EC 2: Structural Analysis

EC 2: Analyse des structures

EC 2: Schnittgrössenermittlung

Josef EIBL
Prof. Dr.
University of Karlsruhe
Karlsruhe, Germany

J. Eibl, born 1936, civil eng. TU München; Assoc. Professor TU Braunschweig; 1974 Professor University Dortmund; since 1982 Professor and Director of the Institute of Concrete Structures and Building Materials, University of Karlsruhe.

SUMMARY

Relevant chapters of EC 2 Part 1 dealing with the structural analysis of concrete structures will be presented and discussed. Particular emphasis is laid on non-linear and plastic methods of analysis, the check of rotation capacity as well as on simplified approaches based on non-linear material behaviour. The presentation covers both the serviceability limit states and ultimate limit states.

RESUME

Les chapitres de la Partie 1 de l'EC 2 qui se rapportent à l'analyse des structures en béton sont présentés et discutés. En particulier, l'importance est attribuée au comportement non-linéaire et plastique, à la vérification de la rotation plastique admissible des sections ainsi qu'aux méthodes d'analyse simplifiées qui sont basées sur un comportement non-linéaire de la structure. En outre, les états-limites ultimes et les états-limites de service sont considérés.

ZUSAMMENFASSUNG

Die einschlägigen Kapitel des EC 2 Teil 1 bezüglich der Schnittgrössenermittlung in Betontragwerken werden angesprochen und erörtert. Der Schwerpunkt liegt hierbei auf nichtlinearen und plastischen Verfahren, der Kontrolle der Rotationsfähigkeit von Querschnitten sowie auf vereinfachten Verfahren auf der Grundlage eines nichtlinearen Materialverhaltens. Die beiden Nachweisbereiche Gebrauchszustand und Versagenszustand werden behandelt.

1 STRUCTURAL ANALYSIS IN EUROCODE 2

For structural analysis four methods are available in EC 2 excluding the regulations for second order effects (buckling):

- The Theory of Elasticity
- The Theory of Elasticity with Redistribution
- The Nonlinear Approach
- The Method of Plasticity

The theory of elasticity is well known to everybody and does not need any further comment. As the method of plasticity, as well as the application of the theory of elasticity with redistribution, is a subgroup of the more general nonlinear method, the latter will be treated in detail in the following, discussing also the mentioned subgroup's applications in EC 2. This presentation is a compendium of several other publications [1],[2],[3],[4] giving more details.

2 WHAT DOES NONLINEAR DESIGN MEAN ?

Design according to the **Theory of Elasticity** with characteristic loads and partial safety coefficients for actions means that one first calculates internal forces and moments using an elastic constitutive law A (Fig. 1)

$$\sigma = \epsilon * E \quad or \quad M = - y''/(EJ), \quad N = u' * EA. \tag{1}$$

With the exception of the modulus of elasticity this constitutive relation is material independent and linear. Therefore the principle of superposition holds as long as geometrical nonlinearities - buckling - are also excluded.

$$\begin{array}{ccc}
\gamma_s \ \boxed{\text{Load}} & & \boxed{\text{Material}} \\
\downarrow & & \downarrow \\
\text{Constitutive Law A} & & \text{Constitutive Law B } /\gamma_M \\
\downarrow & & \downarrow \\
\gamma_s \ \text{Moment} & \leqq & \text{Moment } (\gamma_M)
\end{array}$$

Fig. 1 Current Safety Format

Then in a second step the different cross-sections are designed with realistic nonlinear constitutive laws (Fig. 1 and 2) for steel and concrete, regarding characteristic material parameters and partial safety coefficients for resistance.

In the case of **Nonlinear Design** only one set of realistic constitutive laws is applied (B in Fig. 1) the whole process. This means, when deriving the differential equations - neglecting normal forces - for the beam, that one has to substitute the elastic relation (see also Fig. 3)

$$M_i = - y'' / (EJ) \tag{2}$$

by the nonlinear moment-curvature-relation:

$$M_i = f\,(y''). \tag{3}$$

In this way a complete F-y-diagram can be calculated, increasing the load step by step by numerical means using for example a computer (Fig. 4). Such an investigation starts within the serviceability range accounting also for cracks and realistic deformation and goes on until the computer program reaches a given limiting condition such as e.g. a maximum concrete strain or a limiting steel strain. The so determined load is the bearing capacity of the beam resp. beam system including already redistribution of forces and moments due to the changing stiffness conditions.

Since this method is able to simulate experiments very well, this approach gives the real behaviour and is declared to be the basic method in the CEB-Model Code [5]. Slabs, walls and shells may also be analyzed by this method.

