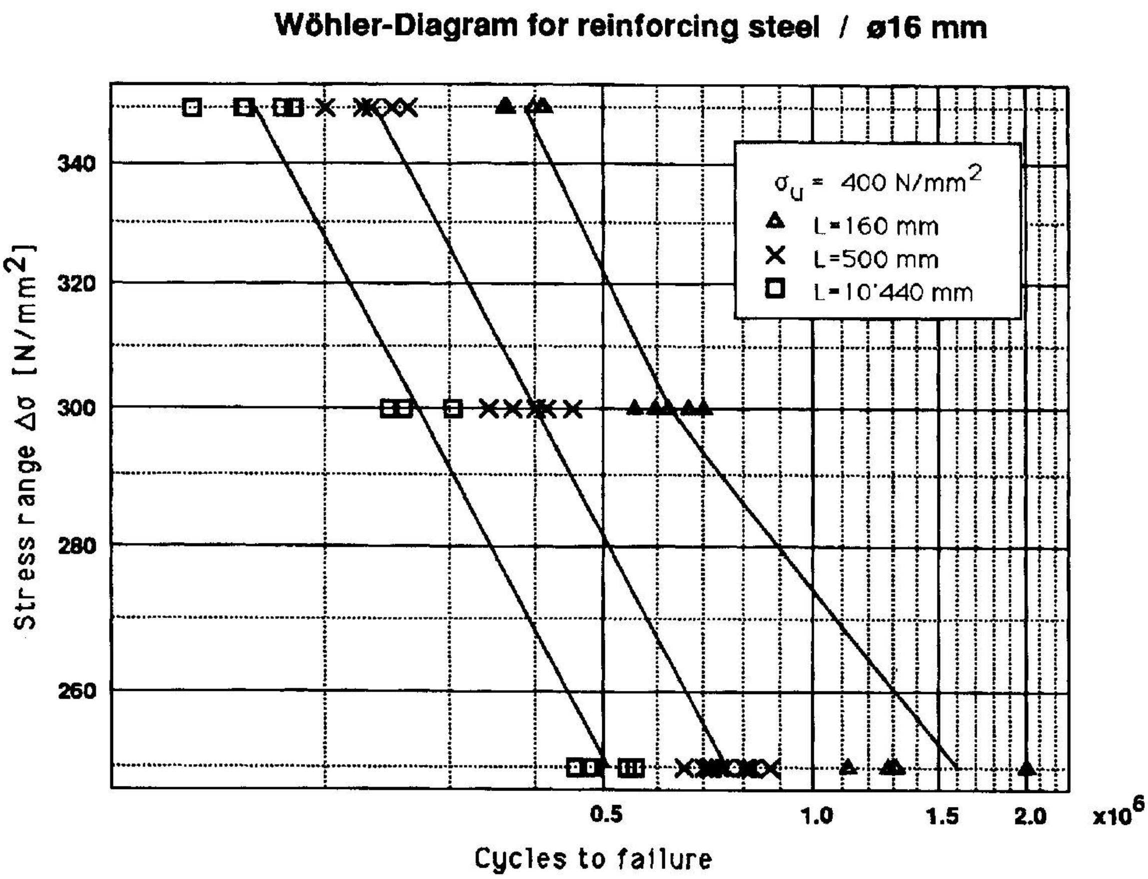


Fig. 5 Length influence for reinforcing steel

The material characteristics and the lack of information concerning failure probability mentioned above can be handled by a statistical model which is presented in the following chapters.

5. STATISTICAL MODEL FOR EVALUATING WÖHLER DATA

5.1. Development of the programme

A mathematical model for analysing data of wires, strands and cables, based on statistical requirements (compatibility, stability and limit conditions) was developed by E. Castillo and co-workers at the **University of Santander**, Spain. The methods, procedures and the theoretical background of the model was published in a IABSE Periodica already 1985 [1]. A computer programme of this model was first implemented for Macintosh but later rewritten for DOS-environments by F. M. Rodrígez at the Technical University of Gijon in 1989. This DOS-programme was sent to EMPA in 1990. For practical reasons this release was adapted and modified at EMPA. Thus, this new version will be discussed in the following chapters.

5.2. Programme modifications

The statistical programme "ZURICH" received from the University of Gijon was written in Turbo Pascal 5.0 and for dialogues, error messages and help texts the Spanish language was used.
In order to understand all the modules and auxiliary files of the programme a translation into the German language was carried out. Additionally an English version was written for more general use. During translation the programme's name was changed to ***"FANOW"*** (**F**atigue **AN**alysis **O**f **W**ires).
Applying the programme to practical fatigue data some of the input routines have been improved by adding default statements or keeping values already entered before. It became also obvious that some input procedures are not needed if the programme is used for more than one data set evaluation. Therefore the general programme's execution was simplified in a way that users may skip input routines for values which shouldn't be altered. In the meantime Turbo Pascal 6.0 was released, so the programme has been adapted to this newest Pascal version in spring 1992.
When trying to output diagrams on a printer the corresponding procedures showed not to work correctly for the used EPSON LQ-500 printer. Modifications within the initialisation and formatting commands solved these problems. In addition to that output of diagrams are also foreseen for pen plotters, but the procedures used to perform this task have not yet been tested.

5.3. Statistical model

This chapter is intended to give just a general idea of the programme's evaluation methods. For detailed theoretical information the reader is referred to ref. [1,5].

Initial for the programme to work correctly, is the availability of numbers of cycles to failure for at least **three** different load levels. Otherwise the programme will crash.

