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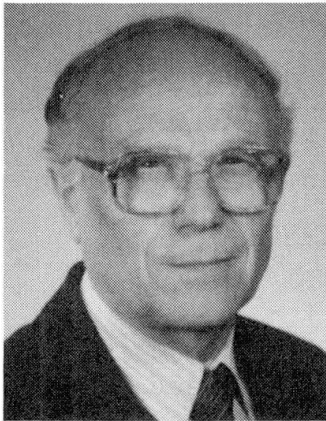
Ductility of Structural Concrete

Ductilité du béton structurel

Duktilität des Konstruktionsbetons

Igor TERTEA

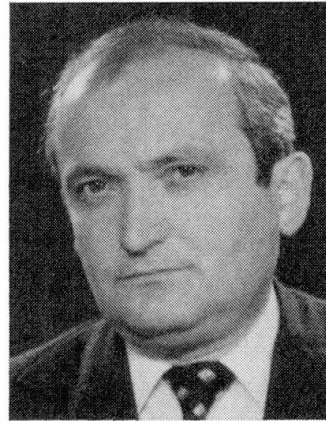
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SUMMARY

The paper presents the ductility computation for B regions and the main parameters influencing the ductility of structural concrete as seen from the correlation of the numerical tests with experimental results.

RÉSUMÉ

Cet article présente une méthode de calcul des zones B, ainsi que les paramètres influençant la ductilité du béton, résultant d'une corrélation entre résultats numériques et expérimentaux.

ZUSAMMENFASSUNG

Der Artikel stellt ein Rechenverfahren für die Duktilität der B-Bereiche vor und zeigt die wesentlichen Einflüsse auf die Duktilität von Konstruktionsbeton auf, die aus Vergleichen von numerischen Berechnungen mit Versuchsergebnissen gewonnen werden.



1. INTRODUCTION

In accordance with the ideas expressed in the introductory reports by J.E. Breen and A.S.G. Bruggeling, as well as with the considerations contained in the lectures of J.G. MacGregor and P. Marti, we should emphasize the fact that one of the fundamental requirements of structural concrete elements design is the provision of a proper ductility. In fact, one of the most important advantages of structural concrete is offered by the possibility to design the required sectional or/and structural ductility, in accordance with the building's emplacement and the nature of actions.

The above assertion is valid only for the portions of the structural elements subjected to bending moments with or without axial load (B regions), for which there are already clear design models permitting a qualitative and especially a quantitative ductility computation [1,2,4].

For the portions subjected to combined action of bending moment and shear force (D regions) design model recently proposed (full-member design procedure) does not refer to ductility but in case of inclined crack width limitation.

2. DUCTILITY COMPUTATION FOR B REGIONS

The design model used by the authors [4,5] does not essentially differ from that proposed by A.S.G. Bruggeling [1], in which the prestressing can simply be regarded as an artificial loading, from the point of view of load capacity.

For the ductility computation the following assumptions are made:

- a) The stress - strain curve of concrete is a parabolic one (Fig.1) and takes into consideration the concrete confinement by transverse reinforcement.
- b) The stress - strain diagram for nonprestressed steel is bilinear (corresponding to elasto-plastic behaviour).
- c) The stress - strain diagram for prestressing steel is linear for $G_p \leq 0,6 f_{pu}$ and five degree parabolic over this value.