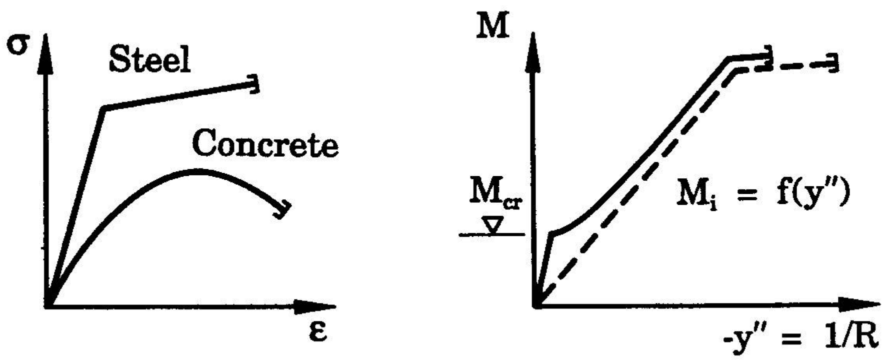

Fig. 2 Constitutive Law and Moment-Curvature-Relation

LINEAR

$$M_i = M_a$$

$$M'_a = Q \;;\; M''_a = -p$$

$$M_i = -y''EJ$$

$$M''_a \longrightarrow \boxed{p = y''''EJ} \longleftarrow M''_i$$

NONLINEAR

$$M_i = M_a$$

$$M'_a = Q \;;\; M''_a = -p$$

$$M_i = f(y'')$$

$$M''_a \longrightarrow \boxed{p = [f(y'')]''} \longleftarrow M''_i$$

Fig. 3 Linear and Nonlinear Design

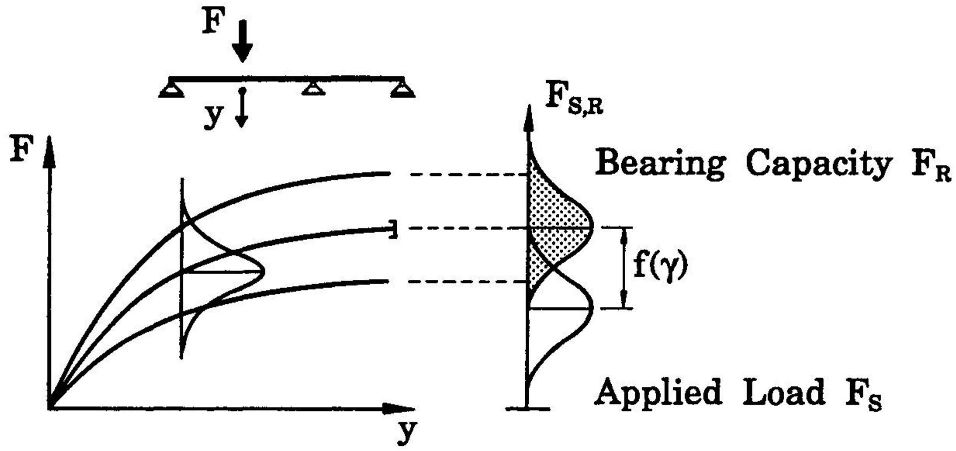

Fig. 4 Proposed Safety Format

With regard to the real behaviour, the **Plastic Method** as addressed in EC 2 is an approximate method derived from this basic method. It allows to concentrate the summed up curvatures along the beam in discrete hinges. This is justified by the fact that big cracks usually appear at special locations of maximum moment and by assuming rigid ranges in between.

It is based on the theory of plasticity with its two limiting theorems for an upper and a lower bound of a system's bearing capacity, stated in engineering terms as:

Theorem 1: A load system which belongs to an admissible stress state, not violating yield conditions, is a lower bound for the bearing capacity of the structure.

Theorem 2: A load system being in equilibrium in a kinematic admissible state of motion is an upper bound for the bearing capacity of the structure.

The so-called **Redistribution of Moments** is an application of this plastic method.

3 ADVANTAGES AND DISADVANTAGES

The theory of elasticity

- Is easy to handle
- Allows superposition to be applied
- Changes of the reinforcement do not influence the internal force and moment distribution
- The serviceability range is rather well approximated
- The ultimate limit state is not covered correctly (Fig. 5).

A complete nonlinear design

- Gives correct results for the serviceability (SLS) as well as for the ultimate limit state (ULS).
- Reinforcement influences the distribution of inner forces and moments.

- The principle of superposition is no longer valid and load combinations have to be considered.
- Demands numerical means and in general a computer.