One of the fundamental theoretical findings is the fact that data in the Wöhler-field follows the Weibull distribution. In a first step the measured data are used to calculate the parameters (constants) of the Weibull-distribution to define its shape, one of the most important value. Beside this, also an endurance limit as well as an asymptotic limit of numbers of cycles to failure results from this calculation. With the known Weibull-distribution the model allows to determine a set of different failure probability curves (hyperbolas, see below), which may give the engineer the required and important design values as shown in Fig. 6.

$$(N-B)\,(\Delta\sigma - C) = D\left[\left\{-\frac{L_0}{L}\log(1-P)\right\}^{\frac{1}{A}} - E\right]$$

For P=¢ and L=¢ ⇒ Hyperbolas

A: Weibull shape or slope parameter
B: Asymptotic N limit
C: Endurance limit
D: Scale fitting parameter obtained for chosen reference length L_0
E: Constant defining S-N threshold curve (zero probability of failure)

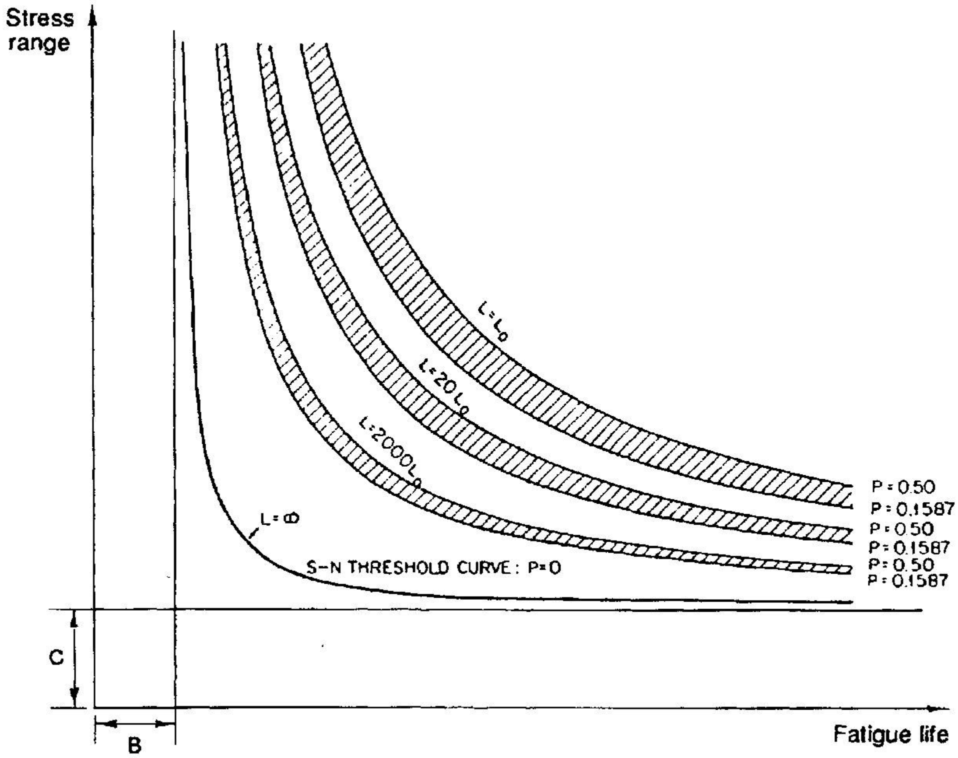


Fig. 6 Statistical model

As already mentioned in chapter 3.1 testing cost may be saved by defining a low runout limit. Information of runouts are incomplete and contain an uncertainty concerning the aspect if the specimen would fail or not. The programme FANOW assumes that the specimen will fail some time later and tries to predict the number of cycles this failure will happen.

The user just has to specify within the data set the number of cycles to failure which represents this runout limit. Using the Weibull-distribution, determined before, the programme extrapolates these runouts by calculating an estimation of a number of cycles to failure for each runout and associates these numbers to the corresponding runouts (Fig. 7).

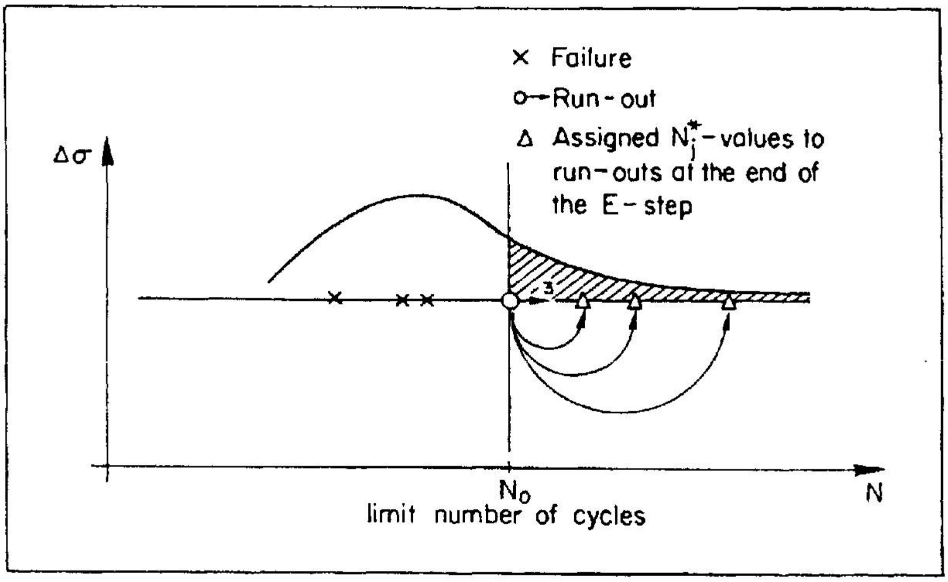


Fig. 7 Considering runouts

6. PRACTICAL EXAMPLE

6.1. Length influence on fatigue data of reinforcing steel

Bars of reinforcing steel (Ø=16mm) have been tested at EMPA for three different length (L=160mm, L=500mm, L=10440mm) at up to five stress levels ($\Delta\sigma$=200N/mm^2, $\Delta\sigma$=220N/mm^2, $\Delta\sigma$=250N/mm^2, $\Delta\sigma$=300N/mm^2, $\Delta\sigma$=350N/mm^2). An upper stress limit was kept constant at σ_u=400N/mm^2. Three test frequencies (f=2.5Hz, f=3.5Hz, f=10Hz) have been used for testing. The influence of the different frequencies was considered as negligible (the differences are small).
In order to evaluate the measured fatigue lives, three different data sets (one for each length) have been created for the programme "FANOW". The results of the analyses are plotted in Fig. 8 to 10.