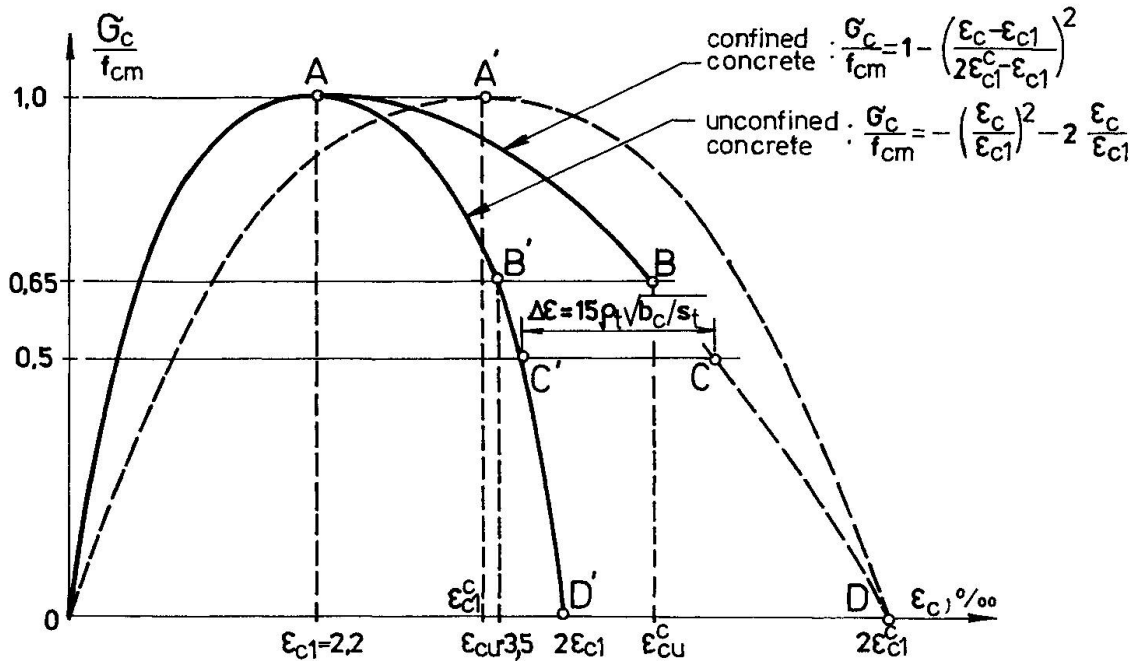


Fig. 1

- d) Active and passive reinforcement yield at the same time.
- e) Plane sections before flexure remain plane after flexure.
- f) The prestressing effect, after losses, is similar to external forces P_{∞} and P'_{∞} (corresponding to the reinforcements A_p and A'_p).
- g) There is a good bond between concrete and reinforcements.
- h) The cross sectional stress and strain distribution at yield point of reinforcement and at fracture of concrete are represented in Fig. 2a and Fig. 2b respectively.

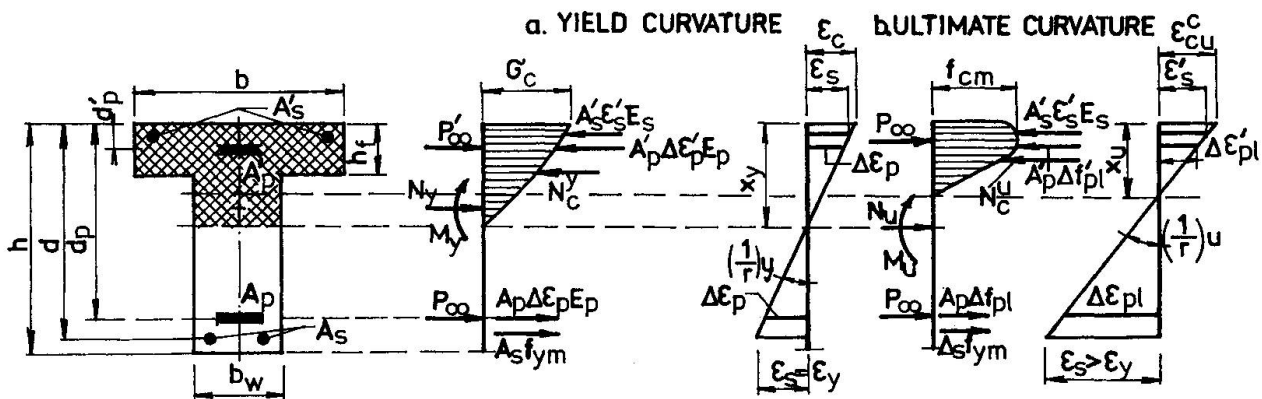


Fig 2



The ductility ratio for a structural concrete section subjected to bending with axial load can be computed as follows:

$$D = \frac{\epsilon_{cu}^c (1 - \xi_y) E_s}{\xi_u f_{ym}} = \frac{\epsilon_{cu}^c (\delta - \xi_y) E_p}{\xi_u (f_{0.2m} - G_{p\infty})} \quad (1)$$

where $\xi_y = \frac{x_y}{d}$ and $\xi_u = \frac{x_u}{d}$.

The values of ξ_y and ξ_u are the solutions of the equations:

$$A \xi_y^3 + B \xi_y^2 + C \xi_y + D = 0 \quad (2)$$

$$E \xi_u^3 + F \xi_u^2 + G \xi_u + H = 0 \quad (3)$$

where the coefficients have the expressions from Appendix, for one of possible situations depending on section characteristics.

Design procedure is programmable. The set of numerical program [4] is providing the possibility to print the diagrams for estimating the ductility ratio depending on different parameters.

3. PARAMETERS INFLUENCING THE STRUCTURAL CONCRETE DUCTILITY

Numerical tests using the above mentioned programs have been correlated with experimental results obtained in Reinforced Concrete Laboratory of Politechnical Institute of Cluj and also with in other laboratories and we got the following conclusions:

- The ductility of structural concrete sections is drastically diminished by increasing the axial forces intensity (external action effects and/or prestressing effect) which accompany the bending moment. The curvature ductility may be improved by reducing the prestressing degree or (at a given prestressing degree) by proper transverse reinforcement [3,5].
- The beams with unbonded prestressing reinforcement have a greater ductility in comparison with those with bonded prestressing reinforcement.



- The passive or active reinforcement of compressive zone has a favourable influence on ductility due to beneficial effect of the concrete confinement.
- The higher the ratio of passive reinforcement (ρ_w) (at the same quantity of the total reinforcement) and the less the quality of this reinforcement, the greater the value of ductility.
- The effect of small number of repeated loading cycles on the ductility was insignificant.