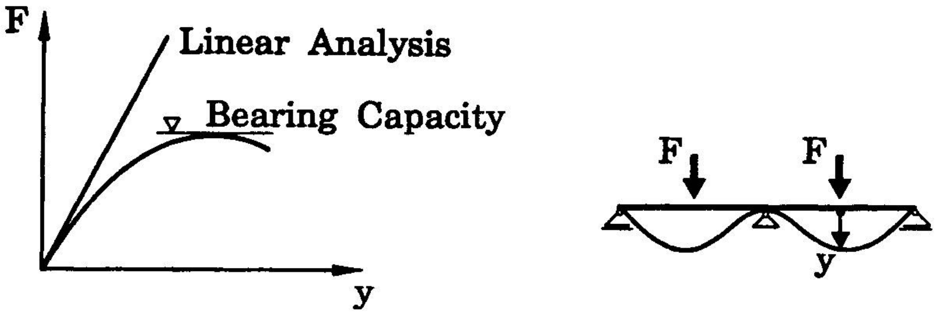

Fig. 5 F-y-Relation for Linear and Nonlinear Design

With regard to the safety format, it has to be mentioned that a safety check on **the Level of Internal Forces and Moments** can never consider system failures correctly, whatever method is used. For example slabs do not fail, when discrete moments caused by loads exceed the design moments. One cross-section failure within a multispan continuous girder may not cause failure of the whole system as it is the case for a similar type failure in a single span girder.

However, with a consistent nonlinear design, where the safety check is done at the level of loads comparing the bearing capacity of the whole structure with the acting loads, automatically regards the different failure behaviour of different systems.

As it is possible to consider the scatter of the F-y-curve (Fig. 4) explicitly, also the scatter of the bearing capacity may be determined. Comparing the density function for the bearing capacity with the density function of the acting loads, the probability of failure of the system may be evaluated and simply controlled.

3 APPLICATION IN EC 2

EC 2 demands the application of the theory of elasticity for the evaluation of internal forces and moments to check the SLS state.

For the ultimate limit state (ULS)

- a design procedure by means of the theory of elasticity and a following cross-section design at the level of internal forces and moments is allowed as well as
- a limited redistribution of elastic moments, if a few requirements are fulfilled garanteeing a minimum of ductility resp. rotation capacity at locations of maximum moments.
- The method of plasticity allows an unlimited redistribution, if in case of beams the rotation is checked and in case of slabs a limit of the relation between support and field moments is regarded. In a recommendation it is further stated that at cases of extreme redistribution only normal ductile steel according to Appendix 2 should be used.

The method of plasticity is very simple and can be applied easily when a calculation by hand is intended. In case of a multispan continuous girder e.g. one just has to assume support moments for loads increased by a partial safety factor and to fulfill the equilibrium conditions between the supports by appropriate field moments before doing a cross-sectional design using characteristic material values and appropriate partial material safety coefficients.

It is a safe method in the sense of theorem 1 of the theory of plasticity as a state of equilibrium is given not violating yield conditions. The so-called Hillerborg strip method for plates belongs to the same class, while the yield line theory according to theorem 2 gives only an upper bound solution, i.e. the real bearing capacity of the plate may be smaller.

Also the use of strut and tie models to investigate walls and in plane loaded plates as well as the so-called D-regions of beams and corbels is a special application of the method of plasticity similar to the Hillerborg strip method. As long as the equlibrium conditions are fullfilled within a properly selected truss system provided with sufficient ductility - according to theorem 1 ''not violating yield conditions'' - a safe solution may be reached.

When applying the method of nonlinear design according to EC 2 also, a primary evaluation of internal forces and moments is required using mean values of material parameters followed by a nonlinear cross-sectional design with partial safety factors and characteristic materials values. This always leads to an oversafe design as the initial assumption of reinforcement on the basis of **mean** values is always increased at the second stage of the cross-sectional design according to partial safety coefficients and **characteristic** material parameters.

The still existent deficiency is the afore mentioned problem that in general a system failure can never be covered realistically for all methods using a cross-sectional design. It will especially underestimate the safety of slabs, walls and shells.

It is more reasonable to compute - nonlinear design by hand is usually not possible - in one single numerical approach the bearing capacity by strain limits, neglecting an evaluation of internal forces and moments and to ensure safety by an appropriate safety concept with slightly changed partial safety coefficients at a comparison level of loads (Fig. 4).

In this case of a consistent nonlinear investigation SLS and ULS may be covered by the **same** method.

REFERENCES

1. EIBL J., Fragen zum Nachweis der Tragsicherheit bei Berücksichtigung nichtlinearer Stoffgesetze. In: Sicherheit von Bauwerken - Günter Breitschaft Symposium. IfBt, Berlin, März 1989;

2. EIBL J., Safety Considerations for Nonlinear Analysis. In: Structural Concrete. IABSE-Colloquium Stuttgart, IABSE-Report Nr. 62;

3. EIBL J., Nichtlineare Traglastermittlung - Bemesssung. In: Vorträge auf dem Deutschen Betontag 1991, (to be printed);

4. EIBL J., Nichtlineare Traglastermittlung / Bemessung. Beton- und Stahlbetonbau 87(1992), (to be printed).

5. CEB-FIP Model Code 1990 - Final Draft. CEB Bulletins No. 203-205. Lausanne 1991