The measured numbers of cycles to failure (test data) are represented in the Wöhler-field by unfilled squares, whereas the filled squares mark the extrapolated runouts. Accordingly, the vertical dotted line shows the chosen runout limit of $2 \cdot 10^6$ cycles [6].

As main result of the programme's evaluation a field of failure probability curves (hyperbolas for P_B=1%, P_B=5%, P_B=50%, P_B=95% and P_B=99%) can be found on these plots.

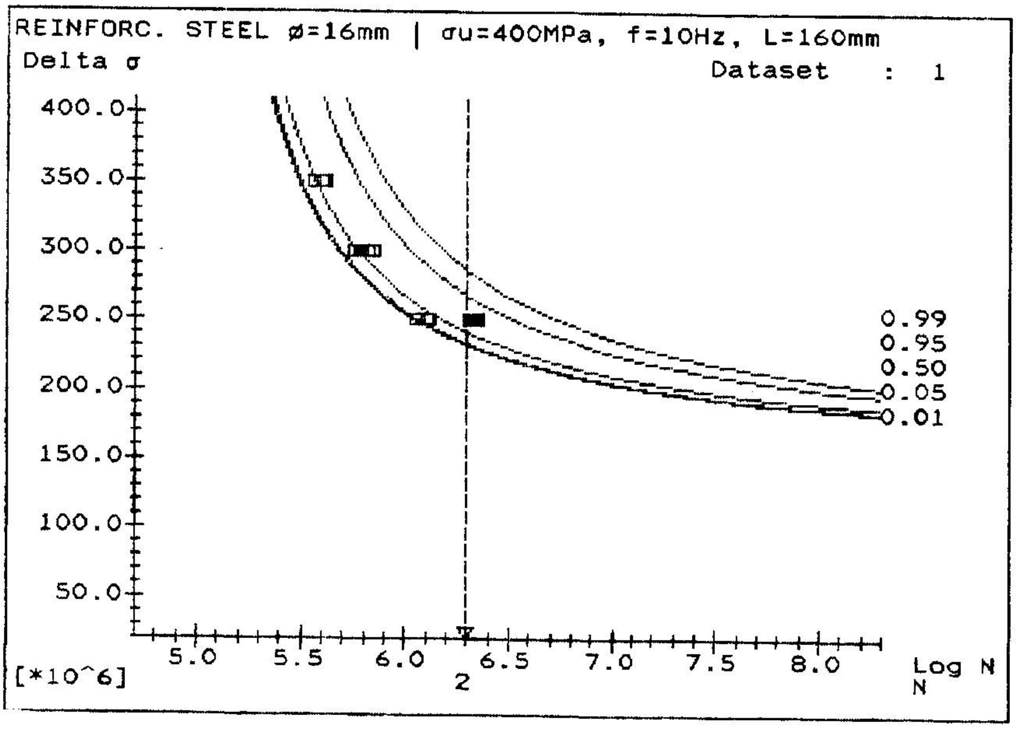


Fig. 8 Evaluation of reinforcing steel, L = 160 mm

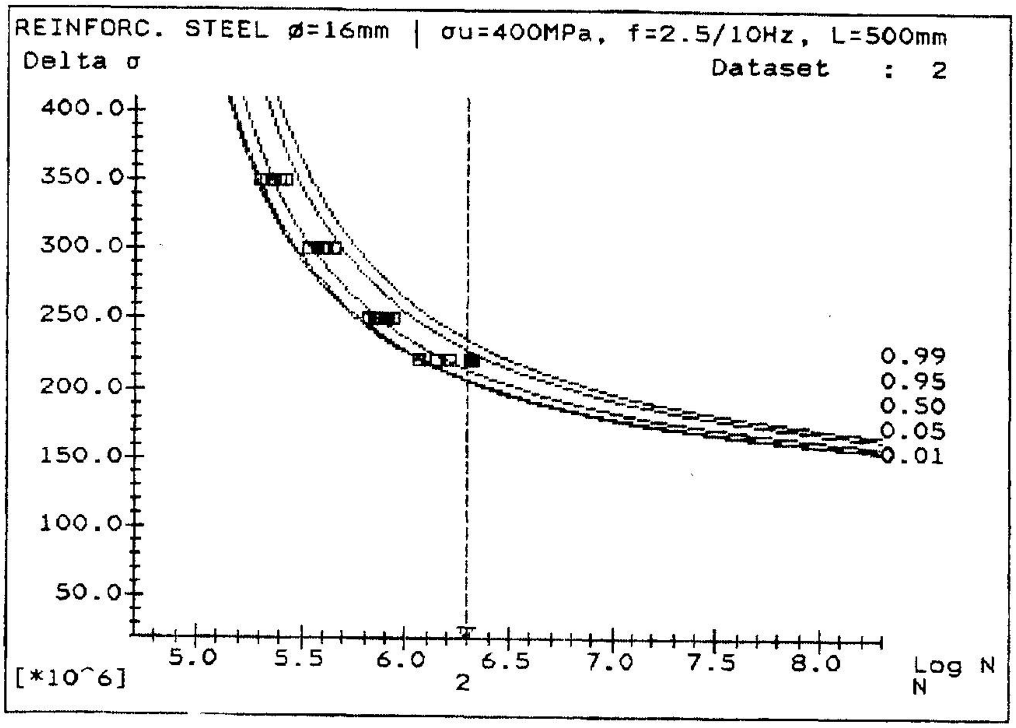


Fig. 9 Evaluation of reinforcing steel, L = 500 mm

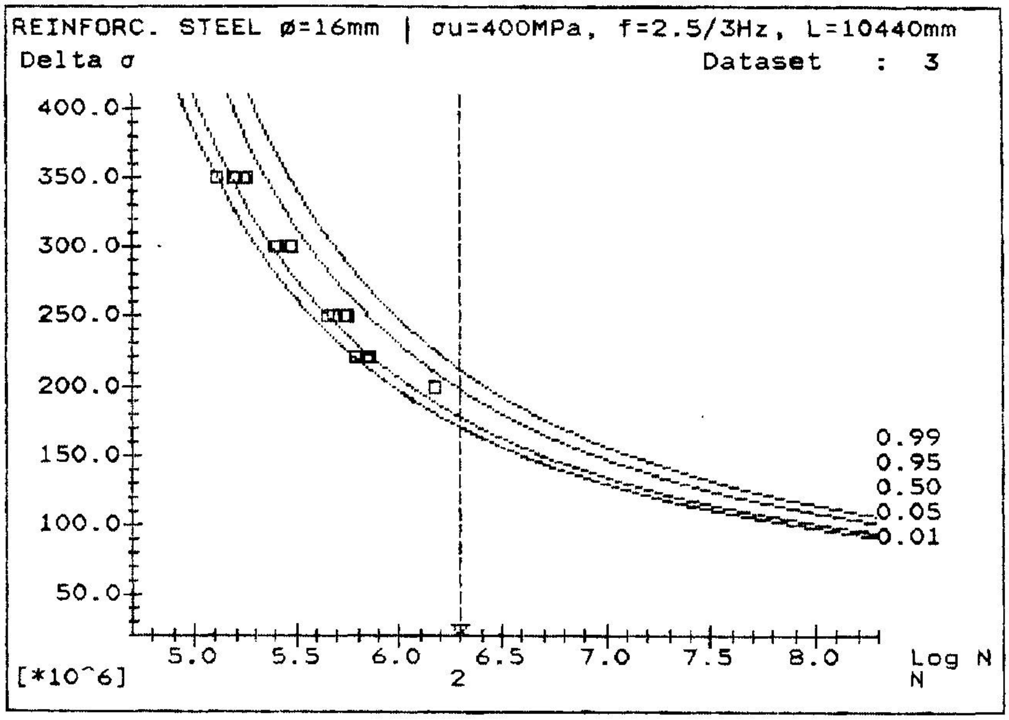


Fig. 10 Evaluation of reinforcing steel, L = 10'440 mm

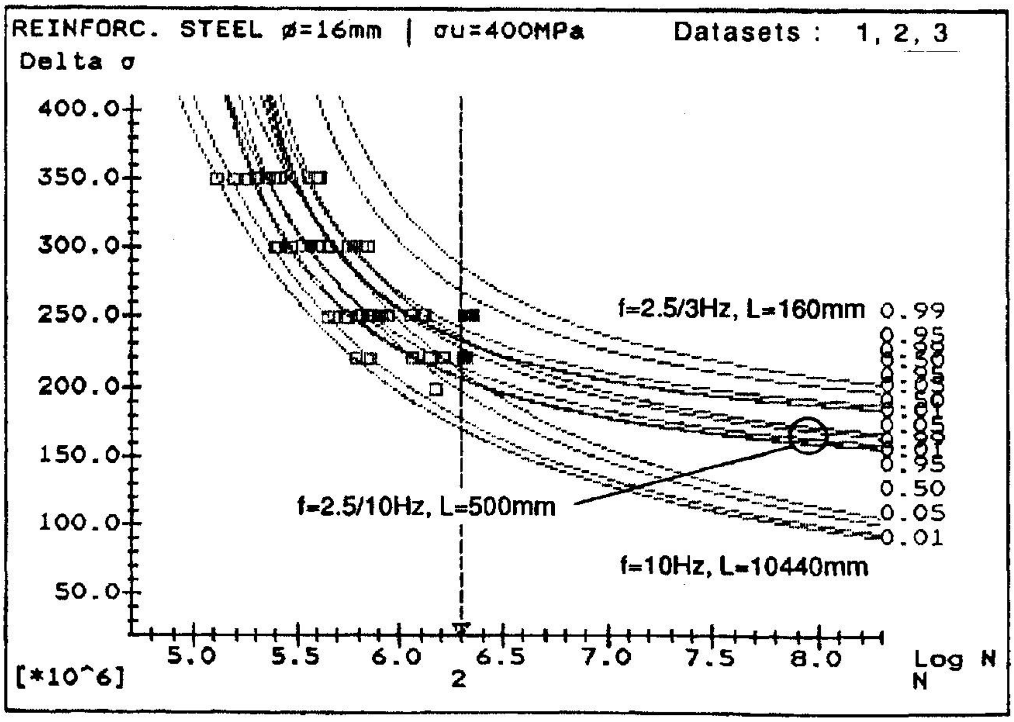


Fig. 11 Length influence for reinforcing steel (L=160mm, 500mm, 10'440mm)

7. CONCLUSIONS

Based on the experience made during evaluation of fatigue data for reinforcing steel this statistical model turned out to be a valuable tool for processing and presentation of Wöhler-test-data. However, it should kept in mind that the model is still in development. As pointed out in the former chapter the user should avoid trying to extrapolate data beyond the range where Wöhler-testing was carried out due to the sensibility of the model to the number of load levels as well as to the choice of the runout limit.
During development of the statistical model an assumption (simplification) was made to derive a solution from a functional equation. This assumption consisted of setting the Weibull-slope-parameter constant for the whole range of loading. One of the future tasks is to discuss in detail the influence of this simplification.
An other open question is the circumstance, that there seems to be no or little influence of the number of tests per load level on the estimated probability curves. In order to demonstrate the difference of just a few test results per load level compared with many tests on the same level a confidence interval for each failure probability curve should be included into the model. The more test results the engineer has for evaluation the narrower the confidence interval for one failure probability curve will be. This problem is worth to pay attention to in future work.
The programme "FANOW" as well is still in development. By the moment evaluations can be carried out just for the actual test length. Future improvements of the code are planned with the object of being able to alter the length and predicting the new Wöhler-field based on the measured data set.

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Length Effect on Fatigue of Wires and Prestressing Steels

Influence de la longueur sur la fatigue des fils et des aciers de précontrainte

Einfluß der Länge auf die Ermüdung von Drähten und Spannstählen

Alfonso FERNANDEZ CANTELI
Prof. Dr.
University of Oviedo
Gijón, Spain

Enrique CASTILLO
Prof. Dr.
University of Cantabria
Santander, Spain

Antonio ARGÜELLES
Mech. Eng.
University of Oviedo
Gijón, Spain

Alfonso Fernández Canteli, born 1945, graduated from the University of Basque Country and obtained his Ph. D. from the Polytechnical University of Madrid.