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APPENDIX

$$\boxed{\varepsilon_s' \geq \varepsilon_y}$$

$$A = \frac{1}{3} \frac{\varepsilon_y}{\varepsilon_{c1}} \left(3 + \frac{\varepsilon_y}{\varepsilon_{c1}} \right)$$

$$B = (\alpha + \alpha_p) - (\alpha' - \alpha'_p) - \frac{\varepsilon_y}{\varepsilon_{c1}} + \frac{\varepsilon_y}{\varepsilon_{c1}} \frac{h_f}{d} \left(\frac{b}{b_w} - 1 \right) \left(2 + \frac{\varepsilon_y}{\varepsilon_{c1}} \right) + n_y$$

$$C = -2\alpha - \alpha_p \left(1 + \frac{d}{d_p} \right) + 2\alpha' - \alpha'_p \left(1 + \frac{d_p'}{d} \right) - \frac{\varepsilon_y}{\varepsilon_{c1}} \frac{h_f}{d} \left(\frac{b}{b_w} - 1 \right) \left(2 + \frac{h_f}{d} + \frac{h_f}{d} \frac{\varepsilon_y}{\varepsilon_{c1}} \right) - 2n_y$$

$$D = \alpha + \alpha_p \frac{d}{d_p} - \alpha' + \alpha'_p \frac{d_p'}{d} + \frac{\varepsilon_y}{\varepsilon_{c1}} \frac{h_f^2}{d^2} \left(\frac{b}{b_w} - 1 \right) \left(1 + \frac{1}{3} \frac{h_f}{d} \frac{\varepsilon_y}{\varepsilon_{c1}} \right) + n_y$$

$$\alpha = \rho_w \frac{f_{ym}}{f_{cm}} ; \alpha' = \rho_w' \frac{f_{ym}}{f_{cm}} ; \alpha_p = \rho_{wp} \frac{\varepsilon_y E_p}{f_{cm}} ; \alpha'_p = \rho_{wp}' \frac{\varepsilon_y E_p}{f_{cm}} ;$$

$$\rho_w = \frac{A_s}{b_w d} ; \rho_w' = \frac{A_s'}{b_w d} ; \rho_{wp} = \frac{A_p}{b_w d} ; \rho_{wp}' = \frac{A_p'}{b_w d} ;$$

$$n_y = \frac{N_y + P_{\infty} + P_{\infty}'}{b_w d f_{cm}} .$$

$$\boxed{\varepsilon_s' \geq \varepsilon_y ; \xi_u > \frac{h_f}{d} > \xi_u \left(1 - \frac{\varepsilon_{c1}}{\varepsilon_{cu}^c} \right)}$$

$$E = \frac{1}{3} \frac{b}{b_w} \frac{\varepsilon_{c1}}{\varepsilon_{cu}^c} \left[1 - \left(\frac{\varepsilon_{c1}}{2\varepsilon_{c1}^c - \varepsilon_{c1}} \right)^2 \right] + \left(\frac{b}{b_w} - 1 \right) \left[\frac{\varepsilon_{cu}^c}{\varepsilon_{c1}} - \frac{1}{3} \left(\frac{\varepsilon_{cu}^c}{\varepsilon_{c1}} \right)^2 \right] - \frac{b}{b_w} \left[1 - \frac{1}{3} \left(\frac{\varepsilon_{cu}^c}{2\varepsilon_{c1}^c - \varepsilon_{c1}} \right)^2 + \frac{\varepsilon_{cu}^c \varepsilon_{c1}}{(2\varepsilon_{c1}^c - \varepsilon_{c1})^2} - \left(\frac{\varepsilon_{c1}}{2\varepsilon_{c1}^c - \varepsilon_{c1}} \right)^2 \right]$$

$$F = \alpha + \alpha_{pu} - \alpha' - \alpha'_{pu} - \frac{h_f}{d} \frac{\varepsilon_{cu}^c}{\varepsilon_y} \left(\frac{b}{b_w} - 1 \right) \left(2 - \frac{\varepsilon_{cu}^c}{\varepsilon_{c1}} \right) + n_u$$

$$G = \frac{\varepsilon_{cu}^c}{\varepsilon_{c1}} \frac{h_f^2}{d^2} \left(\frac{b}{b_w} - 1 \right) \left(1 - \frac{\varepsilon_{cu}^c}{\varepsilon_{c1}} \right)$$

$$H = \frac{1}{3} \frac{h_f^3}{d^3} \left(\frac{b}{b_w} - 1 \right) \left(\frac{\varepsilon_{cu}^c}{\varepsilon_{c1}} \right)^2$$

$$\alpha_{pu} = \rho_{wp} \frac{\Delta f_{pl}}{f_{cm}} ; \alpha'_{pu} = \rho'_{wp} \frac{\Delta f_{pl}}{f_{cm}} ; n_u = \frac{N_u + P_{\infty} + P_{\infty}'}{b_w d f_{cm}} .$$