Enrique Castillo, born 1944, graduated from the Polytechnical University of Madrid in 1969. Awarded a Ph. D. in Civil Engineering from Northwestern University in 1972 and in Mathematics from Complutensis University of Madrid in 1974.

Antonio Argüelles, born 1959, obtained his Mech. Eng. degree at the Open University of Madrid.

SUMMARY
A statistical approach, based on the independence assumption, is proposed here for the consideration of the length effect on the fatigue resistance. As a limiting condition for its applicability the test length must experimentally demonstrate to be longer than a certain threshold length. The validity of the model is currently being tested in two experimental programs on wires and prestressing steels. In spite of the promising results obtained, more research is needed for its general application in design.

RÉSUMÉ
On propose ici un modèle statistique, fondé sur l'hypothèse d'indépendance, pour l'étude de l'effet de la longueur sur la résistance à fatigue. La condition d'applicabilité de cette approche implique de vérifier expérimentalement que la longueur d'essai dépasse celle d'une valeur de seuil. La validité du modèle est actuellement à l'étude dans deux programmes expérimentaux sur des fils at des aciers de précontrainte. En dépit des résultats prometteurs obtenus jusqu'ici, il est nécessaire de continuer la recherche si l'on envisage une application ultérieure dans la pratique du dimensionnnement.

ZUSAMMENFASSUNG
Auf der Grundlage der Unabhängigkeitsvoraussetzung wird ein statistisches Modell zur Berücksichtigung des Längeneinflußes auf die Ermüdungsfestigkeit vorgeschlagen. Als Vorbedingung für dessen Anwendbarkeit muß experimentell nachgewiesen werden, daß die Testlänge einen bestimmten Schwellenwert übersteigt. Die Gültigkeit des Modells wird derzeit in zwei experimentellen Programmen an Drähten und Spannstählen überprüft. Trotz der vielversprechenden, bisher erhaltenen Ergebnisse ist weitere Forschung bis zum Einsatz in der Bemessungspraxis nötig.

1. INTRODUCTION

It is generally accepted that the fatigue life of longitudinal elements is conditioned by the existence of flaws along its surface, derived from the manufacturing process, handling and storage. If the random distribution of the flaws shows no correlation along the element, the fatigue resistance of neighbouring pieces can be assumed to be independent and, as a consequence, the fatigue analysis can be based on the hypothesis of independency.

On the contrary, if such a correlation exists, the dependency effect must not be ignored and a suitable model which takes into account dependency should be used.

2. THE PROBLEM OF EXTRAPOLATION

In order to assess of their fatigue properties, many mechanical and structural elements cannot be tested on a real scale because of the high costs involved or even due to physical impossiblity. This is the case of very long elements such us crane ropes, tendons in cable stay bridges or similar structures. Consequently, prediction of the fatigue resistance for long elements must follow on from extrapolation of test results usually obtained for short or very short specimens. This means that the fatigue life of one element of length s is the minimum fatigue life of s/r independent elements of length r in which the former is hypothetically divided.

Therefore, the survival function for the element of length s can be derived from the survival function of the element of length r (r > s) by means of the expression:

$$G_s(x) = \text{Prob}\,(X_s > x) = [\text{Prob}\,(X_r > x)]^{s/r} = [G_r(x)]^{s/r} \qquad (1)$$

X_s and X_r being the fatigue lives of the two elements.

However, several experimental studies [1, 5, 11] evidence the non-validity of the independence model for extrapolation of fatigue life gained from short specimens in order to obtain the fatigue life of larger elements. To the contrary, extrapolation based on relatively large elements seems to lead to good results [5]. This fact has been theoretically justified since unless strong dependence exists between the strengths of neighbouring pieces, the asymptotic behaviour is that of independence [2,10].

According to [6], the transition between the dependence and independence assumption is governed by a certain threshold value of the length, say s_0, beyond which extrapolation based on Eq. 1 is valid. This will be further discussed in the following section.

3. SUGGESTED MODEL AND ITS JUSTIFICATION

Without theoretical justification, Bogdanoff and Kozin [1] suggest the expression:

$$G_s(x) = [G_r(x)]^{k(s,r)} \qquad (2)$$

for general conversion of the survival function of one specimen of length r into that of another larger one,s, still within the dependence domain. The function k(s,r) is an unknown function to be determined experimentally (see Fig. 1). If independence holds, then k(s,r) = s/r and Eq. (2) becomes Eq. (1).

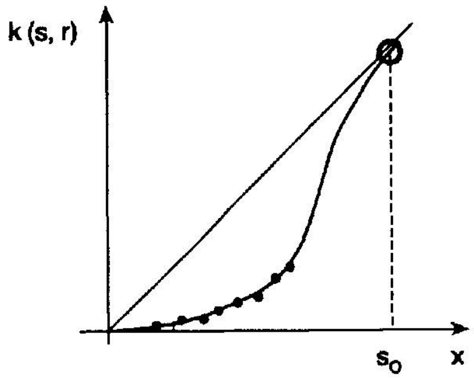

Fig. 1 Function k(s,r)

The validity of Eq. (2), at least within a certain range, has been confirmed by analyzing experimental fatigue results in viscose yarn of differing length [11].

Assuming the number of flaws in a longitudinal element to be a nonstationary Poisson's process of intensity $\lambda(s)$, Castillo et al.[6] developed a statistical model which takes into account the influence of the manufacturing process on the lifetime of the piece from which the survival function is found to be:

$$G_r(x) = |\exp(-F(x))|^{m(r)} \tag{3}$$

where m(r) measures the frequency of appearance of flaws and F(x) represents the lifetime of a single flaw, respectively. The uniqueness of a c.d.f., F(x), as representative of all possible flaws can be reasonably accepted according to the Fracture Mechanics approach.

Consequently, the survival function for a different , s, is governed by

$$G_s(x) = |G_r(x)|^{m(s)/m(r)} \tag{4}$$

By means of the functional equation theory [8]I it can be shown that m(s)/m(r)=k(s, r) and Eq. (4) is identical to Eq.(2).
The model includes the case of independence, for which m(x)=x, and also the case of asymptotic behaviour, in which $F_s(x)$ goes over into that of independence [12] and the following equation holds:

$$m(n\Delta s) / m(\Delta s) = \infty \tag{5}$$

where Δs is the length of a piece; so that

$$G_{n\Delta s}(x) = |G_{\Delta s}(x)|^n \tag{6}$$

The model comprises still the B-model of Bogdanoff-Kozin, the statistical inconsistencies of which have been pointed out in [3].Thus for large n and

$$G_{n\Delta s}(x) = |G_{\Delta s}(x)|^{k} \qquad (8)$$

this means that after some given length the size effect vanishes (the so called saturation effect). However, neither statistical justification for saturation exists, nor experimental evidence for it has been glimpsed in the two test programs reported in the following section.

Because of the asymptotical confluence of k (s,r) into k=s/r, the determination of the threshold value s_0, mentioned in the above section, has only academic significance in research. The extrapolation of the results using equation (2) are hence possible as soon as it can be experimentally proved that k corresponds to the independence domain.

Now a general fatigue model such as the Weibull regression model suggested in [5,7], can be used in order to describe the Wöhler-field as a whole, thus allowing the extrapolation for designing both length and number of cycles to failure (estimation of the endurance limit).

4. EXPERIMENTAL PROGRAM

Two experimental programs have been launched in order to validate the possible use of the independence model for extrapolation by testing long enough elements:

4.1 Prestressing steels

This program has focused on the study of fatigue properties of prestressing steel (wires and stands) with relation to length and started out as a collaboration between the ETH-Zürich, the EMPA Dübendorf, and the Spanish Universities of Cantabria and Oviedo.

The aims of the study are:

- To corroborate the usefulness of the independence model for extrapolation of fatigue test results for both prestressing wire and 7-wire prestressing strands, verifying at the same time that the chosen minimal length hassurpassed the hypothetical threshold length.

- To ascertain that the study of dependence-independence is unrelated to the stress range since the fatigue behaviour is assumed to be conditioned solely by the initial flaw distribution in the element. This would lead to the possibility of recommending a high stress range for testing in order to reduce the test duration.

- To study the influence of the test frequency in order to confirm or reject thepossibility of extrapolating fatigue results obtained from tests at high frequency with very short specimens.

The tests were been carried out at the EMPA Dübendorf (Swiss Federal Laboratories for Material Testing and Research) .Results are given in Tables 1, 2 and 3. A detailed description of testing and devices is given in [9].

Specimen number	Number of cycles to failure (in thousand)	Specimen number	Number of cycles to failure (in thousand)
1	64	1	92
2	70	2	93
3	84	3	109
4	99	4	116
5	105	5	125
6	110	6	129
7	117	7	132
8	133	8	134
9	151	9	135
10	163	10	135
11	199	11	138
12	201	12	188
13	268		
f = 62 Hz		f = 3,5 Hz	

Table 1 Fatigue test results for the study of the influence of the frequency in wires

Specimen number	Number of cycles to failure (in thousand)	Specimen number	Number of cycles to failure (in thousand)	Specimen number	Number of cycles to failure (in thousand)
1	205	1	206	1	143
2	225	2	206	2	179
3	256	3	237	3	200
4	266	4	257	4	209
5	268	5	260	5	210
6	279	6	272	6	216
7	283	7	276	7	218
8	293	8	307	8	222
9	300	9	318	9	223
10	521	10	331	10	225
11	1709	11	365	11	225
12	1762	12	393	12	236
13	2303	13	403	13	279
14	4230	14	413	14	303
		15	824	15	315
				16	325
l=1,04m		l=2,03m		l=10,40m	

Table 2 Fatigue test results for the study of the influence of the length in prestressing strands

Specimen number	Number of cycles to failure (in thousand)	Specimen number	Number of cycles to failure (in thousand)
1	49	1	153
2	53	2	162
3	56	3	165
4	59	4	190
5	60	5	229
6	60	6	235
7	60	7	237
8	60	8	280
9	60	9	297
10	61	10	804
11	62	11	866
12	63	12	1530
		13	1964
		14	2000 runout
		15	2000 runout
		16	2000 runout
$\Delta\sigma$=700 N/mm2		$\Delta\sigma$=400 N/mm2	

Table 3 **Fatigue test results for the study of the influence of the stress range in wires**

4.2 Wires

The initial program on prestressing steels has been extended to another one devoted to the study of length effect on 0.5mm diameter hipo-eutectoid steel wire.

The goals of this experimental research are:

- To develop a testing device for analysising the influence of length in fatigue tests with the purpose of minimizing time and costs.

- To analyze and compare fatigue results for length/ diameter ratios comparable to the ones in real structures. This would apport sufficient data for testing the saturation assumption.

- To explore at the same time the trend of the k(x)-values for large lengths, probably much larger than the critical length, beyond which no dependence effect would be expected.

- To ascertain if the dependence study can be limited to a single level, the conclusions from it being extensive to the whole Wöhler-field

Economical and technical considerations lead to the choice of an electro- mechanical testing device. The authors intention being to test several specimens simultaneously with the possibility of varying the test length within a large range (say up to 20 m). Since such a non-conventional machine is not commercially available it was designed and built at the Dept. of Construction of the University Oviedo (see Fig. 2).

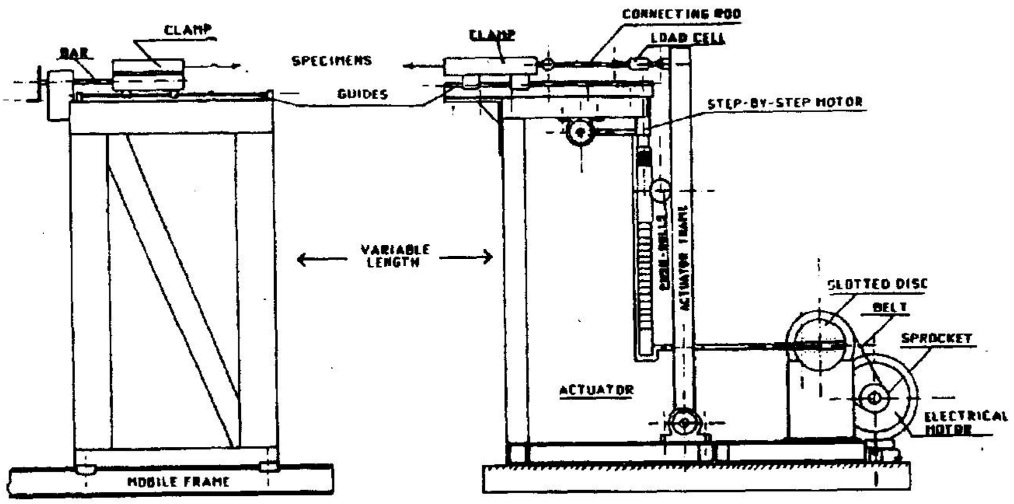

Fig. 2 Testing device for wires

The way-controlled machine permits simultaneous testing of five specimens in horizontal position, moved by a step-by-step electrical motor, that allows a steady variation of the frequency (in the present program frequency was fixed at 2 Hz in order to avoid resonance effects for the thin wire). Depending on the Young modulus of the material, specimens of up to 20 m may be tested.

The relatively small forces required to strecht the small diameter wires permit the fixation of the actuator and the end rig frames directly to the floor of the laboratory.

The stress in the wires is measured individually for each wire by means of extensometric techniques; whilst the reading of the total applied load is made for the set of the five specimens as a whole. The generated load wave can be displayed during the test.

Some of the difficulties found using this kind of wire are:

- The stress-strain curve of this material exhibits a remarkable linearity, practically without any plastic deformation. As a consequence, the actual applied maximum stress level and the strength range, being constant for all the tests, imply, in fact, relative differences in the test conditions for the single wires and propitiate greater scatter for the fatigue results.

- The relative tolerances in the diameter of the tested wire may be not comparable to the ones in the prestressing steels, which incidentally are the main subject of the workshop.

The results for the fatigue life of 2,00 m. and 10.00 long wires are presented in Table 4.

Specimen number	Number of cycles to failure (in thousand)	Specimen number	Number of cycles to failure (in thousand)	Specimen number	Number of cycles to failure (in thousand)	Specimen number	Number of cycles to failure (in thousand)
1	55440	13	222362	1	20250	11	95155
2	80458	14	250155	2	39154	12	101164
3	127127	15	253134	3	61197	13	105107
4	129024	16	271344	4	67868	14	111030
5	129906	17	276612	5	70470	15	121638
6	138600	18	282822	6	76340	16	125086
7	173754	19	315849	7	79206	17	129046
8	190246	20	324955	8	84492	18	134255
9	196329	21	344669	9	94950	19	137057
10	208026	22	351282	10	95030	20	144348
11	221480	23	351282				
12	221495						
l=2.00m				l=10.00m			

Table 4

5. ANALYSIS OF RESULTS

5.1 Prestressing steels

The comparison of fatigue results for two markedly different test frequencies (see Fig. 3) demonstrate that neglecting parameters considered as secondary, such as the frequency, can lead to a groundless rejecting of the independence model, if fatigue results for different test conditions are used for comparison.

The analysis of Fig.4enables us, with certain reservations, to accept the validity of the independence model for strands, if specimens of at least 2m long are used for testing and ulterior extrapolation. The poor prediction arising from the fatigue results for 1m long specimens are not conclusive, since, as reported in [9], surface failures in the outer wires of the strand caused by handling or storage damages are suposed to be determinant for the shorter lifetime measured. Due to the high rate of run-outs obtained in the wire tests, no sound results for length effect of prestressing wires has yet been achieved.

No definitive conclusions can be drawn concerning the question of whether the distributions resulting from different stress levels should be the same (see Fig.5), the study of independency could hence follow for a convenient high stress range.

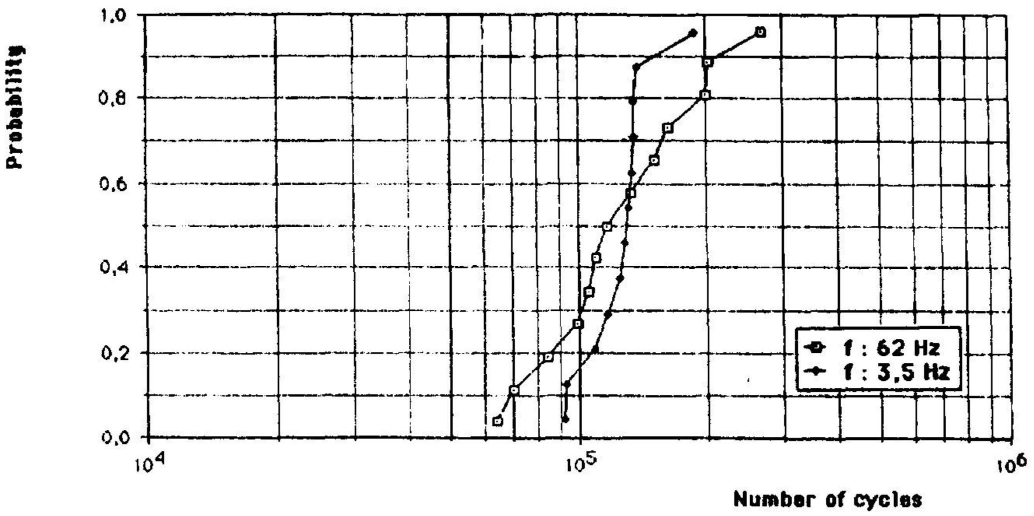

Fig. 3. Fatigue test results on prestressing wire for differing frequencies (l=0,150m, Δσ=700 N/mm²)

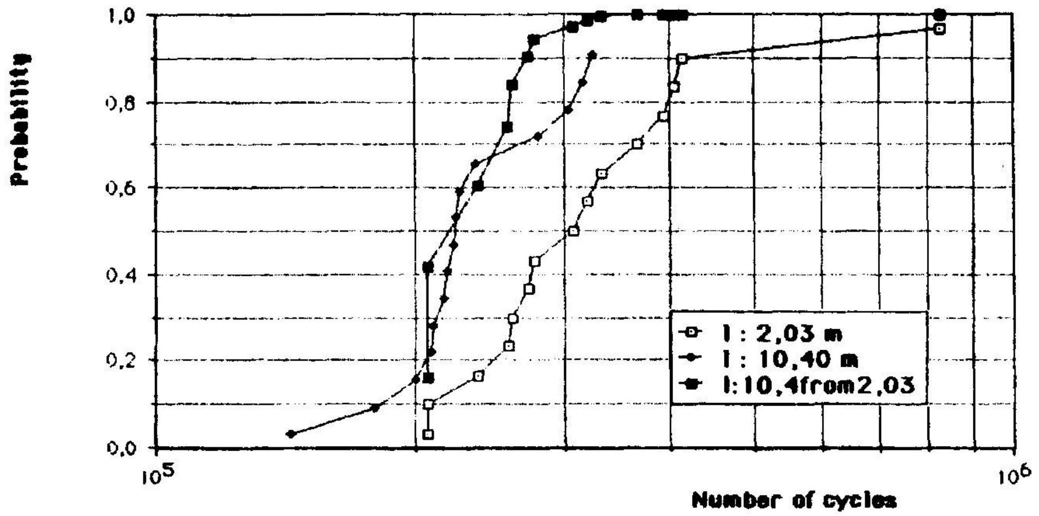

Fig.4. Fatigue test results on prestressing strand for differing length (f=3,5 Hz, Δσ=700 N/mm²)

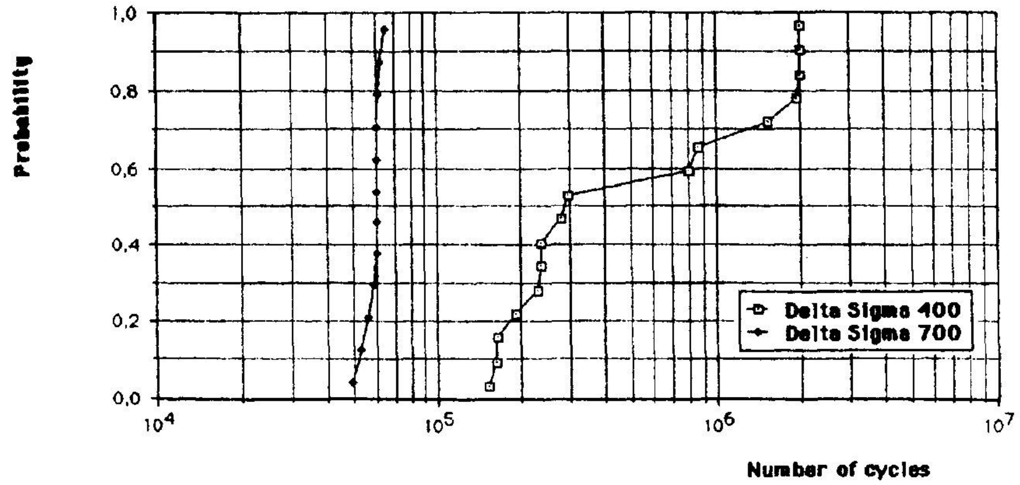

Fig.5. Fatigue test results on prestressing wire for differing stress range (l=0,15m, f=3,5 Hz)

5.2 Wires

The research program is still being followed, so that a comparison can only be made for extrapolation purposes. The fatigue data obtained for 10m long specimens and the regression line derived from the data for 2 m. long specimens show a fair agreement. (Fig.6).

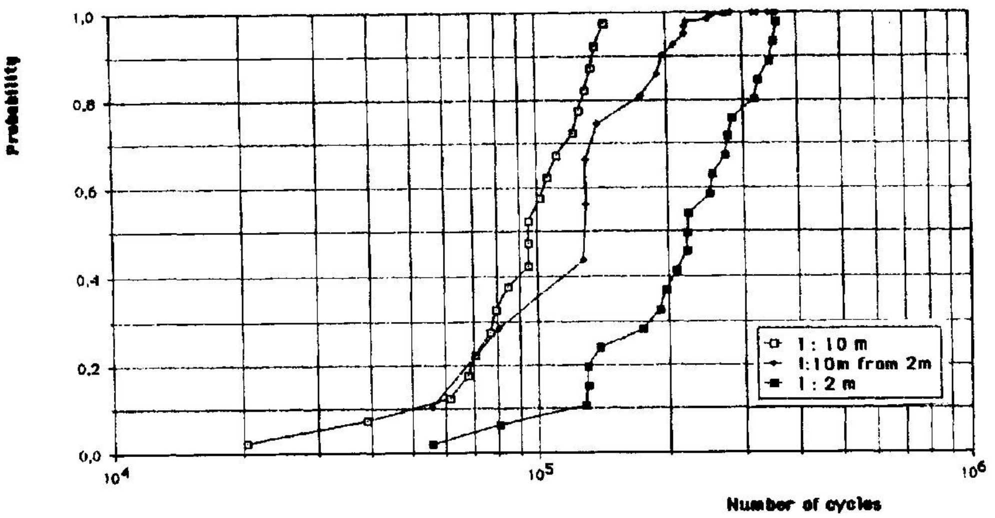

Fig. 6. Fatigue test results on, hipoeutectoid wire for different length (f=2Hz, $\Delta\sigma$=747 N/mm^2)

6. CONCLUSIONS

As general remarks for both experimental programs it can be stated that:

- Since the left tail of the distributions, corresponding to the low probabilities of failure (design region), is determinant for ascertaining or rejecting the initial assumptions, further testing is needed in order to come to reliable and definitive conclusions related to the presented model.Up till then, tproposed model seems to be founded on sound assumptions without physical or statistical inconsistencies, and reasonably well supported experimentally; and herefore acceptable for practical design purposes.

- The graphical representation of the results obtained shows that the different sets of data seem to follow Gumbel rather than Weibull distributions. This should be clarified in the course of theongoingresearch. Nevertheless, since, as a regression model with the Gumbel instead of Weibull assumption proposed in [5]. has been derived by Castillo, (see [2]), and as stated in [4], any Gumbel distribution can be approximated as closely as desired by Weibull distributions.

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